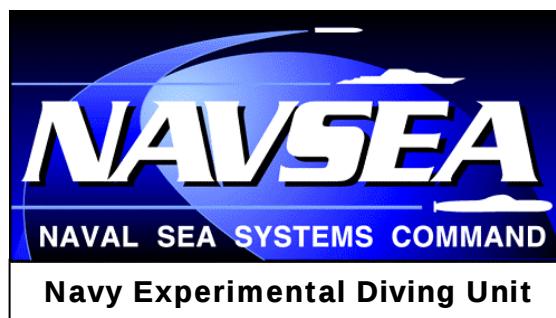


**Navy Experimental Diving Unit
321 Bullfinch Rd.
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**NEDU TR 02-08
August 2002**

THE ANALYSIS OF SODALIME GRANULE SIZE DISTRIBUTIONS



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19. ABSTRACT (Continue on reverse if necessary and identify by block number) NEDU has developed a procedure for translating mesh sizes given in sodalime specifications to those sieve sizes used in testing laboratories. This procedure is used to establish model Gaussian and log-normal distributions that are capable of meeting the specifications. The mesh size distribution measured by sieving a sodalime sample is then compared to the best model. A chi-square goodness of fit test is used to determine whether or not the sample distribution meets the best model distribution. Several MathCad documents are provided to explain the mathematical basis for this procedure.				
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GLOSSARY

ANU	Authorized for Navy Use List (NAVSEAINST 10560.2 series)
bar	Metric unit of pressure conveniently sized for supply pressures. One bar = 100 kPa, or 14.5 psi.
cmH ₂ O	A metric expression of static pressure head. One cmH ₂ O = 0.01 meters of pure water. In pressure equivalents, 1 cmH ₂ O = 0.736 torr, 981.8 Pa, or 0.0982 kPa.
FIO ₂	Fraction inspired O ₂ . The fraction of the inspired gas composed of oxygen.
fsw	Feet of seawater, a unit of pressure. One fsw = 0.3063 msw.
J/L	Joules per liter, unit of measure for "Work of Breathing" normalized for tidal volume. One J/L = 1 kPa.
kPa	Kilopascals or newton/m ² , unit of pressure. One kPa ~ 10.2 cmH ₂ O
kg·m·sec ⁻²	Kilogram meter per second ² , units of force, thrust, or buoyancy. From Newton's second law: $F = m \cdot a$. One lbf (pound force) = 4.448 kg·m·sec ⁻² = 4.448 newton.
msw	Meters of sea water. One msw = 3.2646 fsw.
NAVSEA	Naval Sea Systems Command
NEDU	Navy Experimental Diving Unit
psi	Pounds per square inch, an English measure of pressure. One psi = 6.895 kPa. 1 bar = 14.504 psi.
$\bar{P}_V$	Volume averaged pressure, or resistive effort, otherwise known by the misnomer Work of Breathing (WOB). A computer-derived estimate of total resistive respiratory effort obtained when breathing a UBA with a mechanical breathing simulator.
RMV	Respiratory minute volume with units of L·min ⁻¹ . In scientific publications, this is referred to as expired ventilation ($\dot{V}_E$).
STPD	Standard temperature (0°C), pressure (1 atm abs), dry.
$\dot{V}O_2$	Metabolic oxygen consumption in L·min ⁻¹ at STPD conditions.

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INTRODUCTION

The U.S. Navy Experimental Diving Unit (NEDU) evaluates samples of sodalime CO₂ absorbent for use in diving and hyperbaric applications. The diving Navy currently uses both large and small grain absorbent. The large grain absorbent, with granule sizes ranging from 4 to 8 U.S. mesh, is the traditional diving grade absorbent. High Performance (HP) Sodasorb, manufactured by W.R. Grace; Sofnolime 408 (Molecular Products, Thaxted, U.K.); and Divesorb Pro (Dräger, Germany) are such absorbents. Small grain absorbent granules range from 10 to 14 U.S. mesh (8-12 British mesh size, Table 1), and are used to provide extended canister durations. Molecular Product's Sofnolime 812 is a typical small grain absorbent.

When NEDU evaluates sodalime quality, we determine, among other things, whether or not the absorbent meets specifications for granule size distributions. However, the U.S. Navy buys absorbent from at least three countries (U.S., U.K and Germany) with each supplier having different sodalime specifications. Because of disparities between sieve sizes used in testing and mesh sizes required in various commercial and national specifications, a simple comparison is not always possible.

The purposes of this report are fourfold. We first wish to understand the impact of existing and proposed specifications on sodalimes with varying spreads of granule sizes. Previous reports have dealt only with differences in mean granule size.

Second, we wish to establish a rigorous procedure by which the test operator can, with minimal effort, decide if a sodalime sample meets specifications. This work will show that sodalime specifications are liberal in terms of the allowed distributions of granule sizes. Consequently, to ensure fairness to prospective sodalime suppliers, a method is needed for comparing sodalime samples other than just comparing a new sample with existing data.

A third purpose is to describe how a sodalime sample can be checked against a foreign specification if the foreign sieve sizes are not available to the testing laboratory. We will see that the foreign standards can be converted mathematically to a form any laboratory can use.

NEDU has used the procedure described in this report for several years, and thus the fourth purpose of this report is to document the existing methodology. Although we use examples relevant to the testing sieves available at NEDU, the procedures are general and can be adapted to virtually any assemblage of testing sieves or applied to any national or corporate specification.

I. DISTRIBUTION DETERMINATION

METHODS

To prevent this mathematical analysis from becoming too unwieldy, we must assume that the probable granule size distributions are small in number. We hypothesized that, based on first principles, the distributions should be either normal (Gaussian) or log-normal. This presumption is reasonable due to the random nature of the crushing process used to generate granules from stock material and to the particular applicability of the normal and lognormal distributions to random processes¹⁻⁴.

Although crushed absorbent granules are screened at the factory to produce products of varying size ranges, we anticipated that inefficiencies in the screening process would narrow the granule size distribution rather than sharply truncate it. That is, after screening, the granule size distribution would be likely to remain smooth rather than become discontinuous.

Our first task was to confirm the reasonableness of our assumptions: that is, to measure the actual granule size distributions of various sodalime samples. To our knowledge, this has never been done with the degree of precision required for this analysis. We therefore developed an image analysis technique using off-the-shelf computer software to provide data on granule size. The distributions of the granule sizes were then found by fitting size data to a number of model distributions.

Samples containing a minimum of 800 granules of Sofnolime 812 and 408 (Molecular Products, Thaxted, U.K), small and large granule size absorbents, respectively, were spread on the glass surface of a paper photocopier and copied against a dark background. A ruler was included in the image as a size reference. The result was a high contrast image with white granules against a black background. The image was then scanned by a digital scanner (Hewlett Packard ScanJet 3C) and run by a Hewlett Packard Vectra computer using Image Assistant. 1.12 (Caere Corp.) That image was then imported into a paint program, and partial images of granules on the periphery of the image were manually deleted. The ruler was also removed from the image after calibration marks had been transferred to the main body of the image. Figures 1 and 2 show the result. The calibration marks in the upper right corner of the figures were 20 millimeters (mm) apart.

The images were then subjected to automated image analysis (SigmaScan Pro 4.0, Jandel Scientific). The information obtained from each image was stored in a spreadsheet containing the number of granules as well as the number of pixels and the feret diameter for each granule. The feret diameter is the diameter of a circle that yields the same number of pixels as the actual number of pixels for each granule image. In other words, ferets convert irregularly shaped images into equivalent circles with the same area. The calibration marks were used to convert the feret diameters from numbers of pixels to mm.

After the spreadsheet was imported into the graphing program SigmaPlot (Jandel Scientific), a mathematical transform was used to convert the spreadsheet data into a histogram representing the frequency of occurrence of 50 feret diameter categories. Another Jandel software product, TableCurve 2D v. 4, fit up to 74 peak distributions to the histograms. The curve fits were accomplished by the method of least squares and ranked by the value of the F statistic for the fit.

RESULTS

Sofnolime 812

The sample of fine grain Sofnolime 812 contained 1230 granules (Figure 1). The distribution of feret diameters is shown in Figure 3. The mean granule size was 1.76 mm. The peaks occurring in Figure 3 at very small diameter sizes represent dust and granule fragments. Since we had no interest in the overall bimodal distribution (granules plus dust); we excluded the dust peaks from the distribution fitting process.

The best fit for the fine grain Sofnolime 812 absorbent was to the Complementary Error Function, closely related to the Gaussian. The second best fit was a Gaussian distribution with a mean (μ) of 1.73 ± 0.012 mm (best estimate $\pm$ standard error of the estimate, SEE) and a standard deviation (σ) of 0.401 ± 0.012 mm.

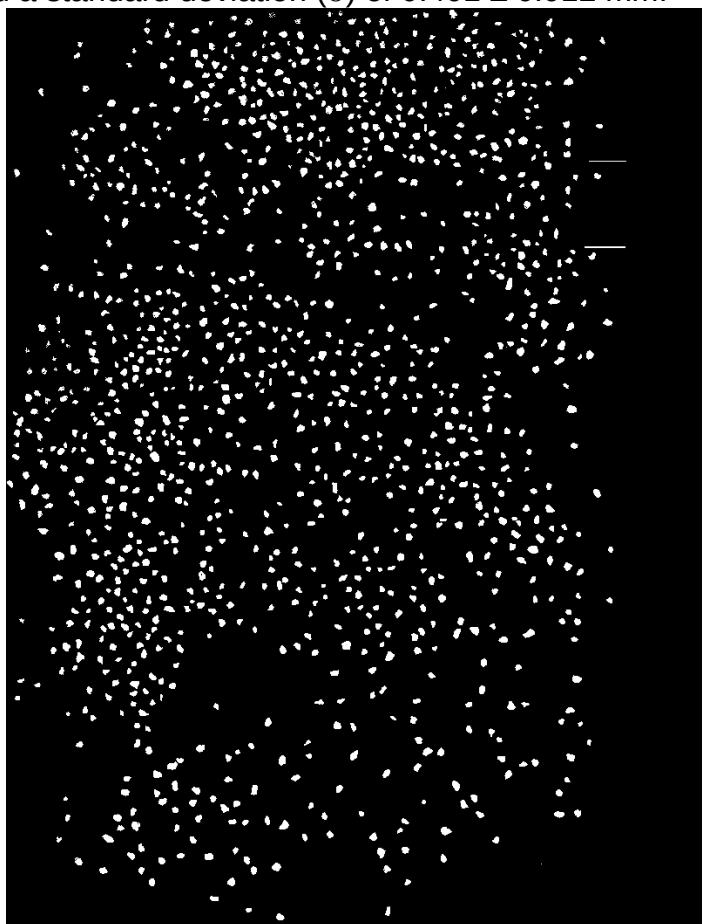


Figure 1. Sofnolime 812 Granules.

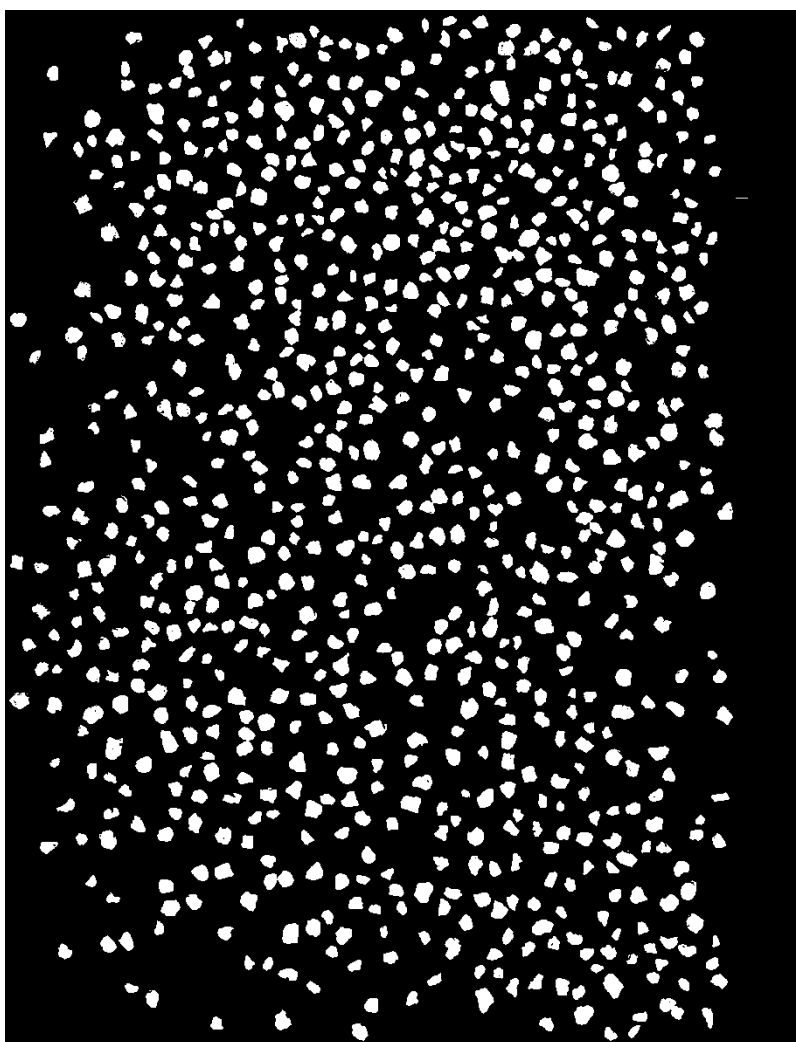


Figure 2. Sofnolime 408 Granules.

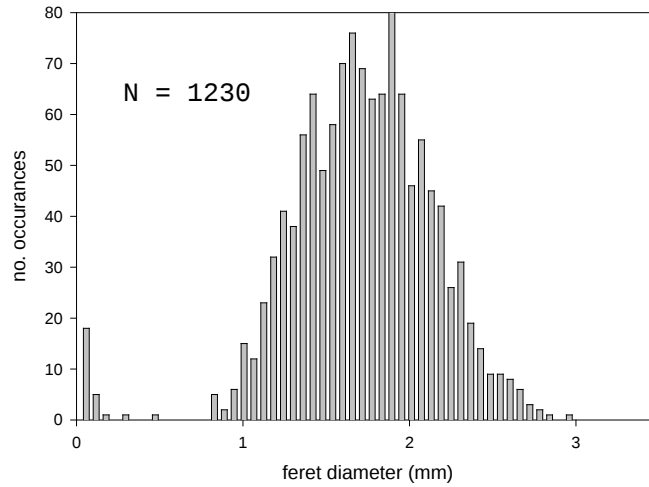


Figure 3. Granule size distributions for Sofnolime 812.

Figure 4 shows the best fit of the feret size frequency distribution to the Gaussian (top panel); the middle panel shows fit residuals. Both 95% confidence and prediction limits for the fit to the Gaussian are in the bottom panel. Comparable graphs for the complementary error function peak are shown in Figure 5.

Gaussian Function

The Gaussian function, Equation (1), is both continuous and integrable.

$$y(x) = e^{(-x)^2} \quad (1)$$

The normal or Gaussian distribution that results from the application of this function is given by

$$y(x) = e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad (2)$$

where x is granule feret diameter in mm, μ is the mean granule diameter in mm, and σ is the standard deviation of the distribution.

When equation (2) is divided by $\sigma\sqrt{2\pi}$, we obtain the so-called standard normal curve described in every statistics book. The area under the standard normal (the integral for all x) is equal to 1.

