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CROPDUSTER: A Model for Evaluating the "Common Relevant Operational Picture (*CROP*)"

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CROPDUSTER: A MODEL FOR EVALUATING THE "COMMON RELEVANT OPERATIONAL PICTURE (*CROP*)"

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ABSTRACT

A high-level, low-resolution (HLLR) stochastic model for evaluating the benefits of a Common Relevant Operational Picture (CROP) is described. The model focuses on the sensor-to-shooter information needed by elements of a joint force to engage time critical Targets. The model includes realistic random delays or latency caused by congestion or system unreliability, random times until Target detection and loss, and a probabilistic representation of the Services' overall capability to detect, classify and shoot at a varying set of different Target types. The model results support the following conclusions. CROP can increase the number of time-critical Targets killed, sometimes considerably; CROP can decrease the mean and variance of the number of weapons expended to kill Targets; the benefits of CROP can be degraded if the CROP process requires more time from Target detection until weapon arrival at the Target.

1. Introduction and Summary

This paper describes and illustrates a high-level, low-resolution (HLLR) stochastic scoping model, CROPDUSTER, or CD, for evaluating the CROP (Common Relevant Operational Picture). The CROP is defined as *a presentation of timely, fused, accurate, assured, and relevant information that can be tailored to meet the requirements of a joint force and is common to every organization and individual involved in a joint operation*; (U.S. Joint Forces Command (2000)). The CROP architecture is attractive since the (Blue) Services (e.g., Army, Air Force, Navy and Marines) have tended to focus on individual "stove-piped" (SP) systems that are, in most cases, not entirely interoperable. The vision for the CROP system is that it provides for all the data, information, and knowledge needs of joint warfare with minimal latency and maximal attainable precision and lethality. The model of this paper focuses on sensor-to-shooter information needed by elements of a joint force to engage enemy forces.

Here CROP capability is compared to the targeting of a variety of Red-force types by several uncoordinated Service forces with their own individual combined Sensor-Shooter systems, *but* which do not share information with other Services, informally called a SP architecture. The CROP capability permits rapid transfer of any detection information to *all* (relevant) Services, and then a quick decision as to which Services actually best Target the detected Red unit. Realistic delays or latency caused by congestion or system unreliability (hardware, software, or human) may be, and are, addressed implicitly by reduction of task processing rates.

The models include representation of the Services' overall capability to detect, classify and shoot at a varying set of different Target types (R in all; a generic type is r, for examples in the paper $R=3$ throughout). In the SP model the Services may each individually Target the same Red unit (e.g., one or more tanks), expending weapons unnecessarily and inefficiently. In the CROP organization this will occur much less frequently, so combat efficiency can be enhanced. Our models permit analytical but probabilistic comparisons of the SP and CROP organizations. Our version of an SP model is tilted somewhat unrealistically towards *too much* individual Service isolation, while the present CROP model may assume *too-efficient* information coordination.

This accentuates the advantage of CROP over SP in general, but the account of stochastic variability tends to put the likely advantage of CROP (for the examples provided) into perspective. The variability between, and within, individual campaigns that use SP or (partial or total) CROP can be substantial.

2. Models and MOEs of UnCoordinated (SP) Sensor-Shooter Systems

2.1 Scenario

A region of a battlespace, $\mathcal{B}$, is under the surveillance of the sensor assets of S ($S \geq 1$, but 2 or more is more interesting) Services. *Assume* for the moment that these systems are completely SP and operate independently, sharing information *within* but without sharing information *between* Services. Information obtained by the myriad of sensor types *within* a Service (e.g., the Army) *is* shared, and can be parameterized as a Service-wide detection rate ξ_{rs} (e.g., unit of measure 1/hrs.) meaning that $1/\xi_{rs}$ is the mean time to detect *one* Red unit of type r, $r \in \{0, 1, \ldots R\}$ by the collaborating sensor assets of Service s, where $s \in \{1, 2, \ldots, S\}$ denotes the s^{th} Service. The random variable T_{rs}, denoting the time to such a detection, is conveniently taken first to be exponentially distributed, although this distributional assumption can often be easily relaxed; if the assumption is relaxed, then let T_{rs} have the general distribution $F_{T_{rs}}(x;\bullet)$, where the solid dot $\bullet$ indicates that other variables may condition F; the default is a "constant" distribution omitting other influential variables. *For the present, all examples are for exponential default conditions.* We define the collection of all sensor assets of a Service s to be its sensor System. Our model also assumes that $\{T_{rs}, r = 0, 1, \ldots, R; s = 1, 2, \ldots S\}$ are all *independent*, a consequence of the assumption that Blue has ample sensor and shooter capability.

Note: Spatial/geographic issues and Red maneuver effects come later; but not in this paper.

2.2 Example Question 1: Time for *System* to Achieve Detection

Suppose a Red unit, type r, enters $\mathcal{B}$ at initial time $t = 0$; all Services' sensor systems are present.

(A) Red Unit Detection by Single Service, *s*

For a Service, s, sensor System, the probability of detection by fixed time $t > 0$, is modeled as

$$\begin{aligned} P\{T_{rs} \le t\} &= 1 - e^{-\xi_{rs} t};\ \xi_{rs} > 0, t \ge 0 \\ &= 0;\ t < 0 \end{aligned} \tag{2.1}$$

In (2.1) the clock starts when an r-type unit first becomes available for detection by Service s. Note that this can occur either by r entering the region in which s is already present, or by the sensors of Service s entering the region where Red units of type r are already present.

Furthermore, T_{rs} is (conditionally on availability time t, and possibly on environmental and terrain conditions) independent of T_{ru}, for different-Service sensor Systems s and u, as stated above.

(B) Red Unit Detection Under Independence of Blue Service Sensor Systems

To complete the probability that all s Services fail to detect a unit of type r in time t after the latter is available in $\mathcal{B}$, we assume all Services are capable of detecting the unit. By independence,

$$\begin{aligned} P\{\overline{T}_r > t\} &= \prod_{s=1}^{S} e^{-\xi_{rs} t} \\ &= e^{-\left(\sum_{s=1}^{S} \xi_{rs}\right)t}, \end{aligned} \tag{2.2}$$

where $\overline{T}_r$ is the time for *some* Service sensor to detect item r, so the time to detection by *some* system is, of course, exponentially distributed with parameter $\xi_r = \sum_{s=1}^{S} \xi_{rs}$. This is equivalent to a detection by the *entire system* under CROP, assuming instant information sharing. Under SP, the probability that first detection is by System s is ξ_{rs}/ξ_r. In this architecture assume that Target prosecution is by System s but also by subsequent Other-than-s, i.e., $\overline{s}$, services acting independently, until the Target is (*i*) lost from track, or (*ii*) targeted, shot at, and abandoned as (apparently) killed.

(C) Summary of Initial Detection Model

Suppose a Target of type r becomes available for detection at time $t = 0$. Then

(a) Under uncoordinated Service architecture (SP) the first Service to detect it carries out initial prosecution, i.e., until (*i*) lost, or (*ii*) declared dead. If not killed, the Target must be re-detected, and this occurs at the same detection rate, ξ_r. As other Services detect, they independently join the attack, which stops when the target is killed.

(b) Under CROP architecture the first Service to detect the Target immediately (negligible latency time initially) shares the information with all other Services. We also study the effect of adding a constant non-zero latency. This allows a cooperative effort to classify the target, and, subject to that joint classification, to shoot at it.

Our model allows the advantage of an idealized CROP to be compared to the more traditional SP architecture. In fact, the contrast provided is a stark caricature, since *some* information sharing is realistically present today. The numbers may therefore tend to exaggerate the CROP advantage.

(D) Generalized Detection Model

The detection rates $\{\xi_{rs}\}$ may vary irregularly ("randomly") and simultaneously from time to time, e.g., day to day, because of say, environmental (weather) changes.

Quick Mixing

To represent such effects one can first replace the exponent $\xi_{rs}t$ in (2.1) conditionally by $\xi_{rs}\boldsymbol{H}(t)$ where $\boldsymbol{H}(t)$ is a *random hazard* (see Gaver (1963), and Cox and Lewis (1966)). Normalize so that the drift $E\left[\boldsymbol{H}(t)\right]=t$ but with adjustable variability; one convenient model is the gamma process, with independent but variable time increments, and

$$P\{\boldsymbol{H}(t)\in dx\}=e^{-\mu x}\frac{(\mu x)^{\mu t-1}}{\Gamma(\mu t)}\mu dx; \tag{2.3}$$

so

$$E\left[e^{-s\boldsymbol{H}(t)}\right]=\left(1+\frac{s}{\mu}\right)^{-\mu t}=e^{-\mu t\ln\left(1+\frac{s}{\mu}\right)} \tag{2.4}$$

and

$$E[\boldsymbol{H}(t)]=t,\quad Var[\boldsymbol{H}(t)]=\frac{t}{\mu}; \tag{2.5}$$

the expression for $Var[\boldsymbol{H}(t)]$ shows that the variance of detection rate per unit time decreases as μ increases. Furthermore, expression (2.4) shows that the marginal probability that an r-Target escapes detection for time t, as in (2.2) remains exponential in form, but with rate parameter $\mu\ln(1+\xi_r/\mu)$. This rate parameter tends to zero if $\mu\to 0$ (so variance of detection rate becomes large), and, if $\mu\to\infty$ tends to ξ_r.

Slow Mixing

An alternative model is obtained by replacing each detection rate parameter by $\xi_{rs}\varepsilon$ where ε is a random perturbing parameter prevailing throughout the campaign. In this case the exponential survival function of (2.2) becomes, conditionally,

$$P\{\overline{\boldsymbol{T}}_r>t|\varepsilon\}=e^{-\xi_r\varepsilon t},$$

and removal of the condition gives

$$P\{\overline{\boldsymbol{T}}_r>t\}=E\left[e^{-\xi_r t\varepsilon}\right]$$

which is the Laplace-Stieltjes transform of the distribution of ε. Two convenient possibilities are the gamma normalized with mean unity and the positive stable law (Feller(1966)).

For the gamma with shape-scale parameter β, $0<\beta$

$$P\{\overline{\boldsymbol{T}}_r>t\}=\left(1+\frac{\xi_r t}{\beta}\right)^{-\beta}$$

and for the stable law with shape parameter $0 < \alpha < 1$,

$$P\{\bar{T}_r > t\} = e^{-(\xi_r t)^\alpha} .$$

Both such models have slower-than-exponential decay, i.e., with long, *Pareto*, tails. They can be used to represent the difficulties caused by terrain features.

3. Shooting, Losses, and Target Survival: The Case of UnCoordinated Sensor-Shooters (SP Architecture)

After a characteristic type r ($r = 0, 1, \ldots R$) potential Red Target is Detected it must next be Classified or Identified, Tracked for some time (and "mensurated"), and perceived-to-be-appropriate weapons shot at it (this may include abstaining if the type is perceived to be $r = 0$, denoting a false target or Decoy). We call this last step a Shot, although the attack may be a sequence of shots, e.g., a salvo, or a quick Shoot-Look-Shoot sequence.

3.1 Losses During Track, and Kill Rate

Assume for the present that the detection rate on the Target for Service s is ξ_{rs}, as before. When the detection time, T_{rs}, terminates in Detection, then Classification and Mensuration occur, followed by one or more Shots; the shot result may be either a Kill or a Miss. If a Kill, the Target is deleted (if a Decoy is correctly identified, it is "removed" with no shots fired, which is equivalent to permanently identified). If a Miss (perhaps after a quick Shoot-Look-Shoot sequence), the Target simply returns to the environment. Note that in the model for the SP case there is negligible probability that any two or more Targets will be prosecuted simultaneously (i.e., during precisely the same engagement period); different Services can prosecute the same Target at different times. However, deliberate simultaneous prosecution *is* possible in the CROP case if it were decided to shoot at a particular Target "simultaneously" with several Weapons, possibly from different Services.

3.2 The Killing Time Process Under SP

Let $\boldsymbol{K}_{rs}$ denote the random time for Service s to kill a type r Red Target. In other words, this time is the first of possibly several attempts (searches, plus classification, plus tracking and mensuration and finally shots) that actually kills the Target; a more realistic model might permit the Target to leave the region alive. This option is developed in an appendix, as is gradual infiltration of the region by Reds.

At the termination of a search time, a tracking and attempted classification event is initiated. While this event is in progress, two competing subevents are in process:

(a) the Target may be lost from track, which occurs at rate υ_{rs}, the parameter of an assumed "fast" (compared to ξ_{rs}) exponential distribution meaning υ_{rs} is large compared to ξ_r; *or*

(b) the Target is classified, and a weapon or attack launched and completed before track loss occurs; this has rate o_{rs}, again the parameter of an independent "fast" exponential.

The consequence is that the rate at which *either* (a) or (b) occurs is $\upsilon_{rs} + o_{rs}$, the parameter of a "faster" exponential distribution that terminates with event (a) with probability $q_{rs} = \frac{\upsilon_{rs}}{(\upsilon_{rs} + o_{rs})}$, and with event (b) with probability $p_{rs} = \frac{o_{rs}}{(\upsilon_{rs} + o_{rs})}$. This latter is the probability that classification and shooting can begin. This is an approximation that can be relaxed if desired, but it is a reasonable and convenient initial approach.

