A Composite Capacitor/Inductor Assembly for Resonant Circuits

J. P. Hull and D. W. Scholfield

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AIR FORCE RESEARCH LABORATORY
Directed Energy Directorate
3550 Aberdeen Ave SE
AIR FORCE MATERIEL COMMAND
KIRTLAND AIR FORCE BASE, NM 87117-5776

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James H. Degnan

James H. Degnan, DR-IV, Ph.D.Physics
Project Manager

FOR THE COMMANDER

R Earl Good

COURTNEY D. HOLMBERG, Lt Col, USAF
Chief, High Power Microwave Division

R. EARL GOOD, SES
Director, Directed Energy

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J.P. Hull, D.W. Scholfield

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14. ABSTRACT Resonant structures are of interest due to their ability to produce oscillatory voltages in circuits. Past resonant structures have typically been designed using a lumped element capacitor for energy storage and a separate inductor. A composite capacitor/inductor assembly has been developed which merges the capacitance utilized for energy storage into the inductor, creating a consolidated electrical component. Composite capacitor/inductor assemblies are of interest due to the ability of these devices to produce resonant responses with one half the number of parts required by more traditional resonant structures. This composite capacitor/inductor could be utilized in applications of frequency band suppression or frequency band pass for frequencies in excess of 100 MHz, or where a resonant circuit is required to reside in an area of minimum space - such as a printed circuit board or an integrated circuit. The device and the mathematical treatment to predict the device's performance are described.

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J. P. Hull, D. W. Scholfield

Directed Energy Directorate
Air Force Research Laboratory
Kirtland AFB, NM 87117

Abstract: Resonant structures are of interest due to their ability to produce oscillatory voltages in circuits. Past resonant structures have typically been designed using a lumped element capacitor for energy storage, and a separate inductor. A composite capacitor/inductor assembly has been developed which merges the capacitance utilized for energy storage into the inductor, creating a consolidated electrical component. Such a device can be manufactured such that it is extremely small and inexpensive. Composite capacitor/inductor assemblies are of interest due to the ability of these devices to produce resonant responses with one half the number of parts required by more traditional resonant structures. This composite capacitor/inductor could be utilized in applications of frequency band suppression or frequency band pass for frequencies in excess of 100 MHz., or where a resonant circuit is required to reside in an area of minimum space - such as a printed circuit board or an integrated circuit.

The device and the mathematical treatment to predict the device's performance are described.

Discussion: Resonant structures are of interest due to their ability to produce oscillatory voltages in circuits such as those utilized in the communications industry[1,2]. Past resonant structures have typically been designed using lumped element capacitors for energy storage, which are separate from the inductor. Composite assemblies are of interest due to the ability of these devices to produce resonant responses with one half the number of parts required to produce the more traditional resonant structure. Previous construction strategies for these composite devices utilized the technique of interweaving the wire coils of inductors[3-5]. The technique which is described in this paper combines the capacitance with the inductance, and employs conducting annuli coaxially positioned such that the annuli function as both capacitor and inductor, refer to Figures 1a and 1b. The annular structure has been chosen to promote inductance. Each annulus of the composite device illustrated in Figures 1a and 1b has a gap located on the circumference, for the positioning of electrical contacts. Conceptually this arrangement forms a parallel plate transmission line with the gap positioned across the opposing ends. A distributed circuit representation for this geometry is shown in Figure 2a. At resonance, the distributed circuit behaves as a lumped inductor and capacitor with the resultant simple equivalent circuit shown in figure 2b.