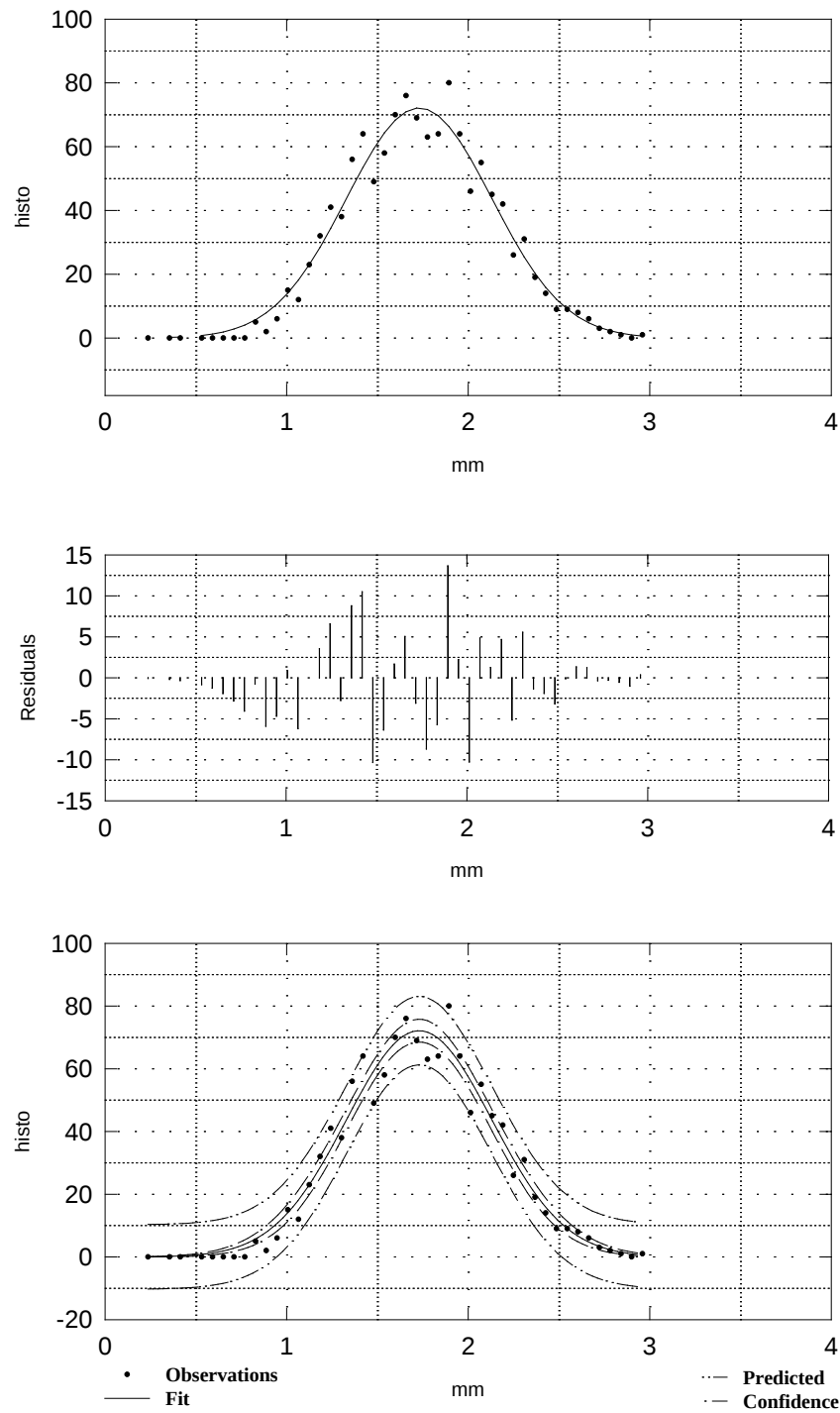


Figure 4. Gaussian fit to Sofnoline 812 granule size distribution. The fit, residual error, and 95% confidence and prediction limits are shown (top to bottom).

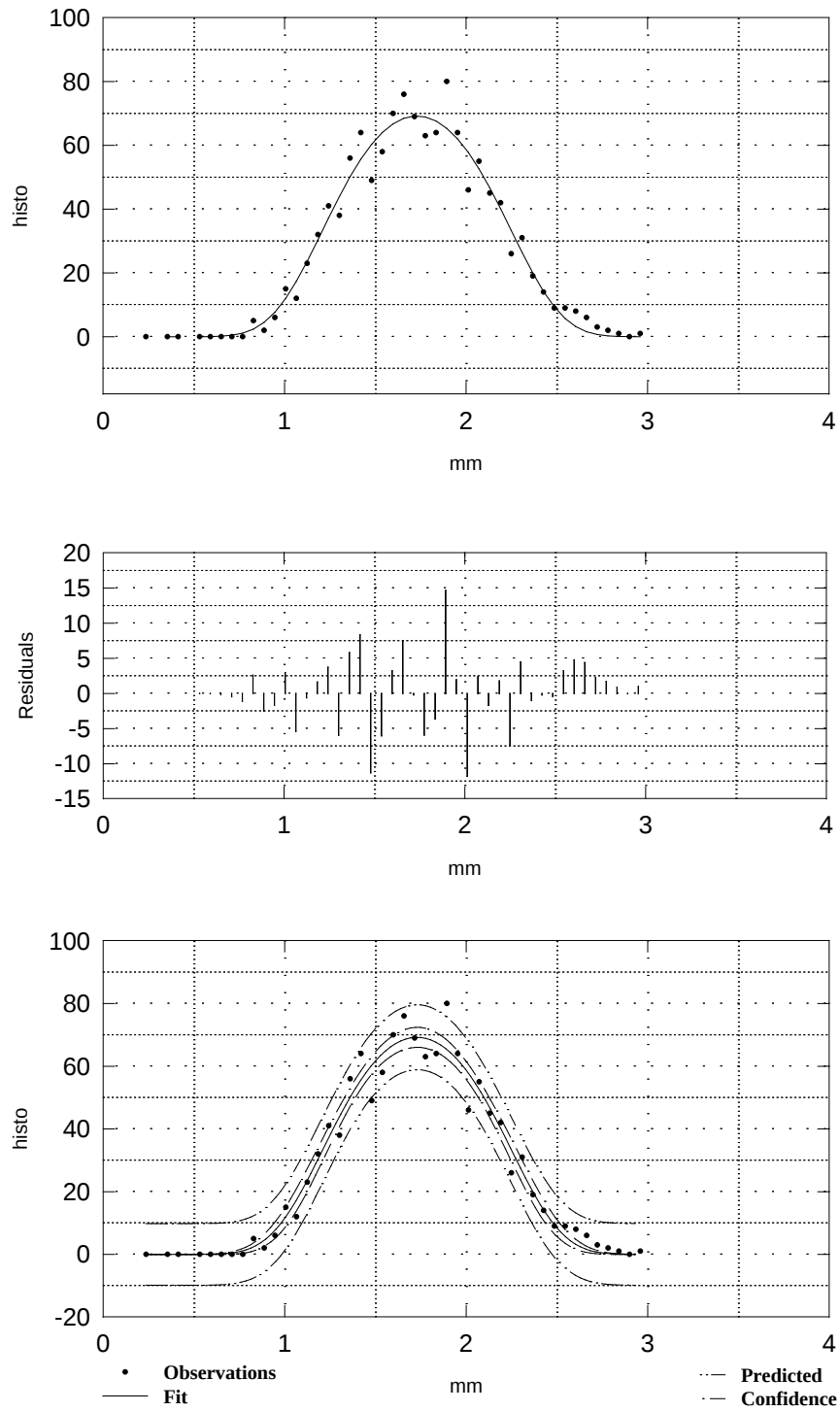


Figure 5. Fit of the peak of the complementary error function to Sofnolime 812 granule distribution.

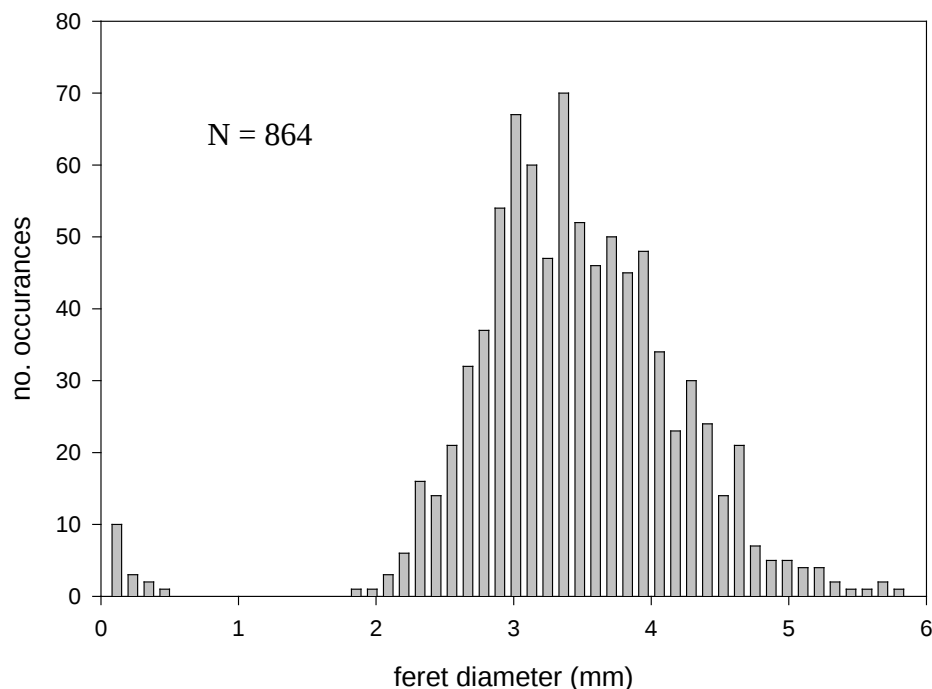


Figure 6. Granule size distribution for Sofnolime 408.

Sofnolime 408

The sample of large grain Sofnolime 408 contained 864 granules, not including dust particles (Figure 2). The distribution of feret diameters is shown in Figure 6; the mean granule size was 3.45 mm, which was approximately twice as large as the 1.76 mm mean for Sofnolime 812. The best fit for the large grain Sofnolime 408 absorbent was a log-normal distribution, with a μ of 3.30 ± 0.022 mm (best estimate $\pm$ standard error of the estimate, SEE) and a σ of 0.195 ± 0.007 mm.

The term log-normal means that the weights of the granule sizes are distributed normally with respect to the logarithms of the respective granule diameters. The equation for the log-normal is similar to that for the Gaussian, with the main difference being that x (granule feret diameter) is replaced by $\ln(x)$.

$$y(x) = e^{\frac{-(\ln(x)-\mu)^2}{2 \cdot \sigma^2}}$$

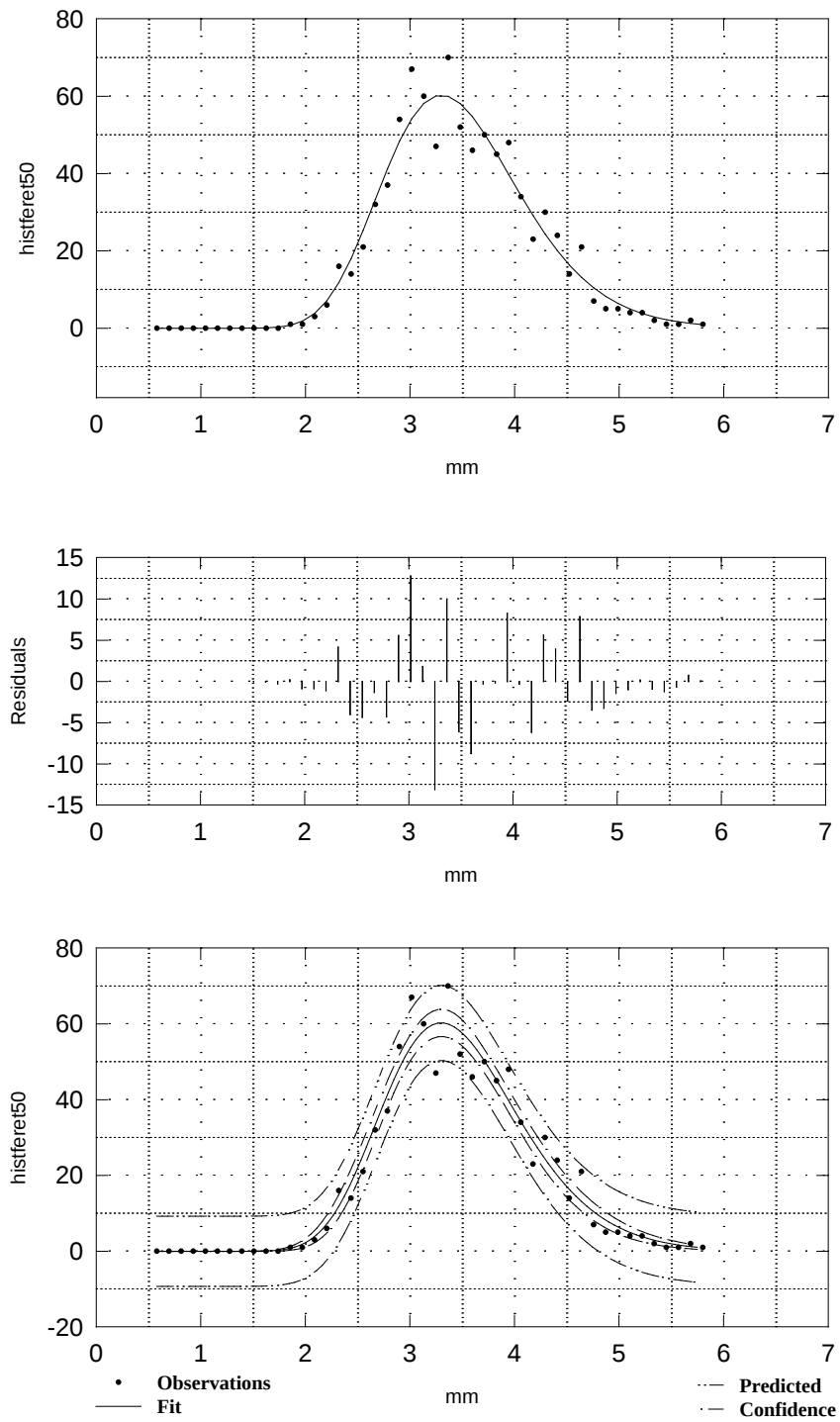


Figure 7. Log-normal fit to Sofnolime 408 granule size distribution.

II. SPECIFICATION TRANSLATION

METHODS

Specifications for sodalime granule size do not describe a particular distribution of sizes, but rather apply bounds to the possible granule size distributions. As an example, according to a British specification for 8-12 U.K mesh range material (10-14 U.S. mesh, Table 1), no more than 20% of the sieved absorbent may be smaller than a U.K. mesh size of 12 (1.40 mm). The specification provides a very general requirement that a number of different granule size distributions could meet.

Table 1. Mesh size conversions.

U.S. Sieve	Tyler Sieve	British (BS)	ISO Standard
3.5	3.5	3	5.60
4	4	4	4.75
5	5	---	4.0
6	6	5	3.35
7	7	6	2.80
8	8	7	2.38
10	--	8	2.00
12	10	10	1.70
14	12	12	1.40
30	28	25	0.600
40	35	36	0.425

The general method we use to determine whether or not a sodalime sample meets specification requirements is by first characterizing the granule size distribution of the sample, not by the laborious image analysis technique described in Section I but by a sieving procedure more suitable to large-scale analyses.

