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PERFORMANCE ANALYSIS OF A CDMA VSAT SYSTEM WITH CONVOLTIONAL AND REED-SOLOMON CODING

by

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September 2002

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PERFORMANCE ANALYSIS OF A CDMA VSAT SYSTEM WITH CONVOLTIONAL AND REED-SOLOMON CODING

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ABSTRACT

The purpose of this thesis is to model a satellite communication system with VSATs, using Spread Spectrum CDMA methods and Forward Error Correction (FEC). Walsh codes and PN sequences are used to generate a CDMA system and FEC is used to further improve the performance. Convolutional and block coding methods are examined and the results are obtained for each different case, including concatenated use of the codes. The performance of the system is given in terms of Bit Error Rate (BER). As observed from the results, the performance is mainly affected by the number of users and the code rates.

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I. INTRODUCTION

A. BACKGROUND

Satellites are an essential part of our daily life, and they have a very large usage ranging from Search and Rescue Operations to Environmental Monitoring. The widest use of satellites is, however, in communication systems. Satellites can cover vast areas on the world; therefore, they are the nodes where all links pass through in a communications network. Many users can access such a network simultaneously while they are widely separated geographically.

New concepts in modern warfare require the world armies to employ satellite communications more than ever before. Since the military requirements are different from the commercial ones, designing and implementing a military satellite system is usually more complex and challenging. Military satellite communication systems must meet both *quantitative* and *qualitative* requirements in order to be deployed into orbit. The capacity needed, in terms of the number of the subscribers within a network and the throughput or the data rate required, is considered quantitative. In contrast, qualitative requirements are quality measures, such as coverage area, protection, survivability, control, security, durability and interoperability [1].

B. OBJECTIVE

The objective of this thesis is to design a new satellite ground segment component for use with military operations and analyze its performance with different data rates and coding structures. Therefore, this work is mostly related with quantitative issues. The ground segment is designed to fit in the description of a Very Small Aperture Terminal (VSAT), which requires antenna dimensions less than 2.4 m. in diameter [6]. The designed system has a wide bandwidth and uses Code Division Multiple Access (CDMA) method.

C. RELATED WORK

The VSAT technology is evolving and more research is being done to further improve the VSATs. Most of the related work in VSATs is about networked use of VSATs as in [2]. The studies in [3] and [4] are about coding systems of VSATs. Studies like [5] are about designing and implementing terminals for VSAT systems and there are studies about frequency aspects of VSATs and Satellite Communications as in [6]. A similar study is done by [21], and Chapter III of this thesis was written jointly. The study in [21] mostly deals with jamming performance of a VSAT system, where this study is about VSAT performances for different coding schemes.

D. ORGANIZATION OF THESIS

In Chapter II, we introduce the Satellite Communication systems. The main elements of a satellite communication, namely, satellites, ground components, frequencies, satellite orbits are briefly described. A detailed explanation of VSATs and their applications is given.

In Chapter III, we explain the properties of Walsh functions, the extended orthogonality, PN sequences and Error Correction Coding (ECC). Different coding architectures like block coding, convolutional coding and concatenated coding and their advantages are examined.

In Chapter IV, we develop the uplink model for the satellite communication and find the Signal-to-Noise plus Interference ratio and the bit-error probability for each different coding scheme.

In Chapter V, we examine the results of the simulations for different systems to find the optimum scheme for various operational scenarios and requirements.

In Chapter VI, we summarize our conclusions.

II. SATELLITE SYSTEMS AND VSATS

A. SATELLITE SYSTEMS

The concept of using satellites dates back to the beginning of the 20th century. A few years after the launch of the first artificial satellite (Sputnik) in 1957, AT&T's Telstar satellite successfully completed a transatlantic telecast, proving satellite communication was possible. By 1965 a commercial satellite (INTELSAT 1) was already in orbit. Today more than 2,700 artificial satellites are in space [8].

1. Basic Satellite System

a. The Space Segment

The space segment of a satellite system contains the satellite and the terrestrial facilities for controlling the satellite. These facilities include Tracking, Telemetry and Command (TT&C) stations and a satellite control center in which all the operations associated with station keeping and assessing the vital functions of the satellite are performed.

A communications satellite has two main duties:

- To amplify the received signal: The carrier power at the receiver input of the satellite is in the order of $100\,pW$ to $1\,nW$. The power that is transmitted back from the satellite is in the order of $10W$ to $100W$. The power gain is, therefore, 100 to 130 dB.

- To change the carrier frequency to avoid interference: Newer satellites have onboard processing capability, which demodulate the received signal, thus correcting its errors and sending the signal back to earth by re-modulating and amplifying the power. This method results in lower bit error probabilities.

A satellite system's space segment components can be listed as below:

- Power Supply

- Altitude Control
- Station Keeping
- Thermal Control
- TT&C Subsystem
- Transponders
- Antenna Subsystem

b. The Ground Segment

The ground segment consists of all the earth stations. These earth stations can either be connected to the end users system directly, as in VSATs, or by terrestrial links. A typical ground system consists of:

- Antenna System
- Feed System
- High Power Amplifiers (HPAs)
- Low Noise Amplifiers (LNAs)
- Up/down Converters
- Controlling and Monitoring Equipment
- Ground Communications Equipment (modems, coders, etc.)

c. Frequency Allocation

Satellite systems use transmission and reception of radio waves to perform their tasks. Since the frequency spectrum is a limited resource, careful planning is required to prevent interference. The frequency band is managed by the *International Telecommunications Union*. Different frequencies are used for uplink and downlink. The main reason for this frequency change is interference. Separation of transmit and receive frequencies helps reduce interference both at the satellite and the ground receiver. In the notation of satellite communication frequency, the uplink frequency is written first. So, 6/4 *GHz* C band satellite communications means 6 *GHz* is the uplink frequency and 4 *GHz* is the downlink frequency. The usual frequencies used for satellite communications is shown in Figure 1.

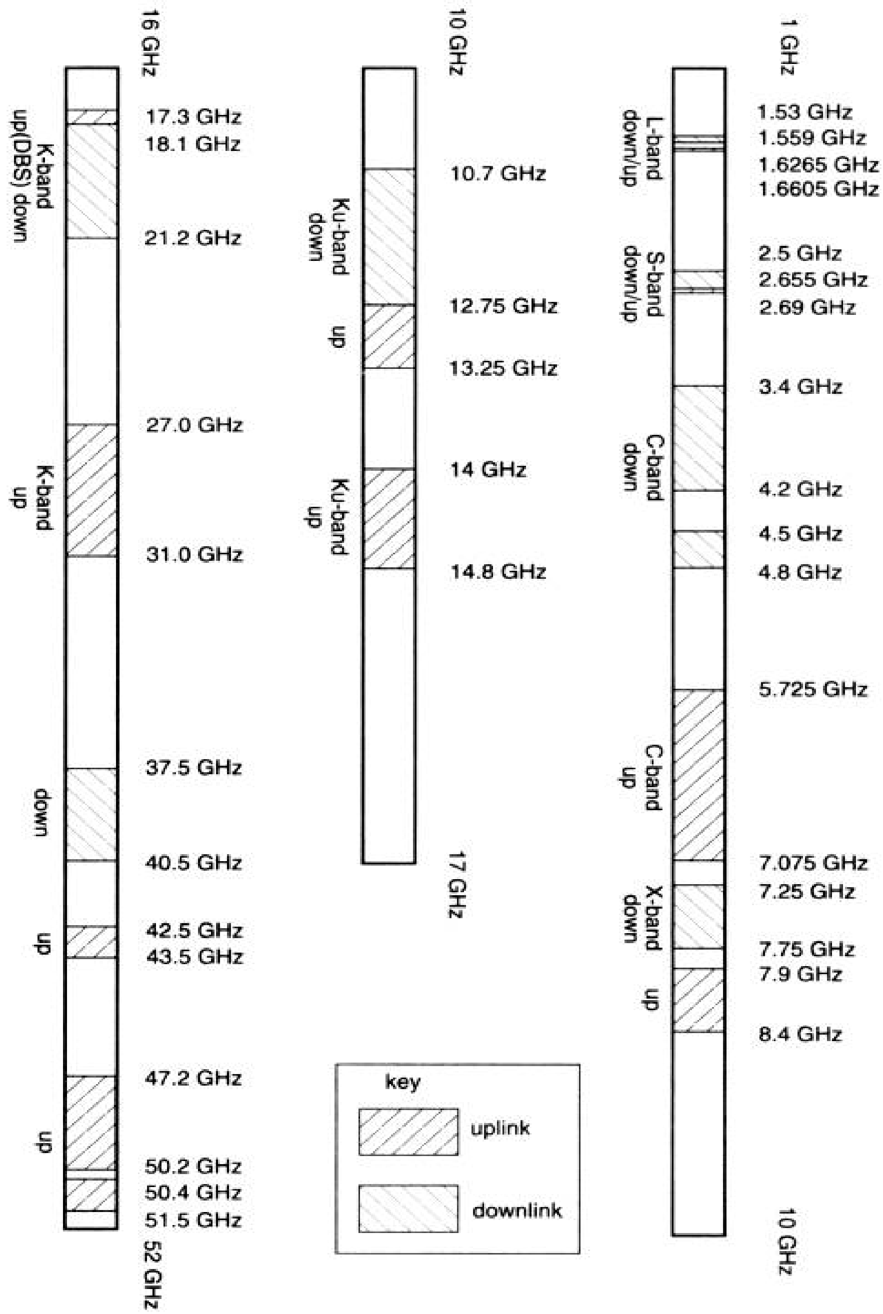

Figure 1. Frequency Allocation (From [9])

2. VSATs

VSAT stands for "Very Small Aperture Terminal." VSATs are used in the earth segment and antenna sizes for a typical VSAT differ between 60 cm. and 2.4 m. in diameter. A VSAT network can provide one-way or two-way data communications, video broadcast, and voice communication. These services can be categorized as point-to-point, broadcast or interactive networks [6].

VSATs provide economical private communication networks. They are reliable and their ability to be used across very distant geographical locations makes them very attractive both for commercial and military organizations. VSATs are flexible in architecture, and they have limited and known components. Therefore diagnosing problems and maintaining the network is easy. A VSAT network can work with different capacities in each direction, thus the nonsymmetrical feature can be an advantage and reduce the costs.

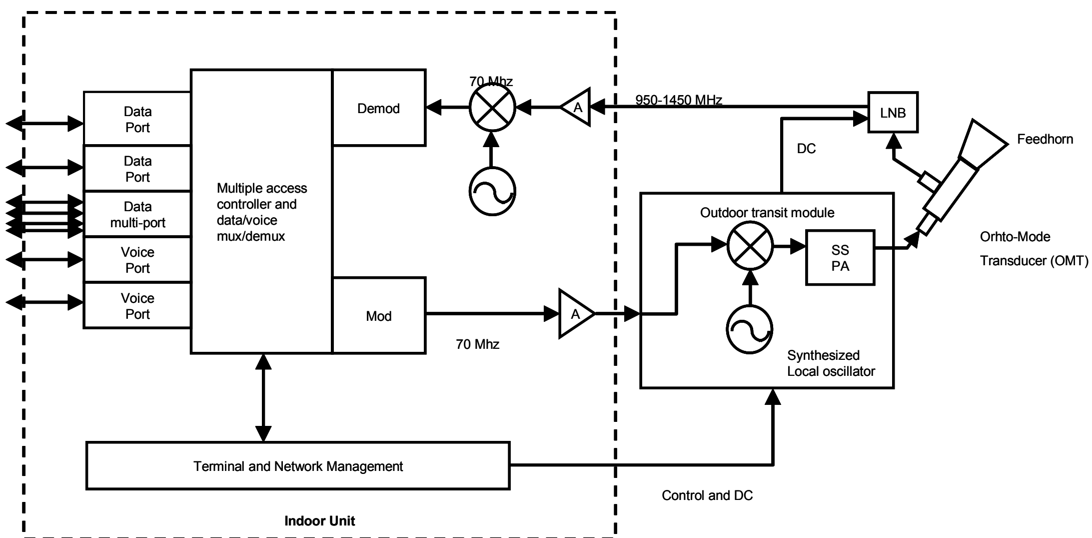

Figure 2. A Typical VSAT Block Diagram (From [14])

VSAT systems are generally configured in star or mesh formations and include a central station known as the *hub*. The other stations are located in different regions, which may include several various countries. The star configuration requires connection of one VSAT to another VSAT through the hub [6]. This results in a double-hop circuit, consequently increasing the propagation delay and requiring twice the satellite capacity. Also the hub is the single point of error in a star configuration, so for critical systems, a physically diverse hub is required. A mesh configuration, on the other hand, enables VSATs to connect to each other through the satellite in a single hop [10].

The important considerations for a VSAT system vary for different types of installations. However, the issues listed below must be carefully examined in any case.

- Orbit Visibility
- Frequency Clearance
- Local Weather Conditions

From the architectural point of view, the network size, network availability, and redundancy are the trade-off areas that determine both the cost and the performance of a VSAT system.

3. Satellite Orbits

Satellites and spacecrafts that orbit the earth follow the same laws that govern the motion of planets around the sun. These laws are known as Kepler's Laws. They arise from Kepler's observation of movement of planets around the sun:

- The planets move in a plane; the orbits described are ellipses with the sun at one focus (1602).
- The vector from sun to the planet sweeps equal areas in equal times (the law of areas, 1605).
- The ratio of the square of the period T of revolution of a planet around the sun to the cube of the semi-major axis a of the ellipse is the same for all planets (1918) [11].