For simplicity we ignore here the actual tracking time with mean $\frac{1}{(\upsilon_{rs} + o_{rs})}$, but maintain the probabilities q_{rs}, and p_{rs}, that must sum to one.

Here is the structure of the killing-time random variable for

$$\boldsymbol{K}_{rs} = \begin{cases} \boldsymbol{T}_{rs} & \textit{with probability } \kappa_{rs}^{*} \\ \boldsymbol{T}_{rs} + \boldsymbol{K}_{rs}' & \textit{with probability } 1 - \kappa_{rs}^{*}, \end{cases} \tag{3.1}$$

where $\boldsymbol{K}_{rs}'$ is a random independent replica of $\boldsymbol{K}_{rs}$ ("start over"). The parameter

$$\kappa_{rs}^{*} = p_{rs} \sum_j c_{rj}(s) \sum_k d_{jk}(s) \kappa_{rk}(s) \tag{3.2}$$

represents the *kill or success* of the entire Track/Classification-Shoot and Re-Shoot process, which here is assumed to be the same at each opportunity. In (3.2) $\kappa_{rk}(s)$ represents the probability that if a k-type weapon is fired by System s at Target type r that the Target is killed.

The decision sequence leading to κ^*_{rs} involves the classification matrix, $[c_{rj}(s)]$, which provides the conditional (on true type, i.e., r) probability that the unit is actually classified and treated as a type $j \in (0, 1, 2, \ldots R)$ when an s-type Service methodology is applied. There is also a decision probability matrix $[d_{jk}(s)]$, and it provides the conditional probability that a unit classified as a j is weaponeered/shot as a $k \in (0, 1, 2, \ldots R)$. Finally, there is the conditional kill probability that a weapon appropriate for a Target of type r, perceived to be of type j, is shot-at by a weapon appropriate for a Target of type $k \in (1, 2, \ldots R)$. For a single-shot engagement by Service s, the rate of kill (per surviving Target of type r) is

$$\eta_{rs}(K) = \xi_{rs} p_{rs} \sum_j c_{rj}(s) \sum_k d_{jk}(s) \kappa_{rk}(s) = \xi_{rs} \kappa^*_{rs}. \qquad (3.3)$$

Note that the probability $d_{jk}(s)$ is desirably near unity if $j = k$ (otherwise zero), meaning that to every Target *perceived* as a type j there is a most appropriate weapon. However, if the supply of j-optimal weapons is low or exhausted, substitution will occur (the finite-supply issue is not yet modeled). If $j = 0$, then the r-type Target is perceived as a decoy, and so *one option* is not to fire (though it may help to shoot, and hence eliminate, decoys).

The above development implies that K_{rs} has the exponential distribution with rate $\eta_{rs}(K) = \xi_{rs}\kappa^*_{rs}$, which is, in words, the rate of s-Service search success at finding an r-type Target, multiplied by the overall probability of a successful kill. This is, the time rate of kill, per surviving Red type r, of Service s, on the remaining type-r Reds.

We use the above in our further development, but recognize that a variety of different modeling possibilities exist and require investigation.

Note: Specific sensor-related properties are here summarized only by (a) rate of detection, and (b) probability of correct—or incorrect—classification (including Battle Damage Assessment

BDA)) skill; all are represented here by $\left[c_{rj}(s)\right]$, $r \in (0, 1, 2, \ldots R)$ and $j \in (0, 1, \ldots R)$. The closer $\left[c_{rj}\right]$ is to a diagonal matrix, $c_{rr}(s) = 1$ for all r, the more skillful the classification. Here both are composites of many within-Service platform-sensor combinations, and both are candidates for improvement. The current CROPDUSTER model enables the analyst to evaluate tradeoffs between such technological changes as well as features such as the architecture and CONOPS for the CROP.

The expression (3.1) results in the (complement of the) distribution of killing time of a particular, surviving Red, of type r, from an arbitrary start time, $t = 0$

$$\bar{G}_{rs}(t) \equiv P\{\boldsymbol{K}_{rs} > t\} = \exp\left[-\eta_{rs}(K)t\right]. \tag{3.4}$$

3.3 UnCoordinated (SP) Survival

This allows calculation of the r-type individual survival probability in the SP architecture:

$$\begin{aligned} \bar{G}_r(t) &= \prod_{s=1}^{S} \exp\left[-\eta_{rs}(K)t\right] = \exp\left[-\sum_{s=1}^{S} \eta_{rs}(K)t\right] \\ &= e^{-\bar{\eta}_r(K)t} \end{aligned} \tag{3.5}$$

where $\bar{\eta}_r(K) = \sum_{s=1}^{S} \eta_{rs}(K)$ is the overall rate of kill of individual Targets of type r. Our assumption is that Blue resources are adequate to locate and kill any and all targets available independently. Of course ξ_{rs} depends on Blue force size, $\boldsymbol{B}_s(t)$. The overall parameter $\bar{\eta}_r(K)$ neatly encapsulates all individual Service classification, weapon choice and effectiveness parameters—even potentially imperfect BDA and re-shooting options. The present model and example assume "perfect BDA," which can easily be rectified. The exponential form is a great convenience, but not essential (however, it is compatible with classical search theory). It is assumed that when an r-type Target is killed (by Service s with probability $\eta_{rs}(K)/\bar{\eta}_r(K)$) all other Services immediately cease shooting at that particular Target.

3.4 Red System Survival, by Unit Type, and Force-Wide

The assumptions of independence imply that $X_r(t)$, the number of r-type Targets surviving out of X_r initially present with no reinforcements, is Binomially distributed with mean

$$E[X_r(t)]=X_r e^{-\bar{\eta}_r(K)t} \tag{3.6}$$

and variance

$$\text{Variance}\,[X_r(t)]=X_r e^{-\bar{\eta}_r(K)t}\left(1-e^{-\bar{\eta}_r(K)t}\right) \tag{3.7,a}$$

so

$$\text{Standard Deviation}\,[X_r(t)]=\sqrt{X_r e^{-\bar{\eta}_r(K)t}\left(1-e^{-\bar{\eta}_r(K)t}\right)} \tag{3.7,b}$$

The first approximation to the distribution of $X_r(t)$ is the Normal; $X_r(t)$ is approximately normal for large X_r.

The total Red-Force-Wide Targets surviving out of an initial number $Y_0=\sum_{r=0}^{R} X_r$

$$Y(t)=\sum_{r=0}^{R} X_r(t) \tag{3.8}$$

is a sum of independent Binomial random variables with mean

$$E[Y(t)]=\sum_{r=0}^{R} X_r e^{-\bar{\eta}_r(K)t} \tag{3.9}$$

and Variance $[Y(t)]$ the sum of the variances of the r-types.

Numerical illustrations are provided in a later section.

3.5 CROP Survival

A different calculation is needed to express killing time when System-first *detection* information is shared. Assume System detection occurs when the *first* Service detection of an r-type occurs; for the present model, the time for this event to occur is exponentially distributed with rate $\xi_r=\sum_{s=1}^{S}\xi_{rs}$. *The CROP architecture assumption is, then, that the detected Target is classified and assigned to the Service Shooter with the highest perceived kill probability after an initial classification:* with probability $\bar{c}_{r\ell}(S)$ it is classified as Target of type ℓ and assigned to the Service, $s(\ell)$, where $s(\ell)$ identifies that Service for which the predicted kill probability, given ℓ, is greatest. At present a Blue Shooter is always available and experiences no logistics restrictions.

Fusion Effect

One way to express this is to suppose that System-level classification is conducted according to a System-Level Uncertainty matrix: $\left[\left(\bar{c}_{r\ell}(S)\right)\right]$. We address later a variety of alternative derivations of $\left[\left(\bar{c}\right)\right]$. A Bayesian methodology for combining data from all Services is described in Appendix B. If the Target is of type r, but is classified by the System as an ℓ (quite possibly incorrectly), then one decision option is to pick that Service to shoot at the Target that has the largest kill probability against an ℓ: $\arg\max_{s} \kappa_{\ell\ell}(s)$; call that Service $s(\ell)$, so the actual kill probability would be $\kappa_{r\ell}\left(s(\ell)\right)$. Then the CROP kill probability against an r (type unknown) is $\kappa_r^* = p_r(C)\sum_{\ell=1}^{S}\bar{c}_{r\ell}(S)\kappa_{r\ell}\left(s(\ell)\right)$, where $p_r(C)$ is the probability that a weapon or attack can be launched and reaches the Target before track loss occurs. Summarize the killing rate under CROP as $\eta_r(C)$.

The above decision algorithm is *just one, possibly naïve and suboptimal,* option, which may be improved upon. In the event the detected Target is a decoy (a low-value Target deliberately introduced by Red), or a false Target such as a commercial vehicle or truck convoy, i.e., that $r = 0$, there is interest in κ_0^* above as the probability of a wasted weapon. Conversely, if $\ell = 0$ when $r \neq 0$ so a Target of value is misclassified as a decoy and not prosecuted there is interest in a passed-up shooting opportunity and possibly a subsequent source of damage to Blue by that omitted, but very possibly lethal, Target. Red's use of decoys can certainly degrade Blue's attrition capability, as the model easily shows quantitatively.

4. Weapons Expenditure

Weapons expenditure is one cost of Red force prosecution. For the present simplest-possible CD model it is possible to provide detailed explicit probabilistic/stochastic information concerning this issue without use of Monte Carlo simulation. More detail can be accommodated by Monte Carlo; see the MS in Operations Research thesis (2002) by Johnson.

Here we summarize results that are formally derived in Appendix A (see expression (A-8)).

4.1 Mean Weapons Expenditure by Service *s* Against Target Type *r* in Time *t*

The result is, stated in words

$$\text{Mean} = \frac{\text{Probability of } s\text{ - Kill} + \text{Probability of } s\text{ - Miss}}{\text{Probability of } s\text{ - Kill} + \text{Probability of Other - than - } s\text{ - Kill}} \tag{4.1}$$

multiplied by a damped exponentially - increasing function of time, t, that approaches one in the long run.

There are only three basic events the occurrence of which determine s-Weapon expenditure: s-Kill, s-Miss, and Other-than-s Kill; an Other-than-s Miss is a non-event in our formulation (the time is neglibible compared to the time between detection of a single Target).

4.2 Variance, and Standard Deviation, of Weapons Expenditure in Time *t*

The result for variance stated in words is as follows.

Variance plus the square of the Mean

$$= \left[1 + 2 \frac{(\text{Probability of } s\text{ - Miss})}{(\text{Probability of } s\text{ - Kill}) + (\text{Probability of Other - than - } s\text{ - Kill})} \right]$$

Multiplied by the Mean (4.2)

plus a term which approaches 0 as t becomes large.

Details can be found in Appendix A.

5. Numerical Examples

In Subsections 5.1 and 5.2, we compare SP to CROP survival for a case with perfect classification and a case with imperfect classification.

There are S=3 Services and R=3 Target types. There are 100 Targets of each type in the region at time 0. No additional Targets arrive.

The loss rate $\upsilon_{rs} = 5$ per hour for all r and s. The tracking and mensuration, etc., rate $o_{rs} = 4$ per hour for each r and s. The detection rates $\xi_{rs} = 1/12$ per hour for each r and s. These parameters are used for both SP and CROP throughout.

5.1 Disparate Probabilities of Kill for Different Target Types for the Services

In this section we present results for an example in which the probabilities of kill for correctly classified Targets are quite different for different Services; that is, the Services are specialized.