Two annuli are required to form one capacitor/inductor. For a stack of N annuli, N-1 capacitor/inductor rings are formed, see Figure 3. This stacking arrangement requires that the magnetic coupling, which exists from annulus to annulus, be taken into account. This can be accomplished via an equivalent circuit for a stack of N annuli, with N-1 capacitor/inductor rings, which has been developed and is shown in Figure 4. A value of L_P is assigned to each inductor (annulus), and each capacitor has an assigned value of C_P. The circuit is assumed to have an initial voltage of V_O. The coupling coefficient between adjacent inductors is annotated as k, a value constrained to be between 0 and 1. The coupling coefficient between nonadjacent inductors will be approximated as the product of the coupling coefficients of the adjacent inductors separating any two nonadjacent inductors. Hence the coupling coefficient between the first and the third lumped inductors will be k^2, the coupling coefficient between the first and the fifth lumped inductors will be k^4, etc. The equations describing the voltages, $V_i(t)$, across each i^{th} inductor after the switch has closed have been developed elsewhere[6] and will not be repeated here. The equations yield the result

$$V(t) = -(\frac{1-k^{N-1}}{1-k})L_P C_P \frac{d^2}{dt^2}V(t)$$

Equation 1.

which is easily recognized as that of a resonant circuit. The effective resonant frequency, ω_{eff}, for Equation 1 has been defined[6] as

$$\omega_{eff}^{2} = \left(\frac{1-k}{1-k^{N-1}}\right)\omega_{O}^{2}$$

Equation 2.

where $\omega_{O} = \frac{1}{\sqrt{L_P C_P}}$. Additional annuli yield an increase in the capacitance of the system by an amount C_P, hence increasing the stored energy. The total capacitance of the device, to be known as the effective capacitance, C_{eff}, is then

$$C_{eff} = (N-1)C_P$$

Equation 3.

For Equations 1 through 3 to remain consistent it is necessary to define a quantity known as the effective inductance[6]

$$L_{eff} = (\frac{1-k^{N-1}}{1-k})\frac{L_P}{(N-1)}$$

Equation 4

For values of k≈1 equation 4 may be approximated as $L_{eff}=L_P$. Equation 2 may now be rewritten as

$$\omega_{eff}^{\ 2} = \frac{1}{L_{eff}C_{eff}} = (\frac{1-k}{1-k^{N-1}})\frac{1}{L_P C_P} = \frac{\omega_O^{\ 2}}{(N-1)}$$

Equation 5

Equation 5 suggests that the effective resonant frequency for a stack of N annuli decreases as the reciprocal of the square root of N-1.

Experimental Results: Four of these composite capacitor/inductor devices have been constructed. The conducting elements for the devices were manufactured from 5 mil (0.127mm) copper tape. The first capacitor/inductor device was assembled with an exterior radius of 1.0 cm. The interior radius was 0.5 cm. The insulator between the each conducting element consisted of a sheet of 0.127mm thick paper, with a dielectric constant of 3.3. A gap 0.08 cm wide was cut into each annulus radially inward to accommodate electrical connections, and to produce the inductive loop. The capacitance and resonant frequency for this device was measured via an HP 8754A Network Analyzer, yielding 34 pF and 865 MHz respectively. Calculated inductance for this device 995 pH. The Q can be calculated via $Q = \frac{\beta}{2\alpha}$, where α is defined to be the attenuation due to dielectric loss

$$\alpha = \alpha_c + \alpha_d = \alpha_c + \frac{k_O \varepsilon_r (\varepsilon_e - 1) \tan \delta}{2\sqrt{\varepsilon_e}(\varepsilon_r - 1)}$$

where $\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}$, α_c =0.075 Np/m, and β

$$\beta = \frac{2\pi f \sqrt{\varepsilon_e}}{c}$$

is the propagation constant. For this device Q has been calculated to be 68.7.

The second capacitor/inductor device was assembled with an exterior radius of 0.5 cm. The interior radius was 0.25 cm. The insulator between the each conducting element consisted of a sheet of 0.127mm thick paper. A gap 0.08 cm wide was cut into each annulus radially inward to accommodate electrical connections, and to produce the inductive loop. The capacitance and resonant frequency for this device was measured, yielding 17 pF and 2500 MHz respectively. Calculated inductance for this device 1090 pH. The Q for this device has been calculated to be 86.5.

The third capacitor/inductor device was assembled with a radius of 1.0 cm. with five annuli separated by approximately 0.32 cm. For this third device the copper tape was positioned to lay in the axial dimension at 1.0 cm. radius. The form to maintain the structure for the conducting elements consisted of two turns of a sheet of 0.127mm thick paper. The capacitance and resonant frequency for this device was measured, yielding 3.7 pF and 350 MHz respectively. Calculated inductance for this device 55.8 nH. The Q for this device has not been calculated.