The third law of Kepler is a highly important one, which helps us determine the geostationary orbit positions. A satellite in a geostationary orbit has the same revolution period with earth, and it always faces the same spot on earth. The third law can also be written in the form:

$$a^3 = \frac{\mu}{n^2} \tag{2.1}$$

a : Semi-major of the ellipse formed by the orbital path

n : Mean motion of the satellite in radians per second

μ : Earth's geocentric gravitational constant

$$\mu = 3.986005 \times 10^{14} \quad {}^{m^3}\!/\!{}_{\sec^2} \tag{2.2}$$

With n in radians per second, the orbital period in seconds is given by:

$$P = \frac{2\pi}{n} \quad \text{seconds.} \tag{2.3}$$

Therefore,

$$a_{GSO} = \left(\frac{\mu \cdot P^2}{4\pi^2} \right)^{\frac{1}{3}} \tag{2.4}$$

To find the distance of a geocentric orbit, we use the period for one siderial day, which is 23 hours, 56 minutes, 4 seconds. Substituting this period with the value in 2.2 into Equation 2.4, we find

$a_{GSO} = 42164\,km.$

When we subtract the equatorial earth radius: $a_E = 6378\,km$, the geostationary orbital height is

$$h_{GSO} = a_{GSO} - a_E = 42164\,km - 6378\,km = 35786\,km.$$

This value is generally approximated at 36,000 km because a precise geostationary orbit cannot be attained. The gravitational fields of the moon and the sun cause a shift of about $0.85°$ per year in inclination. Also, the "equatorial ellipticity" of the earth causes the satellite to drift eastward along the orbit. Therefore, station-keeping maneuvers are performed periodically to keep the geostationary satellites in correct position [12]. Geostationary orbit is considered a natural "resource" and its use is regulated by international agreements.

B. SUMMARY

In this chapter, we briefly examined the satellite system. The Ground Segment and the Space Segment of the satellite system was explained. The commonly used frequencies by the satellite systems were given and the mathematical explanation of the geostationary orbit was presented. A detailed explanation of the VSAT system, which will be the basis of the further chapters in this thesis, was given.

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III. SPREAD SEQUENCE SYSTEMS

Spread Sequence systems are generated by using Pseudorandom Noise (PN) sequences. Selecting different PN sequences for different user groups in a system separates each group from the other. These groups can be cells in cellular communications or VSATs in satellite communication. To separate the individual users in each group, Walsh functions are used. When PN sequences are combined with Walsh functions, a system can be designed that can support high number of users. When Forward Error Correction (FEC) is added, the performance can further be increased.*

A. WALSH FUNCTIONS COMBINED WITH PN SEQUENCES [19]

Walsh functions mean everything for digital DS-CDMA communication systems such as satellite or cellular communication. In cellular communication, each unique Walsh code in a Walsh sequence provides orthogonal cover on the forward traffic channel within each cell to eliminate intracell interference. Each mobile user in the cell has a unique Walsh function assigned, which encodes the traffic coming from the base station to the mobile handset. By applying his unique Walsh function to the traffic coming from the base station, the mobile user despreads and, in effect, decodes only the traffic that is intended for a specified user. When his unique code is applied to the remaining intracell traffic in the channel, the orthogonality between user's code and other user codes zeros out their interference. A simplified representation of this process is depicted in Figure 3.

* This Chapter was written jointly by the author and LTJG. Aras [21]

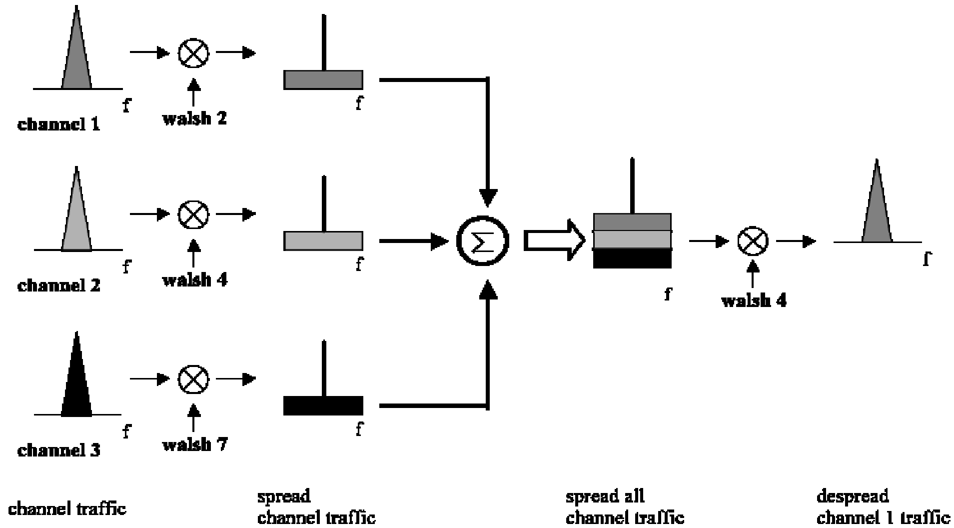

Figure 3. Direct Sequence Spread Spectrum (From [19])

On the other hand, Walsh codes are not the only way to spread the signal. Pseudorandom noise sequences are used to spread the signal as well. Spread Pseudorandom noise is a non-orthogonal function so the coverage supplied by the PN sequence would not be as sufficient as coverage with a Walsh code. The intracell traffic carrying information for other users would remain spread across the channel's frequency band causing interference for the other users in adjacent cells due to imperfect orthogonality between the cell’s PN sequences. How this interference affects the performance of the communication system will be expressed in detail in the next chapter.

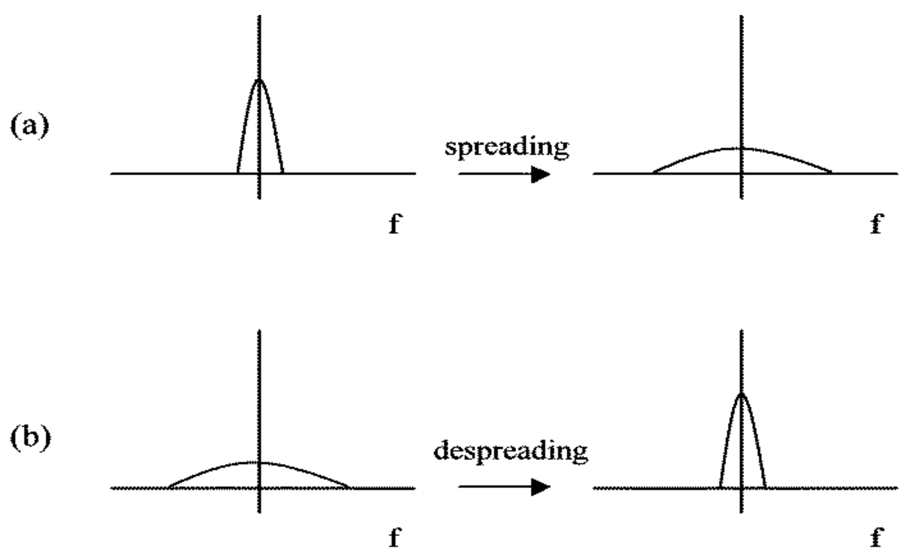

Figure 4. Effect of PN Sequence on Transmit Spectrum

We have introduced an existing example of Walsh codes and PN sequences application, which is cellular communication. Considering the VSAT system, Walsh coding and PN sequences can be applied to satellite communication perfectly with some differences in modeling. While every user, in a cellular communication, communicating with his or her own base station in the same cell with different Walsh codes and the same PN sequence, every VSAT communicates with a single communication satellite. In other words, in a VSAT model every terminal represents cells, which are separated by different PN sequences. Every channel on the single VSAT will be representing users, which are separated by different Walsh Codes.

1. Walsh Codes

Walsh functions have many desirable properties, which we will examine in this part. We will look at autocorrelation functions, and orthogonality property, which may prove useful to the communications engineer.

a. Properties of Walsh Function

Walsh functions are generated from generator matrices, Hadamard matrices, Rademacher functions and Walsh binary index. Orthogonal Walsh functions are defined in order of N as $W_N = \left\{ w_j(t); t \in (0,T), j = 0,1,...,N-1 \right\}$, consisting of $N = 2^k$ elements that are functions of time and that have the following properties [20].

- $w_j(t)$ takes on the values $\{+1,-1\}$ except at a finite number of points of discontinuity; where it is defined to be zero.
- $w_j(t) = 1$ for $j = 0$.
- $w_j(t)$ has precisely j sign changes in the interval $(0,T)$.
- $$\int_0^T w_j(t)\, w_k(t)\, dt = \begin{cases} 0, & \text{if } j \neq k \\ T, & \text{if } j = k \end{cases} \qquad (3.1)$$

In Figure 5, set of Walsh functions of order eight has been illustrated for

$$W_8 = \{w_0(t), w_1(t), w_2(t), w_3(t), w_4(t), w_5(t), w_6(t), w_7(t)\}$$

In this case, the interval $(0,T)$ has been broken into $N = 8$ pieces, each section being $T_c = T / N = T / 8$ time units long.

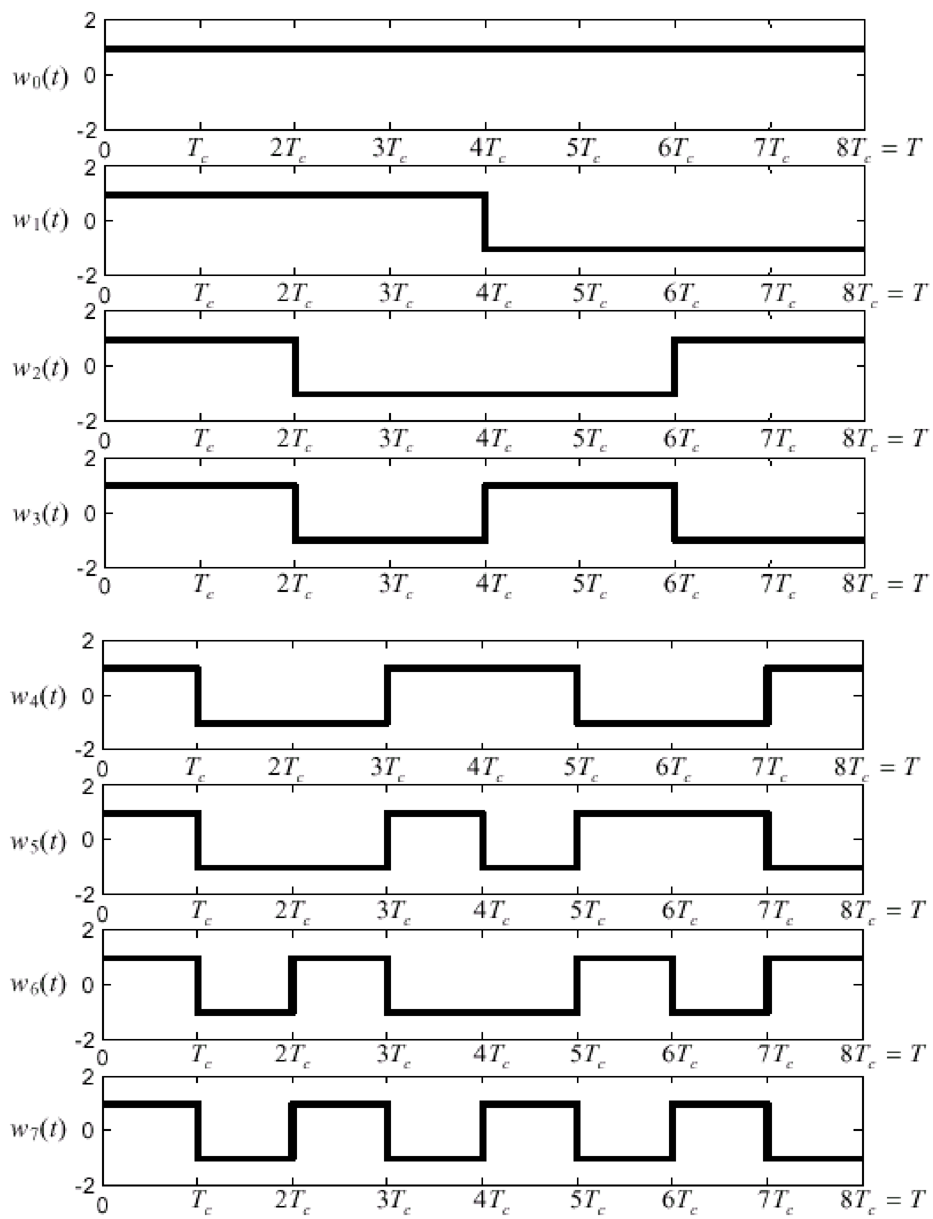

Figure 5. Set of Walsh Function Order Eight (From [19])

The most significant property of the Walsh functions as applied to the DS-CDMA communication systems is that orthogonality between two different Walsh sequences. We will use this property frequently in our performance analysis. Details about analysis will be introduced in the next section. The orthogonality property implies that the integration of the product of any two different Walsh sequences over a period is always zero. An explanation of the forth property is depicted below.

$$\begin{aligned} w_3(t) &= \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \\ w_6(t) &= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \\ w_3(t).w_6(t) &= \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Values of "-1" in Figure 5 indicate *logic zero* in this example where *logic one* is indicated with "+1". The product of $w_3(t).w_6(t)$ is calculated by "*exclusive or* " (XOR) function. XOR function outputs *logic on*e for the different inputs and *logic zero* for same inputs. Since there are equal numbers of ones and zeros in the product matrix of $w_3(t).w_6(t)$, integration over a period is always zero.

b. Extended Orthogonality

Orthogonality may not always be maintained with three Walsh sequences from a set. The result of the integration of two different Walsh sequences may not be orthogonal to the third Walsh sequence.

$$\int_0^T w_i(t)w_j(t)w_k(t) = \int_0^T w_k(t)w_k(t) = T \qquad (3.2)$$

An example for the situation given in equation (3.2) is given below for the Walsh function set in Figure 5. Notice that the integration of $w_6(t).w_7(t).w_1(t)$ over a period is "T"

$$\begin{aligned} w_6(t) &= [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1] \\ w_7(t) &= [1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0] \\ w_6(t).w_7(t) &= [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1] \\ w_1(t) &= [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \\ w_6(t).w_7(t).w_1(t) &= [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \end{aligned}$$

One way around this problem is to eliminate some of the Walsh sequences from the complete set of function W_N and force the equation (3.3) to be valid for every Walsh function in the reduced set $\tilde{W}_N$, which we use to define extended orthogonality.

$$\int_0^T w_i(t) w_j(t) w_k(t) = \int_0^T w_l(t) w_k(t) = 0 \tag{3.3}$$

Besides the advantage of establishing orthogonality for every channel on VSAT, reducing the number of Walsh sequences in a set also reduces the number of channels [19]. In extended orthogonality, we will be limiting the number of channels per VSAT. In the real world some VSAT terminals may require more channels than others do in overall network. In such cases, a greater order of Walsh functions must be used to achieve enough number extended orthogonality Walsh sequences. Therefore, the system can support a sufficient number of channels. The primary central hubs in the star configuration or the auxiliary hubs definitely need more complex Walsh functions than a remote terminal. Methods for generating a maximum set of extended orthogonality Walsh functions are given in reference [19].

c. *Autocorrelation of Walsh Function*

In this section, we will examine the autocorrelation functions for the Walsh functions and average autocorrelation for a set of Walsh functions. In order to explore the autocorrelation function of Walsh functions, we will extend our definition and consider each Walsh function to be periodic with a period of T, which is consistent with their use in practice. Each channel on VSAT has a specific Walsh sequence. A satellite onboard processor encodes the data by applying the channel's entire Walsh sequence. The Walsh functions are periodic signals with a period of T. Accordingly, the Walsh function and the corresponding autocorrelation function is also periodic with period T.