The kill probability matrix for each Service is

Probabilities of Kill

Service 1

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.70	0.125	0.15
2	0.10	0.60	0.10
3	0.10	0.10	0.50

Probabilities of Kill

Service 2

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.15	0.10	0.10
2	0.15	0.75	0.15
3	0.05	0.05	0.15

Probabilities of Kill

Service 3

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.15	0.10	0.10
2	0.10	0.125	0.10
3	0.10	0.10	0.6

5.1.2 Imperfect Classification

The Target classification probabilities for the Stove-piped Services are:

Service 1

Conditional Probability of Classification of Target Type

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	1	0	0
2	0.25	0.50	0.25
3	0.25	0.25	0.50

Service 2

Conditional Probability of Classification of Target Type

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.50	0.25	0.25
2	0	1	0
3	0.25	0.25	0.50

Service 3

Conditional Probability of Classification of Target Type

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.50	0.25	0.25
2	0.25	0.50	0.25
3	0	0	1

CROP

Classification by Fusion

Conditional Probability of Classification of Target Type

	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.80	0.10	0.10
2	0.10	0.80	0.10
3	0.10	0.10	0.80

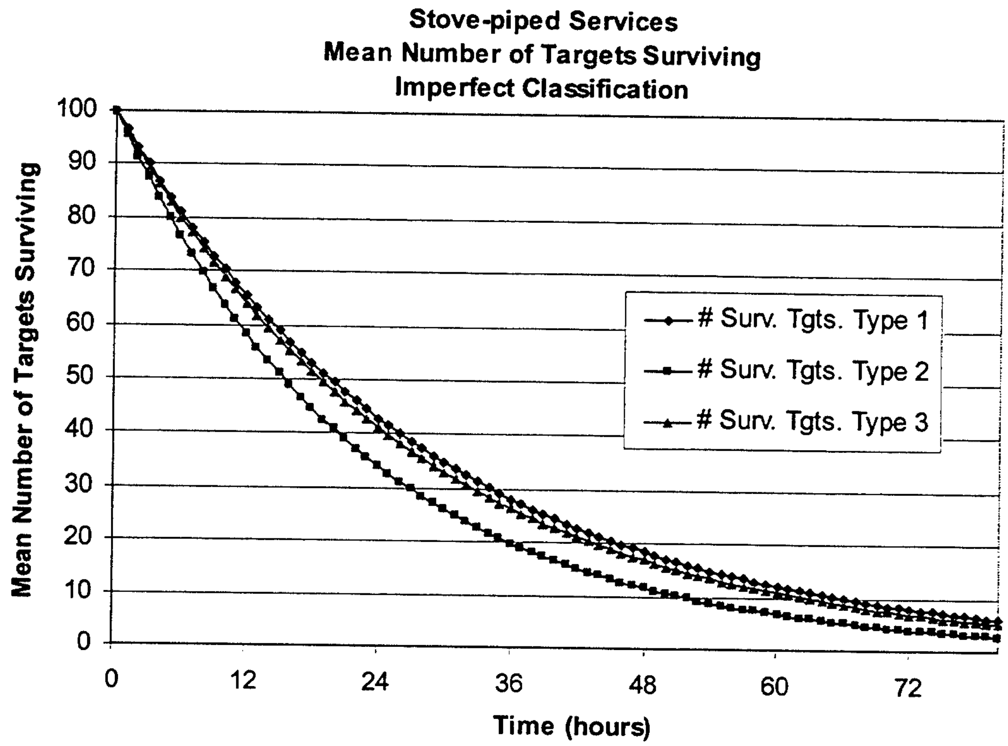
CROP

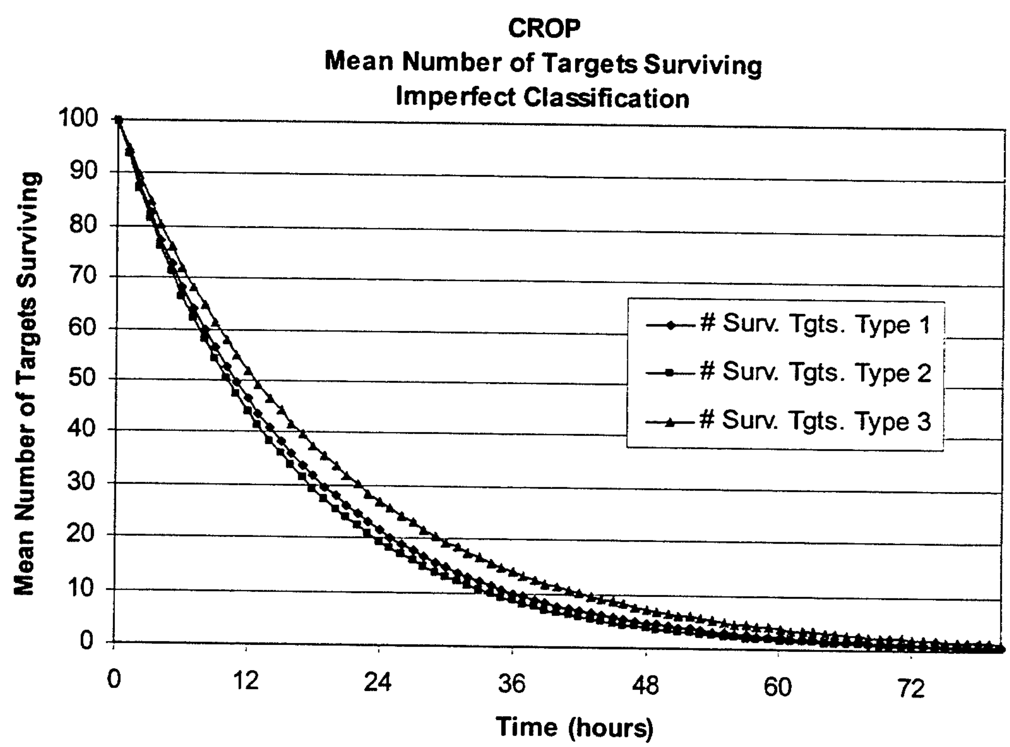
The Service that Detects Does Classification

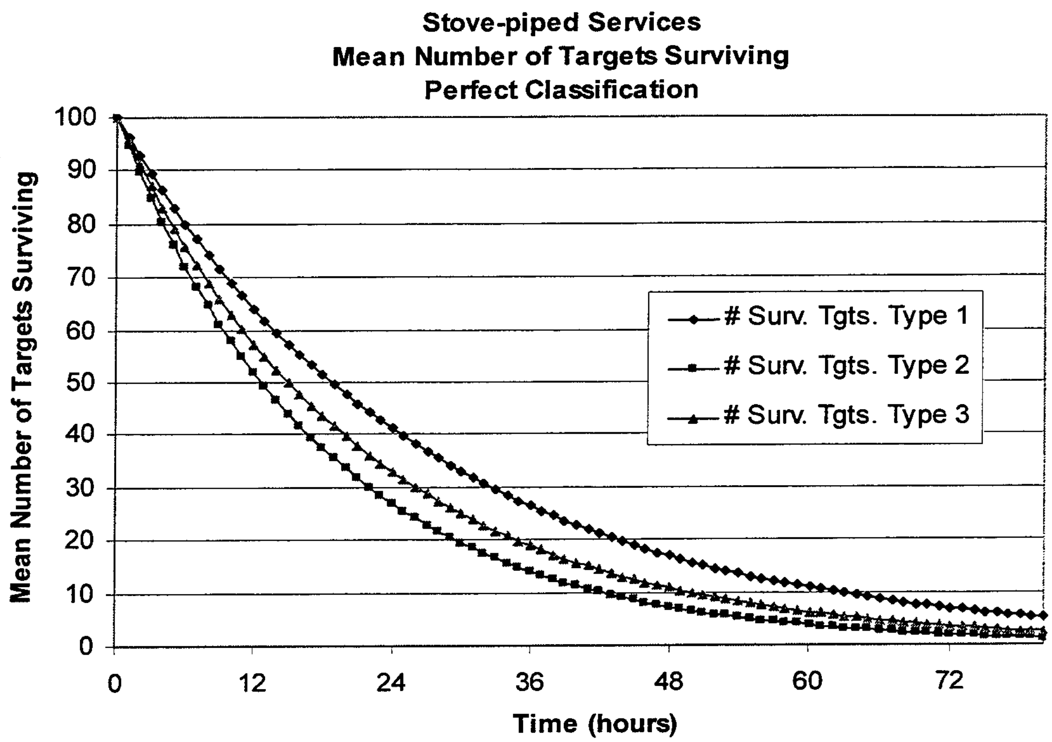
Conditional Probability of Classification of Target Type

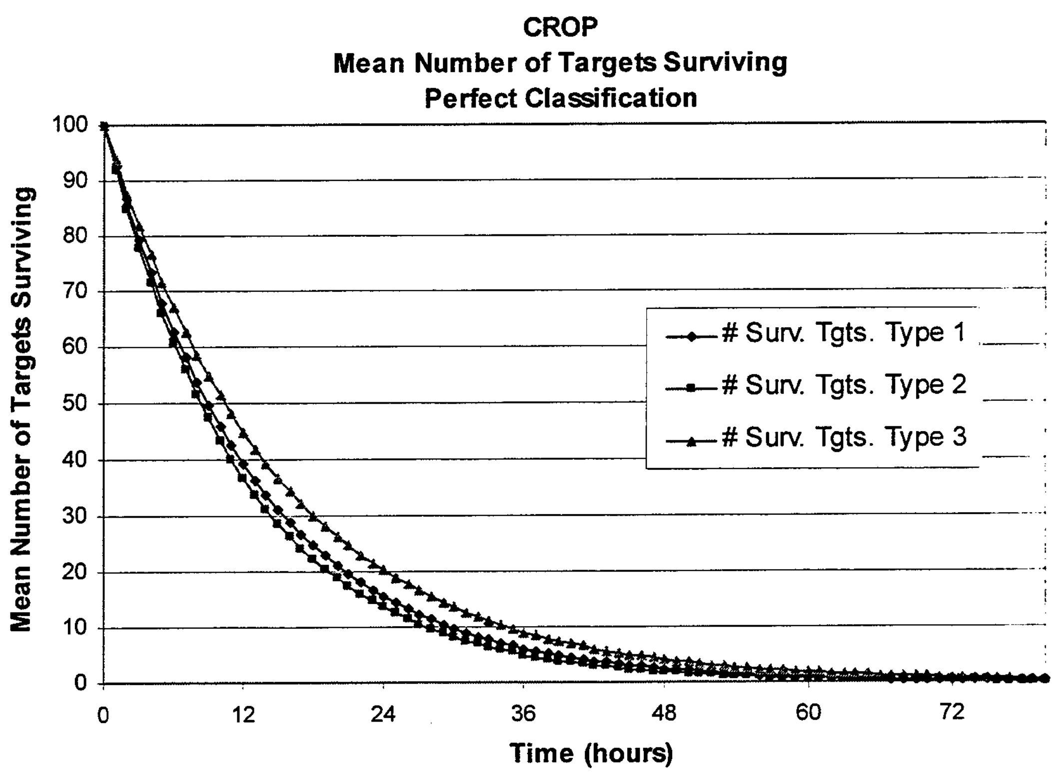
	Classified Target Type		
True Target Type	**1**	**2**	**3**
1	0.66	0.17	0.17
2	0.17	0.66	0.17
3	0.17	0.17	0.66

The mean number of surviving Targets for the Stove-piped Services and CROP with imperfect classification and fused classification are displayed below:






Stove-piped Services
Mean Number of Targets Surviving
Perfect Classification
Mean Number of Targets Surviving
100
90
80
70
60
50
40
30
20
10
0
0
12
24
36
48
60
72
Time (hours)
Surv. Tgts. Type 1
Surv. Tgts. Type 2
Surv. Tgts. Type 3

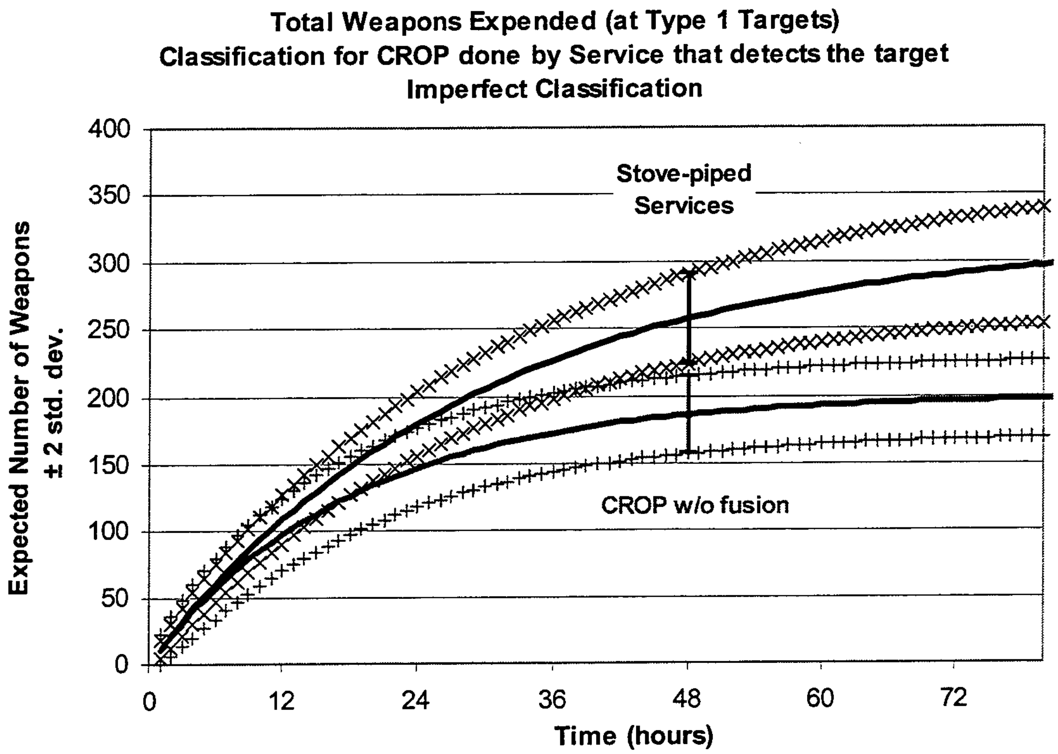

CROP
Mean Number of Targets Surviving
Perfect Classification
Mean Number of Targets Surviving
100
90
80
70
60
50
40
30
20
10
0
0
12
24
36
48
60
72
Time (hours)
Surv. Tgts. Type 1
Surv. Tgts. Type 2
Surv. Tgts. Type 3