The next step is to statistically compare the obtained distribution with a model distribution known to meet the specification. However, since many possible model distributions could meet the specifications, knowing which one to compare with the test sample is not straightforward. An incorrect choice of a model distribution could unfairly penalize a particular test sample. The following procedures and algorithms are designed always to select the best possible model distribution to compare with a particular test sample.

In both the normal (Gaussian) and log-normal distributions, the entire distribution can be described by only two parameters, the mean (μ) and the standard deviation (σ). An additional parameter is often added to define the maximum frequency of occurrence in absolute terms (Appendix A).

We accept the premise that the mean of the granule size distribution should be close to the presumed median of the mesh range: i.e., the average size for absorbent in the 4-8 mesh size range should fall in the vicinity of mesh size 6. In the results section our data supports that premise. Assuming either normal or log-normal distributions, we generate a range of candidate distributions with various values of μ and σ that center on the expected median granule size. A range of σ s are examined that range from extraordinarily small up to a σ larger than would be expected in actual samples.

The following material, including the appendixes, describe procedures for working with the Gaussian distribution. Analogous methods are used with the log-normal distribution (Appendix C).

The percentage of each candidate distribution that lies within particular mesh ranges can be found by integrating Equation (2) over definite limits representing the extremes of the respective size ranges expressed as granule diameter in millimeters (mm) rather than mesh size. Therefore, the limits of integration for each integral represent granule diameters at the extreme of each mesh range cited in the specifications.

Appendices A-C are documents generated by MathCad (MathSoft, Inc., Princeton, NJ) detailing the steps taken to select the few model distributions that meet sodalime specifications from numerous candidate distributions. For both large grain (4-8 U.S. mesh) and small grain (10-14 U.S. mesh) absorbent, we simulated more than 90 candidate distributions, each defined by a separate μ and σ . For the example of normally distributed 10-14 mesh sodalime, we simulated 99 distributions and tested them; means were assumed to vary between 1.45 and 1.95 mm in diameter, with increments of 0.05 mm. Standard deviations varied from 0.05 to 0.45 mm, also in increments of 0.05 mm.

Using Table 1 for mesh size to mm equivalences, we defined four integral equations with definite limits corresponding to mesh ranges called out in the specifications. The first step in assessing the candidate distributions was to evaluate each of the four integral equations for all candidate distributions. Each integral was then tested against the logical conditions listed in the relevant sodalime specification. For example, referring to Appendix A for fine grain sodalime, and the matrix **A**, the integral equation for the Gaussian between limits of 2.8 and 20 mm (sizes larger than #7 mesh), most but not all candidate distributions yielded sample fractions less than 1% (p. A2). That is, less than 1% of the sodalime sample would be larger than 2.0 mm in diameter. Those distributions that met the specification requirement were marked by a 1 in the matrix of μ and σ , and those that did not meet the specification were indicated by a 0 (array **XA**, binary outcome).

When either 1 or 0 had been applied to each cell in the (μ , σ) array for each integral, the matrices were added (xtot, p. A6). Those cells with a sum of 4 (1 for each integral) were by definition cells that met all four specification requirements. Those cells were then further identified with a 1 while all others were assigned a 0 value in the "Final" array,

forming a triangular pattern of ones among a field of zeros. Of 99 potential distributions, only 46 met the complete specification and therefore could be classified as model distributions (“Final” array, p. B-6).

NEDU Sieves

The next step was to reevaluate the normal integrals over limits defining the sieve sizes available at NEDU (Fig. 8). Six definite integrals representing six size ranges were defined (3.5 mesh size and larger, 3.5 - 5 mesh, 5 - 8 mesh, 8 - 10 mesh, 10-14 mesh, and 14-30 mesh). Each of the six integrals was applied to all of the 99 potential distributions for a total of 594 integrations. Obviously, many of those results were of no interest since they applied to distributions that did not meet the specification. Those integrals were eliminated from view by applying a masking function or filter consisting of the Final Array. The result was a two-dimensional array of μ and σ with a value of either zero or the integral for each of the six mesh ranges (Tables 2 and 3).



FIGURE 8. ROTAP SHAKER WITH 5 SIEVES IN PLACE.

Three μ and σ combinations in Table 2 represent the extremes of distributions meeting the specifications for 10-14 mesh sodalime. Those values in the top left, bottom left, and mid-far right portions of the table are in bold font for clarity. They represent small granules with a tight distribution of granule sizes, larger granules with an equally tight distribution, or mid-size granules with a wide distribution, respectively.

The non-zero numbers in Table 2 tell us that if absorbents meeting the specifications are sieved, the #14 mesh sieve would collect anywhere from 59% to 100% of the test sample, depending on which of the 46 model distributions best applied to the sample. Both the shapes of these three distributions and their expected sieve retentions are shown in Figure 9.

Table 3 shows similar results for large grain, 4-8 U.S. mesh sodalime. Anywhere from 61% to 79% of the absorbent could be trapped on the #8 U.S. mesh sieve and still meet specifications.

Table 2. The “Final” array for 10-14 mesh sodalime, #14 sieve.

	Distribution Standard Deviation (σ, mm)								
μ (mm)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
1.45	0.84	0	0	0	0	0	0	0	0
1.50	0.98	0.84	0	0	0	0	0	0	0
1.55	1	0.93	0.84	0	0	0	0	0	0
1.60	1	0.98	0.91	0.83	0	0	0	0	0
1.65	1	0.99	0.95	0.87	0.79	0	0	0	0
1.70	1	1	0.97	0.9	0.81	0.73	0.65	0	0
1.75	1	1	0.97	0.9	0.81	0.73	0.65	0.59	0
1.80	1	1	0.95	0.88	0.8	0.72	0.64	0	0
1.85	1	0.98	0.92	0.84	0.76	0	0	0	0
1.90	1	0.95	0.86	0	0	0	0	0	0
1.95	0.99	0	0	0	0	0	0	0	0

Table 3. The “Final” array for 4-8 mesh sodalime, #8 sieve.

	Distribution Standard Deviation (σ, mm)							
μ (mm)	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.0
3.02	0	0	0	0	0	0	0	0
3.08	0.79	0	0	0	0	0	0	0
3.15	0.79	0.76	0.73	0	0	0	0	0
3.21	0.79	0.76	0.73	0.69	0	0	0	0
3.28	0.79	0.75	0.72	0.69	0.66	0	0	0
3.34	0.78	0.75	0.71	0.69	0.66	0.63	0.61	0
3.41	0.77	0.73	0.70	0.68	0.66	0.62	0	0
3.47	0.75	0.72	0.69	0.66	0.64	0	0	0
3.54	0.73	0.70	0.67	0.63	0	0	0	0
3.60	0.70	0.68	0.65	0	0	0	0	0
3.67	0.67	0.65	0	0	0	0	0	0
3.73	0.64	0	0	0	0	0	0	0

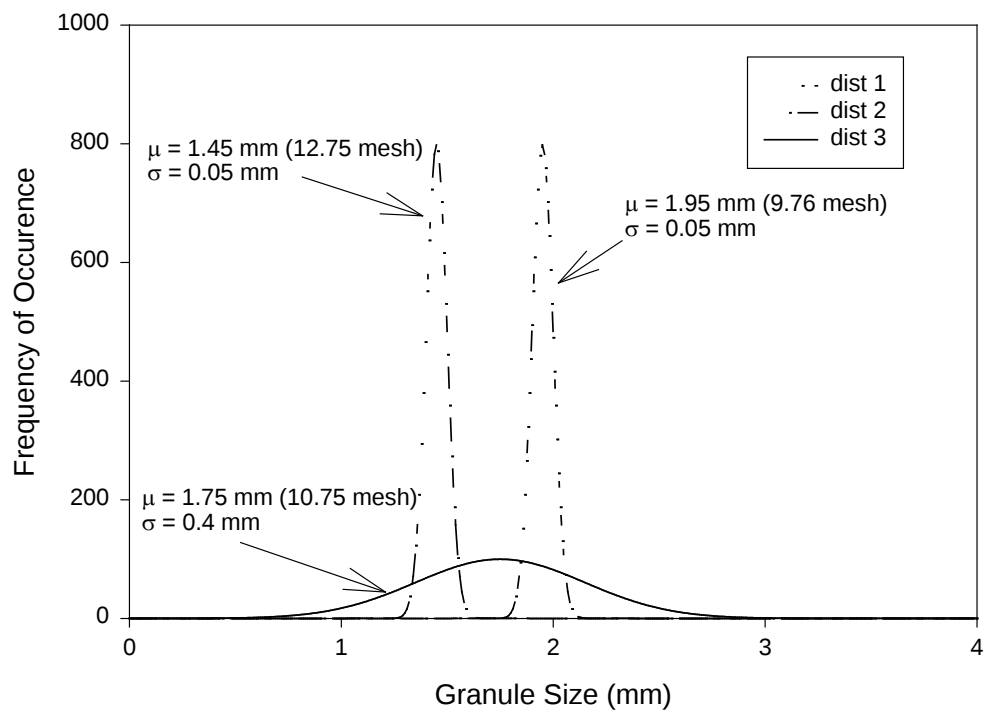
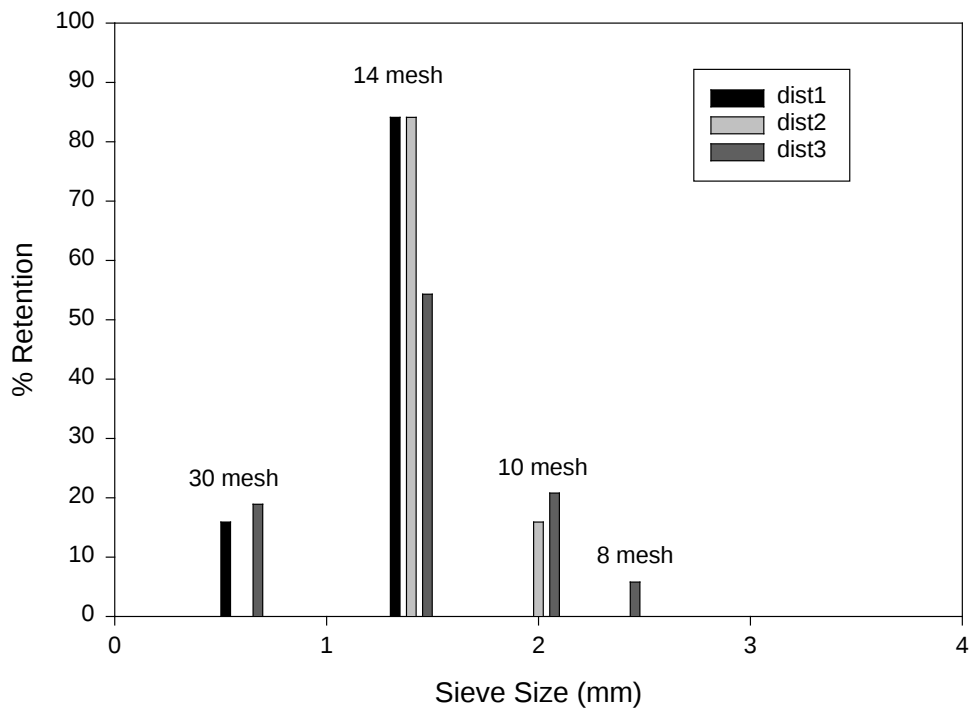


Figure 9. Three granule size distributions (bottom panel) and corresponding sieve results (top panel).

MeshFit

It is possible that a test sample will match none of the model distributions particularly well. Fortunately, the degree of match or mismatch can be measured and used as a basis for evaluating the test sample. NEDU uses the *MeshFit* software developed in-house to enter the sieve data from the test sample and to locate the one model distribution that best agrees with the test sample data. The goodness of fit between the test data and the best model distribution is then found through two mechanisms, one being simply the sum of the absolute differences between actual and ideal retained fractions and the other being the classical chi square statistical test for goodness of fit. *MeshFit* was written in Visual Basic Version 6 for Windows 98, 32-bit operating system. The most relevant portions of code are provided in Appendix D.

As explained in Appendix G, the user selects whether the test sample is from a 4-8 U.S. mesh sample or 10-14 U.S. mesh sample. The user then enters the mean retained percentages for all sieve screens obtained from the sieving of five or more sodalime samples. The software next identifies the best fit to a model distribution and plots bar graphs for the mesh distribution of both the sample and the best model. Finally, the chi square goodness of fit statistic is computed, with its significance being determined from the chi square and the degrees of freedom for the test.

Chi square Goodness of Fit

The chi square test enables observed versus expected values for multiple data groupings to be tested. In this particular case, we compare the observed fraction retained by a particular sieve screen against the expected fraction for the closest distribution, either normal or log-normal, meeting the specifications.

Although the expected values may be real numbers, to be valid the chi square test requires that all observations be expressed as integers. Furthermore, to be included in the test, the minimum value observed must be 5 or greater. Data groupings; i.e., mesh sizes, may be pooled to meet the requirement of 5 observations. To satisfy the above requirements, when the chi square test is run retained fractions are expressed as percentages of the total sample.

RESULTS

MeshFit

Figures 10 and 11 are *MeshFit* program printouts illustrating the analysis of test results from two large grain sodalime samples. In Figure 10 the sample mesh distribution was statistically equivalent to the best model distribution for 4-8 mesh absorbent. In Figure 11 the sample distribution departed significantly from the expected mesh distribution. The probability that a sample distribution would depart as far as it did from the model if in fact the sodalime met the best model distribution was < 0.001 . Statistical significance is assumed at a probability (P) less than 0.05.

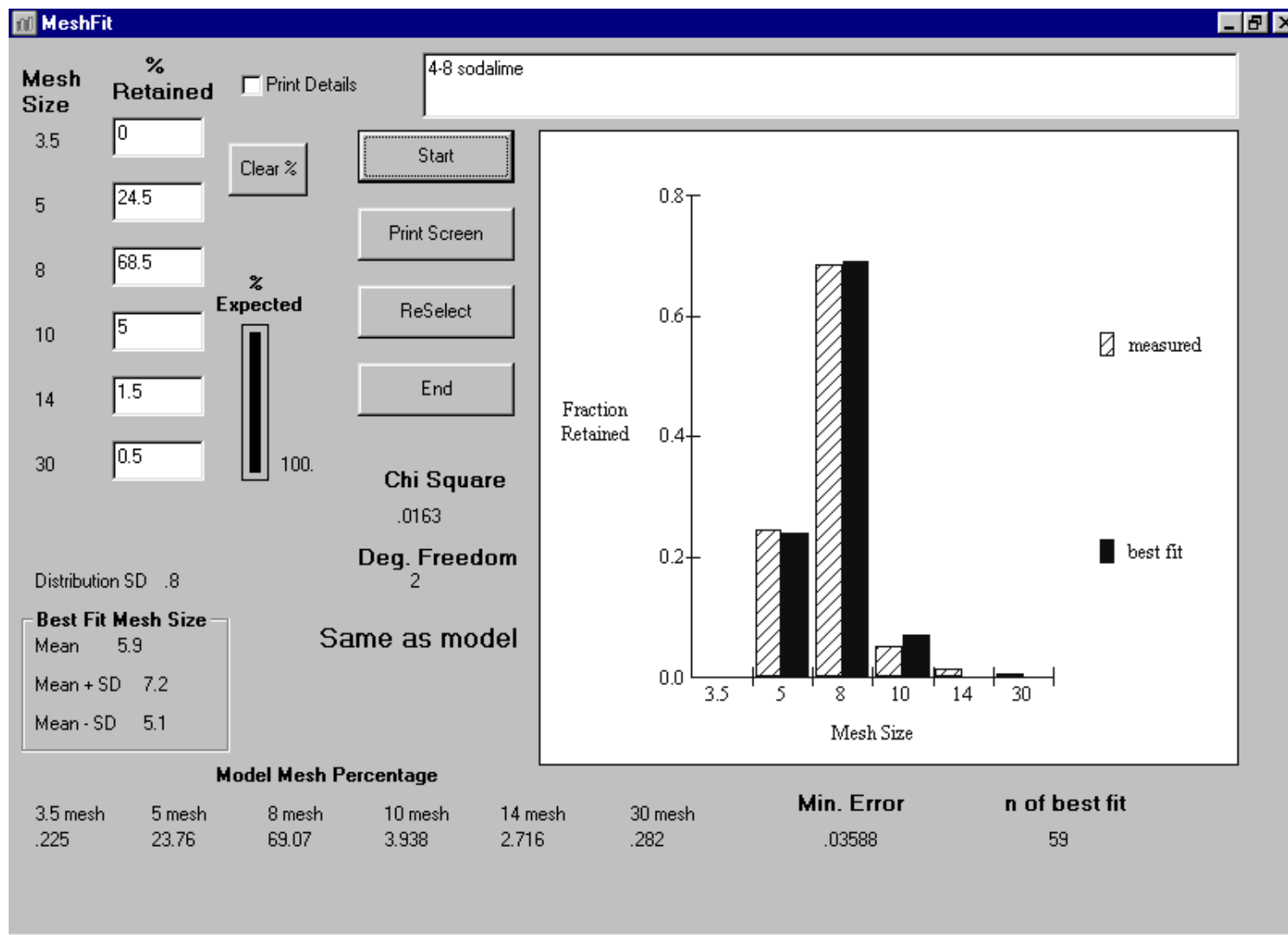


Figure 10. Screen print from *MeshFit*; 4-8 mesh sodalime data. The measured and model distribution characteristics are similar.