We define the normalized autocorrelation function, $\alpha_i(u)$, for any periodic Walsh function $w_i(t) \in W_N$ by

$$\alpha_i(u) = \frac{1}{T}\int_0^T w_i(t) w_i(t-u) dt. \tag{3.4}$$

Although Equation (3.4) is used for continuous signals, we can consider Walsh sequences as if they were discrete signals. This allows us to generate the autocorrelation of the Walsh functions easily. In spread spectrum systems, the chip duration, T_c, is defined to be the bit duration divided by the number of chips per bit.

$$T_c = \frac{T}{N} \tag{3.5}$$

We will consider T_c as the time increments in discrete Walsh sequences to build the autocorrelation functions. The Walsh function presented in Figure 5 is now presented as the autocorrelation function in Figure 6.

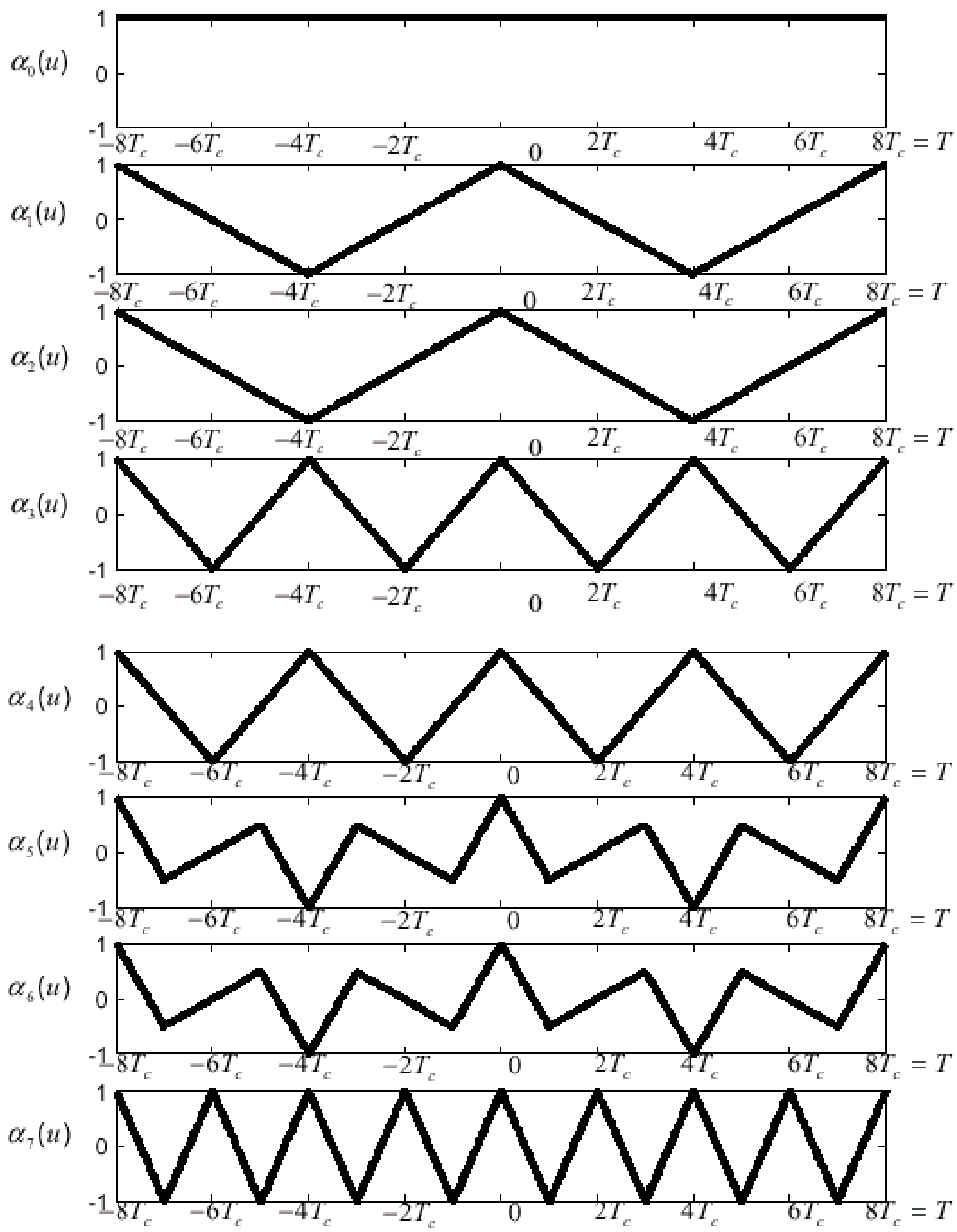

Figure 6. Set of Autocorrelation Function for W_8 (From [19])

The superposition of all the autocorrelation sequences in Figure 6 divided by N gives the "Average Autocorrelation Function" as:

$$A_N(u) = \frac{1}{N}\sum_{i=0}^{N-1}\alpha_i(u) \tag{3.6}$$

where $\alpha_i(u)$ was defined in (3.4). The average autocorrelation function of W_8 is generated from (3.6).

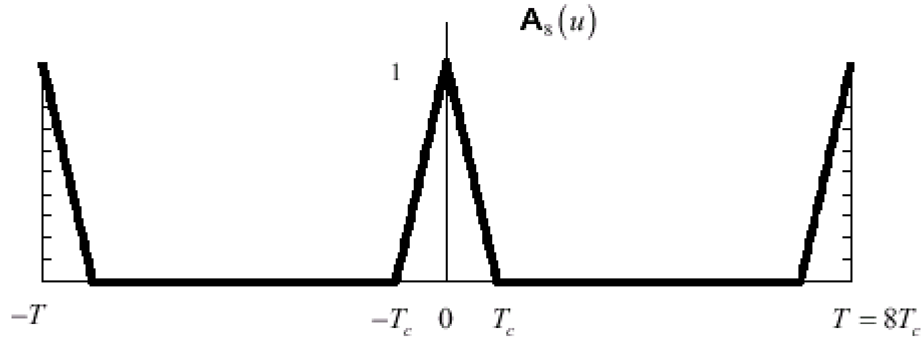

Figure 7. Average Autocorrelation Function, $A_8(u)$ for the Set W_N (From [19])

We find that the form of $A_8(u)$ is similar to the form of the autocorrelation function for a random binary signal. A random binary signal is generated from an infinitely long binary random sequence in which the bits are independent and identically distributed random variables. The resulting normalized autocorrelation function for the random binary signal is defined after [20] by

$$\beta(u) = \begin{cases} 1 - \dfrac{|u|}{T_c}, & |u| \le T \\ 0 & otherwise \end{cases} \tag{3.7}$$

2. PN Sequences

A pseudorandom noise (PN) sequence is a sequence of binary numbers, which appears to be random, but is in fact perfectly deterministic. The PN spreading on the uplink provides near-orthogonality and, hence, minimal interference between signals from each channels sets from different VSATs. This allows reuse of the band of frequencies available, a major advantage of the CDMA. The Direct Sequence Spread Spectrum (DSSS) uses a secondary modulation, faster than the information bit rate, to spread the frequency domain content over a larger band. The spreading process not only assures the system to be more jammer resistant, but also ensures it not to be detected by enemy interceptors. The Low Probability Intercept (LPI) systems use spread spectrum as well. In DSSS, each data bit is modulated by a Pseudo Noise (PN) sequence that accomplishes the spectral spreading. The PN sequence consists of random-like plus and minus ones, which are called “chips.” Each data bit is modulated with at least 11 to 16 chips. Therefore modulated data bits seem like PN sequences in time domain after being modulated.

a. *Properties of PN Sequences*

The ratio of data bit intervals to the chip durations are known as processing gain. The higher the processing gain, the better the autocorrelation properties, and hence, the better the ability to reject narrowband interference. In Figure 8-a, a PN sequence modulates the data bits, 1 0 1 0, (Figure 8-b), resulting in the modulated data bits in Figure 8-c.

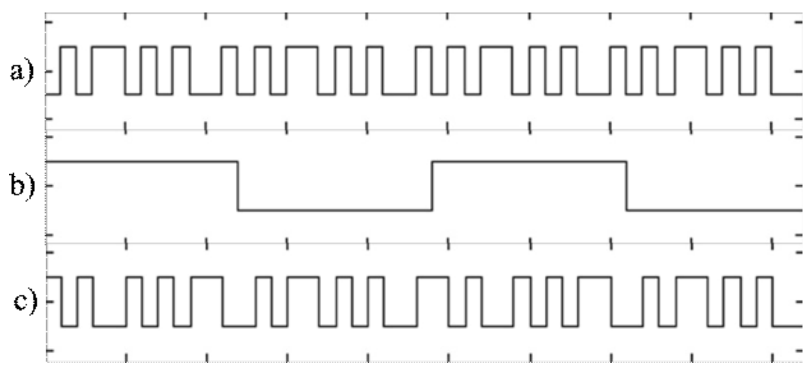

Figure 8. Modulation with PN Sequence

In this example the processing gain is calculated as 13 because there are thirteen chips per bit. The demodulation process is established in the satellite receiver by applying the exact PN sequence at the correct time (Figure 9). The timing process will be introduced in the following section.

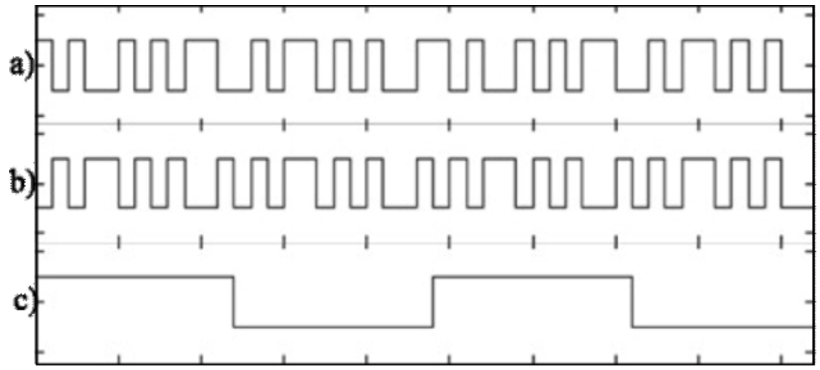

Figure 9. Correct Demodulation with PN Sequence

Data bits (1 0 1 0) were recovered after the applying of the PN sequence to the modulated data bits. Figure 8 illustrates the time domain characteristics of a DS Spread Spectrum signal. The frequency domain representation is given in Figure 10. The over modulation of the data signal leads to a lower power spectral density covering a larger frequency band. If the spread signal is transmitted in the presence of a narrowband jammer, the despreading operation at the receiver will take the wide band spread spectrum signal and collapse it back to the original data bandwidth. The receiver will also act on the narrowband jammer so that its spectrum is spread and causes much lower interference to the despread signal. This is known as jamming resistance or the natural interference immunity of spread spectrum signals.

We can see that DS Spread Spectrum is bandwidth inefficient in that it uses N chips to transmit a single bit of information. Without spreading the spectrum, we could have transmitted N bits in the same bandwidth. This inefficiency is the tradeoff to achieve interference rejection, or the ability to have reliable communications even in the presence of an interfering signal, such as a jammer. It also reduces the power spectral density of the transmitted signal so that its transmission causes less interference in other systems operating at the same time on the same frequency band.