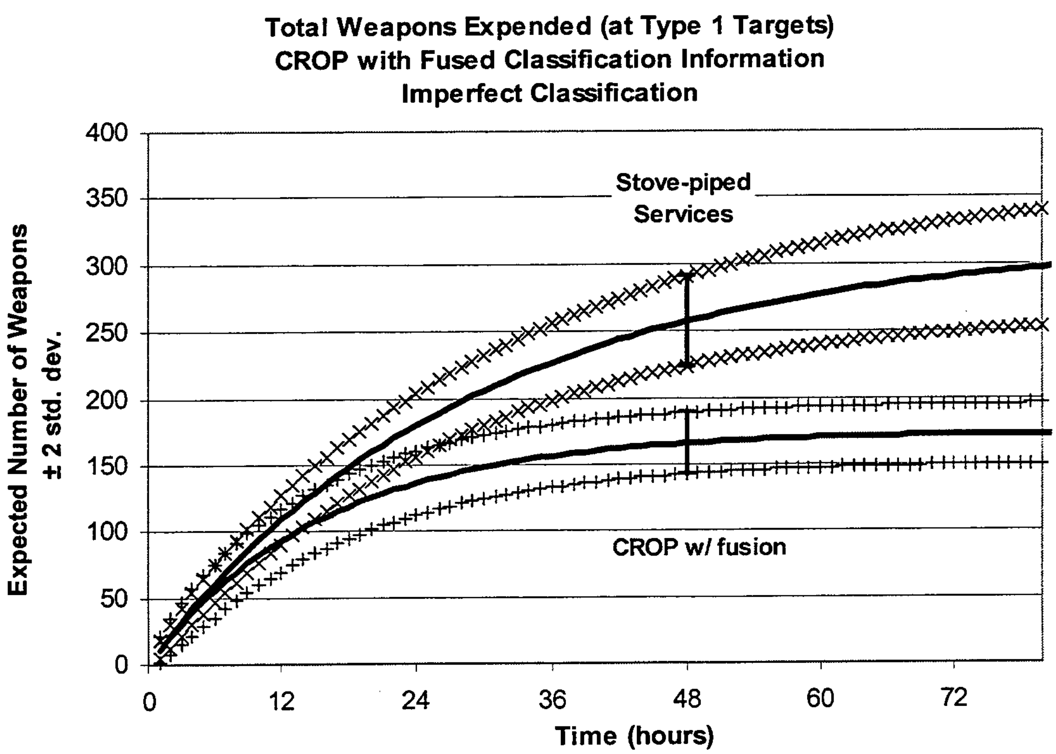
Discussion

In this example, Services 2 and 3 each have a particular Target type for which the probability of kill is larger than that for the other Services. Services 2 and 3 are relatively ineffective against other Target types. Hence, perfect classification does not decrease the mean number of surviving Targets over imperfect classification by much for the Stove-pipe (SP) case. The Stove-piped Services do not have the weapon capability that can take advantage of perfect Target classification. Since at least one Service has a weapon that is effective against each kind of Target, the ability of CROP to optimize weapon type to Target type results in a substantial decrease in the number of Targets surviving from that of Stove-piped Services. Note that the probability that a Target placed on the CROP targeting list is lost is the same as that for each of the individual Services. Hence, it is not surprising that CROP, either under imperfect classification or perfect classification, results in average numbers of Targets surviving, which are smaller than those for the Stove-piped Services architecture at any time. If the probability that a Target is lost before it can be fired upon is increased for CROP, the advantage of CROP will diminish.

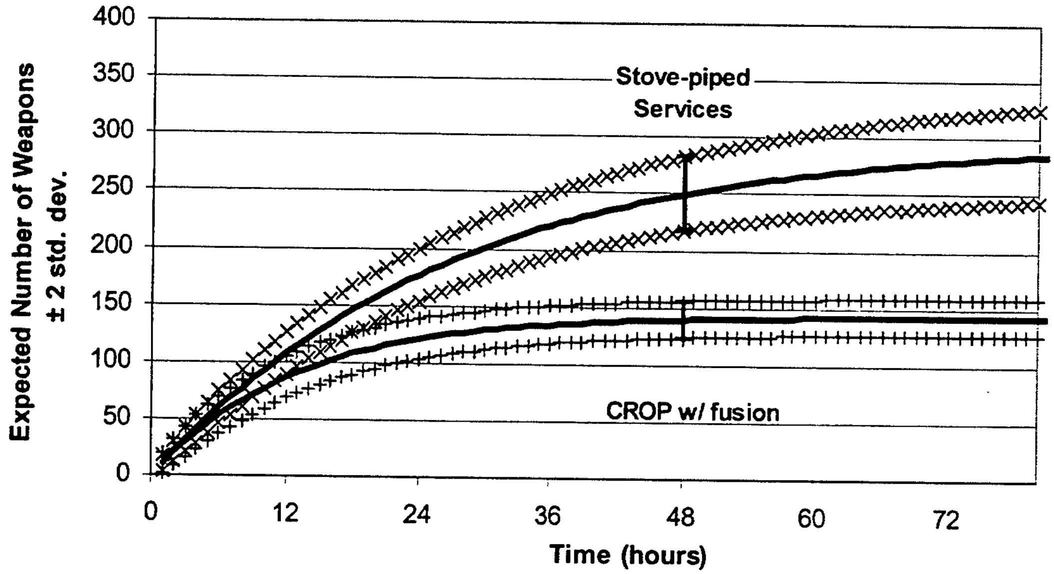
5.1.3 The Number of Weapons Fired at a Target

Expressions for moments for the number of weapons fired at a Target until it is killed are given in Appendix A. Graphs of the numbers of weapons expended against all Targets of type 1 are displayed on the following pages.

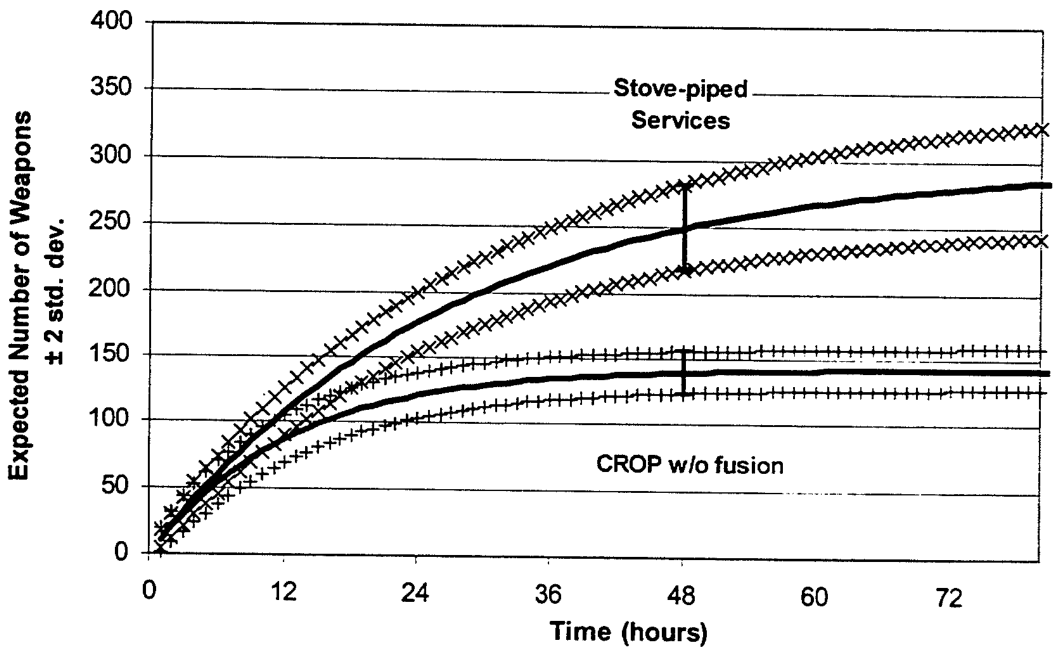

Total Weapons Expended (at Type 1 Targets)
Classification for CROP done by Service that detects the target
Imperfect Classification
Expected Number of Weapons ± 2 std. dev.
400
350
300
250
200
150
100
50
0
0
12
24
36
48
60
72
Time (hours)
Stove-piped Services
CROP w/o fusion


Total Weapons Expended (at Type 1 Targets)
CROP with Fused Classification Information
Imperfect Classification
Expected Number of Weapons ± 2 std. dev.
400
350
300
250
200
150
100
50
0
0
12
24
36
48
60
72
Time (hours)
Stove-piped Services
CROP w/ fusion

Total Weapons Expended (at Type 1 Targets)
CROP with Fused Classification Information
Perfect Classification



Total Weapons Expended (at Type 1 Targets)
Classification for CROP done by Service that detects the target
Perfect Classification

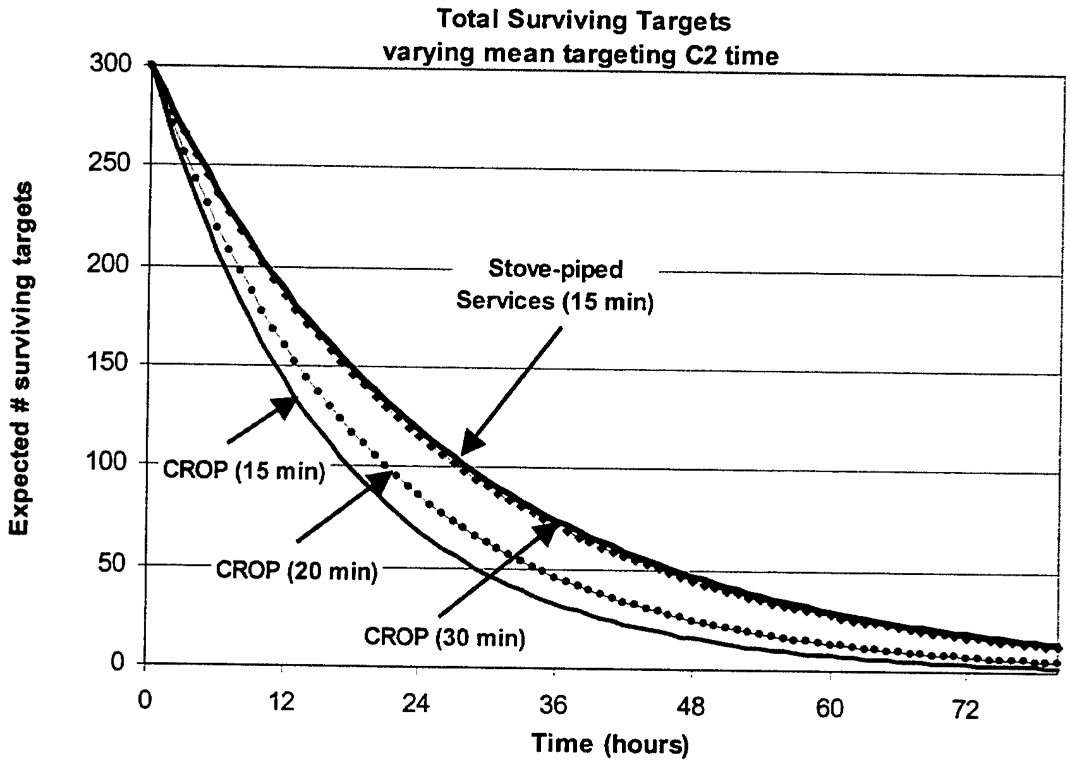


Discussion

Once again the disparate abilities of the Stove-piped Services do not allow them to take advantage of perfect Target classification. The ability of CROP to "optimize" weapon to Target type results in a smaller mean number of weapons fired and smaller variability on the number of weapons fired. This CROP ability also allows it to take advantage of perfect Target classification.

5.1.4 Variation of the Mean Targeting C2 Time

In this section we present results to study the effect of varying the mean targeting C2 time; the mean targeting C2 time is 1/(mensuration, etc., rate). The other parameters are as above. The figure on the next page displays the mean number of Targets surviving under Stove-piped Service architecture and CROP under imperfect Target classification for a baseline case (mean targeting C2 time = 15 minutes), and the cases in which the mean targeting C2 time is 33% longer (mean targeting C2 time = 20 minutes), and 100% longer (mean targeting C2 time = 30 minutes). CROP classification is done with fused classification information. Not surprisingly, the mean number of surviving Targets is quite sensitive to the mean targeting C2 time. Since the Stove-piped Services do not have weapons that are equally effective against all the Target types, the ability of CROP to assign the best weapon to a Target results in a considerable advantage.