A physically larger version of these capacitor/inductor devices has been constructed for other applications[7]. The conducting elements of this last capacitor/inductor were manufactured from 5 mil (0.127mm) aluminum foil with an interior radius of 13.65 cm. and an exterior radius of 23.22 cm. A section of each annulus was formed to accommodate the electrical connection required for a gaseous switch. The insulator between the conducting elements consisted of 20 sheets of 5 mil (0.127mm) kraft paper for a total thickness of approximately 0.254 cm. The insulator had an interior radius of 13.02 cm. and an exterior radius of 23.5 cm. A gap 6.35 cm wide was cut into the kraft paper radially inward. The calculated value for each capacitance of this assembly was approximately 0.985 nF. The Q for this device has been calculated to be 81.3.

Measurements of the effective capacitance and the effective resonant frequency of the device as additional annuli are added, are tabulated in Table 1 below. A plot of Inductance versus Number of Annuli data from Table 1 is shown in Figure 5, along with a plot of the theoretical inductance calculated as per Grover[8]. The experimental and theoretical values are in agreement on average within 104 nH. The 21 annuli listed in Table 1 form 20 capacitors. The value C_P can be approximated by the average capacitance value yielded by the device *in toto*. Thus, $C_P = (19.653\text{ nF})/20 = 0.983$ nF per capacitor. Setting the resonant frequency for two annuli equal to ω_O yields $\omega_O = 2\pi(9.26\text{MHz})$. Thus, L_P may now be estimated to be 300 nH. The measured resonant frequency produced by this device with each incremental plate is taken to be ω_{eff}. For two plates $\omega_{eff} = \omega_O$. For three plates $\omega_{eff} = \omega_O \sqrt{\frac{1-k}{1-k^{3-1}}} = \omega_O \sqrt{\frac{1}{1+k}}$, as per Equation 5. A semi-log plot of $\frac{1-k}{1-k^{N-1}}$ versus the number of annuli, N, is shown in Figure 6 for various values of the coupling coefficient, k. Also shown in Figure 6 is a graph of $\frac{\omega_{eff}^2}{\omega_O^2}$ versus N. Figure 6 suggests that the values of obtained for $\frac{\omega_{eff}^2}{\omega_O^2}$ are in good agreement for a coupling coefficient of $k=0.999$, validating the approximation of $k \approx 1.0$ stated earlier.

The performance of these devices could, of course, be significantly improved by utilization of the advanced construction techniques currently available in the capacitor industry. Energy density could be increased, Q could be improved and hence bandwidth decreased, with the employment of more advanced materials. While this particular type of device does represents a departure from the traditional method of construction of resonant structures, the commonly used mathematical techniques such as those outlined in Grover still provide good estimates of the inductance values.

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Table 1. Experimental Data Obtained From the Construction of a Capacitor/Inductor Device

Number of Annuli	Measured Capacitance (nF)	Measured Frequency (MHz)	Calculated Inductance (nH)
2	0.6585	9.26	448
3	1.474	6.33	429
4	2.270	5.32	394
5	3.139	4.50	398
6	3.936	4.08	386
7	4.786	3.75	377
8	5.728	3.40	382
9	6.219	3.31	372
10	7.120	3.11	369
11	8.385	2.92	353
12	9.212	2.78	356
13	10.033	2.65	360
14	10.783	2.60	348
15	11.582	2.52	346
16	12.793	2.41	341
17	13.392	2.37	338
18	14.361	2.29	338
19	14.952	2.25	336
20	16.022	2.19	331
21	19.653	1.98	329

Figure 1a. Oblique View of Coaxial Annuli.