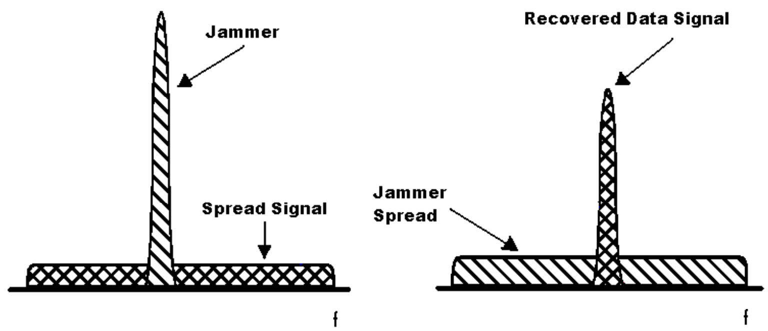

Figure 10. Direct Sequence Spread Spectrum

b. Autocorrelation of PN Sequence

PN sequences have good autocorrelation properties to allow the receiver to recover bit timing and to synchronize the receiver with the uplink. In satellite communication, we expect the users to transmit spontaneously without any synchronization between platforms. In such a case, a satellite receiver must apply the Walsh Sequence and PN sequence at the exact time not to allow a phase difference between the incoming signal and the demodulation signals. The satellite receiver decides the perfect timing by using the autocorrelation of the incoming signal. The autocorrelation function outputs the peak value at the moment of the correct timing. Figure 8 illustrates the recovering data signal by applying the correct PN sequence without any phase difference. If the receiver would not have applied the correct timing to demodulate the incoming signal, the data would not have been recovered.

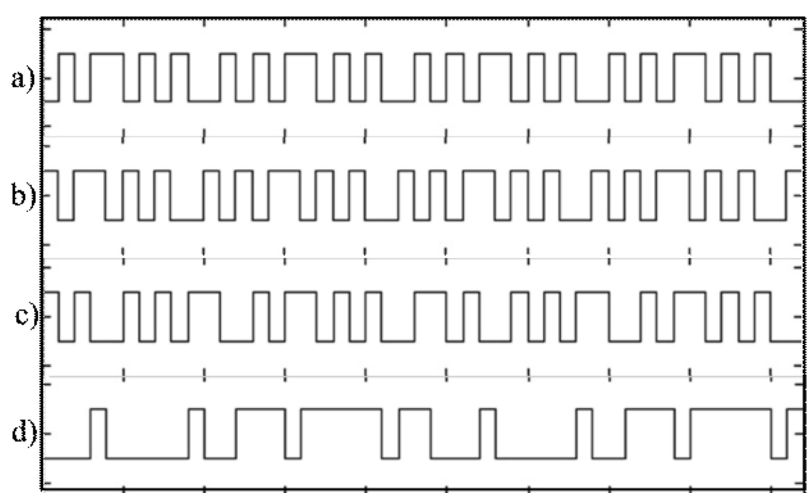

Figure 11. Incorrect Demodulation of Data Signal

An illustration of such a case is given in Figure 11. In part (a) is the PN sequence that is used in Figure 8. Instead of multiplying the PN sequence (Figure 11-a) and modulated data bits (Figure 11-c), we multiply the shifted version of the PN sequence (Figure 11-b). The shifted version of the PN sequence was derived by shifting the PN sequence one chip left (early). The result was plotted in Figure 11-d. The decision mechanism after the PN demodulation is different from the demodulation with the Walsh functions. Due to perfect orthogonality, the integration of the demodulated signal over a period has given either zero or one as the decision statistic. However, the PN sequences do not support full orthogonal coverage. Therefore, we must use a decision mechanism other than the one in the Walsh function. We check the after demodulation-integration results with a predefined threshold, which is zero, to decide on the data bit. Although the transmitted and modulated data sequence was (1 0 1 0), the decision is made as (0 1 0 1) after the integration in the example given in Figure 8.

B. ERROR CORRECTION CODING

The digital communications itself is prone to errors. The information transferred between the satellite and the earth stations is affected by external factors and depending on the signal strength, errors are introduced. The amount of error that can be tolerated depends on the application. For voice communications, bit error rates (BER) up to 10^{-3} can be acceptable, but data communications require BER of at least 10^{-6}. Using higher signal power can increase the performance, but this itself alone is not enough to correct all the errors. Also, the high power may not be available everywhere and the non-linearities in amplifiers limit the output power. For these reasons, error correction mechanisms are used.

The principle of error correction coding is adding redundant bits to the information bits and using these redundant bits to detect and to correct the errors at the receiver.

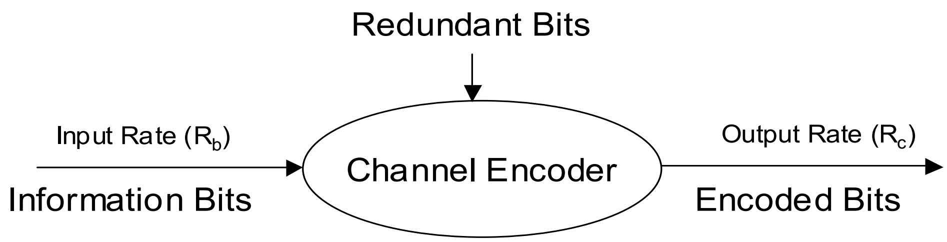

Figure 12. Error Correction Coding

The code rate is defined as:

$$r = \frac{k}{n} \tag{3.8}$$

n : Number of encoded bits

k : Information Bits

Therefore, in a coded system, there are $(n-k)$ redundant bits. The coding is usually introduced before the modulation in the transmitter, and decoding is done after the demodulator at the receiver. In 1948, Shannon demonstrated that by proper encoding, errors in the received sequence can be reduced to a desired level without sacrificing the rate of information transfer [13]. For an AWGN channel, Shannon's capacity formula is

$$C = B \cdot \log_2\left(1 + \frac{P}{N_0 B}\right) = B \cdot \log_2\left(1 + \frac{S}{N}\right) \tag{3.9}$$

C : Channel capacity (bits per second)

B : Transmission bandwidth (Hz)

P : Received signal power (W)

N_0 : Single-sided noise power density (W/Hz)

Since $P = E_b R_b$, Equation (3.9) can be rewritten as

$$\frac{C}{B} = \log_2\left(1 + \frac{R_b E_b}{B N_0}\right) \tag{3.10}$$

C/B is the bandwidth efficiency.

There are two methods for coding: Convolutional Coding and Block Coding. The block codes are linear and calculate the output frame by depending on the current input frame only, so each block is coded independently of the others, and it has no memory. The convolutional codes store the memory of previous input frames and use this to encode the current input frame. Convolutional codes have finite memory and they are also linear.

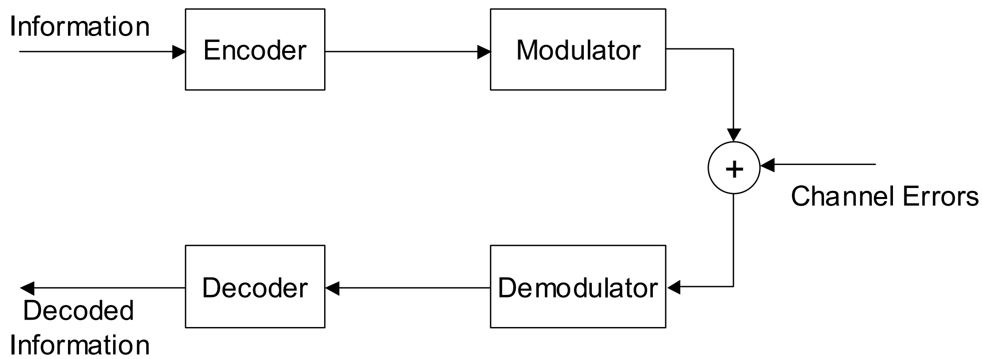

Figure 13. A Communication System Link with Error Correction Coding

The errors can be corrected once they are detected. The detection of errors is achieved by using the redundant bits. Detected errors can be corrected in several ways. The most common methods are "Error Concealment," "Automatic Repeat Request" (ARQ) and "Forward Error Correction" (FEC).

The FEC tries to recover the original information from the received data. A simple example of a (5, 2) block code can be used to demonstrate the FEC:

Information	Code
00	00000
01	01011
10	10101
11	11110

Table 1. (5,2) Block Code

A look-up table or a logic construction can be used to implement the values in the table. In Table 1, at least three bits would have to change in any code sequence to produce another code sequence. Therefore, this code has a minimum distance of three.

When a code sequence is received, it is verified for its correctness. If there are errors in this received sequence, the code compares it with other sequences and tries to correct it. Clearly, the code can easily correct 1-bit errors. For example, if the transmitted sequence was 01011 and it is received as 01001, the received sequence will have a difference of one bit from 01011, two bits from 00000, and three bits from 10101 and 11110. Thus, it can be corrected as 01011. On the other hand, 2 bits of error will be closer to another sequence, and the decoder will miscorrect the error. In some cases, the detector can recognize that more than one error has occurred and declare that there are uncorrectable errors. The FEC can be used with other correction mechanisms in conjunction, resulting in even lower error rates.

The cost of error correction coding is the decrease in the information bandwidth. If we represent the uncoded bit rate as R_b and the coded bit rate as R_{coded}, the code rate is

$$\frac{R_b}{R_{coded}} = r_c \tag{3.11}$$

r_c is always less than 1. For constant carrier power, the bit energy is inversely proportional to bit rate. Therefore,

$$\frac{E_b}{E_{coded}} = r_c \tag{3.12}$$

For BPSK modulation, the BER is

$$P_{e,uncoded} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{3.13}$$

For a coded bit stream

$$P_{e,coded} = Q\left(\sqrt{\frac{2r_cE_b}{N_0}}\right) \tag{3.14}$$

This shows that $P_{e,coded}$ is larger than the $P_{e,uncoded}$, i.e. the probability of bit error with coding is worse than without coding. However, this is the error rate at the input of the decoder. When the demodulated bit streams are fed into the decoder, some of these errors will be corrected and there will be a coding gain.

1. Convolutional Codes

A convolutional code is generated by passing the information bits through a finite state shift register. The shift register consists of N (k -bit) stages and n linear algebraic function generators, as shown in Figure 14. The input data is shifted into and along the register, k bits at a time. The number of output bits for each k bit input data sequence is n bits. The N is called the "constraint length" and indicates the number of input data bits the current output is dependent upon. The constraint length determines the complexity and the power of the code [15][16]. Convolutional codes can be described by their generator matrices, tree diagrams, trellis diagrams or state diagrams.

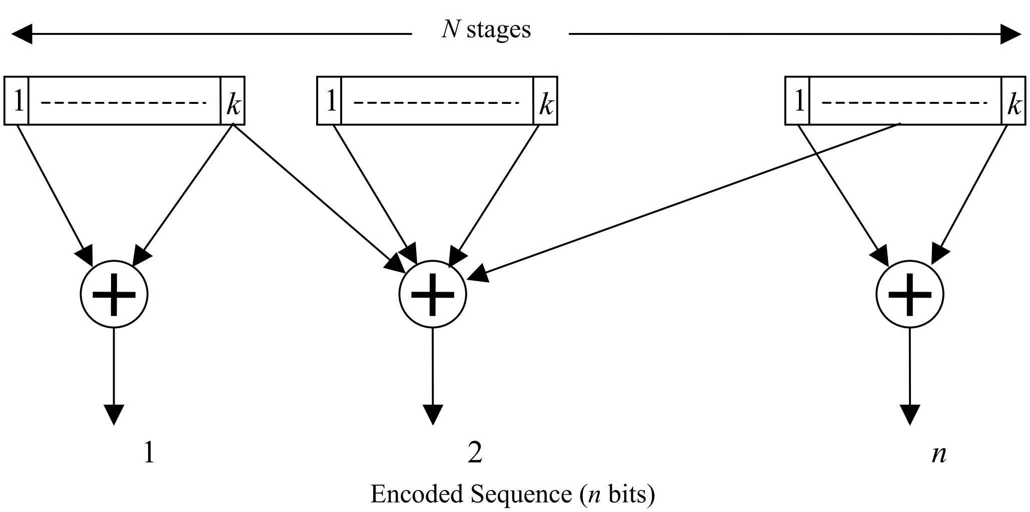

Figure 14. Convolutional Encoder (From [15])

Decoding of convolutional codes is a more difficult problem than encoding. The function of a convolutional decoder is estimating the encoded input information using a method that results in the minimum possible number of errors. Unlike a block code, a convolutional code is a finite state machine. Therefore, the output decoder is a "maximum likelihood estimator" and optimum decoding is done by searching through the trellis for the most probable sequence. Depending on whether hard decision or soft decision decoding is used, either the Hamming or Euclidian metric is used, respectively.

Convolutional coding can be decoded with several different algorithms. The Viterbi algorithm is the most commonly used. Rappaport has given the following definition for the Viterbi algorithm, which is valid for both hard and soft decision decoding [15]:

> Let the trellis node corresponding to state S_j at time i be denoted $S_{j,i}$. Each node in the trellis is to be assigned a value $V(S_{j,i})$ based on a metric. The node values are computed in the following manner:
>
> 1. Set $V(S_{0,0}) = 0$ and $i = 1$
> 2. At time *i*, compute the partial path metrics for all paths entering each node.
> 3. Set $V(S_{j,i})$ equal to the smallest partial path metric entering the node corresponding to state S_j at time *i*. Ties can be broken by previous node choosing a path randomly. The non-surviving branches are deleted from the trellis. In this manner, a group of minimum paths is created from $S_{0,0}$.
> 4. If $i < L + m$, where *L* is the number of input code segments (*k* bits for each segment) and *m* is the length of the longest shift register in the encoder, let $i = i + 1$ and go back to step 2.
>
> Once all the node values have been computed, start at state S_0, time $i = L + m$, and follow the surviving branches backward through the trellis.
>
> The resultant path is the decoded output for the input stream.

2. Reed-Solomon Codes

Reed Solomon (RS) codes are best known for their burst error correction capabilities, their importance in concatenated systems, and their use in compact disc audio technology. RS codes are non-binary BCH codes that use the input and output alphabets, which have 2^m symbols. The block length of a RS code is

$$n = 2^m - 1$$

RS codes can correct t errors within a block of n symbols. The number of errors that can be corrected is a function of the minimum distance of the code [17]:

$$d_{\min} = 2t + 1$$

RS codes achieve the longest possible $d_{\min}$ of any linear code [15].