5.2 Comparable Probabilities of Kill for Each Service

In this example, the probabilities of correct classification under imperfect classification will remain the same, but each Service will have roughly comparable kill probabilities. The probabilities of kill are as follows:

Probabilities of Kill

Service 1

True Target Type	Classified Target Type 1	2	3
1	0.70	0.125	0.15
2	0.10	0.60	0.10
3	0.10	0.10	0.50

Probabilities of Kill

Service 2

True Target Type	Classified Target Type: 1	2	3
1	0.50	0.10	0.10
2	0.15	0.75	0.15
3	0.05	0.05	0.50

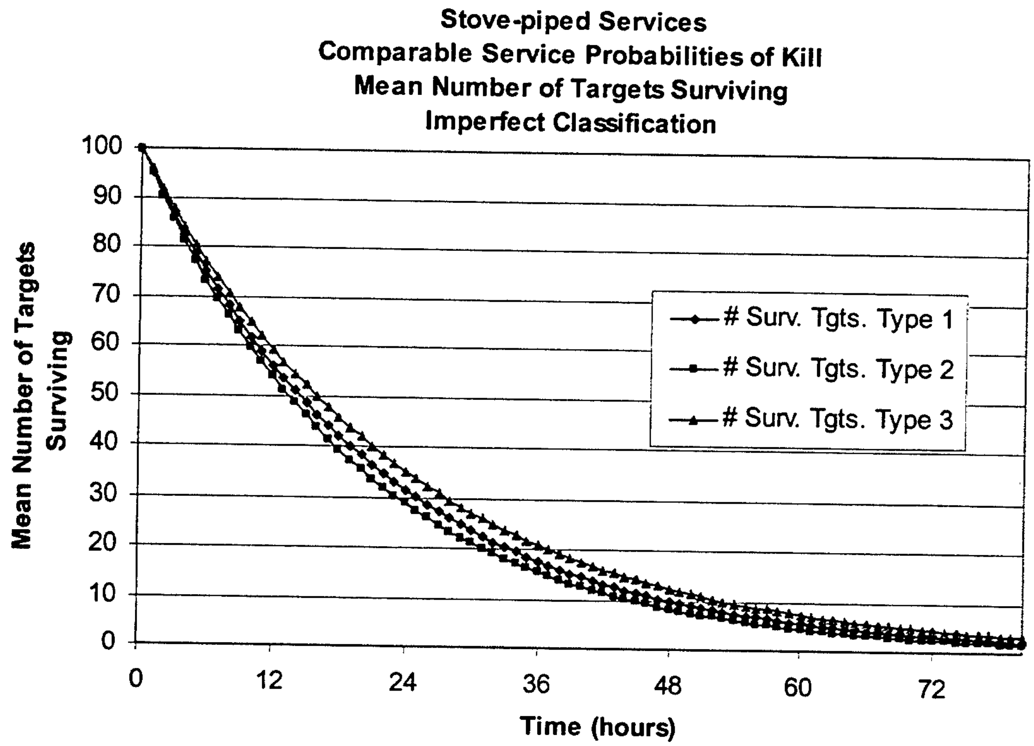
Probabilities of Kill

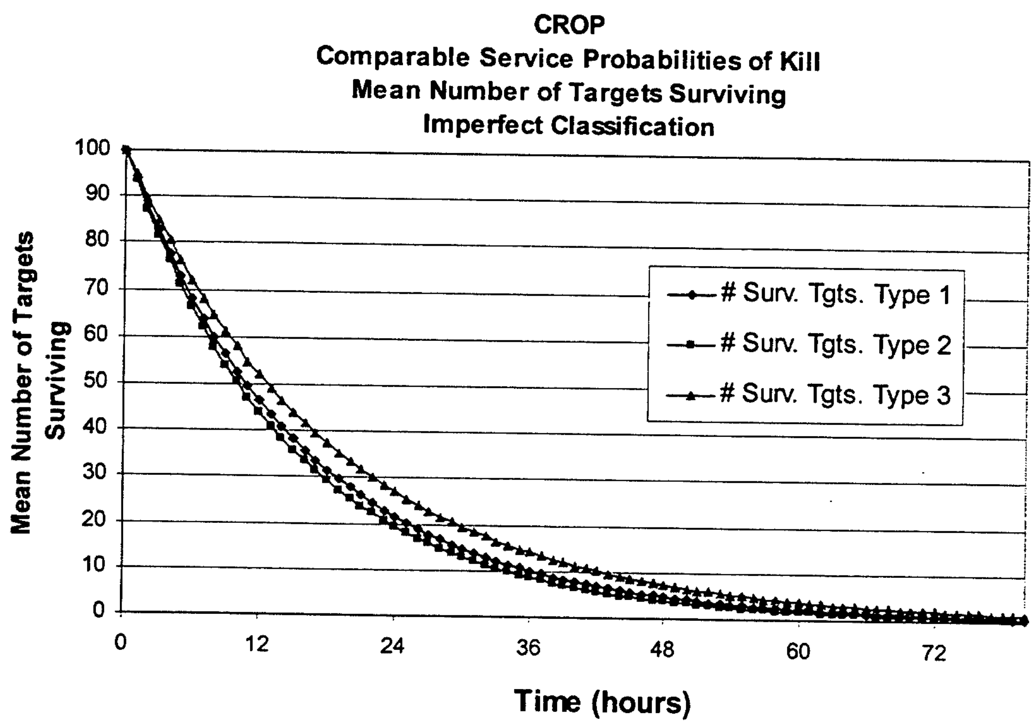
Service 3

True Target Type	Classified Target Type: 1	2	3
1	0.50	0.10	0.10
2	0.10	0.50	0.10
3	0.10	0.10	0.60

5.2.1 Imperfect Classification

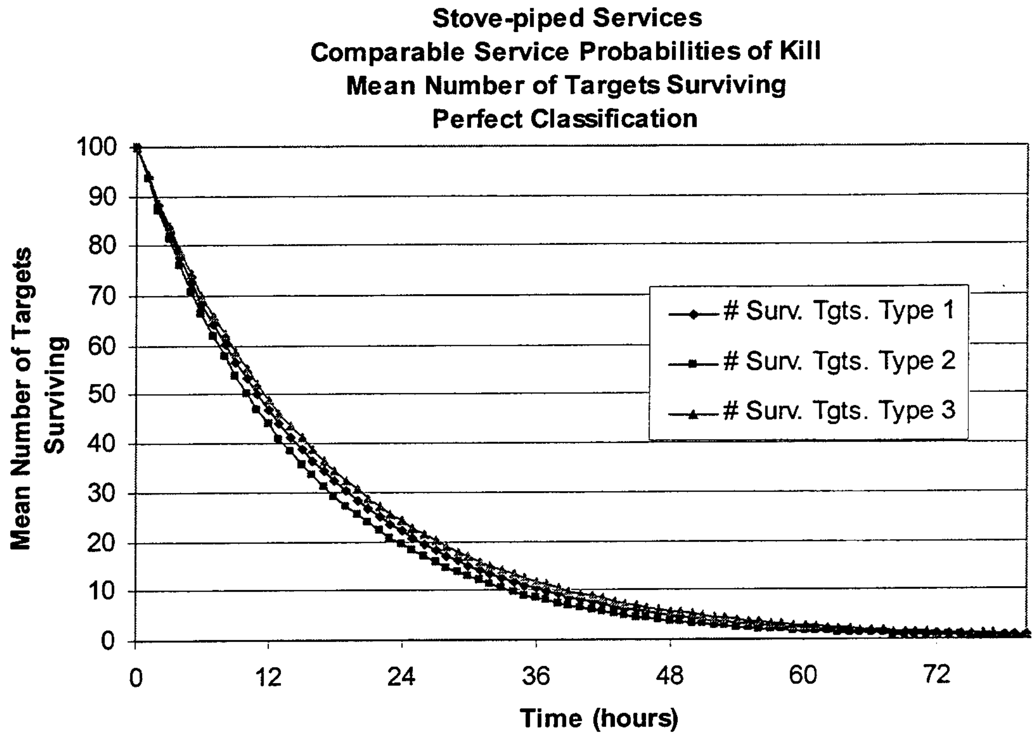
The figures on the next page display the expected number of Targets surviving for the UnCoordinated Services architecture, SP, and for CROP with imperfect Target classification.

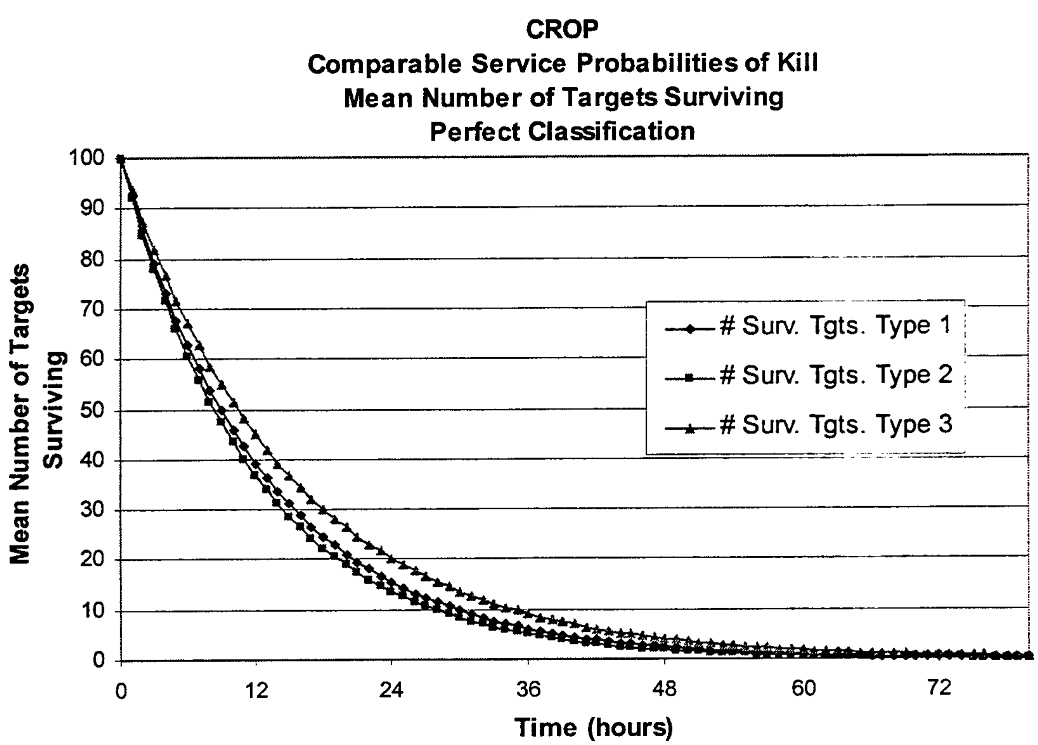

Stove-piped Services
Comparable Service Probabilities of Kill
Mean Number of Targets Surviving
Imperfect Classification
Mean Number of Targets Surviving
100
90
80
70
60
50
40
30
20
10
0
0
12
24
36
48
60
72
Time (hours)
Surv. Tgts. Type 1
Surv. Tgts. Type 2
Surv. Tgts. Type 3


CROP
Comparable Service Probabilities of Kill
Mean Number of Targets Surviving
Imperfect Classification
Mean Number of Targets Surviving
100
90
80
70
60
50
40
30
20
10
0
0
12
24
36
48
60
72
Time (hours)
Surv. Tgts. Type 1
Surv. Tgts. Type 2
Surv. Tgts. Type 3

5.2.2 Perfect Classification

The figures below display the expected number of Targets surviving for the Stove-piped Services architecture and for CROP with perfect Target classification.