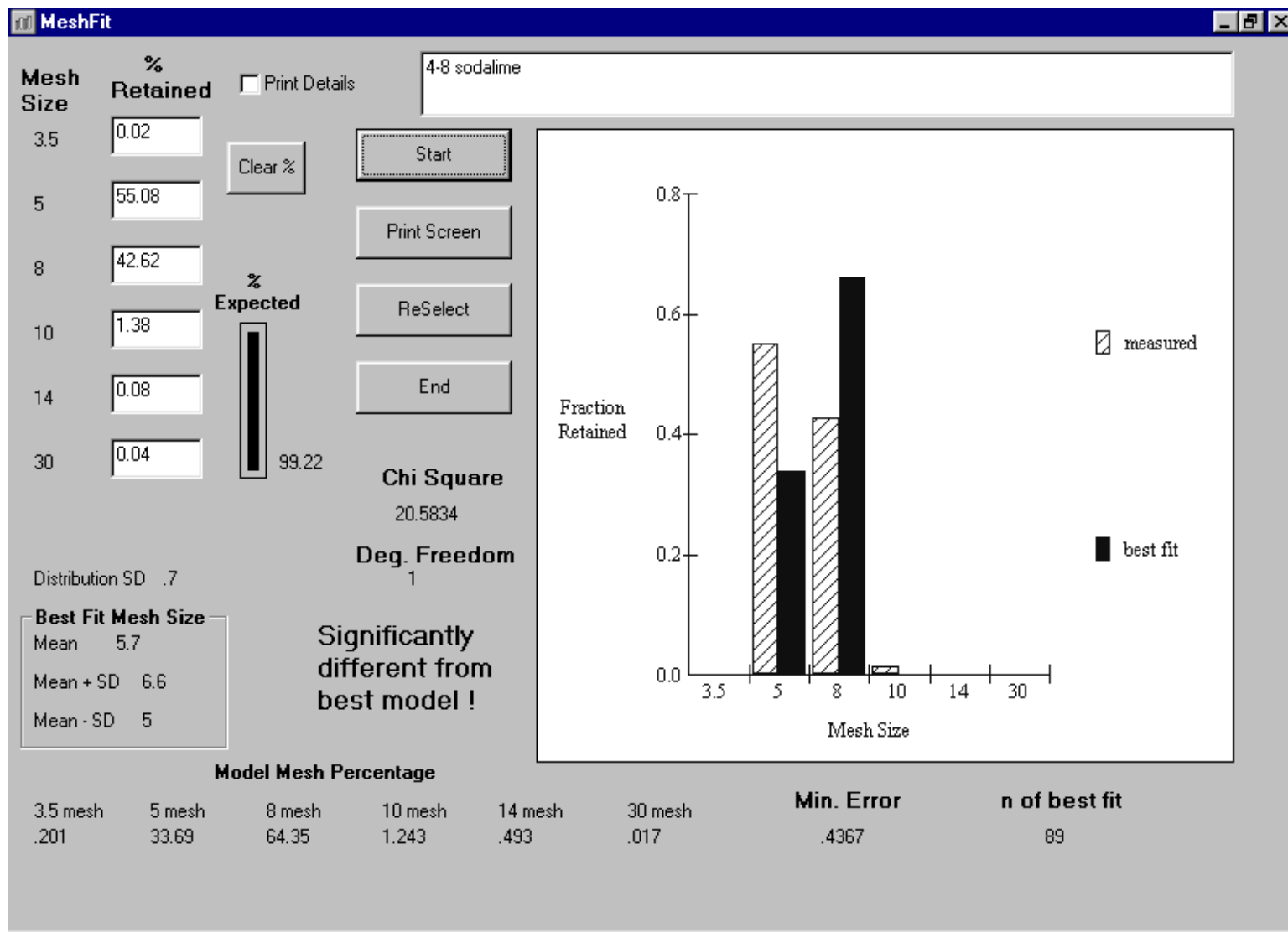


Figure 11. MeshFit screen print; 4-8 mesh data not matching any of the allowed distributions. The closest match is displayed.

Appendix D results from printing “details” when running the *MeshFit* program for the sample in Figure 10. The appendix lists the results of fitting the sample distribution to each of the 96 candidate distributions. The first column records the number of the candidate distribution, the second presents the error of the fit, and the remaining columns show the expected fractions of retained sodalime on the 3.5-, 5-, 8-, 10-, 14-, and 30-mesh screens. Only the distributions that have a non-zero value in two or more of the retained fraction columns serve as model distributions. As shown in Figure 10, the best fit is obtained with model distribution number 59. The expected retained fractions in the Details printout for row 59 (marked in Appendix D by a shaded line) are also displayed in Figure 10 under the heading “Model Mesh Fractions”. As expected, the mean for the best model distribution corresponds to a mesh size of 5.9, approximately midway between the mesh sizes of 4 and 8. The majority of the granules have a mesh size of 5.1 (mean - SD) – 7.2 (mean + SD).

At the end of the “Details” listing is an across the board comparison between the observed results (obs) and the model distribution (modl). Retained fractions from both sample and model populations have been pooled as necessary to meet the requirements of the chi square test: to have a minimum of “5” in any of the compared groupings.

In the example of Figure 10, only three pooled mesh sizes (n) can be used for the chi square test. The resulting degrees of freedom for the test are n-1, or 2, as reported in Figure 10. The probability of obtaining a chi square goodness of fit statistic of 0.0163 from a distribution that is the same as the model distribution is 0.998. Since that probability is larger than 0.05, we accept the null hypothesis that the tested sample and the model are equivalent. That information is displayed on the *MeshFit* screen by the wording “Same as model”, or in an alternate version of the software (Appendix G), “Meets spec.”

Appendix E is the “Detail” listing for Figure 11. The test sample fit candidate distribution 89 (row 89, shaded) the best. This time after pooling, only 2 mesh sizes could be tested by the chi square; therefore, the test had 1 degree of freedom. The probability of obtaining a chi square statistic of 20.65 from a test sample that matches the model is less than 0.001. Therefore, we reject the null hypothesis that the test sample matches the best modeled distribution; that is, the sample differs significantly from the model and therefore does not meet specifications. That information is displayed on the *MeshFit* screen by the wording “Significantly different from best model!” or in Appendix G, “Does not meet spec!”

DISCUSSION

Image analysis of absorbent granules show that the assumptions required for the *MeshFit* program are reasonable. Both normal and log-normal distributions described the absorbent granule size distributions of our two samples (Figures 1–7). Furthermore, the mean granule size did lie near the middle of the mesh range of the absorbent.

The NEDU image analysis technique for determining granule size distributions is too time consuming for routine use. The standard sieve test with Rotap shaker is far more efficient. However, armed with the insights gained from image analysis, we can mathematically analyze sieve data to achieve our goal of establishing a rigorous procedure for qualifying or disqualifying a sodalime sample while requiring minimal decisions by the test operator. The means to that end is NEDU's *MeshFit* software, which requires as input both the mean data from multiple sieving runs and the data files generated by MathCad (Appendices A-C).

One benefit from this approach is that this method reduces unfairness in comparing a sodalime sample to specifications. The approach appreciates that specifications are fairly general. If *MeshFit* says that a sodalime sample does not meet specification, it is satisfying to know that the failing sample has been compared to virtually all possible distributions meeting the specification. *MeshFit*'s accept or reject decision is based on a comparison with the test sample's best match.

One of the aims of this work was to learn how existing and proposed specifications for sodalimes would be affected by varying spreads of granule sizes. As shown in Figure 9, current specifications for fine grain absorbent allow widely varying distributions of granule sizes. Narrow distributions can differ considerably in their mean value and still meet the specification. Since mean granule size is inversely related to absorbent activity, which in turn dramatically affects canister duration, specifications for absorbents with narrowly defined granule sizes should have much tighter restrictions on allowable size than specifications for absorbents with widely dispersed granule sizes.

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APPENDIX A. Normally Distributed 10-14 Mesh Sodalime

Problem: A) Find a Gaussian (normal) distribution that fulfills the requirements of the original specification for 8-12 (10-14 US) mesh size CO₂ absorbent (sodalime).

B) Find allowable % retention of absorbent for sieve sizes used in the NEDU T&E laboratory.

ORIGIN= 1

j := 1..11 i := 1..9 Iteration range

Gaussian (normal) distribution parameters

Mean Standard deviation

$\mu_j := 1.40 + .05 \cdot j$ $\sigma_i := i \cdot .05$

Normal distribution:
$$y(x, \mu, \sigma) := \left(\frac{1}{\sqrt{2 \cdot \pi \cdot \sigma}} \right) \cdot e^{-\frac{(x-\mu)^2}{2 \cdot (\sigma)^2}}$$

$$\mu = \begin{pmatrix} 1.45 \\ 1.5 \\ 1.55 \\ 1.6 \\ 1.65 \\ 1.7 \\ 1.75 \\ 1.8 \\ 1.85 \\ 1.9 \\ 1.95 \end{pmatrix} \quad \sigma = \begin{pmatrix} 0.05 \\ 0.1 \\ 0.15 \\ 0.2 \\ 0.25 \\ 0.3 \\ 0.35 \\ 0.4 \\ 0.45 \end{pmatrix}$$

Integral equations relevant to specifications for 10-14 mesh absorbent

All mesh sizes expressed in diameter of mesh screen (in mm).

<7 mesh

$$A_{j,i} := \int_{2.80}^{20} y(x, \mu_j, \sigma_i) dx$$

7-10 mesh

$$B_{j,i} := \int_{2.0}^{2.8} y(x, \mu_j, \sigma_i) dx$$

10-14 mesh

$$C_{j,i} := \int_{1.4}^{2.0} y(x, \mu_j, \sigma_i) dx$$

14-40 mesh

$$D_{j,i} := \int_{0.425}^{1.4} y(x, \mu_j, \sigma_i) dx$$

The proposed draft sodalime specification calls for no more than 1% larger than 7 mesh, no more than 30% in the 7-10 mesh size range, no less than 48% in the 10-14 range, no more than 20% between 14 and 40 mesh.

Out of 99 candidate distributions, we found 46 that met all criteria for the specification, and were therefore classified as model distributions. The manner in which these 46 distributions were identified are indicated on the following pages. Of those 46, 6 are listed on the last two pages.

Fraction of absorbent larger than #7 mesh

Matrix of absorbent fraction greater than #7 mesh (2.8 mm) in size for various distribution j 's (means) and i 's (standard deviations).

$$A = \begin{pmatrix} 0 & 0 & 0 & 4.17 \times 10^{-11} & 1.22 \times 10^{-7} & 8.73 \times 10^{-6} & 1.1 \times 10^{-4} & 3.85 \times 10^{-4} & 1.35 \times 10^{-3} \\ 0 & 0 & 0 & 2.19 \times 10^{-10} & 3.51 \times 10^{-7} & 1.82 \times 10^{-5} & 1.89 \times 10^{-4} & 5.97 \times 10^{-4} & 1.93 \times 10^{-3} \\ 0 & 0 & 0 & 1.08 \times 10^{-9} & 9.74 \times 10^{-7} & 3.7 \times 10^{-5} & 3.19 \times 10^{-4} & 8.89 \times 10^{-4} & 2.74 \times 10^{-3} \\ 0 & 0 & 5.52 \times 10^{-15} & 4.98 \times 10^{-9} & 2.59 \times 10^{-6} & 7.31 \times 10^{-5} & 3.29 \times 10^{-4} & 1.35 \times 10^{-3} & 3.83 \times 10^{-3} \\ 0 & 0 & 7.51 \times 10^{-14} & 2.16 \times 10^{-8} & 6.64 \times 10^{-6} & 1.4 \times 10^{-4} & 5.44 \times 10^{-4} & 2.02 \times 10^{-3} & 5.3 \times 10^{-3} \\ 0 & 0 & 9.15 \times 10^{-13} & 8.82 \times 10^{-8} & 1.63 \times 10^{-5} & 2.63 \times 10^{-4} & 8.37 \times 10^{-4} & 2.98 \times 10^{-3} & 7.25 \times 10^{-3} \\ 0 & 0 & 9.97 \times 10^{-12} & 3.38 \times 10^{-7} & 3.86 \times 10^{-5} & 4.77 \times 10^{-4} & 1.35 \times 10^{-3} & 4.33 \times 10^{-3} & 9.82 \times 10^{-3} \\ 0 & 0 & 9.73 \times 10^{-11} & 1.22 \times 10^{-6} & 8.77 \times 10^{-5} & 4.89 \times 10^{-4} & 2.14 \times 10^{-3} & 6.21 \times 10^{-3} & 0.01 \\ 0 & 0 & 8.49 \times 10^{-10} & 4.12 \times 10^{-6} & 1.91 \times 10^{-4} & 7.72 \times 10^{-4} & 3.32 \times 10^{-3} & 8.77 \times 10^{-3} & 0.02 \\ 0 & 0 & 6.63 \times 10^{-9} & 1.31 \times 10^{-5} & 4.01 \times 10^{-4} & 1.35 \times 10^{-3} & 5.06 \times 10^{-3} & 0.01 & 0.02 \\ 0 & 0 & 4.64 \times 10^{-8} & 3.91 \times 10^{-5} & 4.31 \times 10^{-4} & 2.3 \times 10^{-3} & 7.58 \times 10^{-3} & 0.02 & 0.03 \end{pmatrix} \quad (A1)$$

Binary Outcome

Specific Specification Criterion

$$XA_{j,i} := \text{if}(A_{j,i} < .01, 1, 0)$$

$$XA = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

1 = meets specification
0 = does not meet specification

Each "1" identifies a theoretical distribution that meets the portion of the specification that requires no more than 1% of the absorbent can be larger than #7 mesh.