RS codes are decoded by calculating a syndrome from the received block and known structure of the code. The syndromes are used to determine an error locator polynomial, which is then solved to find the specific error estimates. Once the errors are corrected, the information is sent out.

3. Concatenated Codes

RS codes are not very efficient against randomly distributed errors. However, together with convolutional codes in a concatenated system, they complement each other very nicely. The Viterbi decoder does not have any problems accepting soft decisions from the channel, and it delivers short bursts of errors. The short error bursts do not affect the RS decoder, as long as they are within a q bit symbol of a RS decoder [18].

D. CONCLUSION

This chapter contained brief explanations of Spread Spectrum Systems, Walsh Sequences and Orthogonality, PN sequences and Error Correction Coding. When all of them are combined, a robust and high performance system can be designed. The next chapter analyses a system that utilizes all these components. A block diagram of the system is given in Figure 15.

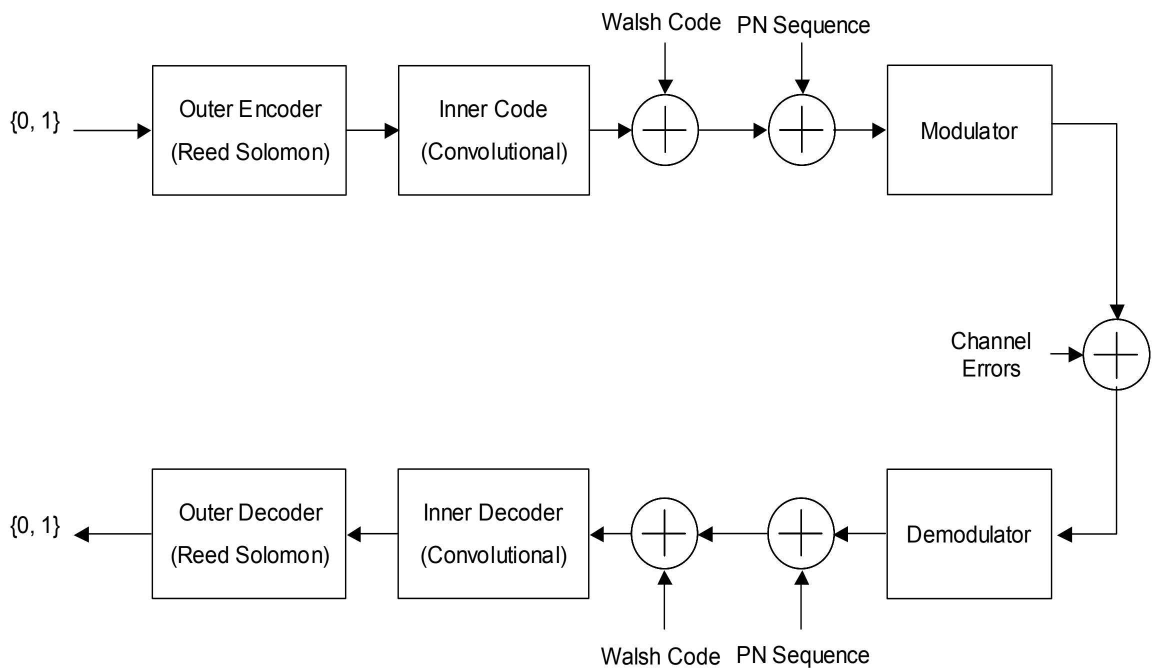

Figure 15. The Block Diagram of the overall Satellite Communication System

IV. UPLINK MODEL

A. PERFORMANCE ANALYSIS OF A DS-CDMA SYSTEM

This analysis is based on a satellite communication system with a total number of i VSATs carrying j channels. Although each VSAT can carry a certain amount of channels, all of these channels are not used at the same time. The total number of active channels in all of the VSATs cannot exceed the satellite capacity. The amount of channels allocated for each VSAT is determined according to the operational requirements.

The received information bits $b_1(t)$ from a specified channel ($k=1$) and a VSAT ($i=1$) will be used for the analysis. In order to form the decision statistic, Y, the received spread signal will be despread, demodulated and integrated over a one-bit duration.

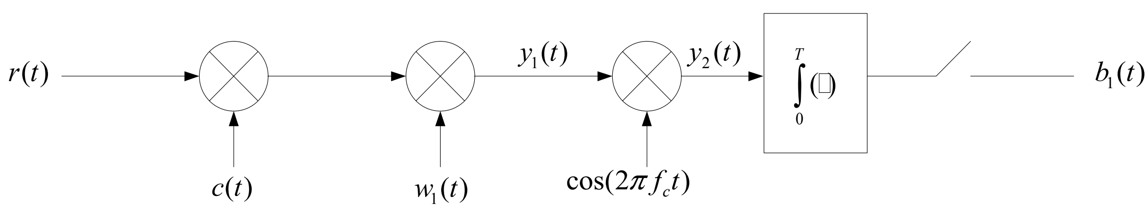

Figure 16. Block Diagram of the Receiver

The received signal can be expressed as:

$$r(t) = S_0(t) + n(t) + \zeta(t) \tag{4.1}$$

$S_0(t)$: Information + Co-channel Interference

$n(t)$: Additive White Gaussian Noise

$\zeta(t)$: Inter-VSAT Interference

1. The Despread Signal, $y_1(t)$

The signal is despread by applying the appropriate PN sequence. The PN sequence must be synchronized with the transmitting VSAT. After dispreading, the Walsh sequence is applied to select the desired channel and remove all the other channels' information. The despread signal, $y_1(t)$ is

$$y_1(t) = r(t)\,c(t)\,w_1(t) \tag{4.2}$$

$$y_1(t) = S_0(t)\,c(t)\,w_1(t) + n(t)\,c(t)\,w_1(t) + \varsigma(t)\,c(t)\,w_1(t) \tag{4.3}$$

$S_0(t)$ contains both information and co-channel interference:

$$S_0 = I_0 + \gamma_0$$

$$y_1(t) = I_1(t) + \gamma_1(t) + n_1(t) + \varsigma_1(t)$$

$n_1(t)$: Additive White Gaussian Noise for 1^{st} channel

$\varsigma_1(t)$: Inter-VSAT Interference for 1^{st} channel

$$I_1(t) + \gamma_1(t) = \sum_{k=0}^{K-1}\left(2P_k b_k(t) w_k(t) \cos(2\pi f_c t)\right) c(t) w_1(t) \tag{4.4}$$

P_k : Received power from k^{th} channel

$b_k(t)$: Information bits for k^{th} channel $\{+1,-1\}$

$w_k(t)$: Walsh Code for k^{th} channel $\{+1,-1\}$

In Equation (4.4), $k = 1$ is the index of the despread information bits we want to recover. All the other values of k ($k \neq 1$) will be considered as co-channel interference.

$$\begin{aligned} I_1(t) &= \sqrt{2P_1}\, b_1(t) w_1(t) c(t) \cos(2\pi f_c t) c(t) w_1(t) \\ &= \sqrt{2P_1}\, b_1(t) \cos(2\pi f_c t) \end{aligned} \tag{4.5}$$

$$\gamma_1(t) = \left(\sum_{\substack{k=0 \\ k\neq 1}}^{K-1} \sqrt{2P_k}\, b_k(t) w_k(t) c(t) \cos(2\pi f_c t) \right) c(t) w_1(t)$$
$$= \sum_{\substack{k=0 \\ k\neq 1}}^{K-1} \sqrt{2P_k}\, b_k(t) w_k(t) w_1(t) \cos(2\pi f_c t) \tag{4.6}$$

Additive White Gaussian Noise term:

$$n_1(t) = n(t)c(t)w_1(t) \tag{4.7}$$

Each VSAT terminal has its own PN sequence. But the quasi-orthogonal nature of PN sequences results in interference. This interference level increases as the number of VSATs increases. The VSAT and the satellite are synchronized; therefore, a phase difference (φ_i), with the interfering terminals is present.

$$\varsigma_1(t) = \left(\sum_{i=1}^{M} \sum_{k=0}^{K_i-1} \sqrt{2P_{ik}}\, b_{ik}(t+\tau_i)\, w_{ik}(t+\tau_i)\, c_i(t+\tau_i) \cos(2\pi f_c t + \varphi_i) \right) c(t)\, w_1(t)$$
$$= \sum_{i=1}^{M} \sum_{k=0}^{K_i-1} \sqrt{2P_{ik}}\, b_{ik}(t+\tau_i)\, w_{ik}(t+\tau_i) w_1(t)\, c_i(t+\tau_i) c(t) \cos(2\pi f_c t + \varphi_i) \tag{4.8}$$

2. The Demodulated Signal, $y_2(t)$

The demodulated signal is obtained by applying the modulation tone, which is coherent detection.

$$\begin{aligned} y_2(t) &= y_1(t)\cos(2\pi f_c t) \\ &= \left(I_1(t) + \gamma_1(t) + n_1(t) + \varsigma_1(t)\right)\cos(2\pi f_c t) \\ &= I_2(t) + \gamma_2(t) + n_2(t) + \varsigma_2(t) \end{aligned}$$

In order to obtain the decision statistic, Y, we need to calculate each of the components separately.

Despread and modulated information bits, $I_2(t)$:

$$I_2 = \sqrt{2P_1}\, b_1(t) \cos(2\pi f_c t) \cos(2\pi f_c t)$$
$$= \frac{\sqrt{2P_1}}{2} b_1(t)\left(1 + \cos(4\pi f_c t)\right) \tag{4.9}$$

where

$\cos^2(2\pi f_c t) = \frac{1}{2}\left(1+\cos(4\pi f_c t)\right)$

Despread and modulated co-channel interference, $\gamma_2(t)$:

$$\gamma_2(t) = \sum_{k=2}^{K-1} \frac{\sqrt{2P_k}}{2} b_k(t)\, w_k(t)\, w_1(t)\left(1+\cos(4\pi f_c t)\right) \tag{4.10}$$

Despread and modulated AWGN

$$n_2(t) = n(t)\, c(t)\, w_1(t)\cos(2\pi f_c t) \tag{4.11}$$

Despread and demodulated inter-VSAT interference

$$\varsigma_2(t) = \sum_{i=1}^{M}\sum_{k=0}^{K_i-1} \frac{\sqrt{2P_{ik}}}{2} b_{ik}(t+\tau_i)\, w_{ik}(t+\tau_i) w_1(t)\, c_i(t+\tau_i) c(t)\cos(2\pi f_c t+\varphi_i)\cos(2\pi f_c t)$$

where

$\cos(2\pi f_c t+\varphi_i)\cos(2\pi f_c t) = \frac{1}{2}\left[\cos\varphi_i - \cos(4\pi f_c t)\cos\varphi_i + \sin(4\pi f_c t)\sin\varphi_i\right]$

$$\begin{aligned}\varsigma_2(t) = \sum_{i=1}^{M}\sum_{k=0}^{K_i-1} \frac{\sqrt{2P_{ik}}}{2} b_{ik}(t+\tau_i)\, w_{ik}(t+\tau_i) w_1(t)\, c_i(t+\tau_i) c(t) \\ \left[\cos\varphi_i - \cos(4\pi f_c t)\cos\varphi_i + \sin(4\pi f_c t)\sin\varphi_i\right]\end{aligned} \tag{4.12}$$

M : Total number of VSATs.
K_i : The total number of users in the i^{th} VSAT.

3. The Decision Statistic

The decision statistic, Y, is modeled as a Gaussian distribution with a mean value of $\overline{Y}$ and variance of ξ.

$$Y = \int_0^T y_2(t)\, dt$$

$$Y = \int_0^T I_2(t)\, dt + \underbrace{\int_0^T \gamma_2(t)\, dt}_{\gamma} + \underbrace{\int_0^T n_2(t)\, dt}_{n} + \underbrace{\int_0^T \varsigma_2(t)\, dt}_{\varsigma} \tag{4.13}$$

γ : Co-channel Interference,

$$\gamma = \int_0^T \sum_{k=2}^{K-1} \frac{\sqrt{2P_k}}{2} b_k(t)\, w_k(t)\, w_1(t) \left(1 + \cos(4\pi f_c t)\right) dt \tag{4.14}$$

n : Additive White Gaussian Noise

$$\eta = \int_0^T n(t)\, c(t)\, w_1(t) \cos(2\pi f_c t)\, dt \tag{4.15}$$

ς : Inter-VSAT Interference

$$\begin{aligned}
\varsigma &= \int_0^T \varsigma_2(t) dt \\
&= \int_0^T \sum_{i=1}^{M} \sum_{k=0}^{K_i-1} \frac{\sqrt{2P_{ik}}}{2} b_{ik}(t+\tau_i)\, w_{ik}(t+\tau_i) w_1(t)\, c_i(t+\tau_i) c(t) \\
&\qquad \left[\cos\varphi_i - \cos(4\pi f_c t)\cos\varphi_i + \sin(4\pi f_c t)\sin\varphi_i\right] dt \\
&= \sum_{i=1}^{M} \sum_{k=0}^{K_i-1} \cos\varphi_i \frac{\sqrt{2P_{ik}}}{2} \int_0^T b_{ik}(t+\tau_i) w_{ik}(t+\tau_i) w_1(t)\, c_i(t+\tau_i) c(t)\, dt
\end{aligned}$$

In reference [19] $a_{ik}(t+\tau_i)$ and $d_1(t)$ are given as

$$\begin{aligned}
a_{ik}(t+\tau_i) &= b_{ik}(t+\tau_i) w_{ik}(t+\tau_i)\, c_i(t+\tau_i) \\
d_1(t) &= w_1(t)\, c(t)
\end{aligned} \tag{4.16}$$

$$\varsigma = \sum_{i=1}^{M} \sum_{k=0}^{K_i-1} \frac{\sqrt{2P_{ik}}}{2} \cos\varphi_i \int_0^T a_{ik}(t+\tau_i) d_1(t)\, dt \tag{4.17}$$

a. Mean Value of Decision Statistic, Y

$$\overline{Y} = E\left\{\int_0^T I_2(t)\,dt + \int_0^T \gamma_2(t)\,dt + \int_0^T n_2(t)\,dt + \int_0^T \varsigma_2(t)\,dt + \int_0^T J_2(t)\,dt\right\} \tag{4.18}$$

$$\begin{aligned}\overline{Y} &= E\left\{\int_0^T I_2(t)\,dt\right\} + \cancel{E\{\gamma\}} + \cancel{E\{n\}} + \cancel{E\{\varsigma\}} + \cancel{E\{J\}} \\ &= E\left\{\int_0^T I_2(t)\,dt\right\}\end{aligned}$$

$$\begin{aligned}\overline{Y} &= \int_0^T \frac{\sqrt{2P_1}}{2} b_1(t)\left(1+\cos(4\pi f_c t)\right)dt \\ &= \pm\frac{\sqrt{2P_1}}{2}T\end{aligned} \tag{4.19}$$

The number of zeros and ones are assumed to be equal, so the integral of $b_1(t)$ over a period is equal to ± 1. Also, f_c is a integer multiple of $1/T$, therefore, integrating a cosine function over *m* periods is always zero.