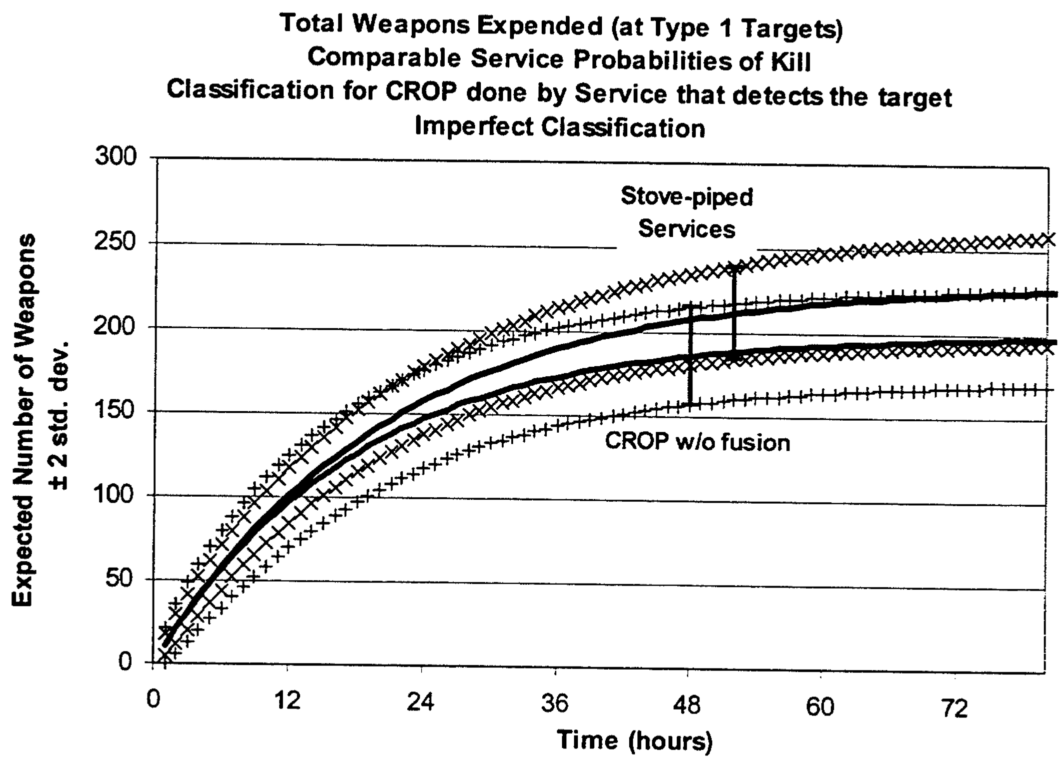


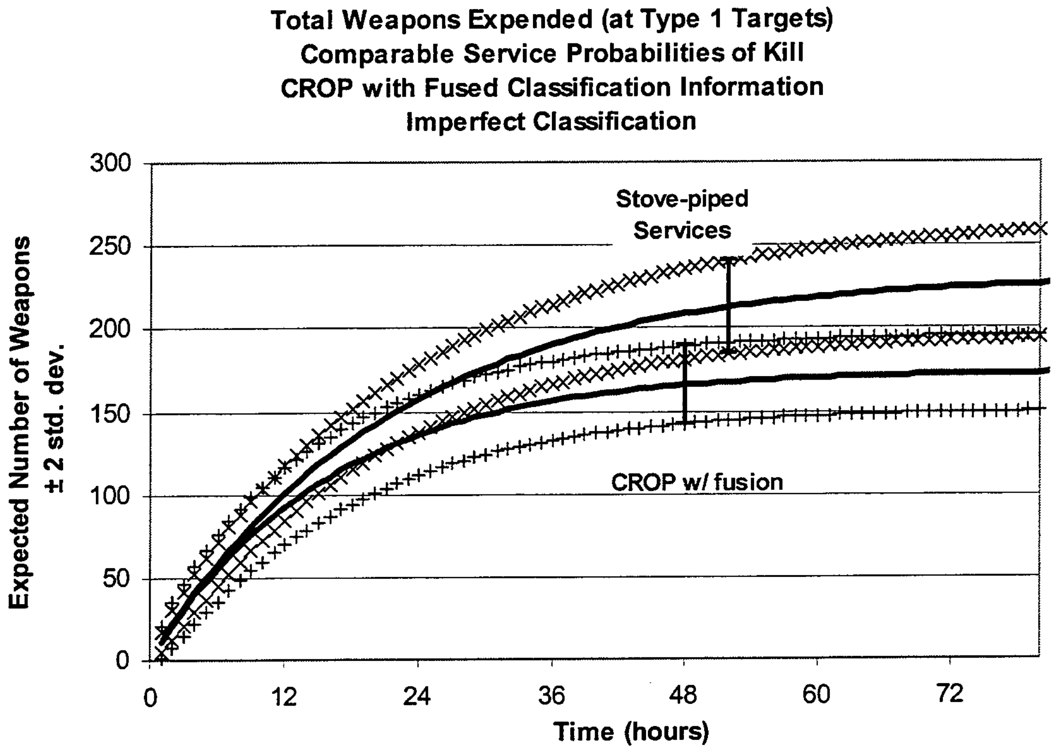
Discussion

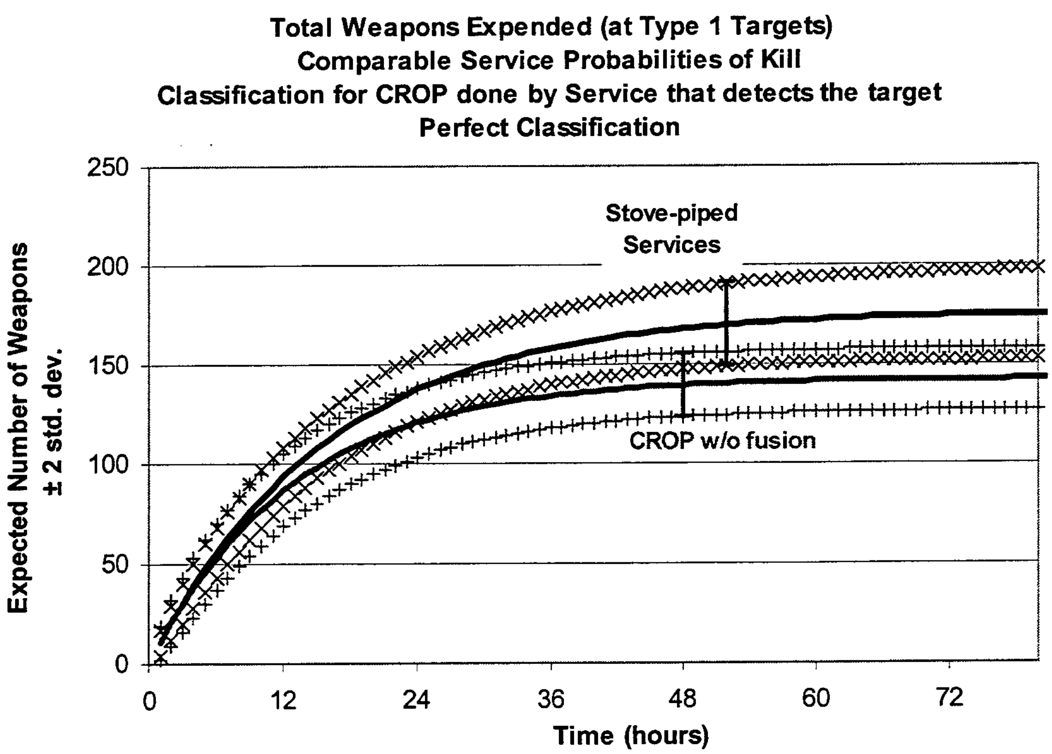
Comparison of the figure for the Stove-piped Services architecture with perfect classification with that for CROP under imperfect classification shows that the mean numbers of Targets surviving for the Stove-piped Services architecture with perfect information are smaller than those for CROP with imperfect information. Since the probability that a Target is lost under CROP is the same as that for the Stove-piped Services architecture, the mean numbers of Targets surviving are the smallest for CROP with perfect information. However, since each Service has reasonably large probabilities of kill against all the Target types, the advantage of CROP is not as large. Once again the advantage to CROP can be decreased if there is an increased targeting C2 time associated with CROP.

5.2.3 Number of Weapons Fired

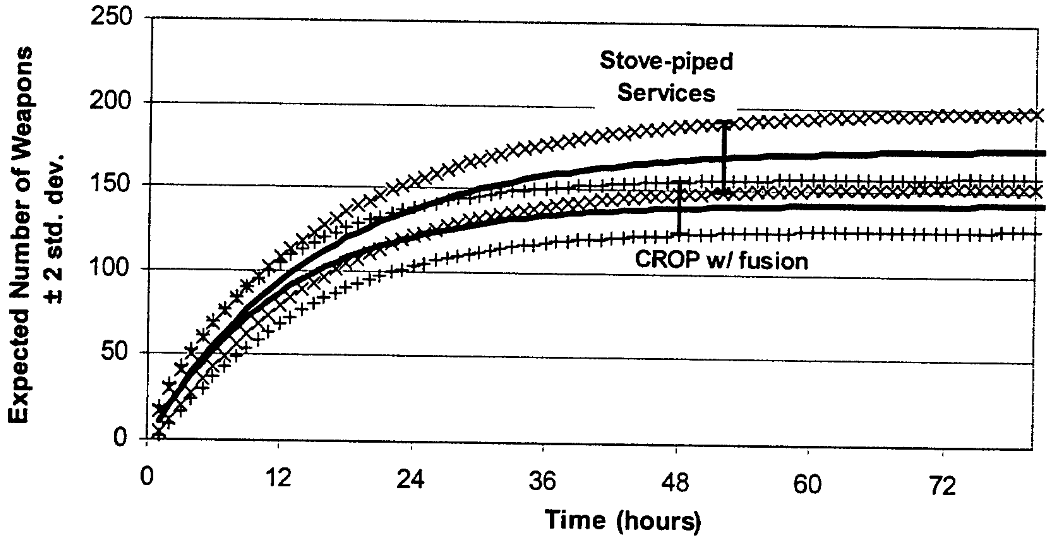
Expressions for the mean and variance of the number of weapons fired at a particular Target are given in Appendix A.




Total Weapons Expended (at Type 1 Targets)
Comparable Service Probabilities of Kill
CROP with Fused Classification Information
Imperfect Classification
Stove-piped Services
CROP w/ fusion
Expected Number of Weapons ± 2 std. dev.
Time (hours)
0
50
100
150
200
250
300
0
12
24
36
48
60
72


Total Weapons Expended (at Type 1 Targets)
Comparable Service Probabilities of Kill
Classification for CROP done by Service that detects the target
Perfect Classification
Stove-piped Services
CROP w/o fusion
Expected Number of Weapons ± 2 std. dev.
Time (hours)
0
50
100
150
200
250
0
12
24
36
48
60
72

Total Weapons Expended (at Type 1 Targets)
Comparable Service Probabilities of Kill
CROP with Fused Classification Information
Perfect Classification

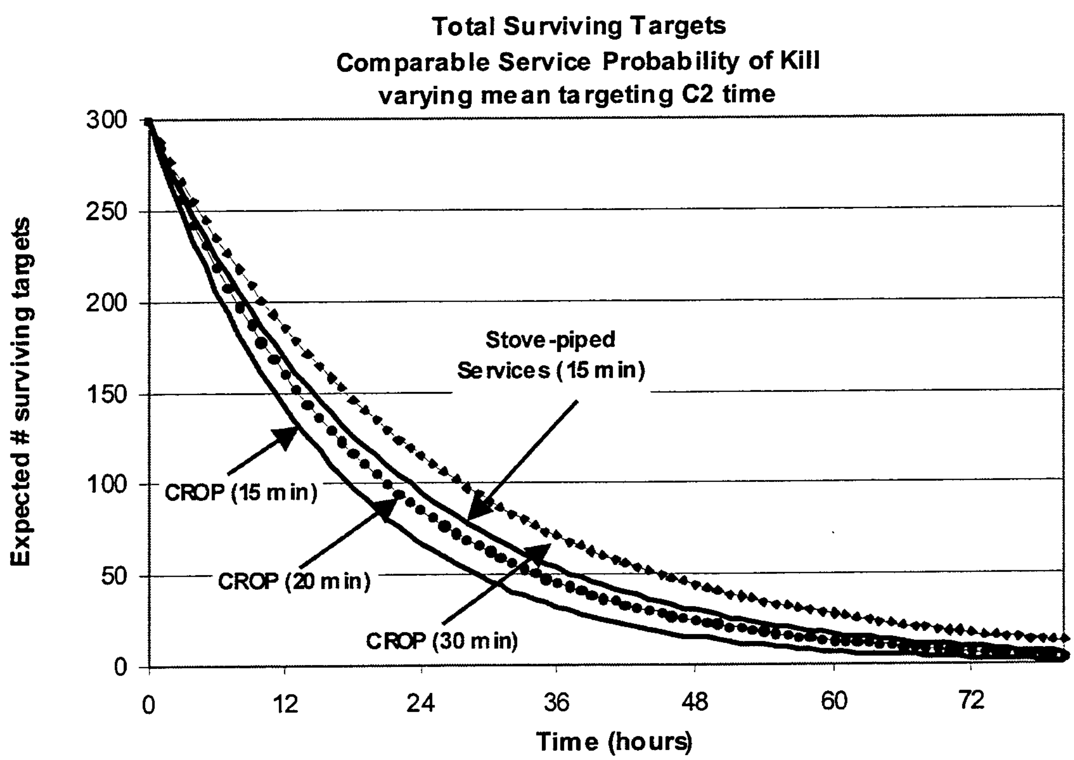


Discussion

When Stove-piped Services' Target classification is perfect, CROP Target classification is perfect. Perfect Target classification decreases the mean number of weapons shot and the variability of the number of weapons shot by about 1/3 over imperfect Target classification. Note that the Stove-piped Services architecture with perfect classification has a mean number of weapons expended, which is smaller than those for CROP with imperfect Target classification. Perfect classification does not mean as much for CROP in this case because the Services' probabilities of kill against a particular correctly classified Target type are roughly comparable. Thus, there is not much gained by allocating a particular Target to a "best" Service.