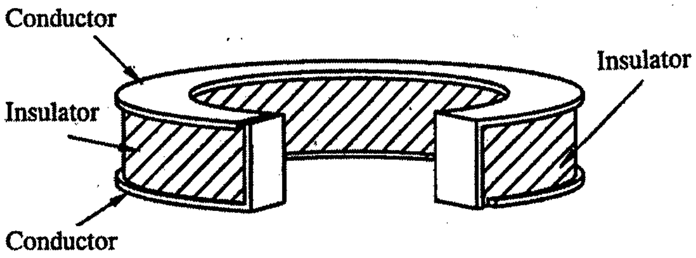


Figure 1b. Detail Front View of Coaxial Annuli.

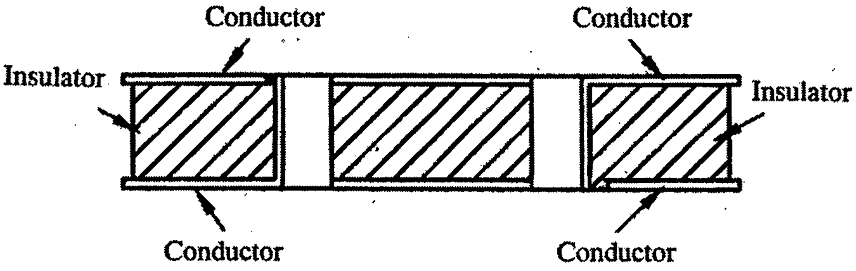


Figure 2a. Distributed Circuit for Coaxial Annulus Transmission Line.

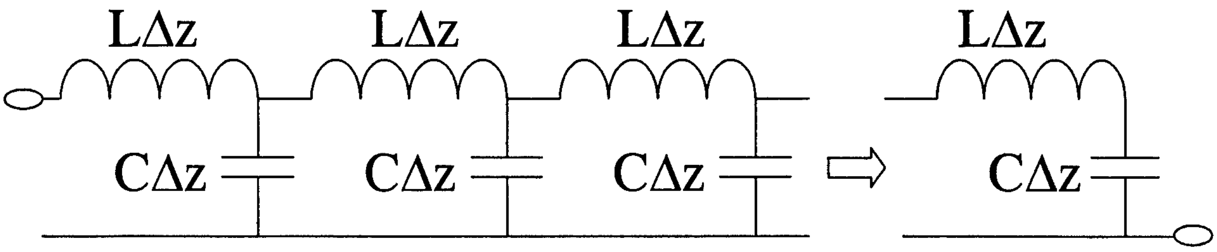


Figure 2b. Equivalent Lumped Circuit for Coaxial Annulus Transmission Line at Resonance.

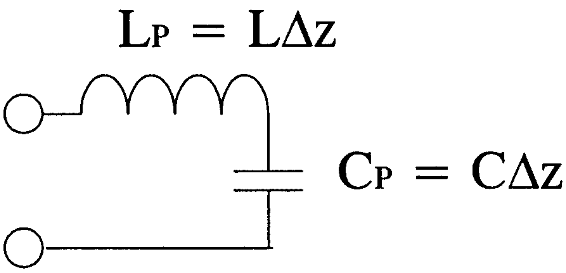


Figure 3a. Oblique View of N=7 Coaxial Annuli.

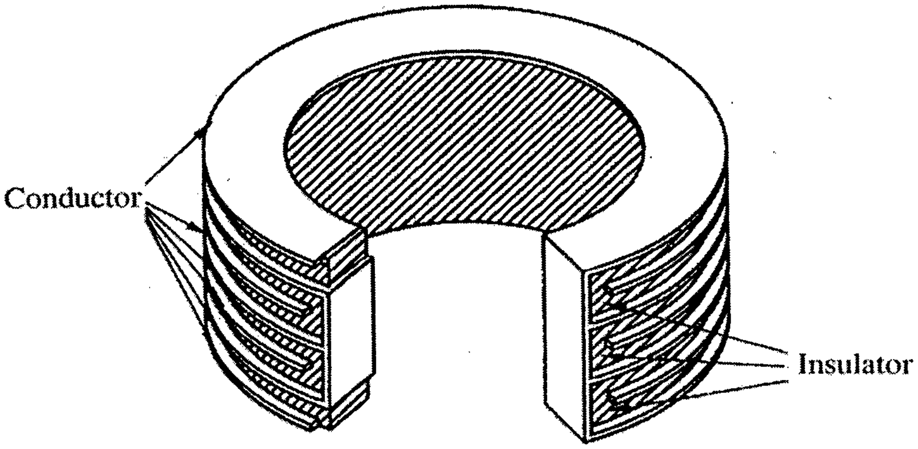


Figure 3b. Detail Front View of N=7 Coaxial Annuli.