4. Variance of Total Interference plus Noise

Since the co-channel interference and noise variances are independent, the variance of total interference and noise is the summation of each variance.

$$\begin{aligned}Var\{\xi\} &= Var\{\gamma\} + Var\{n\} + Var\{\varsigma\} \\ &= E\{\gamma^2\} + E\{n^2\} + E\{\varsigma^2\}\end{aligned} \tag{4.20}$$

a. Variance of Co-channel Interference

$$\begin{aligned}Var\{\gamma\} &= E\{\gamma^2\} \\ &= E\left\{\left(\int_0^T \sum_{\substack{k=0 \\ k\neq 1}}^{K-1} \frac{\sqrt{2P_k}}{2} b_k(t)\,w_k(t)\,w_1(t)\left(1+\cos(4\pi f_c t)\right)dt\right)^2\right\} \\ &= 0\end{aligned} \tag{4.21}$$

Because $\int_0^T w_1(t) w_k(t) dt = 0$ where $k \neq 1$.

This is why we use Walsh functions for CDMA.

b. Variance of AWGN

$$Var\{n\} = E\{n^2\}$$
$$= E\left\{ \int_0^T n(t)\, c(t)\, w_1(t) \cos(2\pi f_c t) dt \int_0^T n(\lambda)\, c(\lambda)\, w_1(\lambda) \cos(2\pi f_c \lambda)\, d\lambda \right\}$$

$$= E\left\{ \int_0^T \int_0^T n(t) n(\lambda)\, c(t)\, c(\lambda) w_1(t) w_1(\lambda) \cos(2\pi f_c t) \cos(2\pi f_c \lambda) dt\, d\lambda \right\} \tag{4.22}$$

We assume that $n(t)$ is a wide-sense stationary white noise process. The autocorrelation of the process was defined in [16] as

$$E\{n(t) n(\lambda)\} = \frac{N_0}{2} \delta(t - \lambda). \tag{4.23}$$

so,

$$\begin{aligned} Var\{n\} &= \frac{N_0}{2} \int_0^T c^2(t)\, w_1^2(t) \frac{1}{2}\left(1 + \cos(4\pi f_c t)\right) dt \\ &= \frac{N_0 T}{4} \end{aligned} \tag{4.24}$$

c. Variance of the Inter-VSAT Interference

$$Var\{\varsigma\} = E\{\varsigma^2\}$$
$$Var\{\varsigma\} = \sum_{i=1}^{M} \sum_{k=0}^{K_i - 1} E\left\{ \left(\frac{\sqrt{2P_{ik}}}{2} \cos(\varphi_i) \int_0^T a_{ik}(t + \tau_i) d_1(t)\, dt \right)^2 \right\}$$

$$Var\{\varsigma\} = \sum_{i=1}^{M} \sum_{k=0}^{K_i - 1} E\left\{ \frac{P_{ik}}{2} \cos^2(\varphi_i) \int_0^T a_{ik}(t + \tau_i) d_1(t)\, dt \int_0^T a_{ik}(\lambda + \tau_i) d_1(\lambda)\, d\lambda \right\}$$

$$Var\{\varsigma\} = \frac{P_{ik}}{4}\sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\int_0^T\int_0^T \underbrace{E\{a_{ik}(t+\tau_i)a_{ik}(\lambda+\tau_i)\}}_{\beta(t-\lambda)}\underbrace{E\{d_1(t)\,d_1(\lambda)\}}_{\beta(t-\lambda)}dtd\lambda$$

$$Var\{\varsigma\} = \frac{P_{ik}}{4}\sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\int_0^T\int_0^T \beta^2(t-\lambda)dt\,d\lambda$$

Applying the transformation of limits and variables for the integral

$$u = t-\lambda \qquad \rightarrow u+v = 2t$$
$$v = t+\lambda \qquad \rightarrow u-v = -2\lambda$$
$$t = \frac{1}{2}(u+v)$$
$$\lambda = \frac{1}{2}(v-u)$$
$$J_{t\lambda} = \det\begin{bmatrix}\frac{\partial t}{\partial u} & \frac{\partial \lambda}{\partial u}\\ \frac{\partial t}{\partial v} & \frac{\partial \lambda}{\partial v}\end{bmatrix} = \det\begin{bmatrix}\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix} = 1/2$$

After transformation,

$$\begin{aligned}Var\{\varsigma\} &= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{4}\int_{-T}^{T}\int_{|u|}^{2T-|u|}\beta^2(u)J_{t\lambda}du\,dv\\ &= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{4}\int_{0}^{T}\int_{|u|}^{2T-|u|}\beta^2(u)du\,dv\\ &= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{4}\int_{0}^{T}\beta^2(u)\left(2T-2u\right)du\end{aligned}$$

$$\beta(u) = \left\{\begin{matrix}1-\frac{Nu}{T} & |u|\le T_c\\ 0 & otherwise\end{matrix}\right\}$$

$$
\begin{aligned}
Var\{\varsigma\} &= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{4}\int_{0}^{T/N}\left(1-\frac{2Nu}{T}+\frac{N^2u^2}{T^2}\right)(2T-2u)\,du \\
&= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{4}2\left(Tu-Nu^2+\frac{N^2u^3}{3T}-\frac{u^2}{2}+\frac{2Nu^3}{3T}-\frac{N^2u^4}{4T^2}\right)\Bigg|_{0}^{T/N} \\
&= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{4}\left(\frac{2T^2}{3N}-\frac{T^2}{6N^2}\right) \\
&= \sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{2}\left(\frac{T^2}{3N}\right) \qquad for\ N \gg 1
\end{aligned}
\tag{4.25}
$$

5. Signal-to-Noise plus Interference Ratio

$$
\begin{aligned}
Var\{\xi\} &= Var\{\eta\}+Var\{\varsigma\} \\
&= \frac{N_0T}{4}+\sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}T^2}{6N}
\end{aligned}
\tag{4.26}
$$

$$
SNIR = \frac{\overline{Y^2}}{Var\{\xi\}}
$$

$$
= \frac{\dfrac{P_1T^2}{2}}{\dfrac{N_0T}{4}+\displaystyle\sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}T^2}{6N}}
$$

$$
SNIR = \frac{1}{\dfrac{N_0}{2TP_1}+\displaystyle\sum_{i=1}^{M}\sum_{k=0}^{K_i-1}\frac{P_{ik}}{3NP_1}} \tag{4.27}
$$

The footprint of the geostationary satellite is very small compared to its distance to the earth. Therefore, all the received signal powers, P_{ik} and P_1 can be considered equal.

$$SNIR = \frac{1}{\frac{N_0}{2TP_1} + \frac{M(K-1)}{3N}} \tag{4.28}$$

$$P_1 T = E_b$$

$$\frac{E_b}{N_0} = SNR\,(Signal\ to\ Noise\ Ratio)$$

$$SNIR = \frac{1}{\frac{1}{2 \cdot SNR} + \frac{M(K-1)}{3N}} \tag{4.29}$$

Equation (3.13) gives the bit error probability of a BPSK system. We need to replace the SNR expression in that formula with the Signal-to-Noise plus Interference Ratio found in Equation (4.29). The bit error probability for BPSK modulation with spreading is:

$$P_{e,uncoded} = Q\left(\sqrt{2.SNIR}\right) = Q\left(\sqrt{\frac{2}{\frac{1}{2 \cdot SNR} + \frac{K_1}{3N}}}\right) \tag{4.30}$$

where

$$K_1 = M(K-1)$$

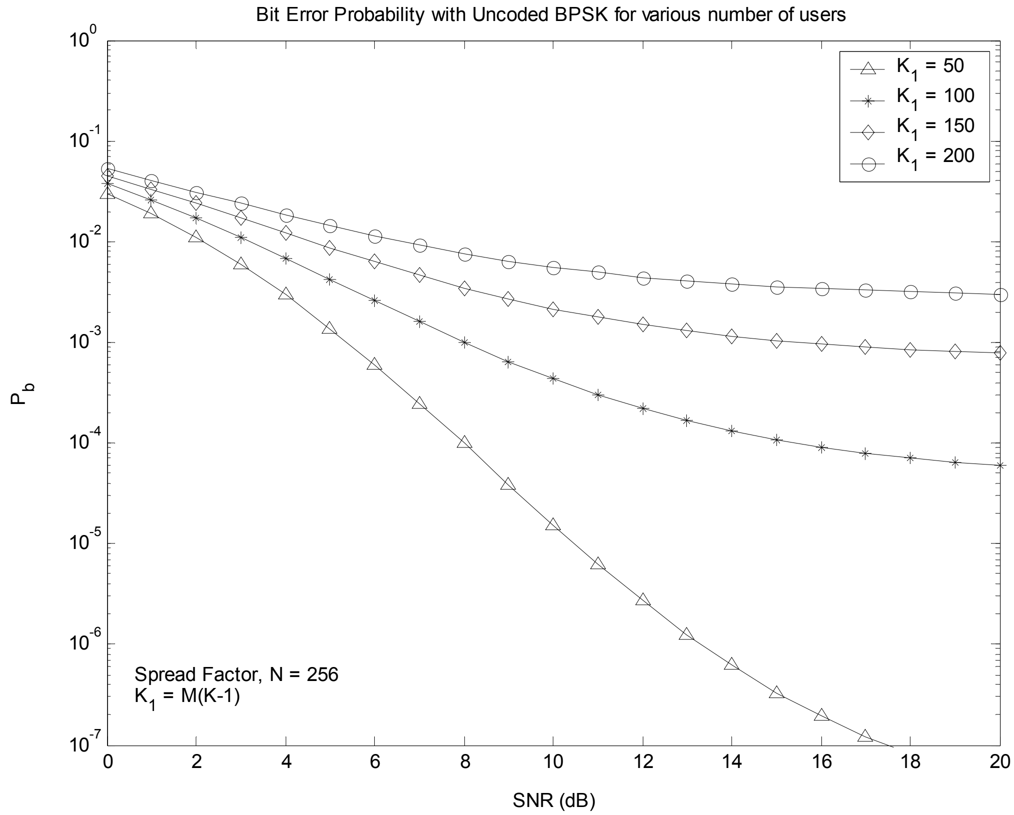

Figure 17. Bit Error Probability for Uncoded BPSK

As seen in Figure 17, the performance is not very good without coding, and it degrades rapidly as the total number of users increase.

B. PERFORMANCE ANALYSIS WITH FORWARD ERROR CORRECTION CODING

1. Performance Analysis with Convolutional Encoding

The bit error probability of a communication system with convolutional coding is given in [16] as:

$$P_b < \sum_{d=d_{free}}^{\infty} \beta_d P_2(d)$$

β_d : The code distance spectrum.

$P_2(d)$: The probability of error in the pair wise comparison of metrics.