5.2.4 Variation of the Mean Targeting C2 Time

In this section we present results to study the effect of varying the mean targeting C2 time; the mean targeting C2 time is 1/(mensuration, etc., rate). The other parameters are as above. The figure below displays the mean number of Targets surviving under Stove-piped Service architecture and CROP under imperfect Target classification for a baseline case (mean targeting C2 time = 15 minutes), and the cases in which the mean targeting C2 time is 33% longer (mean targeting C2 time = 20 minutes), and 100% longer (mean targeting C2 time = 30 minutes). CROP classification is done with fused classification information. Not surprisingly, the mean number of surviving Targets is quite sensitive to the mean targeting C2 time. Since the Services' probabilities of kill against different Target types are comparable, the advantage of CROP's ability to assign the best weapon to a Target does not result in as large a decrease in the mean number of Targets surviving as it has when the Services have different capabilities.



6. Summary and Conclusions

This paper describes and illustrates use of a high-level, low-resolution (HLLR) stochastic model for evaluating the benefits of a Common Relevant Operational Picture (CROP). The model focuses on the sensor-to-shooter information needed by elements of a joint force to engage time critical Targets. CROP capability is compared to the targeting ability of several uncoordinated Service forces with their own individual combined Sensor-Shooter systems, but which do not share information with other Services, informally called Stove-Piped (SP) architecture. The models include realistic random delays or latency caused by congestion or system unreliability, random times until Target detection and loss, and a probabilitistic representation of the Services' overall capability to detect, classify and shoot at a varying set of different Target types. Our version of a SP model is tilted somewhat unrealistically towards *too much* individual Service isolation, while the CROP model may assume *too-efficient* information coordination.

In summary, the models' results support the following conclusions:

- CROP can increase the number of time-critical Targets killed, sometimes considerably.
- CROP can decrease the mean and variance of the number of weapons expended to kill Targets.
- The benefits of CROP can be degraded if the CROP process requires more time from Target detection until weapon arrival at the Target.

One CROP capability is the ability to "optimize" weapon assignment across Services against a perceived Target type. This ability can result in a smaller mean and variance of the number of weapons fired. The size of the decrease depends on the effectiveness of each of the SP Services against different perceived Target types. If a Service in the SP architecture is relatively ineffective against some perceived Target types, then CROP's ability to assign these Targets to another more effective Service can result in significantly fewer weapons expended and more Targets killed. However, if the Services in the SP architecture are equally capable against all

Target types, then the benefit of CROP's optimal weapon assignment is less. Further, if CROP's optimal weapon assignment requires additional time from Target detection until weapon arrival at the Target, then there may be no benefit to the ability to assign a Target to a "best" Service.

Another CROP capability is the ability to allocate sensor assets, fuse sensor information, and pass common targeting information to all Shooters. This can result in shorter Target detection times and more accurate Target type classification. The increase in ability to correctly classify a Target type decreases the mean and variance of the number of weapons expended for both the SP architecture and CROP. However, if this increased capability comes at the price of an increase in the C2 targeting time, there may be no benefit.

In general, the time critical targeting benefit of CROP is sensitive to the time from Target detection until weapon arrival at the Target. Even a modest increase in this time can nullify the time critical targeting benefits of CROP over the SP architecture.

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Cox, D. R. and Lewis, P. A. W., *The Statistical Analysis of Series of Events*, Methuen & Co., Ltd., London, 1966.

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Gaver, D. P., "Random hazard in reliability problems," *Technometrics*, Vol. 5, 1963, pp. 211-226.

Gaver, D. P. and Jacobs, P. A., "Probability Models for Battle Damage Assessment (Simple Shoot-Look-Shoot and Beyond)," Naval Postgraduate School Technical Report, NPS-OR-97-014, Monterey, CA, 1997.

Johnson, C., "Common relevant operational picture: an analysis of effects on the prosecution of time-critical targets," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, March 2002.

Ross, S. M., *Introduction to Probability Models*, Seventh Edition, Academic Press, New York, NY, 2000.

U.S. Joint Forces Command, J92 Concepts Division, *A White Paper for Common Relevant Operational Picture*, Version 1.1, 21 April 2000.

Appendix A

Weapons Expenditure Under SP

Here is a simple version of the weapons expenditure problem:

Number of s-Weapons Fired at an r-Target in Time t =

(a) Number of s-Weapons fired in Time t when kill is by s-Weapon Before t +

(b) Number of s-Weapons Fired in Time t When kill is by $\bar{s}$, Other-Than-s Weapon Before t +

(c) Number of s-Weapons Fired in Time t when Target Survives (until t)

This viewpoint provides one-variable ("marginal") information concerning one type of Weapon fired at one member of a Target type. Because of the independence assumed in the CD (present version) statistical summaries such as mean, standard deviations and distribution transforms such as the generating function can be derived by conditioning.

Mathematical Analysis

Notation:

$$\begin{aligned}
\eta_{rs}(K)\,dt &= \text{probability that a live } r\text{-Target is } \textit{killed} \text{ by an} \\
&\quad s\text{-Weapon in time interval } dt; \\
\eta_{rs}(M)\,dt &= \text{probability the } r\text{-Target is } \textit{missed} \text{ by an} \\
&\quad s\text{-Weapon in time interval } dt; \\
\eta_{ro}(K)\,dt &= \text{probability an } r\text{-Target is } \textit{killed} \text{ by an} \\
&\quad \text{Other-Than-}s\text{-Shooter (denoted } \bar{s}\text{), in time interval } dt; \\
\eta_{ro}(M)\,dt &= \text{probability an } r\text{-Target is } \textit{missed} \text{ by an} \\
&\quad \bar{s}\text{, in time interval } dt;
\end{aligned} \tag{A-1}$$

The η-parameters are summaries of detection and Weapon choice and Shooting (killing, and missing) parameters, *for example*, but not exclusively (there are *many alternatives*, such as Shoot-Look-Shoot with imperfect BDA; see Gaver, *et al* (1997)).

In either of the competitive kill cases it is possible that, say, the time until another Service kills the Target, τ_o is censored because an s-Kill occurs first, $\tau_s < \tau_o$; but also vice versa. In finite time, killing may not occur, so τ_s and τ_o may both be censored.

$$\begin{aligned}
\eta_{rs}(K) &= \xi_{rs} p_{rs} \sum_j \sum_k c_{rj}(s) d_{jk}(s) \kappa_{rk}(s) \\
\eta_{rs}(M) &= \xi_{rs} p_{rs} \sum_j \sum_k c_{rj}(s) d_{jk}(s)\left[1-\kappa_{rk}(s)\right] \\
\eta_{ro}(s;K) &= \sum_{l \neq s} \sum_j \sum_k \xi_{rl} p_{rl} c_{rj}(l) d_{jk}(l) \kappa_{rk}(l) \\
\eta_{ro}(s;M) &= \sum_{l \neq s} \sum_j \sum_k \xi_{rl} p_{rl} c_{rj}(l) d_{jk}(l)\left[1-\kappa_{rk}(l)\right]
\end{aligned} \qquad \text{(A-2)}$$

In the following it is convenient to abbreviate:

$$\begin{aligned}
\eta_s(K) &\equiv \eta_{rs}(K) \\
\eta_s(M) &\equiv \eta_{rs}(M) \\
\eta_o(K) &\equiv \eta_{ro}(s;K) \\
\eta_o(M) &\equiv \eta_{ro}(s;M)
\end{aligned} \qquad \text{(A-3)}$$

Let

$$\begin{aligned}
\boldsymbol{M}_{rs}(t) &= \text{Number of } s\text{-Weapons expended on} \\
&\quad\ \text{an } r\text{-Target by time } t. \\
\boldsymbol{M}_{rs}(\infty) &= \text{same as above, but until } r\text{-Target kill.}
\end{aligned} \qquad \text{(A-4)}$$

Abbreviate (since r is fixed)

$$\boldsymbol{M}_s(t) \equiv \boldsymbol{M}_{rs}(t) \text{ and } \boldsymbol{M}_s \equiv \boldsymbol{M}_{rs}(\infty)$$

Generating Functions

Generating functions "generate moments" and also can reveal the forms of distributions. If $M_s(t)$ represents the random number of s-Weapons expended by time t against a particular r-Target (that Target can be either alive or dead by t), and $M_s(\infty)$ is the number of shots required to finally kill a particular r-Target, then let the generating functions (*g.f.*) be

$$g(z,t) = E\left[z^{M_s(t)}\right],$$

$$g(z,\infty) = E\left[z^{M_s(\infty)}\right]; \qquad \text{(A-5)}$$

for convenience abbreviate as g, where $g = g_s$, is the *g.f.* of s-Weapons.

Backward Equations

Note these mutually exclusive possibilities:

(a) Kill by Other - than - s in $(0, dt)$, so

$$E\left[z^{M_s(t+dt)}, (a)\right] = z^0 \eta_O(K)dt + o(dt)$$

(b) Kill by s - Weapon in $(0, dt)$, so

$$E\left[z^{M_s(t+dt)}, (b)\right] = z\eta_s(K)dt + o(dt)$$

(A-6)

(c) Miss by s - Weapon in $(0, dt)$, so

$$E\left[z^{M_s(t+dt)}, (c)\right] = E\left[z^{1+M_s(t)}\right]\eta_s(M)dt + o(dt)$$

(d) No relevant event in $(0, dt)$, so

$$E\left[z^{M_s(t+dt)}, (c)\right] = E\left[z^{M_s(t)}\right]\left[1 - \left(\eta_O(K) + \eta_s(K) + \eta_s(M)\right)dt\right]$$

Note that for this objective we can eliminate reference to Missing by Other-than-s.

Using (a) through (d) produces a first-order differential equation:

$$\frac{d}{dt} g(z,t) = -(\eta_o(K) + \eta_s(K) + \eta_s(M)) g(z,t) + \eta_s(M) z g(z,t) + \\ + z\eta_s(K) + \eta_o(K) \tag{A-7}$$

Its solution, subject to $g(z,0) = 1$, and putting $E_1(z) \equiv \eta_o(K) + \eta_s(K) + (1-z)\eta_s(M)$

$$g(z,t) = \exp(-E_1(z)t) + (1 - \exp(-E_1(z))\,t) \frac{\eta_o(K) + z\eta_s(K)}{\eta_o(K) + \eta_s(K) + (1-z)\eta_s(M)}.$$

The mean number of s-Weapons expended against the r-Target can be found by differentiation of g at $z = 1$ or a direct argument.

(e) The *mean* number of s - Weapons expended against the r - Target in $(0,\mathrm{t})$

$$m_s(t) \equiv E[M_s(t)] = \frac{p_s(K) + p_s(M)}{p_o(K) + p_s(K)} (1 - \exp(-E(1)t)) \tag{A-8}$$

$$\equiv \frac{p_s(K) + p_s(M)}{p_o(K) + p_s(K)} \{1 - \exp(-(p_o(K) + p_s(K))\eta t)\}$$

where $p_s(K) = \eta_s(K)/\eta$, $p_s(M) = \eta_s(M)/\eta$, $p_o(K) = \eta_o(K)/\eta$ defining

$$\eta = \eta_o(K) + \eta_s(K) + \eta_s(M),$$

the overall relevant event rate (of Target type r, implicitly). The probabilities sum to one/unity.

(f) Manipulation of the generating function, or a direct argument (backward equations) provides that the variance of the number of s - Weapons expended at an r - Target in $(0,t)$ is

(A-9)

$$v_s(t)=\frac{N(1)}{E(1)}\left[1-\exp\left(-E(1)t\right)\right]+2\frac{\eta_s(M)}{\eta_s(K)+\eta_o(K)}\left[\frac{N(1)}{E(1)}\left[1-\exp\left(-E(1)t\right)\right]\right.$$

$$\left.-N(1)\left(t\exp\left(-E(1)t\right)\right)\right]-\left[\frac{N(1)}{E(1)}\left[1-\exp\left(-E(1)t\right)\right]\right]^2$$

where $N(1)=\eta_s(K)+\eta_s(M)$ and $E(1)=\eta_s(K)+\eta_o(K)$.

Letting time, t, become large

$$v_s(\infty)=\frac{N(1)}{E(1)}\left[1+2\frac{p_s(M)}{p_s(K)+p_o(K)}\right]-\left(\frac{N(1)}{E(1)}\right)^2 \qquad \text{(A-10)}$$

Note 1: The formula (A-8) shows that, as t increases indefinitely if $p_s(M)$, the probability of s-Weapon Miss exceeds $p_o(K)$, the probability of Other-than-s Weapon Kill, then the mean number of s-Weapons expended until Target kill *exceeds* one, whereas if the inequality is reversed the mean numbers of weapons to kill is less than one.

Note 2: The probabilities of first-kill and miss, $p_s(M)$, depend only upon the *ratios* of detection rates or sums thereof. This means that if rates are simultaneously affected by common "environmental" conditions, interpreted broadly, the probabilities, and hence the mean, and also variance of weapons expenditure needed to kill a particular Target is relatively insensitive to common environmental conditions. Of course this does not extend to the time to kill, or the number killed in a fixed time.

Appendix B

A Bayesian Model for Data Fusion Under CROP

Here is a Bayesian methodology representing the fused system-level classification conducted by CROP.