Fraction of absorbent between #7 and #10 mesh
(2.8 mm to 2.0 mm)

$$\mathbf{B} = \begin{pmatrix} 0 & 1.91 \times 10^{-8} & 1.23 \times 10^{-4} & 2.98 \times 10^{-3} & 0.01 & 0.03 & 0.06 & 0.08 & 0.11 \\ 0 & 2.87 \times 10^{-7} & 4.29 \times 10^{-4} & 6.21 \times 10^{-3} & 0.02 & 0.05 & 0.08 & 0.11 & 0.13 \\ 0 & 3.4 \times 10^{-6} & 1.35 \times 10^{-3} & 0.01 & 0.04 & 0.07 & 0.1 & 0.13 & 0.16 \\ 0 & 3.17 \times 10^{-5} & 3.83 \times 10^{-3} & 0.02 & 0.05 & 0.09 & 0.13 & 0.16 & 0.18 \\ 1.57 \times 10^{-12} & 2.33 \times 10^{-4} & 9.82 \times 10^{-3} & 0.04 & 0.08 & 0.12 & 0.16 & 0.19 & 0.21 \\ 1.12 \times 10^{-9} & 1.35 \times 10^{-3} & 0.02 & 0.07 & 0.12 & 0.16 & 0.19 & 0.22 & 0.25 \\ 3.06 \times 10^{-7} & 6.21 \times 10^{-3} & 0.05 & 0.11 & 0.16 & 0.2 & 0.24 & 0.26 & 0.28 \\ 3.24 \times 10^{-5} & 0.02 & 0.09 & 0.16 & 0.21 & 0.25 & 0.28 & 0.3 & 0.32 \\ 1.35 \times 10^{-3} & 0.07 & 0.16 & 0.23 & 0.27 & 0.31 & 0.33 & 0.35 & 0.35 \\ 0.02 & 0.16 & 0.25 & 0.31 & 0.34 & 0.37 & 0.38 & 0.39 & 0.39 \\ 0.16 & 0.31 & 0.37 & 0.4 & 0.42 & 0.43 & 0.44 & 0.43 & 0.43 \end{pmatrix} \quad (\text{A2})$$

Binary Outcome

Specific Specification Criterion

$$\mathbf{XB}_{j,i} := \text{if}(\mathbf{B}_{j,i} < .30, 1, 0)$$

$$\mathbf{XB} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Fraction of absorbent between #10 and #14 mesh
(2.0 mm to 1.4 mm)

$$C = \begin{pmatrix} 0.84 & 0.69 & 0.63 & 0.6 & 0.57 & 0.53 & 0.5 & 0.47 & 0.43 \\ 0.98 & 0.84 & 0.75 & 0.69 & 0.63 & 0.58 & 0.54 & 0.49 & 0.45 \\ 1 & 0.93 & 0.84 & 0.76 & 0.69 & 0.62 & 0.57 & 0.52 & 0.47 \\ 1 & 0.98 & 0.9 & 0.82 & 0.73 & 0.66 & 0.59 & 0.53 & 0.48 \\ 1 & 0.99 & 0.94 & 0.85 & 0.76 & 0.68 & 0.6 & 0.54 & 0.49 \\ 1 & 1 & 0.95 & 0.87 & 0.77 & 0.68 & 0.61 & 0.55 & 0.5 \\ 1 & 0.99 & 0.94 & 0.85 & 0.76 & 0.68 & 0.6 & 0.54 & 0.49 \\ 1 & 0.98 & 0.9 & 0.82 & 0.73 & 0.66 & 0.59 & 0.53 & 0.48 \\ 1 & 0.93 & 0.84 & 0.76 & 0.69 & 0.62 & 0.57 & 0.52 & 0.47 \\ 0.98 & 0.84 & 0.75 & 0.69 & 0.63 & 0.58 & 0.54 & 0.49 & 0.45 \\ 0.84 & 0.69 & 0.63 & 0.6 & 0.57 & 0.53 & 0.5 & 0.47 & 0.43 \end{pmatrix} \quad (A3)$$

Binary Outcome

Specific Specification Criterion

$$XC_{j,i} := \text{if}(C_{j,i} > .48, 1, 0)$$

$$XC = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Fraction of absorbent between #14 and #30 mesh
(1.40 mm to 0.60 mm)

$$D = \begin{pmatrix} 0.16 & 0.31 & 0.37 & 0.4 & 0.42 & 0.43 & 0.44 & 0.45 & 0.44 \\ 0.02 & 0.16 & 0.25 & 0.31 & 0.34 & 0.37 & 0.39 & 0.4 & 0.4 \\ 1.35 \times 10^{-3} & 0.07 & 0.16 & 0.23 & 0.27 & 0.31 & 0.33 & 0.35 & 0.36 \\ 3.36 \times 10^{-5} & 0.02 & 0.09 & 0.16 & 0.21 & 0.25 & 0.28 & 0.31 & 0.32 \\ 3.28 \times 10^{-7} & 6.21 \times 10^{-3} & 0.05 & 0.11 & 0.16 & 0.2 & 0.24 & 0.26 & 0.29 \\ 1.24 \times 10^{-9} & 1.35 \times 10^{-3} & 0.02 & 0.07 & 0.12 & 0.16 & 0.2 & 0.23 & 0.25 \\ 1.79 \times 10^{-12} & 2.33 \times 10^{-4} & 9.82 \times 10^{-3} & 0.04 & 0.08 & 0.12 & 0.16 & 0.19 & 0.22 \\ 0 & 3.17 \times 10^{-5} & 3.83 \times 10^{-3} & 0.02 & 0.05 & 0.09 & 0.13 & 0.16 & 0.19 \\ 0 & 3.41 \times 10^{-6} & 1.35 \times 10^{-3} & 0.01 & 0.04 & 0.07 & 0.1 & 0.13 & 0.16 \\ 0 & 2.88 \times 10^{-7} & 4.29 \times 10^{-4} & 6.21 \times 10^{-3} & 0.02 & 0.05 & 0.08 & 0.11 & 0.13 \\ 0 & 1.92 \times 10^{-8} & 1.23 \times 10^{-4} & 2.98 \times 10^{-3} & 0.01 & 0.03 & 0.06 & 0.08 & 0.11 \end{pmatrix} \quad (A4)$$

Binary Outcome

Specific Specification Criterion

$$XD_{j,i} := \text{if}(D_{j,i} < .20, 1, 0)$$

$$XD = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Sum of binary outcomes

$$XTOT_{j,i} := (XA_{j,i} + XB_{j,i} + XC_{j,i} + XD_{j,i})$$

$$XTOT = \begin{pmatrix} 4 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 \\ 4 & 4 & 3 & 3 & 3 & 3 & 3 & 3 & 2 \\ 4 & 4 & 4 & 3 & 3 & 3 & 3 & 3 & 2 \\ 4 & 4 & 4 & 4 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 3 & 2 \\ 4 & 4 & 4 & 4 & 4 & 3 & 3 & 3 & 1 \\ 4 & 4 & 4 & 3 & 3 & 3 & 3 & 2 & 1 \\ 4 & 3 & 3 & 3 & 3 & 3 & 3 & 1 & 1 \end{pmatrix}$$

$$FINAL_{j,i} := \text{if}(XTOT_{j,i} = 4, 1, 0)$$

1's identify distributions that simultaneously meet all four specification requirements. That is, these are good model distributions.

This matrix is used below as a mask or filter to select valid values for retained fractions with NEDU's mesh screens.

$$FINAL = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The following definite integrals define the fraction of the normal distribution falling within the confines of the six sieves available at NEDU.

3.5 sieve

$$E_{j,i} := \int_{5.6}^{20} y(x, \mu_j, \sigma_i) dx$$

5 sieve

$$F_{j,i} := \int_{4.0}^{5.6} y(x, \mu_j, \sigma_i) dx$$

8 sieve

$$G_{j,i} := \int_{2.36}^{4.0} y(x, \mu_j, \sigma_i) dx$$

10 sieve

$$H_{j,i} := \int_{2.06}^{2.36} y(x, \mu_j, \sigma_i) dx$$

14 sieve

$$K_{j,i} := \int_{1.4}^{2.06} y(x, \mu_j, \sigma_i) dx$$

30 sieve

$$L_{j,i} := \int_{.6}^{1.4} y(x, \mu_j, \sigma_i) dx$$

3.5 sieve

Fractions retained by a #3.5 mesh sieve.

$$E = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Results of binary filtering

Apply binary filter

$$EANS_{j,i} := E_{j,i} \cdot FINAL_{j,i}$$

Write results to a data file for use by *MeshFit* software.

$$WRITEPRN("mesh1014.prn") := EANS$$

$$EANS = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5 sieve

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1.6 \times 10^{-13} & 9.15 \times 10^{-11} & 7.28 \times 10^{-9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.57 \times 10^{-13} & 2.05 \times 10^{-10} & 1.38 \times 10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.28 \times 10^{-12} & 4.53 \times 10^{-10} & 2.6 \times 10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.51 \times 10^{-12} & 9.87 \times 10^{-10} & 4.82 \times 10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 2.38 \times 10^{-15} & 9.45 \times 10^{-12} & 2.11 \times 10^{-9} & 8.84 \times 10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 8.85 \times 10^{-15} & 2.49 \times 10^{-11} & 4.46 \times 10^{-9} & 1.6 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 3.2 \times 10^{-14} & 6.44 \times 10^{-11} & 9.28 \times 10^{-9} & 2.87 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 1.12 \times 10^{-13} & 1.63 \times 10^{-10} & 1.9 \times 10^{-8} & 5.07 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 3.85 \times 10^{-13} & 4.05 \times 10^{-10} & 3.83 \times 10^{-8} & 8.86 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 1.28 \times 10^{-12} & 9.87 \times 10^{-10} & 7.61 \times 10^{-8} & 1.53 \times 10^{-6} \\ 0 & 0 & 0 & 0 & 0 & 4.15 \times 10^{-12} & 2.35 \times 10^{-9} & 1.49 \times 10^{-7} & 2.61 \times 10^{-6} \end{pmatrix}$$

Results of Binary filtering

$$FANS = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8.85 \times 10^{-15} & 2.49 \times 10^{-11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.2 \times 10^{-14} & 6.44 \times 10^{-11} & 9.28 \times 10^{-9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.12 \times 10^{-13} & 1.63 \times 10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$FANS_{j,i} := F_{j,i} \cdot FINAL_{j,i}$$

Append results to the data file.

$$APPENDPRN("mesh1014.prn") := FANS$$

Maximum integral

$$\max(FANS) = 9.28 \times 10^{-9}$$

8 sieve

$$G = \begin{pmatrix} 0 & 0 & 6.73 \times 10^{-10} & 2.69 \times 10^{-6} & 1.36 \times 10^{-4} & 1.21 \times 10^{-3} & 4.66 \times 10^{-3} & 0.01 & 0.02 \\ 0 & 0 & 5.04 \times 10^{-9} & 8.55 \times 10^{-6} & 2.91 \times 10^{-4} & 2.07 \times 10^{-3} & 7 \times 10^{-3} & 0.02 & 0.03 \\ 0 & 0 & 3.39 \times 10^{-8} & 2.56 \times 10^{-5} & 5.98 \times 10^{-4} & 3.47 \times 10^{-3} & 0.01 & 0.02 & 0.04 \\ 0 & 1.94 \times 10^{-14} & 2.05 \times 10^{-7} & 7.24 \times 10^{-5} & 1.18 \times 10^{-3} & 5.65 \times 10^{-3} & 0.01 & 0.03 & 0.05 \\ 0 & 7.83 \times 10^{-13} & 1.11 \times 10^{-6} & 1.93 \times 10^{-4} & 2.26 \times 10^{-3} & 8.97 \times 10^{-3} & 0.02 & 0.04 & 0.06 \\ 0 & 2.48 \times 10^{-11} & 5.44 \times 10^{-6} & 4.83 \times 10^{-4} & 4.15 \times 10^{-3} & 0.01 & 0.03 & 0.05 & 0.07 \\ 0 & 6.14 \times 10^{-10} & 2.39 \times 10^{-5} & 1.14 \times 10^{-3} & 7.34 \times 10^{-3} & 0.02 & 0.04 & 0.06 & 0.09 \\ 0 & 1.2 \times 10^{-8} & 9.46 \times 10^{-5} & 2.55 \times 10^{-3} & 0.01 & 0.03 & 0.05 & 0.08 & 0.11 \\ 0 & 1.84 \times 10^{-7} & 3.37 \times 10^{-4} & 5.39 \times 10^{-3} & 0.02 & 0.04 & 0.07 & 0.1 & 0.13 \\ 0 & 2.23 \times 10^{-6} & 1.08 \times 10^{-3} & 0.01 & 0.03 & 0.06 & 0.09 & 0.13 & 0.15 \\ 0 & 2.13 \times 10^{-5} & 3.13 \times 10^{-3} & 0.02 & 0.05 & 0.09 & 0.12 & 0.15 & 0.18 \end{pmatrix}$$

$$GANS = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.39 \times 10^{-8} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.94 \times 10^{-14} & 2.05 \times 10^{-7} & 7.24 \times 10^{-5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 7.83 \times 10^{-13} & 1.11 \times 10^{-6} & 1.93 \times 10^{-4} & 2.26 \times 10^{-3} & 0 & 0 & 0 & 0 \\ 0 & 2.48 \times 10^{-11} & 5.44 \times 10^{-6} & 4.83 \times 10^{-4} & 4.15 \times 10^{-3} & 0.01 & 0.03 & 0 & 0 \\ 0 & 6.14 \times 10^{-10} & 2.39 \times 10^{-5} & 1.14 \times 10^{-3} & 7.34 \times 10^{-3} & 0.02 & 0.04 & 0.06 & 0 \\ 0 & 1.2 \times 10^{-8} & 9.46 \times 10^{-5} & 2.55 \times 10^{-3} & 0.01 & 0.03 & 0.05 & 0 & 0 \\ 0 & 1.84 \times 10^{-7} & 3.37 \times 10^{-4} & 5.39 \times 10^{-3} & 0.02 & 0 & 0 & 0 & 0 \\ 0 & 2.23 \times 10^{-6} & 1.08 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter
 $GANS_{j,i} := G_{j,i} \cdot FINAL_{j,i}$

Maximum integral
 $\max(GANS) = 0.06$

10 sieve

$$\mathbf{H} = \begin{pmatrix} 0 & 5.3 \times 10^{-10} & 2.38 \times 10^{-5} & 1.14 \times 10^{-3} & 7.21 \times 10^{-3} & 0.02 & 0.04 & 0.05 & 0.07 \\ 0 & 1.07 \times 10^{-8} & 9.45 \times 10^{-5} & 2.55 \times 10^{-3} & 0.01 & 0.03 & 0.05 & 0.06 & 0.08 \\ 0 & 1.7 \times 10^{-7} & 3.37 \times 10^{-4} & 5.36 \times 10^{-3} & 0.02 & 0.04 & 0.06 & 0.08 & 0.09 \\ 0 & 2.11 \times 10^{-6} & 1.08 \times 10^{-3} & 0.01 & 0.03 & 0.06 & 0.08 & 0.1 & 0.11 \\ 0 & 2.07 \times 10^{-5} & 3.13 \times 10^{-3} & 0.02 & 0.05 & 0.08 & 0.1 & 0.11 & 0.12 \\ 3.02 \times 10^{-13} & 1.59 \times 10^{-4} & 8.19 \times 10^{-3} & 0.04 & 0.07 & 0.1 & 0.12 & 0.13 & 0.14 \\ 2.83 \times 10^{-10} & 9.68 \times 10^{-4} & 0.02 & 0.06 & 0.1 & 0.13 & 0.15 & 0.16 & 0.16 \\ 9.97 \times 10^{-8} & 4.66 \times 10^{-3} & 0.04 & 0.09 & 0.14 & 0.16 & 0.17 & 0.18 & 0.18 \\ 1.33 \times 10^{-5} & 0.02 & 0.08 & 0.14 & 0.18 & 0.2 & 0.2 & 0.2 & 0.19 \\ 6.87 \times 10^{-4} & 0.05 & 0.14 & 0.2 & 0.23 & 0.23 & 0.23 & 0.22 & 0.21 \\ 0.01 & 0.14 & 0.23 & 0.27 & 0.28 & 0.27 & 0.26 & 0.24 & 0.22 \end{pmatrix}$$

$$\mathbf{HANS} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.07 \times 10^{-8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.7 \times 10^{-7} & 3.37 \times 10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.11 \times 10^{-6} & 1.08 \times 10^{-3} & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.07 \times 10^{-5} & 3.13 \times 10^{-3} & 0.02 & 0.05 & 0 & 0 & 0 & 0 \\ 3.02 \times 10^{-13} & 1.59 \times 10^{-4} & 8.19 \times 10^{-3} & 0.04 & 0.07 & 0.1 & 0.12 & 0 & 0 \\ 2.83 \times 10^{-10} & 9.68 \times 10^{-4} & 0.02 & 0.06 & 0.1 & 0.13 & 0.15 & 0.16 & 0 \\ 9.97 \times 10^{-8} & 4.66 \times 10^{-3} & 0.04 & 0.09 & 0.14 & 0.16 & 0.17 & 0 & 0 \\ 1.33 \times 10^{-5} & 0.02 & 0.08 & 0.14 & 0.18 & 0 & 0 & 0 & 0 \\ 6.87 \times 10^{-4} & 0.05 & 0.14 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter
 $\mathbf{HANS}_{j,i} := \mathbf{H}_{j,i} \cdot \mathbf{FINAL}_{j,i}$