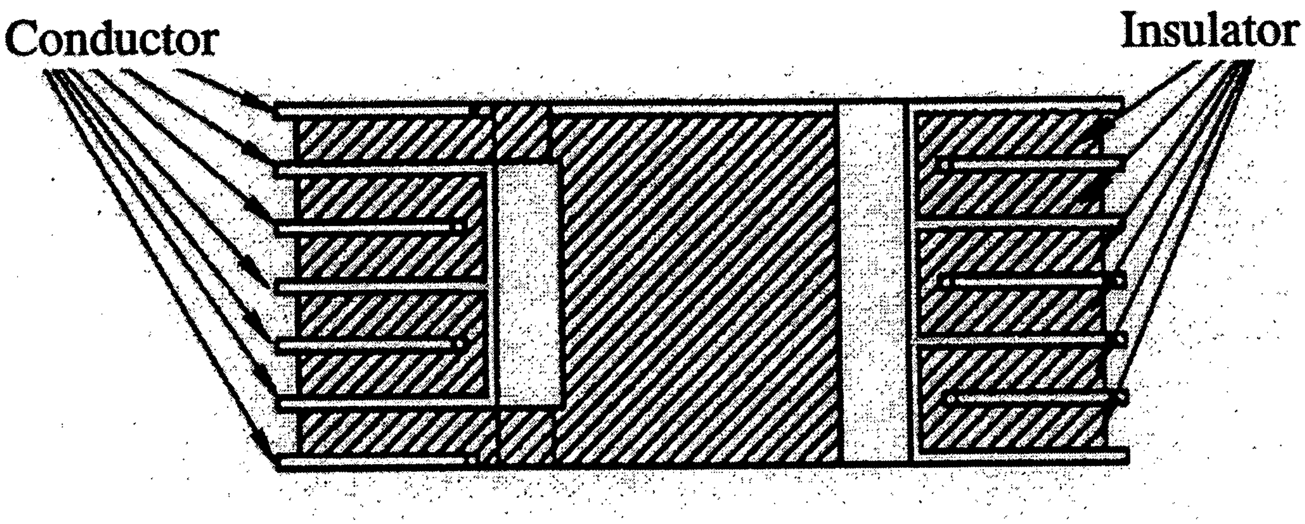


Figure 4. Equivalent Lumped Circuit
for N Coaxial Annuli at Resonance

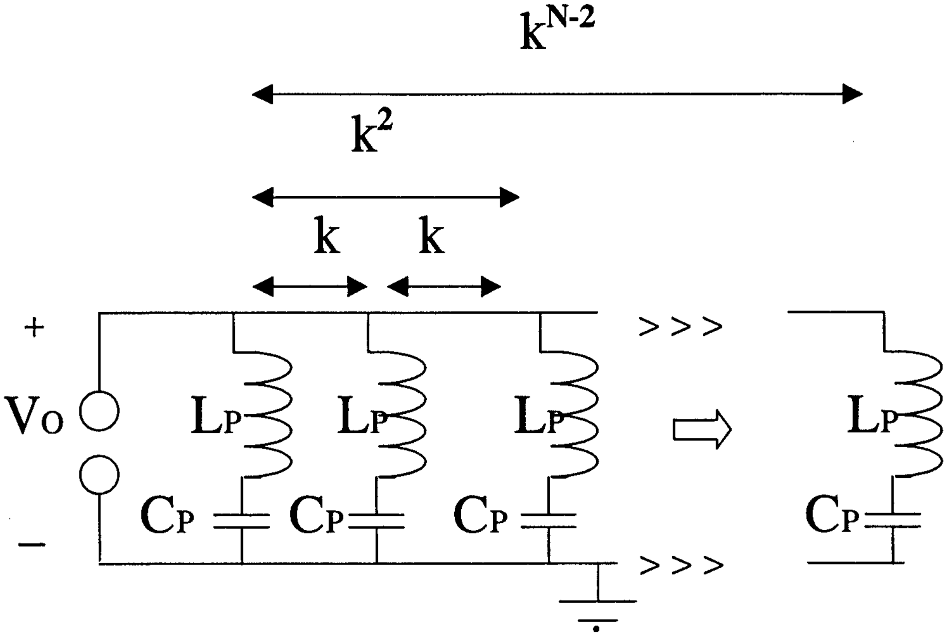


Figure 5. Plot of Inductance vs. N (Number of Annuli) from Table 1.