Consider a single Target. Let $\boldsymbol{Z} \in \{0,1,...R\}$ denote its type. Let $\boldsymbol{C}(s)$ be the classification of Target type by the sensors of Service s, $s \in \{1,...S\}$. The conditional distribution of $\boldsymbol{C}(s)$ given $\boldsymbol{Z}$ is $c_{r,j(s)}(s) \equiv P\{\boldsymbol{C}(s) = j(s) \mid \boldsymbol{Z} = r\}$. The joint probability distribution of the identifications made (independently) by the various Services is

$$
\begin{aligned}
&P\{\boldsymbol{C}(1) = j(1), \boldsymbol{C}(2) = j(2), ..., \boldsymbol{C}(S) = j(S) \mid \boldsymbol{Z} = r\} \\
&= \prod_{s=1}^{S} c_{r,j(s)}(s) \equiv c_{r,\underline{j}(S)}(S),
\end{aligned}
\qquad \text{(B-1)}
$$

where $\underline{j}(S)$ is the vector of individual classifications, and which assumes conditional independence between classifications by Services; (the Services do their own internal fusion).

B.1 Joint/Mutual Classification, Based on Combined Data

Bayes' formula evaluates of the probability that a Target is type k, *given* observations (Target classifications) $j(1),...,j(s),...,j(S)$ by the S Services: If $\{\pi_k; k = 0,...I\}$ denotes a fixed prior on the type of Target, the conditional posterior probability a Target is of type k given the Service classifications $j(1),...,j(s),...,j(S)$ is

$$
\begin{aligned}
&\pi_k(j(1),...,j(S)) \equiv \pi_k(\underline{j}(S)) \\
&= \pi_k c_{k,\underline{j}(S)}(S) / \sum_{i=1}^{I} \pi_i c_{i,\underline{j}(S)}(S)
\end{aligned}
\qquad \text{(B-2)}
$$

for k=1,2,...,I. A non-informative prior assigns probability $1/R$ to each Target type.

Targets are detected; the probability that a target is type r is α_r. Each Service classifies the Target. On the basis of these classifications (observations), Bayes is used to find the probability that the Target is a type k. The conditional probability a Target of type r is assessed as a type k under CROP, based on the observations (Target classifications) of all the Services, is

$$\bar{c}_{r,k}(S) = \frac{1}{\alpha_r} \sum_{\underline{j}(S)} \underbrace{\alpha_r c_{r,\underline{j}(S)}(S)}_{\substack{\text{prob. tgt.} \\ \text{is of type } r \\ \text{and is class.} \\ \text{as } \underline{j}(S)}} \underbrace{\pi_k\left(\underline{j}(S)\right)}_{\substack{\text{conditional} \\ \text{prob. tgt. is} \\ \text{of type } k \text{ given} \\ \text{it is class. as} \\ \underline{j}(S)}}$$

$$= \pi_k \sum_{\underline{j}(S)} \frac{\prod_{s=1}^{S} c_{r,j(s)}(s) \prod_{s=1}^{S} c_{k,j(s)}(s)}{\sum_{i=1}^{R} \pi_i \prod_{s=1}^{S} c_{i,j(s)}(s)} \tag{B-3}$$

$$= \pi_k \sum_{\underline{j}(S)} \frac{\prod_{s=1}^{S} c_{r,j(s)}(s)\, c_{k,j(s)}(s)}{\sum_{i=1}^{R} \pi_i \prod_{s=1}^{S} c_{i,j(s)}(s)}$$

Example 1: There are three Target types for each Service, and there are three Services. Assume, for one example, that each Service can perfectly classify one type of Target and cannot distinguish the other types of Targets. The classification matrices for the different Services are as follows.

	Service 1			**Service 2**			**Service 3**		
Classified Tgt. Type	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
True Tgt. Type									
1	1	0	0	0.5	0	0.5	0.5	0.5	0
2	0	0.5	0.5	0	1	0	0.5	0.5	0
3	0	0.5	0.5	0.5	0	0.5	0	0	1

With the non-informative prior (the probabilities a Target is of type i, i=1,2,3, are 1/3), the Bayesian classification probabilities for each Service separately are the same as the conditional classification probabilities. *The Bayesian classification probabilities for the three Services combined ("fused") result in perfect classification.*

All Three Services

Assessed Tgt. Type	1	2	3
True Tgt. Type			
1	1.00	0.00	0.00
2	0.00	1.00	0.00
3	0.00	0.00	1.00

Example 2: Alternatively, there are three types of Targets and each Service has the following classification probabilities:

	Service 1			Service 2			Service 3		
Classified Tgt. Type	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
True Tgt. Type									
1	1	0	0	0.50	0.25	0.25	0.50	0.25	0.25
2	0.25	0.5	0.25	0	1	0	0.25	0.50	0.25
3	0.25	0.25	0.50	0.25	0.25	0.50	0	0	1

In this example, there are positive probabilities that Service 1 will misclassify Targets of types 2 and 3 as type 1 Targets. Assume the prior probability that a Target is of type i is 1/3 for i=1,2,3.

Using only information from Service 1, the conditional probability a target is classified as a type j, given it is a type i, is as follows:

Service 1

Assessed Tgt. Type	1	2	3
True Tgt. Type			
1	0.67	0.17	0.17
2	0.17	0.46	0.38
3	0.17	0.38	0.46

Similar calculations for the other Services results in the following.

Service 2

Assessed Tgt. Type	1	2	3
True Tgt. Type			
1	0.46	0.17	0.38
2	0.17	0.67	0.17
3	0.38	0.17	0.46

Service 3

Assessed Tgt. Type	1	2	3
True Tgt. Type			
1	0.46	0.38	0.17
2	0.38	0.46	0.17
3	0.17	0.17	0.67

Combining information from all three Services using the above non-informative prior distribution results in the following conditional probability of Target classification given Target type.

All Three Services

Assessed Tgt. Type	1	2	3
True Tgt. Type			
1	0.79	0.10	0.10
2	0.10	0.79	0.10
3	0.10	0.10	0.79

In general, the application of Bayes with an uninformative prior appears superior to the classification of one Service alone. If valid information concerning $\{\alpha_r\}$ were available, and if, say, one class of Target greatly predominates, then results superior to those equally-likely priors should result.

Appendix C

Non-Stationary Poisson Arrivals to the Battlespace

Targets of type r arrive to a region according to a Poisson process with intensity function $\lambda_r(t)$. Assume targets can also leave the region. Let $U_{i,r}$ be the time the ith Target spends in the region if it is not killed. Assume $U_{i,r}$ are independent identically distributed random variables with distribution function F_r independent of the actions of the Services and the other Targets. Let (a) $N_r(t)$ be the number of type r live Targets in the region at time t; (b) $D_r(t)$ be the number of type r Targets killed in $(0,t]$; (c) $A_r(t)$ be the number of type r Targets to depart the region alive in $(0,t]$. The random variables $N_r(t)$, $D_r(t)$, $A_r(t)$ are independent and each Poisson. The probability a Target leaves the region alive is $\int_0^\infty e^{-\bar{\eta}_r(K)w} F_r(dw)$ where $\bar{\eta}_r(K) = \eta_{rs}(K) + \eta_{ro}(K)$, the rate at which a Target of type r is killed, in the notation of Appendix A.

$$(a')\ E\left[N_r(t)\right] \equiv a_N(t) = \int_0^t e^{-\bar{\eta}_r(K)(t-w)}\left(1 - F_r(t-w)\right)\lambda(w)\,dw \qquad \text{(C-1)}$$

$$(b')\ E\left[D_r(t)\right] \equiv a_D(t) = \int_0^t \left[\int_0^{t-s} \bar{\eta}_r(K) e^{-\bar{\eta}_r(K)u}\left[1 - F_r(u)\right]du\right]\lambda(w)\,dw \qquad \text{(C-2)}$$

$$(c')\ E\left[A_r(t)\right] \equiv a_A(t) = \int_0^t \left[\int_0^{t-w} e^{-\bar{\eta}_r(K)u} F_r(du)\right]\lambda(w)\,dw \qquad \text{(C-3)}$$

Let $M_{rs}(t)$ be the total number of weapons fired by Service s at Targets of type r during $(0,t]$. Expressions for the mean, $m_{rs}(t)$, and variance, $v_{rs}(t)$, of the number of weapons fired by Service s at one Target of type r that is initially present and never leaves the region until a kill occurs appear in Appendix A. Similar expressions can be obtained in a similar manner for the mean and variance of the total number of weapons fired by all Services in the SP and CROP models.

Generating Functions

The generating function of the total number of weapons fired at Targets type r (that is available for detection since $t=0$) by Service s is $E\left[z^{M_{rs}(t)}\right] \equiv g_{rs}(z,t)$; see (A-8). For convenience, write $g(z,t) = g_{rs}(z,t)$.

Departure from Region Time U_r

A unit of type r may well leave the battlespace, $\mathcal{B}$, either because of presumed threat, inferred from Blue attack frequency, or because endurance limits are exceeded. Here model the available time in $\mathcal{B}$ as U_r, measured from a unit entry instant, with distribution $F_r(u)$; let all versions of U_r for different individuals be independent and identically distributed. By conditioning, then, the modified (A-8) generating function is

$$\tilde{g}_{rs}(z,t) = g_{rs}(z,t)\bar{F}_r(t) + \int_0^t g_{rs}(z,u)F_r(du) \tag{C-4}$$

again drop subscripts. Differentiation at $z=1$ produces moments.

Generating Function of Total Weapons Fired (By Blue Service s at Red Unit Types that Have Occupied $\mathcal{B}$ during $(0,t]$)

Let $G_{rs}(z;u,t)$ denote the generating function of all weapons fired by Service s at Red Targets of type r that have ever entered, and still remain in $\mathcal{B}$ during $(u,t]$; if $u=0$ then put $G_{rs}(z;t) = G_{rs}(z;0,t)$ for short. A straightforward backward equation argument, shows that

$$G(z;t) = exp\left\{-\Lambda(t)\left[1 - \int_0^t \tilde{g}(z,t-u)p(u,t)du\right]\right\} \tag{C-5}$$

where $\Lambda(t) = \int_0^t \lambda(u)\, du$ and $p(u,t) = \lambda(u)/\Lambda(t)$, for $0 \le u \le t$.

Moments: Mean and Variance of Weapons Expended

Since, by, (C-5)

$$ln\, G(z;t) = -\Lambda(t)\left[1 - \int_0^t \tilde{g}(z,t-u)p(u,t)du\right] \tag{C-6}$$

$$\frac{\partial ln\, G(z;t)}{\partial z} = \Lambda(t)\int_0^t \frac{\partial \tilde{g}}{\partial z}(z,t-u)p(u,t)du \tag{C-7}$$

and at $z=1$ the expection/mean of the total number of weapons fired at type r Red Targets during $(0,t]$, $\bar{M}_r(t) = \sum_s \bar{M}_{rs}(t)$, is

$$E\left[\bar{M}_r(t)\right] = \Lambda(t)\int_0^t \tilde{m}_{1r}(t-u)p_r(u)du \tag{C-8}$$

where $\tilde{m}_{1r}(t)$ is the mean number of Blue weapons shot during $(0,t]$ at one type r Red Target that is in the region at time 0.

Since we can show that

$$\frac{\partial}{\partial z}\left[z\frac{\partial ln\, G(z,t)}{\partial z}\right]\Bigg|_{z=1} = Var\left[\bar{M}_r(t)\right], \tag{C-9}$$

then from (C-7)

$$Var\left[\bar{M}_r(t)\right] = \Lambda(t)\int_0^t \tilde{m}_{2r}(t-u)p_r(u)du \tag{C-10}$$

where $\tilde{m}_{2r}(t)$ is the second moment of the number of Blue weapons fired during $(0,t]$ at a type r Red Target that is in the region at time 0; it can be derived from the first and second derivatives of $\tilde{g}(z,t)$ at $z=1$.

By straightforward (tedious) backward equation arguments one gets

$$\tilde{m}_{1r}(t) = E\left[\sum_s M_{rs}(t)\right] = \left[\bar{\eta}_r(K) + \bar{\eta}_r(M)\right]\left(1 - e^{-\bar{\eta}_r(K)t}\right) \tag{C-11}$$

$$\tilde{m}_{2r}(t) = E\left[\left(\sum_s M_{rs}(t)\right)^2\right] = \left[\frac{\bar{\eta}_r(K)+\bar{\eta}_r(M)}{\bar{\eta}_r(K)}\right]\left(1-e^{-\bar{\eta}_r(K)t}\right)$$

$$+\ 2\bar{\eta}_r(M)\frac{\left[\bar{\eta}_r(K)+\bar{\eta}_r(M)\right]}{\bar{\eta}_r(K)}\left(\frac{1-\left(1+\bar{\eta}_r(K)t\right)e^{-\bar{\eta}_r(K)t}}{\bar{\eta}_r(K)}\right) \tag{C-12}$$

where $\bar{\eta}_r(M) = \sum_s \eta_{rs}(M)$ in the notation of Appendix A.

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