Maximum integral
 $\max(\mathbf{HANS}) = 0.18$

14 sieve

$$K = \begin{pmatrix} 0.84 & 0.69 & 0.63 & 0.6 & 0.57 & 0.55 & 0.52 & 0.49 & 0.46 \\ 0.98 & 0.84 & 0.75 & 0.69 & 0.64 & 0.6 & 0.56 & 0.52 & 0.48 \\ 1 & 0.93 & 0.84 & 0.77 & 0.71 & 0.65 & 0.59 & 0.55 & 0.5 \\ 1 & 0.98 & 0.91 & 0.83 & 0.76 & 0.68 & 0.62 & 0.57 & 0.52 \\ 1 & 0.99 & 0.95 & 0.87 & 0.79 & 0.71 & 0.64 & 0.58 & 0.53 \\ 1 & 1 & 0.97 & 0.9 & 0.81 & 0.73 & 0.65 & 0.59 & 0.54 \\ 1 & 1 & 0.97 & 0.9 & 0.81 & 0.73 & 0.65 & 0.59 & 0.54 \\ 1 & 1 & 0.95 & 0.88 & 0.8 & 0.72 & 0.64 & 0.58 & 0.53 \\ 1 & 0.98 & 0.92 & 0.84 & 0.76 & 0.69 & 0.63 & 0.57 & 0.52 \\ 1 & 0.95 & 0.86 & 0.78 & 0.72 & 0.66 & 0.6 & 0.55 & 0.51 \\ 0.99 & 0.86 & 0.77 & 0.71 & 0.66 & 0.61 & 0.57 & 0.52 & 0.49 \end{pmatrix}$$

$$KANS = \begin{pmatrix} 0.84 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.98 & 0.84 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.93 & 0.84 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.98 & 0.91 & 0.83 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.99 & 0.95 & 0.87 & 0.79 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0.97 & 0.9 & 0.81 & 0.73 & 0.65 & 0 & 0 \\ 1 & 1 & 0.97 & 0.9 & 0.81 & 0.73 & 0.65 & 0.59 & 0 \\ 1 & 1 & 0.95 & 0.88 & 0.8 & 0.72 & 0.64 & 0 & 0 \\ 1 & 0.98 & 0.92 & 0.84 & 0.76 & 0 & 0 & 0 & 0 \\ 1 & 0.95 & 0.86 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.99 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$KANS_{j,i} := K_{j,i} \cdot FINAL_{j,i}$$

Maximum integral

$$\max(KANS) = 1$$

30 sieve

$$\mathbf{L} = \begin{pmatrix} 0.16 & 0.31 & 0.37 & 0.4 & 0.42 & 0.43 & 0.44 & 0.43 & 0.43 \\ 0.02 & 0.16 & 0.25 & 0.31 & 0.34 & 0.37 & 0.38 & 0.39 & 0.39 \\ 1.35 \times 10^{-3} & 0.07 & 0.16 & 0.23 & 0.27 & 0.31 & 0.33 & 0.35 & 0.35 \\ 3.24 \times 10^{-5} & 0.02 & 0.09 & 0.16 & 0.21 & 0.25 & 0.28 & 0.3 & 0.32 \\ 3.06 \times 10^{-7} & 6.21 \times 10^{-3} & 0.05 & 0.11 & 0.16 & 0.2 & 0.24 & 0.26 & 0.28 \\ 1.12 \times 10^{-9} & 1.35 \times 10^{-3} & 0.02 & 0.07 & 0.12 & 0.16 & 0.19 & 0.22 & 0.25 \\ 1.57 \times 10^{-12} & 2.33 \times 10^{-4} & 9.82 \times 10^{-3} & 0.04 & 0.08 & 0.12 & 0.16 & 0.19 & 0.21 \\ 0 & 3.17 \times 10^{-5} & 3.83 \times 10^{-3} & 0.02 & 0.05 & 0.09 & 0.13 & 0.16 & 0.18 \\ 0 & 3.4 \times 10^{-6} & 1.35 \times 10^{-3} & 0.01 & 0.04 & 0.07 & 0.1 & 0.13 & 0.16 \\ 0 & 2.87 \times 10^{-7} & 4.29 \times 10^{-4} & 6.21 \times 10^{-3} & 0.02 & 0.05 & 0.08 & 0.11 & 0.13 \\ 0 & 1.91 \times 10^{-8} & 1.23 \times 10^{-4} & 2.98 \times 10^{-3} & 0.01 & 0.03 & 0.06 & 0.08 & 0.11 \end{pmatrix}$$

$$\mathbf{LANS} = \begin{pmatrix} 0.16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0.16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.35 \times 10^{-3} & 0.07 & 0.16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.24 \times 10^{-5} & 0.02 & 0.09 & 0.16 & 0 & 0 & 0 & 0 & 0 \\ 3.06 \times 10^{-7} & 6.21 \times 10^{-3} & 0.05 & 0.11 & 0.16 & 0 & 0 & 0 & 0 \\ 1.12 \times 10^{-9} & 1.35 \times 10^{-3} & 0.02 & 0.07 & 0.12 & 0.16 & 0.19 & 0 & 0 \\ 1.57 \times 10^{-12} & 2.33 \times 10^{-4} & 9.82 \times 10^{-3} & 0.04 & 0.08 & 0.12 & 0.16 & 0.19 & 0 \\ 0 & 3.17 \times 10^{-5} & 3.83 \times 10^{-3} & 0.02 & 0.05 & 0.09 & 0.13 & 0 & 0 \\ 0 & 3.4 \times 10^{-6} & 1.35 \times 10^{-3} & 0.01 & 0.04 & 0 & 0 & 0 & 0 \\ 0 & 2.87 \times 10^{-7} & 4.29 \times 10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$\mathbf{LANS}_{j,i} := \mathbf{L}_{j,i} \cdot \mathbf{FINAL}_{j,i}$$

Maximum integral

$$\max(\mathbf{LANS}) = 0.19$$

Examples of expected retained fractions across NEDU mesh sizes for distributions meeting the overall specifications. The output file of this MathCad program contains 46 such distributions. The Visual Basic program "MeshFit" fits NEDU's data from the sieve analysis to these distributions. The model distribution representing the best fit to the test sample is then compared to the sample distribution. That comparison is quantified by assessments of "Goodness of Fit" using either a simple summed error, or the Chi Square statistical test (see MeshFit documentation).

Definition of one of the 46 model distributions - down selected from 135 candidate distributions.

$$j := 3 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 12.75$$

$$\text{mmMean} = 1.45$$

$$i := 1 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.05$$

Fractions retained in the NEDU sieves

<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>	<u>30 mesh</u>
$\text{GANS}_{j,i} = 0$	$\text{HANS}_{j,i} = 0$	$\text{KANS}_{j,i} = 1$	$\text{LANS}_{j,i} = 1.35 \times 10^{-3}$

Another model distribution

$$j := 11 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 10.23$$

$$\text{mmMean} = 1.85$$

$$i := 1 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.05$$

<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>	<u>30 mesh</u>
$\text{GANS}_{j,i} = 0$	$\text{HANS}_{j,i} = 0.01$	$\text{KANS}_{j,i} = 0.99$	$\text{LANS}_{j,i} = 0$

Yet another model distribution

$$j := 9 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 10.75$$

$$\text{mmMean} = 1.75$$

$$i := 8 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.4$$

$\text{GANS}_{j,i} = 0$	$\text{HANS}_{j,i} = 0$	$\text{KANS}_{j,i} = 0$	$\text{LANS}_{j,i} = 0$
-------------------------	-------------------------	-------------------------	-------------------------

$$j := 6 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 11.66$$

$$\text{mmMean} = 1.6$$

$$i := 4 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.2$$

$\text{GANS}_{j,i} = 4.83 \times 10^{-4}$	$\text{HANS}_{j,i} = 0.04$	$\text{KANS}_{j,i} = 0.9$	$\text{LANS}_{j,i} = 0.07$
---	----------------------------	---------------------------	----------------------------

$$j := 11 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 10.23$$

$$\text{mmMean} = 1.85$$

$$i := 4 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.2$$

$$\text{GANS}_{j,i} = 0 \quad \text{HANS}_{j,i} = 0 \quad \text{KANS}_{j,i} = 0 \quad \text{LANS}_{j,i} = 0$$

$$j := 9 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 10.75$$

$$\text{mmMean} = 1.75$$

$$i := 4 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.2$$

$$\text{GANS}_{j,i} = 5.39 \times 10^{-3} \quad \text{HANS}_{j,i} = 0.14 \quad \text{KANS}_{j,i} = 0.84 \quad \text{LANS}_{j,i} = 0.01$$

APPENDIX B. Normally Distributed 4-8 Mesh Sodalime

Problem: A) Find a Gaussian (normal) distribution that fulfills the requirements of the original specification for 4-8 mesh size CO₂ absorbent (sodalime).

B) Find allowable % retention of absorbent for sieve sizes used in the NEDU T&E laboratory.

ORIGIN= 1

j := 1..15 i := 1..8 Iteration range

Gaussian (normal) distribution parameters

Mean Standard deviation

$\mu_j := 3.0 + .05 \cdot j$ $\sigma_i := i \cdot .05 + 0.60$

Normal distribution:
$$y(x, \mu, \sigma) := \left(\frac{1}{\sqrt{2 \cdot \pi \cdot \sigma}} \right) \cdot e^{-\frac{(x-\mu)^2}{2 \cdot (\sigma)^2}}$$

$$\mu = \begin{pmatrix} 3.05 \\ 3.1 \\ 3.15 \\ 3.2 \\ 3.25 \\ 3.3 \\ 3.35 \\ 3.4 \\ 3.45 \\ 3.5 \\ 3.55 \\ 3.6 \\ 3.65 \\ 3.7 \\ 3.75 \end{pmatrix} \quad \sigma = \begin{pmatrix} 0.65 \\ 0.7 \\ 0.75 \\ 0.8 \\ 0.85 \\ 0.9 \\ 0.95 \\ 1 \end{pmatrix}$$

Integral equations relevant to specifications for 10-14 mesh absorbent

All mesh sizes expressed in diameter of mesh screen (in mm).

<4 mesh

$$A_{j,i} := \int_{4.76}^{20} y(x, \mu_j, \sigma_i) dx$$

4-8 mesh

$$B_{j,i} := \int_{2.36}^{4.76} y(x, \mu_j, \sigma_i) dx$$

8-30 mesh

$$C_{j,i} := \int_{0.6}^{2.36} y(x, \mu_j, \sigma_i) dx$$

The proposed draft sodalime specification calls for no more than 7% larger than 4 mesh, no less than 77% in the 4-8 mesh range, no more than 15% between 8 and 30 mesh.

Out of 120 candidate distributions, we found 57 that met all criteria for the specification, and were therefore classified as model distributions. The manner in which these 57 distributions were identified are indicated on the following pages. Of those 57, 7 are listed on the last two pages.

Fraction of absorbent larger than #4 mesh

Matrix of absorbent fraction greater than #4 mesh (4.76 mm) in size for various distribution j's (means) and i's (standard deviations).

$$A = \begin{pmatrix} 4.26 \times 10^{-3} & 7.29 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0.04 \\ 5.33 \times 10^{-3} & 8.86 \times 10^{-3} & 0.01 & 0.02 & 0.03 & 0.03 & 0.04 & 0.05 \\ 6.63 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0.05 & 0.05 \\ 8.2 \times 10^{-3} & 0.01 & 0.02 & 0.03 & 0.03 & 0.04 & 0.05 & 0.06 \\ 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 \\ 0.01 & 0.02 & 0.03 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 \\ 0.02 & 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 \\ 0.02 & 0.03 & 0.03 & 0.04 & 0.05 & 0.07 & 0.08 & 0.09 \\ 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.1 \\ 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.09 & 0.1 \\ 0.03 & 0.04 & 0.05 & 0.07 & 0.08 & 0.09 & 0.1 & 0.11 \\ 0.04 & 0.05 & 0.06 & 0.07 & 0.09 & 0.1 & 0.11 & 0.12 \\ 0.04 & 0.06 & 0.07 & 0.08 & 0.1 & 0.11 & 0.12 & 0.13 \\ 0.05 & 0.06 & 0.08 & 0.09 & 0.11 & 0.12 & 0.13 & 0.14 \\ 0.06 & 0.07 & 0.09 & 0.1 & 0.12 & 0.13 & 0.14 & 0.16 \end{pmatrix} \quad (B1)$$

Binary Outcome

Specific Specification Criterion

$$XA_{j,i} := \text{if}(A_{j,i} < .07, 1, 0)$$

$$XA = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1 = meets specification
0 = does not meet specification

Each "1" identifies a theoretical distribution that meets the portion of the specification that requires no more than 1% of the absorbent can be larger than #4 mesh.