$P_2(d)$ is defined differently for Hard and Soft Decision Decoding.

a. Hard Decision Decoding

$$P_2(d) = \begin{cases} \displaystyle\sum_{i=\frac{(d+1)}{2}}^{d} \binom{d}{i} p^i (1-p)^{d-i} & , d \text{ odd} \\ \displaystyle\frac{1}{2}\binom{d}{d/2} p^{d/2}(1-p)^{d/2} + \sum_{i=\frac{d}{2}+1}^{d} \binom{d}{i} p^i (1-p)^{d-i} & , d \text{ even} \end{cases} \tag{4.31}$$

$$p = Q\left(\sqrt{SNIR \cdot R}\right) = Q\left(\sqrt{\frac{R}{\frac{N_0}{2 \cdot SNR} + \frac{M(K-1)}{3N}}}\right) \tag{4.32}$$

R : Code Rate

b. Soft Decision Decoding

$$P_2(d) = Q\left(\sqrt{d \cdot SNIR \cdot R}\right) \tag{4.33}$$

$$= Q\left(\sqrt{\frac{d \cdot R}{\frac{N_0}{2 \cdot SNR} + \frac{M(K-1)}{3N}}}\right)$$

$$= Q\left(\sqrt{\frac{d}{\frac{N_0}{2 \cdot SNR \cdot R} + \frac{M(K-1)}{3 \cdot N \cdot R}}}\right)$$

$$P_b < \sum_{d=d_{free}}^{\infty} \beta_d Q\left(\sqrt{\frac{d}{\frac{N_0}{2R \cdot SNR} + \frac{M(K-1)}{3 \cdot N \cdot R}}}\right) \tag{4.34}$$

R : Code Rate

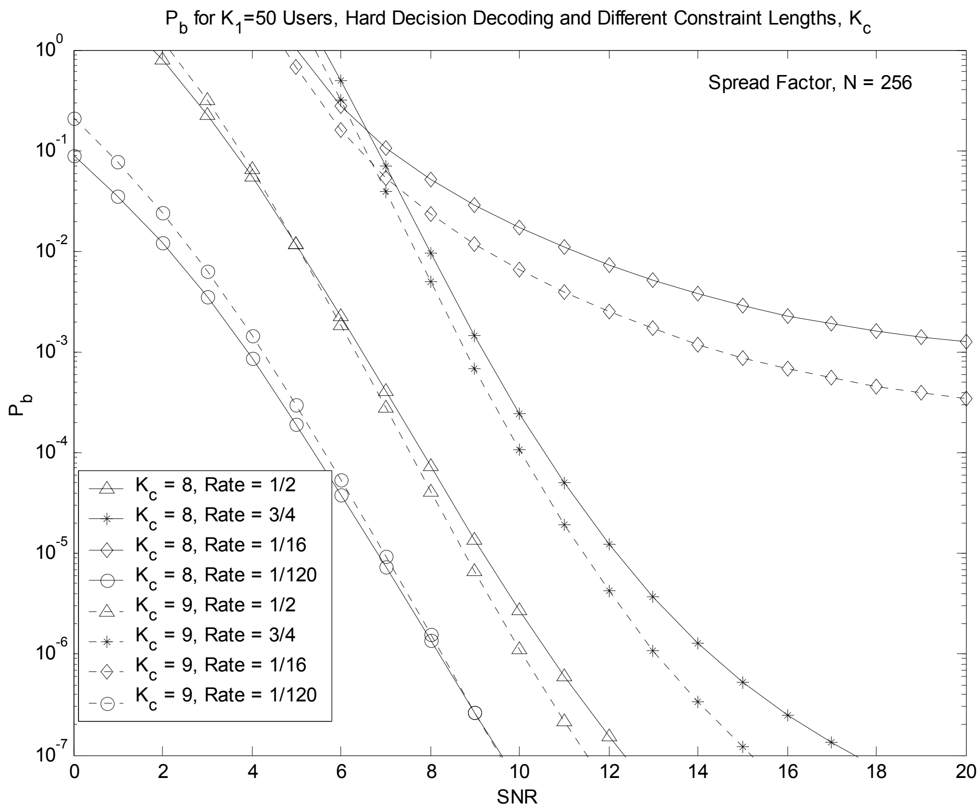

Figure 18. BER for 50 Users and Hard Decision Convolutional Decoding with Different Rates and Constraint Lengths (K_c)

Lower bit error rates can be achieved by higher code rates, as seen in Figure 18. However, higher rates decrease the information bit rate dramatically. Rate 1/2 performs almost as well as rate 1/120 and it does not affect the information bit rate as much. Therefore, in situations where extreme reliability is required, lower rates can be used, but for normal usage, rate 1/2 gives sufficient performance. The spectrum distances algorithm used to obtain the β_d coefficients gives perfect results for rates 1/4, 1//20, 1/40, 1/60, 1/120 [22]. Because of these perfect spectral distance coefficients, rate 1/20 performs better than rate 1/16. Also, higher constraint lengths, K_c, improve the performance, but this makes the decoder hardware more complex. An increase in the K_c from 8 to 9 improves the performance about 1 dB, except rate 1/120, where the improvement is less.

The number of users greatly affects the performance of the system. As seen in Figure 19, increasing the number of users from 50 to 200 degrades the bit error probability substantially. The optimum number of users can be around 100 because it gives the additional channel capacity without excessively degrading the performance.

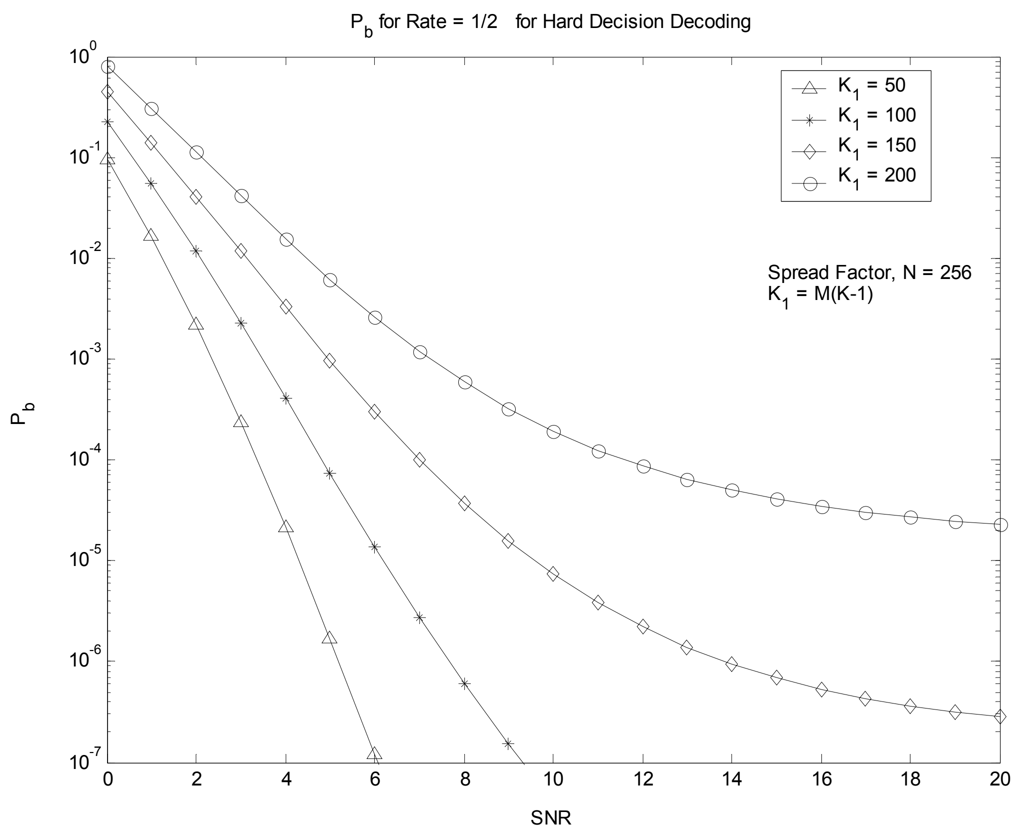

Figure 19. BER for Rate 1/2 Hard Decision Decoding with Different Number of Users

Figure 20 shows the difference in the performances of hard decision and soft decision decoding algorithms. The soft decision performs better than the hard decision decoding; however, its hardware is more complex.

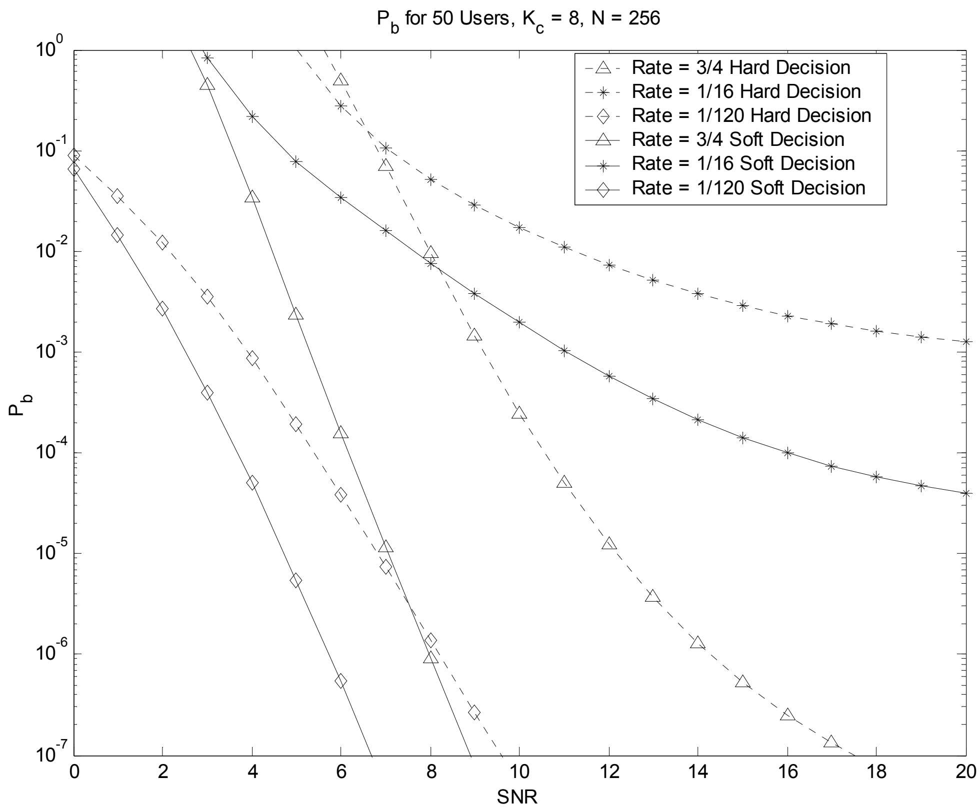

Figure 20. Differences Between Hard Decision and Soft Decision Decoding

2. Performance Analysis with Reed-Solomon Encoding [23]

Code Rate, $R = \frac{k}{n}$

$n = 2^m - 1, \qquad n - k = 2t$

$$P_b < \frac{2^{(m-1)}}{2^m - 1} \sum_{j=t+1}^{n} \frac{j+t}{n} \binom{n}{j} p_s{}^j \left(1 - p_s\right)^{n-j}$$

$$P_b < \frac{n+1}{2n} \sum_{j=t+1}^{n} \frac{j+t}{n} \binom{n}{j} p_s{}^j \left(1 - p_s\right)^{n-j}$$

$$p_s = 1 - \left(1 - p\right)^m$$

From Equation (4.32)

$$p = Q\left(\sqrt{\frac{R}{\frac{N_0}{2 \cdot SNR} + \frac{M(K-1)}{3N}}}\right)$$

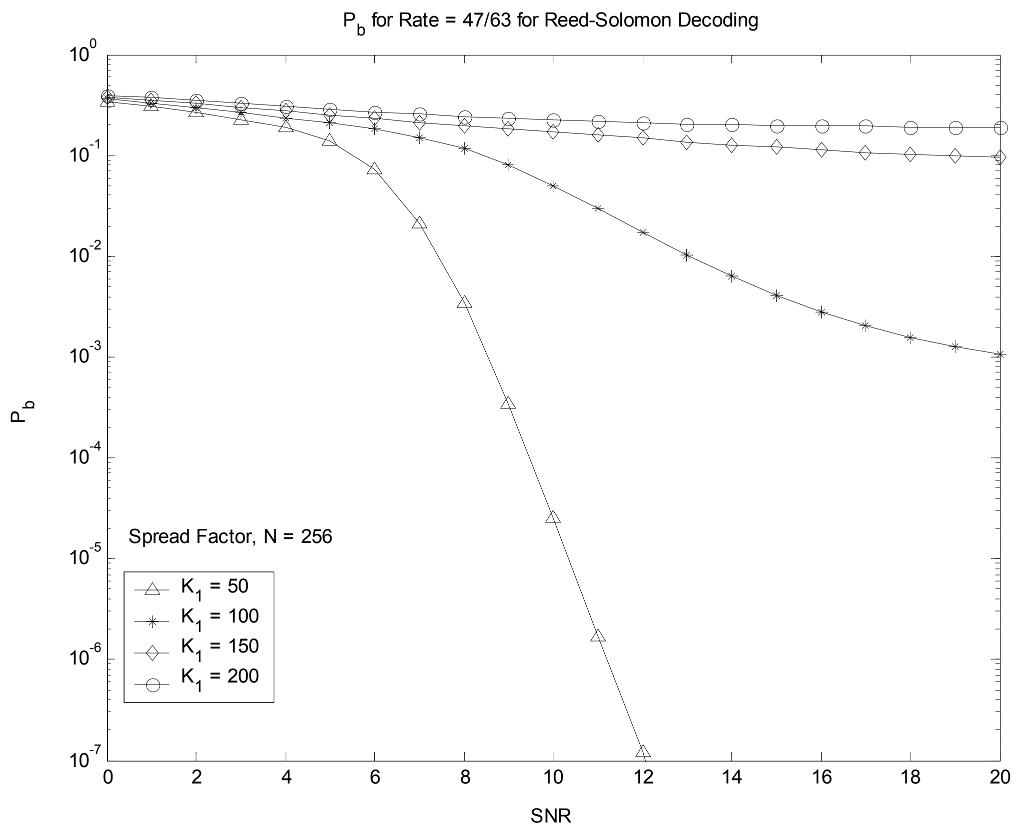

Figure 21. P_b for Reed-Solomon Decoding

As mentioned in Chapter III, Reed-Solomon codes are not very efficient for normal distribution of errors. RS codes are more effective against bursts of errors. Figure 21 shows that the performance of RS decoding is not as effective as convolutional coding when compared to Figure 19.