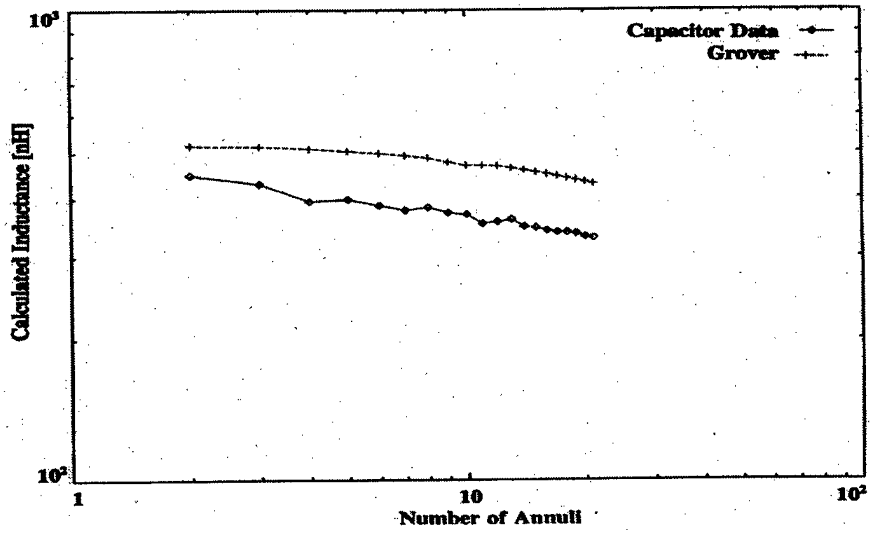
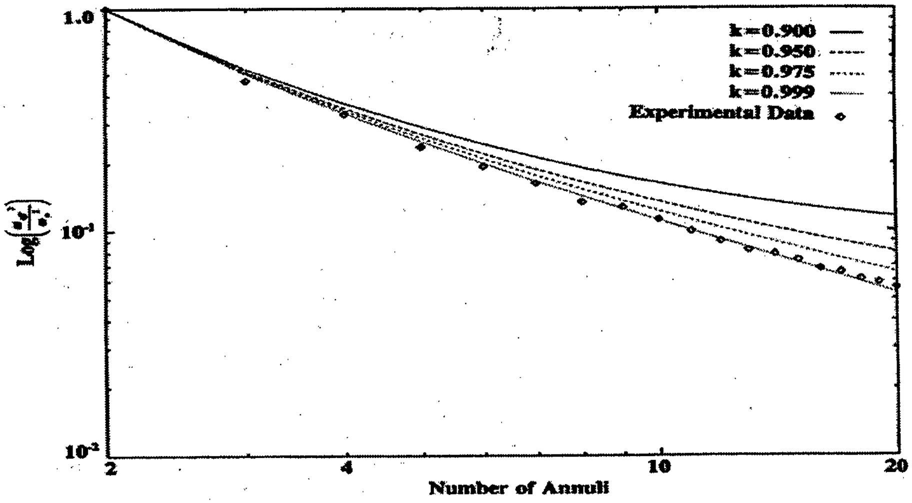


Figure 6. Plot of Frequency vs N (Number of Annuli) for various Coupling Coefficient Values.



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