Fraction of absorbent between #4 and #8 mesh
(4.76 mm to 2.36 mm)

$$\mathbf{B} = \begin{pmatrix} 0.85 & 0.83 & 0.81 & 0.79 & 0.77 & 0.75 & 0.73 & 0.71 \\ 0.87 & 0.85 & 0.82 & 0.8 & 0.78 & 0.76 & 0.74 & 0.72 \\ 0.88 & 0.86 & 0.84 & 0.82 & 0.79 & 0.77 & 0.75 & 0.73 \\ 0.89 & 0.87 & 0.85 & 0.83 & 0.81 & 0.78 & 0.76 & 0.74 \\ 0.9 & 0.88 & 0.86 & 0.84 & 0.81 & 0.79 & 0.77 & 0.75 \\ 0.91 & 0.89 & 0.87 & 0.85 & 0.82 & 0.8 & 0.78 & 0.75 \\ 0.92 & 0.9 & 0.88 & 0.85 & 0.83 & 0.81 & 0.78 & 0.76 \\ 0.93 & 0.91 & 0.88 & 0.86 & 0.83 & 0.81 & 0.79 & 0.76 \\ 0.93 & 0.91 & 0.89 & 0.86 & 0.84 & 0.81 & 0.79 & 0.77 \\ 0.93 & 0.91 & 0.89 & 0.87 & 0.84 & 0.82 & 0.79 & 0.77 \\ 0.94 & 0.91 & 0.89 & 0.87 & 0.84 & 0.82 & 0.79 & 0.77 \\ 0.93 & 0.91 & 0.89 & 0.87 & 0.84 & 0.82 & 0.79 & 0.77 \\ 0.93 & 0.91 & 0.89 & 0.86 & 0.84 & 0.82 & 0.79 & 0.77 \\ 0.93 & 0.91 & 0.88 & 0.86 & 0.84 & 0.81 & 0.79 & 0.77 \\ 0.92 & 0.9 & 0.88 & 0.86 & 0.83 & 0.81 & 0.78 & 0.76 \end{pmatrix} \quad (\text{B2})$$

Binary Outcome

$$\mathbf{XB} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Specific Specification Criterion

$$\mathbf{XB}_{j,i} := \text{if}(\mathbf{B}_{j,i} > 0.77, 1, 0)$$

Fraction of absorbent between #8 and #30 mesh
(2.36 mm to 0.6 mm)

$$C = \begin{pmatrix} 0.14 & 0.16 & 0.18 & 0.19 & 0.21 & 0.22 & 0.23 & 0.24 \\ 0.13 & 0.15 & 0.16 & 0.18 & 0.19 & 0.2 & 0.21 & 0.22 \\ 0.11 & 0.13 & 0.15 & 0.16 & 0.17 & 0.19 & 0.2 & 0.21 \\ 0.1 & 0.11 & 0.13 & 0.15 & 0.16 & 0.17 & 0.19 & 0.2 \\ 0.09 & 0.1 & 0.12 & 0.13 & 0.15 & 0.16 & 0.17 & 0.18 \\ 0.07 & 0.09 & 0.1 & 0.12 & 0.13 & 0.15 & 0.16 & 0.17 \\ 0.06 & 0.08 & 0.09 & 0.11 & 0.12 & 0.13 & 0.15 & 0.16 \\ 0.05 & 0.07 & 0.08 & 0.1 & 0.11 & 0.12 & 0.14 & 0.15 \\ 0.05 & 0.06 & 0.07 & 0.09 & 0.1 & 0.11 & 0.12 & 0.14 \\ 0.04 & 0.05 & 0.06 & 0.08 & 0.09 & 0.1 & 0.11 & 0.13 \\ 0.03 & 0.04 & 0.06 & 0.07 & 0.08 & 0.09 & 0.1 & 0.12 \\ 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.1 & 0.11 \\ 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.08 & 0.09 & 0.1 \\ 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.09 \\ 0.02 & 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 \end{pmatrix} \quad (B3)$$

Binary Outcome

$$XC = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Specific Specification Criterion

$$XC_{j,i} := \text{if}(C_{j,i} < 0.15, 1, 0)$$

$$\text{XTOT} = \begin{pmatrix} 3 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 2 & 2 & 1 & 1 & 1 \\ 3 & 3 & 3 & 2 & 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 3 & 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 3 & 3 & 2 & 1 & 1 \\ 3 & 3 & 3 & 3 & 3 & 3 & 2 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 & 2 & 1 \\ 3 & 3 & 3 & 3 & 3 & 2 & 2 & 1 \\ 3 & 3 & 3 & 3 & 3 & 2 & 2 & 1 \\ 3 & 3 & 3 & 3 & 2 & 2 & 2 & 1 \\ 3 & 3 & 3 & 2 & 2 & 2 & 2 & 1 \\ 3 & 3 & 3 & 2 & 2 & 2 & 2 & 1 \\ 3 & 3 & 2 & 2 & 2 & 2 & 2 & 1 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \end{pmatrix}$$

Sum of binary outcomes

$$\text{XTOT}_{j,i} := (\text{XA}_{j,i} + \text{XB}_{j,i} + \text{XC}_{j,i})$$

$$\text{FINAL}_{j,i} := \text{if}(\text{XTOT}_{j,i} = 3, 1, 0)$$

1's identify distributions that simultaneously meet all three specification requirements. That is, these are good model distributions.

This matrix is used below as a mask or filter to select valid values for retained fractions with NEDU's mesh screens.

$$\text{FINAL} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The following definite integrals define the fraction of the normal distribution falling within the confines of the six sieves available at NEDU.

3.5 sieve

5 sieve

8 sieve

$$E_{j,i} := \int_{5.6}^{20} y(x, \mu_j, \sigma_i) dx$$

$$F_{j,i} := \int_{4.0}^{5.6} y(x, \mu_j, \sigma_i) dx$$

$$G_{j,i} := \int_{2.36}^{4.0} y(x, \mu_j, \sigma_i) dx$$

10 sieve

14 sieve

30 sieve

$$H_{j,i} := \int_{2.06}^{2.36} y(x, \mu_j, \sigma_i) dx$$

$$K_{j,i} := \int_{1.4}^{2.06} y(x, \mu_j, \sigma_i) dx$$

$$L_{j,i} := \int_{.6}^{1.4} y(x, \mu_j, \sigma_i) dx$$

3.5 sieve

Fractions retained by a #3.5 mesh sieve.

$$E = \begin{pmatrix} 4.79 \times 10^{-5} & 1.42 \times 10^{-4} & 3.45 \times 10^{-4} & 7.25 \times 10^{-4} & 1.35 \times 10^{-3} & 2.3 \times 10^{-3} & 3.63 \times 10^{-3} & 5.39 \times 10^{-3} \\ 6.53 \times 10^{-5} & 1.86 \times 10^{-4} & 4.38 \times 10^{-4} & 8.97 \times 10^{-4} & 1.63 \times 10^{-3} & 2.74 \times 10^{-3} & 4.25 \times 10^{-3} & 6.21 \times 10^{-3} \\ 8.85 \times 10^{-5} & 2.42 \times 10^{-4} & 5.55 \times 10^{-4} & 1.1 \times 10^{-3} & 1.97 \times 10^{-3} & 3.24 \times 10^{-3} & 4.95 \times 10^{-3} & 7.14 \times 10^{-3} \\ 1.19 \times 10^{-4} & 3.15 \times 10^{-4} & 6.99 \times 10^{-4} & 1.35 \times 10^{-3} & 2.37 \times 10^{-3} & 3.83 \times 10^{-3} & 5.76 \times 10^{-3} & 8.2 \times 10^{-3} \\ 1.6 \times 10^{-4} & 4.07 \times 10^{-4} & 8.64 \times 10^{-4} & 1.65 \times 10^{-3} & 2.85 \times 10^{-3} & 4.51 \times 10^{-3} & 6.69 \times 10^{-3} & 9.39 \times 10^{-3} \\ 2.14 \times 10^{-4} & 5.24 \times 10^{-4} & 1.08 \times 10^{-3} & 2.02 \times 10^{-3} & 3.41 \times 10^{-3} & 5.3 \times 10^{-3} & 7.74 \times 10^{-3} & 0.01 \\ 2.83 \times 10^{-4} & 6.71 \times 10^{-4} & 1.35 \times 10^{-3} & 2.46 \times 10^{-3} & 4.06 \times 10^{-3} & 6.21 \times 10^{-3} & 8.93 \times 10^{-3} & 0.01 \\ 3.74 \times 10^{-4} & 8.36 \times 10^{-4} & 1.68 \times 10^{-3} & 2.98 \times 10^{-3} & 4.82 \times 10^{-3} & 7.25 \times 10^{-3} & 0.01 & 0.01 \\ 4.91 \times 10^{-4} & 1.07 \times 10^{-3} & 2.07 \times 10^{-3} & 3.6 \times 10^{-3} & 5.71 \times 10^{-3} & 8.45 \times 10^{-3} & 0.01 & 0.02 \\ 6.41 \times 10^{-4} & 1.35 \times 10^{-3} & 2.55 \times 10^{-3} & 4.33 \times 10^{-3} & 6.74 \times 10^{-3} & 9.82 \times 10^{-3} & 0.01 & 0.02 \\ 8.06 \times 10^{-4} & 1.7 \times 10^{-3} & 3.13 \times 10^{-3} & 5.2 \times 10^{-3} & 7.94 \times 10^{-3} & 0.01 & 0.02 & 0.02 \\ 1.05 \times 10^{-3} & 2.14 \times 10^{-3} & 3.83 \times 10^{-3} & 6.21 \times 10^{-3} & 9.31 \times 10^{-3} & 0.01 & 0.02 & 0.02 \\ 1.35 \times 10^{-3} & 2.67 \times 10^{-3} & 4.66 \times 10^{-3} & 7.39 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 \\ 1.73 \times 10^{-3} & 3.32 \times 10^{-3} & 5.65 \times 10^{-3} & 8.77 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 \\ 2.21 \times 10^{-3} & 4.11 \times 10^{-3} & 6.82 \times 10^{-3} & 0.01 & 0.01 & 0.02 & 0.03 & 0.03 \end{pmatrix}$$

Apply binary filter

$$EANS_{j,i} := E_{j,i} \cdot FINAL_{j,i}$$

$$EANS = \begin{pmatrix} 4.79 \times 10^{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6.53 \times 10^{-5} & 1.86 \times 10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 8.85 \times 10^{-5} & 2.42 \times 10^{-4} & 5.55 \times 10^{-4} & 0 & 0 & 0 & 0 & 0 \\ 1.19 \times 10^{-4} & 3.15 \times 10^{-4} & 6.99 \times 10^{-4} & 1.35 \times 10^{-3} & 0 & 0 & 0 & 0 \\ 1.6 \times 10^{-4} & 4.07 \times 10^{-4} & 8.64 \times 10^{-4} & 1.65 \times 10^{-3} & 2.85 \times 10^{-3} & 0 & 0 & 0 \\ 2.14 \times 10^{-4} & 5.24 \times 10^{-4} & 1.08 \times 10^{-3} & 2.02 \times 10^{-3} & 3.41 \times 10^{-3} & 5.3 \times 10^{-3} & 0 & 0 \\ 2.83 \times 10^{-4} & 6.71 \times 10^{-4} & 1.35 \times 10^{-3} & 2.46 \times 10^{-3} & 4.06 \times 10^{-3} & 6.21 \times 10^{-3} & 8.93 \times 10^{-3} & 0 \\ 3.74 \times 10^{-4} & 8.36 \times 10^{-4} & 1.68 \times 10^{-3} & 2.98 \times 10^{-3} & 4.82 \times 10^{-3} & 7.25 \times 10^{-3} & 0 & 0 \\ 4.91 \times 10^{-4} & 1.07 \times 10^{-3} & 2.07 \times 10^{-3} & 3.6 \times 10^{-3} & 5.71 \times 10^{-3} & 0 & 0 & 0 \\ 6.41 \times 10^{-4} & 1.35 \times 10^{-3} & 2.55 \times 10^{-3} & 4.33 \times 10^{-3} & 6.74 \times 10^{-3} & 0 & 0 & 0 \\ 8.06 \times 10^{-4} & 1.7 \times 10^{-3} & 3.13 \times 10^{-3} & 5.2 \times 10^{-3} & 0 & 0 & 0 & 0 \\ 1.05 \times 10^{-3} & 2.14 \times 10^{-3} & 3.83 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 \\ 1.35 \times 10^{-3} & 2.67 \times 10^{-3} & 4.66 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 \\ 1.73 \times 10^{-3} & 3.32 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.21 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5 sieve

$$F = \begin{pmatrix} 0.07 & 0.09 & 0.1 & 0.12 & 0.13 & 0.14 & 0.16 & 0.17 \\ 0.08 & 0.1 & 0.11 & 0.13 & 0.14 & 0.16 & 0.17 & 0.18 \\ 0.1 & 0.11 & 0.13 & 0.14 & 0.16 & 0.17 & 0.18 & 0.19 \\ 0.11 & 0.13 & 0.14 & 0.16 & 0.17 & 0.18 & 0.19 & 0.2 \\ 0.12 & 0.14 & 0.16 & 0.17 & 0.19 & 0.2 & 0.21 & 0.22 \\ 0.14 & 0.16 & 0.17 & 0.19 & 0.2 & 0.21 & 0.22 & 0.23 \\ 0.16 & 0.18 & 0.19 & 0.21 & 0.22 & 0.23 & 0.24 & 0.25 \\ 0.18 & 0.19 & 0.21 & 0.22 & 0.24 & 0.25 & 0.25 & 0.26 \\ 0.2 & 0.21 & 0.23 & 0.24 & 0.25 & 0.26 & 0.27 & 0.28 \\ 0.22 & 0.24 & 0.25 & 0.26 & 0.27 & 0.28 & 0.29 & 0.29 \\ 0.24 & 0.26 & 0.27 & 0.28 & 0.29 & 0.3 & 0.3 & 0.31 \\ 0.27 & 0.28 & 0.29 & 0.3 & 0.31 & 0.32 & 0.32 & 0.32 \\ 0.29 & 0.31 & 0.32 & 0.32 & 0.33 & 0.33 & 0.34 & 0.34 \\ 0.32 & 0.33 & 0.34 & 0.35 & 0.35 & 0.35 & 0.35 & 0.35 \\ 0.35 & 0.36 & 0.36 & 0.37 & 0.37 & 0.37 & 0.37 & 0.37 \end{pmatrix}$$

Results of Binary filtering

$$FANS = \begin{pmatrix} 0.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.08 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.11 & 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0.11 & 0.13 & 0.14 & 0.16 & 0 & 0 & 0 & 0 \\ 0.12 & 0.14 & 0.16 & 0.17 & 0.19 & 0 & 0 & 0 \\ 0.14 & 0.16 & 0.17 & 0.19 & 0.2 & 0.21 & 0 & 0 \\ 0.16 & 0.18 & 0.19 & 0.21 & 0.22 & 0.23 & 0.24 & 0 \\ 0.18 & 0.19 & 0.21 & 0.22 & 0.24 & 0.25 & 0 & 0 \\ 0.2 & 0.21 & 0.23 & 0.24 & 0.25 & 0 & 0 & 0 \\ 0.22 & 0.24 & 0.25 & 0.26 & 0.27 & 0 & 0 & 0 \\ 0.24 & 0.26 & 0.27 & 0.28 & 0 & 0 & 0 & 0 \\ 0.27 & 0.28 & 0.29 & 0 & 0 & 0 & 0 & 0 \\ 0.29 & 0.31 & 0.32 & 0 & 0 & 0 & 0 & 0 \\ 0.32 & 0.33 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.35 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$FANS_{j,i} := F_{j,i} \cdot FINAL_{j,i}$$