3. Performance Analysis with Concatenated Coding: RS-RS Codes

$$(n_1, k_1)/(n_2, k_2)$$

$$n_1 = 2^{m_1} - 1, \quad n_1 - k_1 = 2t_1$$

$$n_2 = 2^{m_2} - 1, \quad n_2 - k_2 = 2t_2$$

Inner Code Rate, $\quad r_1 = \dfrac{k_1}{n_1}$

Outer Code Rate, $\quad r_2 = \dfrac{k_2}{n_2}$

Concatenated Code Rate $\quad R = r_1 r_2 = \dfrac{k_1 k_2}{n_1 n_2}$

Symbol Error Probability at output of inner decoder:

$$P_{s_1} < \sum_{j=t_1+1}^{n_1} \frac{j+t_1}{n_1} \binom{n_1}{j} p_s^{\,j} \left(1-p_s\right)^{n_1-j}$$

$$p_s = 1 - \left(1-p\right)^{m_1}$$

$$p = Q\left(\sqrt{\frac{R}{\dfrac{N_0}{2 \cdot SNR} + \dfrac{M(K-1)}{3N}}}\right)$$

Bit error probability at output of outer decoder (i.e. Concatenated bit error probability)

$$P_b < \frac{n_2+1}{2n_2} \sum_{j=t_2+1}^{n_2} \frac{j+t_2}{n_2} \binom{n_2}{j} P_{s_1}^{\,j} \left(1-P_{s_1}\right)^{n_2-j}$$

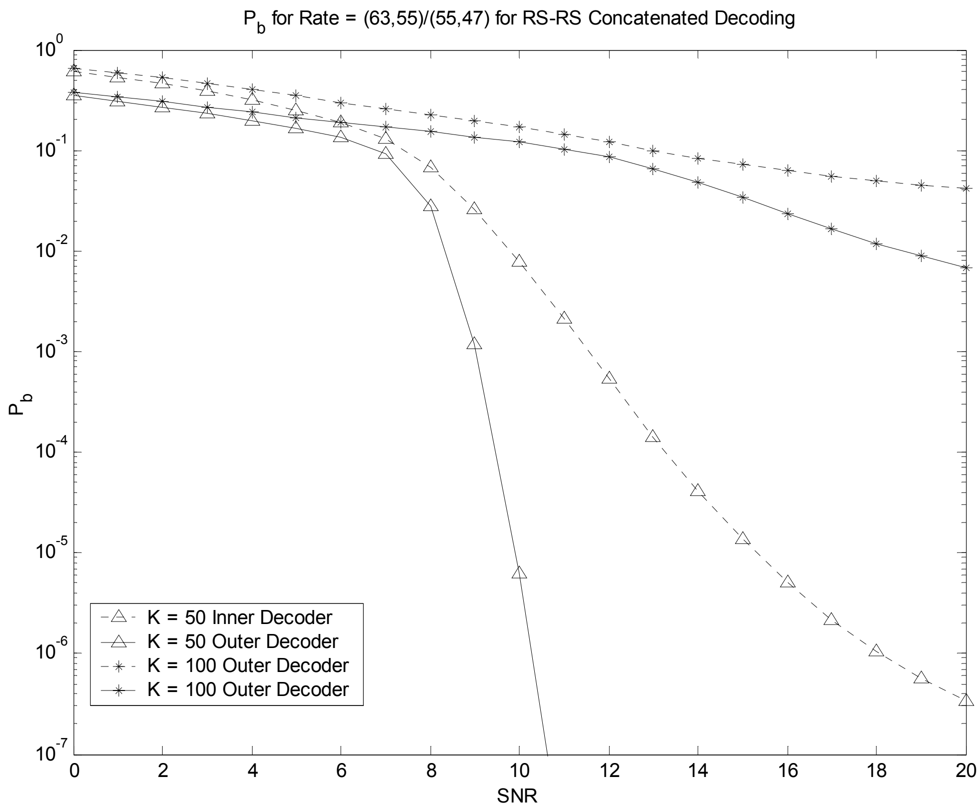

Figure 22. Concatenated RS-RS Codes

Concatenated RS-RS codes do not perform very well up to a certain SNR. After that point, the performance increases rapidly with slight increases in SNR. The decrease in the performance becomes faster as the number of users increases.

4. Performance Analysis with Concatenated Coding: Convolutional-RS Codes

$$(n_1, k_1)/(n_2, k_2)$$

$$n_1 = 2^{m_1} - 1, \quad n_1 - k_1 = 2t_1$$

$$n_2 = 2^{m_2} - 1, \quad n_2 - k_2 = 2t_2$$

Inner Code Rate, $r_1 = \dfrac{k_1}{n_1}$

Outer Code Rate, $r_2 = \dfrac{k_2}{n_2}$

Concatenated Code Rate $R = r_1 r_2 = \dfrac{k_1 k_2}{n_1 n_2}$

The bit error probability at the output of the inner convolutional decoder:

$$P_{b_1} < \sum_{d=dfree}^{\infty} \beta_d P_2(d)$$

The performance of only soft decision inner decoding will be examined in this thesis.

$$P_2(d) = Q\left(\sqrt{d \cdot SNIR \cdot R}\right)$$

The symbol error probability at the input of outer RS decoder:

$$P_{s_1} = 1 - \left(1 - P_{b_1}\right)^{m_2}$$

The bit error probability at the output of outer decoder (Concatenated Bit Error Probability):

$$P_b < \frac{n_2 + 1}{2n_2} \sum_{j=t_2+1}^{n_2} \frac{j + t_2}{n_2} \binom{n_2}{j} P_{s_1}{}^{j} \left(1 - P_{s_1}\right)^{n_2 - j}$$

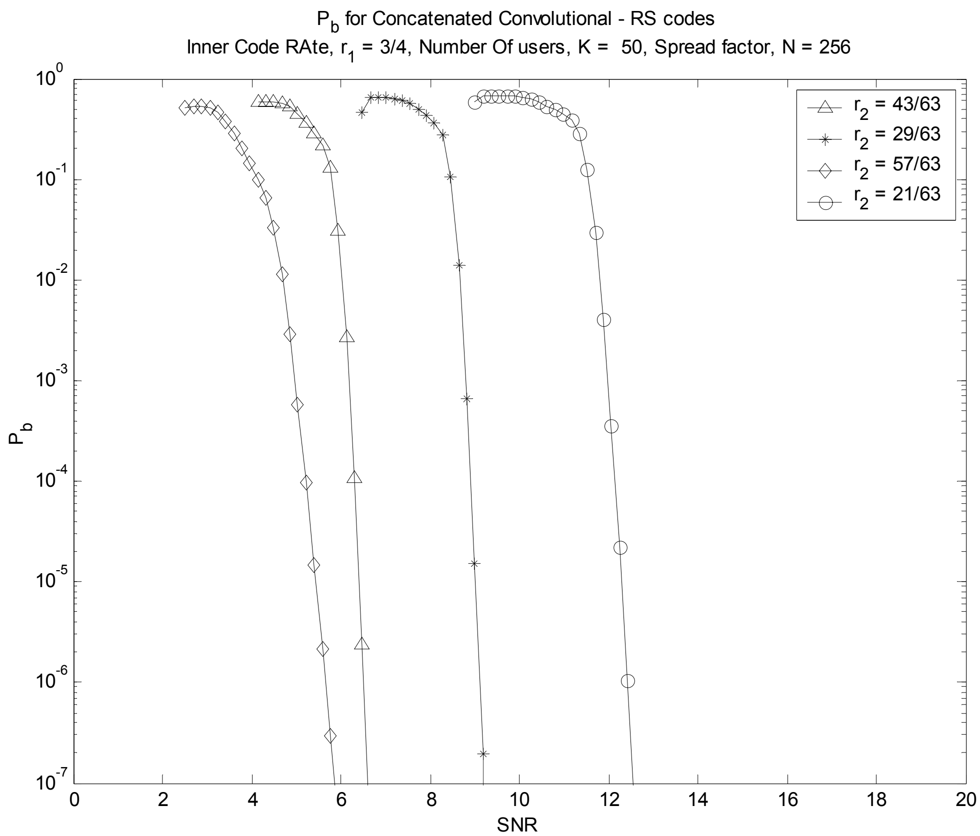

Figure 23. Concatenated Convolutional – RS codes

Concatenated systems do not always perform better than the single coding structures. When Figure 23 is compared to Figure 19, we can clearly see that rate 1/2 convolutional coding performs better than the concatenated system which has an inner code rate of 3/4 and outer code rate of 43/63. The overall code rate is 0.5119, which is very close to rate 1/2. However, the concatenated system can correct both normally distributed errors and burst errors. The concatenated Convolutional - Reed-Solomon codes are affected by the increase in the number of users worse than the other codes as seen in Figure 24.

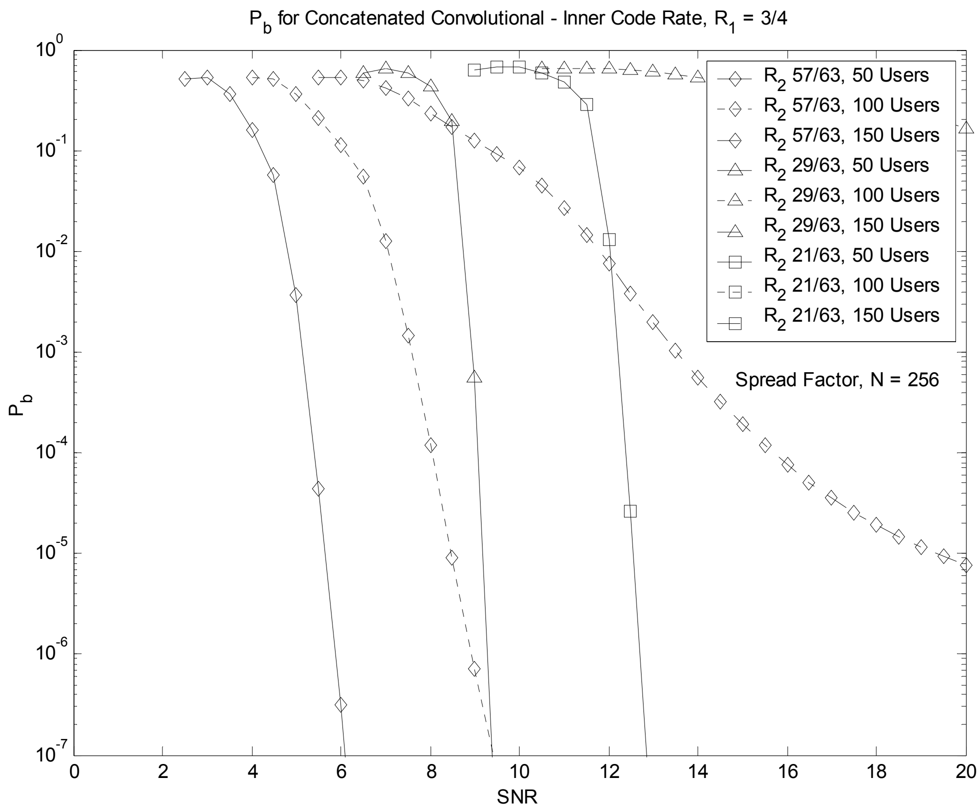

Figure 24. P_b for Concatenated Convolutional-RS codes with different number of users

D. CONCLUSION

We have given a detailed analysis of a spread-spectrum communication system with various coding schemes. Each analysis has different parameters that can be changed in order to find the optimum satellite communication system. From the results of this analysis, the suggested VSAT system, which utilizes CDMA, will use concatenated convolutional-RS coding, support 100 users and still provide low error rates. Actually, a system with 50 users performs better, but the system capacity is very low in that case. The concatenated system does not perform as well with the convolutional coding alone, but concatenation provides extra protection for burst errors, where convolutional coding is effective only against normally distributed errors. Finally, a system with inner convolutional code rate of 3/4 and outer RS code rate of 57/63 gives the best result of P_b for any $\frac{E_b}{N_0}$.

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V. CONCLUSIONS

In this study, a satellite communication system, which uses VSATs as the ground terminals, was examined. The system utilizes CDMA with Forward Error Correction. The CDMA system uses PN sequences and Walsh Codes to generate a spread signal. Each VSAT uses a different PN sequence. Each user in a VSAT uses different Walsh Codes. Since Walsh codes are orthogonal, interference among the users is eliminated. PN sequences are not completely orthogonal; therefore interference exists due to other VSATs. To reduce the effects of interference and channel errors, the spread signal is encoded with different error correction mechanisms and its performance was analyzed. The results for bit error probabilities are upper bounds, not the exact results. Particularly, Convolutional and Reed-Solomon codes were examined.

Convolutional codes provide low bit error rates and they are flexible. Good convolutional codes provide the maximum coding gain. The results for different code rates show that higher code rates result in lower bit error rates. Rate 1/120 has the best performance among all, but this rate decreases the information rate dramatically. On the other hand, rate 1/2 performs almost as good as rate 1/120 and it does not reduce the information rate that much. In cases where extremely reliable communications are needed, lowering the data rate and increasing the code rate can satisfy the users. However, lowering the data rate makes voice communication impossible after a certain level. In these cases, only data communication can be established.

Reed-Solomon (RS) codes are used in many modern communication systems. They are effective for bursts of errors, and they are not very good in correcting the random errors. The increase in the error rate of RS codes is more than convolutional codes for the same amount of increase in the number of users. The block length for the RS code in this study is 63 ($2^6 - 1$). For lower bit error rates, longer block lengths can be selected.

The optimum results are obtained by using concatenated codes. Concatenated codes are used for deep space missions by the NASA. Usually, the inner codes are

Viterbi decoded convolutional codes and the outer codes are Reed-Solomon block codes. The inner code rate used in this study was always ¾, because this rate has a good performance and it does not decrease the information rate very much. The outer code can be changed according to the operational requirements. An inner code rate of 3/4 together with an outer code rate code rate 43/63 give a concatenated code rate 0.5119. When this result is compared to rate 1/2 convolutional code, the bit error probability of the former is not very good. For higher Signal-to-Noise ratios, concatenated codes have a greater advantage over convolutional codes.

The VSAT system that can be proposed based on this study is a DS-CDMA system with rate 3/4 Viterbi decoded inner convolutional code and rate 57/63 outer RS code. Using an interleaver between the inner and outer codes can further improve the burst error performance. Also using different code rates both for the inner and the outer codes can help to find the lowest bit error rate for a given requirement.

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