Maximum integral

$$\max(FANS) = 0.35$$

8 sieve

$$G = \begin{pmatrix} 0.78 & 0.75 & 0.72 & 0.69 & 0.66 & 0.63 & 0.61 & 0.58 \\ 0.79 & 0.76 & 0.72 & 0.69 & 0.66 & 0.64 & 0.61 & 0.59 \\ 0.79 & 0.76 & 0.73 & 0.69 & 0.67 & 0.64 & 0.61 & 0.59 \\ 0.79 & 0.76 & 0.73 & 0.69 & 0.67 & 0.64 & 0.61 & 0.59 \\ 0.79 & 0.76 & 0.72 & 0.69 & 0.66 & 0.64 & 0.61 & 0.59 \\ 0.79 & 0.75 & 0.72 & 0.69 & 0.66 & 0.63 & 0.61 & 0.58 \\ 0.78 & 0.74 & 0.71 & 0.68 & 0.66 & 0.63 & 0.6 & 0.58 \\ 0.77 & 0.74 & 0.71 & 0.68 & 0.65 & 0.62 & 0.6 & 0.58 \\ 0.75 & 0.72 & 0.7 & 0.67 & 0.64 & 0.62 & 0.59 & 0.57 \\ 0.74 & 0.71 & 0.68 & 0.66 & 0.63 & 0.61 & 0.59 & 0.56 \\ 0.72 & 0.7 & 0.67 & 0.64 & 0.62 & 0.6 & 0.58 & 0.56 \\ 0.7 & 0.68 & 0.65 & 0.63 & 0.61 & 0.59 & 0.57 & 0.55 \\ 0.68 & 0.66 & 0.64 & 0.62 & 0.6 & 0.58 & 0.56 & 0.54 \\ 0.66 & 0.64 & 0.62 & 0.6 & 0.58 & 0.56 & 0.54 & 0.53 \\ 0.63 & 0.62 & 0.6 & 0.58 & 0.56 & 0.55 & 0.53 & 0.52 \end{pmatrix}$$

$$GANS = \begin{pmatrix} 0.78 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.79 & 0.76 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.79 & 0.76 & 0.73 & 0 & 0 & 0 & 0 & 0 \\ 0.79 & 0.76 & 0.73 & 0.69 & 0 & 0 & 0 & 0 \\ 0.79 & 0.76 & 0.72 & 0.69 & 0.66 & 0 & 0 & 0 \\ 0.79 & 0.75 & 0.72 & 0.69 & 0.66 & 0.63 & 0 & 0 \\ 0.78 & 0.74 & 0.71 & 0.68 & 0.66 & 0.63 & 0.6 & 0 \\ 0.77 & 0.74 & 0.71 & 0.68 & 0.65 & 0.62 & 0 & 0 \\ 0.75 & 0.72 & 0.7 & 0.67 & 0.64 & 0 & 0 & 0 \\ 0.74 & 0.71 & 0.68 & 0.66 & 0.63 & 0 & 0 & 0 \\ 0.72 & 0.7 & 0.67 & 0.64 & 0 & 0 & 0 & 0 \\ 0.7 & 0.68 & 0.65 & 0 & 0 & 0 & 0 & 0 \\ 0.68 & 0.66 & 0.64 & 0 & 0 & 0 & 0 & 0 \\ 0.66 & 0.64 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.63 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$GANS_{j,i} := G_{j,i} \cdot FINAL_{j,i}$$

Maximum integral

$$\max(GANS) = 0.79$$

10 sieve

$$\mathbf{H} = \begin{pmatrix} 0.08 & 0.08 & 0.09 & 0.09 & 0.09 & 0.09 & 0.09 & 0.08 \\ 0.07 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 \\ 0.07 & 0.07 & 0.07 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 \\ 0.06 & 0.06 & 0.07 & 0.07 & 0.07 & 0.07 & 0.07 & 0.07 \\ 0.05 & 0.06 & 0.06 & 0.06 & 0.07 & 0.07 & 0.07 & 0.07 \\ 0.05 & 0.05 & 0.06 & 0.06 & 0.06 & 0.06 & 0.07 & 0.07 \\ 0.04 & 0.05 & 0.05 & 0.05 & 0.06 & 0.06 & 0.06 & 0.06 \\ 0.04 & 0.04 & 0.05 & 0.05 & 0.05 & 0.06 & 0.06 & 0.06 \\ 0.03 & 0.04 & 0.04 & 0.05 & 0.05 & 0.05 & 0.05 & 0.06 \\ 0.03 & 0.03 & 0.04 & 0.04 & 0.04 & 0.05 & 0.05 & 0.05 \\ 0.02 & 0.03 & 0.03 & 0.04 & 0.04 & 0.04 & 0.05 & 0.05 \\ 0.02 & 0.02 & 0.03 & 0.03 & 0.04 & 0.04 & 0.04 & 0.05 \\ 0.02 & 0.02 & 0.03 & 0.03 & 0.03 & 0.04 & 0.04 & 0.04 \\ 0.01 & 0.02 & 0.02 & 0.03 & 0.03 & 0.03 & 0.04 & 0.04 \\ 0.01 & 0.02 & 0.02 & 0.02 & 0.03 & 0.03 & 0.03 & 0.04 \end{pmatrix}$$

$$\mathbf{HANS} = \begin{pmatrix} 0.08 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.07 & 0.08 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.07 & 0.07 & 0.07 & 0 & 0 & 0 & 0 & 0 \\ 0.06 & 0.06 & 0.07 & 0.07 & 0 & 0 & 0 & 0 \\ 0.05 & 0.06 & 0.06 & 0.06 & 0.07 & 0 & 0 & 0 \\ 0.05 & 0.05 & 0.06 & 0.06 & 0.06 & 0.06 & 0 & 0 \\ 0.04 & 0.05 & 0.05 & 0.05 & 0.06 & 0.06 & 0.06 & 0 \\ 0.04 & 0.04 & 0.05 & 0.05 & 0.05 & 0.06 & 0 & 0 \\ 0.03 & 0.04 & 0.04 & 0.05 & 0.05 & 0 & 0 & 0 \\ 0.03 & 0.03 & 0.04 & 0.04 & 0.04 & 0 & 0 & 0 \\ 0.02 & 0.03 & 0.03 & 0.04 & 0 & 0 & 0 & 0 \\ 0.02 & 0.02 & 0.03 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0.02 & 0.03 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$\mathbf{HANS}_{j,i} := \mathbf{H}_{j,i} \cdot \mathbf{FINAL}_{j,i}$$

Maximum integral

$$\max(\mathbf{HANS}) = 0.08$$

14 sieve

$$K = \begin{pmatrix} 0.06 & 0.07 & 0.08 & 0.09 & 0.1 & 0.1 & 0.11 & 0.11 \\ 0.05 & 0.06 & 0.07 & 0.08 & 0.09 & 0.09 & 0.1 & 0.1 \\ 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.09 & 0.09 & 0.1 \\ 0.04 & 0.05 & 0.06 & 0.06 & 0.07 & 0.08 & 0.09 & 0.09 \\ 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.07 & 0.08 & 0.08 \\ 0.03 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.07 & 0.08 \\ 0.02 & 0.03 & 0.04 & 0.05 & 0.05 & 0.06 & 0.07 & 0.07 \\ 0.02 & 0.03 & 0.03 & 0.04 & 0.05 & 0.06 & 0.06 & 0.07 \\ 0.02 & 0.02 & 0.03 & 0.04 & 0.04 & 0.05 & 0.06 & 0.06 \\ 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0.04 & 0.05 & 0.06 \\ 0.01 & 0.02 & 0.02 & 0.03 & 0.03 & 0.04 & 0.05 & 0.05 \\ 8.56 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0.04 & 0.05 \\ 6.95 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.03 & 0.04 & 0.04 \\ 5.62 \times 10^{-3} & 9.06 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.03 & 0.04 \\ 4.51 \times 10^{-3} & 7.49 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.03 & 0.04 \end{pmatrix}$$

$$KANS = \begin{pmatrix} 0.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0.06 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.04 & 0.05 & 0.06 & 0 & 0 & 0 & 0 & 0 \\ 0.04 & 0.05 & 0.06 & 0.06 & 0 & 0 & 0 & 0 \\ 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0 & 0 & 0 \\ 0.03 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0 & 0 \\ 0.02 & 0.03 & 0.04 & 0.05 & 0.05 & 0.06 & 0.07 & 0 \\ 0.02 & 0.03 & 0.03 & 0.04 & 0.05 & 0.06 & 0 & 0 \\ 0.02 & 0.02 & 0.03 & 0.04 & 0.04 & 0 & 0 & 0 \\ 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0 & 0 & 0 \\ 0.01 & 0.02 & 0.02 & 0.03 & 0 & 0 & 0 & 0 \\ 8.56 \times 10^{-3} & 0.01 & 0.02 & 0 & 0 & 0 & 0 & 0 \\ 6.95 \times 10^{-3} & 0.01 & 0.02 & 0 & 0 & 0 & 0 & 0 \\ 5.62 \times 10^{-3} & 9.06 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 4.51 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$KANS_{j,i} := K_{j,i} \cdot FINAL_{j,i}$$

Maximum integral

$$\max(KANS) = 0.07$$

30 sieve

$$L = \begin{pmatrix} 5.49 \times 10^{-3} & 8.98 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.04 & 0.04 \\ 4.4 \times 10^{-3} & 7.4 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.03 & 0.04 \\ 3.5 \times 10^{-3} & 6.07 \times 10^{-3} & 9.48 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.03 \\ 2.78 \times 10^{-3} & 4.96 \times 10^{-3} & 7.93 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 & 0.03 \\ 2.19 \times 10^{-3} & 4.03 \times 10^{-3} & 6.61 \times 10^{-3} & 9.91 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 \\ 1.72 \times 10^{-3} & 3.26 \times 10^{-3} & 5.49 \times 10^{-3} & 8.41 \times 10^{-3} & 0.01 & 0.02 & 0.02 & 0.03 \\ 1.34 \times 10^{-3} & 2.63 \times 10^{-3} & 4.54 \times 10^{-3} & 7.1 \times 10^{-3} & 0.01 & 0.01 & 0.02 & 0.02 \\ 1.04 \times 10^{-3} & 2.11 \times 10^{-3} & 3.74 \times 10^{-3} & 5.98 \times 10^{-3} & 8.82 \times 10^{-3} & 0.01 & 0.02 & 0.02 \\ 8 \times 10^{-4} & 1.68 \times 10^{-3} & 3.06 \times 10^{-3} & 5.01 \times 10^{-3} & 7.54 \times 10^{-3} & 0.01 & 0.01 & 0.02 \\ 6.13 \times 10^{-4} & 1.33 \times 10^{-3} & 2.5 \times 10^{-3} & 4.19 \times 10^{-3} & 6.42 \times 10^{-3} & 9.18 \times 10^{-3} & 0.01 & 0.02 \\ 4.68 \times 10^{-4} & 1.05 \times 10^{-3} & 2.03 \times 10^{-3} & 3.49 \times 10^{-3} & 5.45 \times 10^{-3} & 7.93 \times 10^{-3} & 0.01 & 0.01 \\ 3.54 \times 10^{-4} & 8.27 \times 10^{-4} & 1.65 \times 10^{-3} & 2.89 \times 10^{-3} & 4.62 \times 10^{-3} & 6.82 \times 10^{-3} & 9.49 \times 10^{-3} & 0.01 \\ 2.67 \times 10^{-4} & 6.47 \times 10^{-4} & 1.33 \times 10^{-3} & 2.39 \times 10^{-3} & 3.89 \times 10^{-3} & 5.86 \times 10^{-3} & 8.27 \times 10^{-3} & 0.01 \\ 2 \times 10^{-4} & 5.04 \times 10^{-4} & 1.06 \times 10^{-3} & 1.97 \times 10^{-3} & 3.27 \times 10^{-3} & 5.01 \times 10^{-3} & 7.19 \times 10^{-3} & 9.76 \times 10^{-3} \\ 1.49 \times 10^{-4} & 3.9 \times 10^{-4} & 8.51 \times 10^{-4} & 1.61 \times 10^{-3} & 2.74 \times 10^{-3} & 4.28 \times 10^{-3} & 6.23 \times 10^{-3} & 8.57 \times 10^{-3} \end{pmatrix}$$

Apply binary filter

Maximum integral

$$\text{LANS}_{j,i} := L_{j,i} \cdot \text{FINAL}_{j,i}$$

$$\max(\text{LANS}) = 0.02$$

$$\text{LANS} = \begin{pmatrix} 5.49 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4.4 \times 10^{-3} & 7.4 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.5 \times 10^{-3} & 6.07 \times 10^{-3} & 9.48 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 \\ 2.78 \times 10^{-3} & 4.96 \times 10^{-3} & 7.93 \times 10^{-3} & 0.01 & 0 & 0 & 0 & 0 \\ 2.19 \times 10^{-3} & 4.03 \times 10^{-3} & 6.61 \times 10^{-3} & 9.91 \times 10^{-3} & 0.01 & 0 & 0 & 0 \\ 1.72 \times 10^{-3} & 3.26 \times 10^{-3} & 5.49 \times 10^{-3} & 8.41 \times 10^{-3} & 0.01 & 0.02 & 0 & 0 \\ 1.34 \times 10^{-3} & 2.63 \times 10^{-3} & 4.54 \times 10^{-3} & 7.1 \times 10^{-3} & 0.01 & 0.01 & 0.02 & 0 \\ 1.04 \times 10^{-3} & 2.11 \times 10^{-3} & 3.74 \times 10^{-3} & 5.98 \times 10^{-3} & 8.82 \times 10^{-3} & 0.01 & 0 & 0 \\ 8 \times 10^{-4} & 1.68 \times 10^{-3} & 3.06 \times 10^{-3} & 5.01 \times 10^{-3} & 7.54 \times 10^{-3} & 0 & 0 & 0 \\ 6.13 \times 10^{-4} & 1.33 \times 10^{-3} & 2.5 \times 10^{-3} & 4.19 \times 10^{-3} & 6.42 \times 10^{-3} & 0 & 0 & 0 \\ 4.68 \times 10^{-4} & 1.05 \times 10^{-3} & 2.03 \times 10^{-3} & 3.49 \times 10^{-3} & 0 & 0 & 0 & 0 \\ 3.54 \times 10^{-4} & 8.27 \times 10^{-4} & 1.65 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 \\ 2.67 \times 10^{-4} & 6.47 \times 10^{-4} & 1.33 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 \\ 2 \times 10^{-4} & 5.04 \times 10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.49 \times 10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Examples of expected retained fractions across NEDU mesh sizes for distributions meeting the overall specifications. The output file of this MathCad program contains 57 such distributions. The Visual Basic program "MeshFit" fits NEDU's data from the sieve analysis to these distributions. The model distribution representing the best fit to the test sample is then compared to the sample distribution. That comparison is quantified by assessments of "Goodness of Fit" using either a simple summed error, or the Chi Square statistical test (see MeshFit documentation).

Definition of seven of the 57 model distributions - down selected from 120 candidate distributions.

Example B1

$$j := 1 \quad \text{mmMean} := 2.95 + (0.065 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 6.7$$

$$\text{mmMean} = 3.02$$

$$i := 1 \quad \text{mmSD} := 0.05 \cdot i + 0.60 \quad \text{mmSD} = 0.65$$

Fractions retained in the NEDU sieves

<u>3.5 mesh</u>	<u>5 mesh</u>	<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>
$\text{EANS}_{j,i} = 4.79 \times 10^{-5}$	$\text{FANS}_{j,i} = 0.07$	$\text{GANS}_{j,i} = 0.78$	$\text{HANS}_{j,i} = 0.08$	$\text{KANS}_{j,i} = 0.06$

Example B2

$$j := 7 \quad \text{mmMean} := 2.95 + (0.065 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 6.06$$

$$\text{mmMean} = 3.41$$

$$i := 1 \quad \text{mmSD} := 0.05 \cdot i + 0.60 \quad \text{mmSD} = 0.65$$

<u>3.5 mesh</u>	<u>5 mesh</u>	<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>
$\text{EANS}_{j,i} = 2.83 \times 10^{-4}$	$\text{FANS}_{j,i} = 0.16$	$\text{GANS}_{j,i} = 0.78$	$\text{HANS}_{j,i} = 0.04$	$\text{KANS}_{j,i} = 0.02$

Example B3

$$j := 15 \quad \text{mmMean} := 2.95 + (0.065 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 5.4$$

$$\text{mmMean} = 3.93$$

$$i := 1 \quad \text{mmSD} := 0.05 \cdot i + 0.60 \quad \text{mmSD} = 0.65$$

<u>3.5 mesh</u>	<u>5 mesh</u>	<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>
$\text{EANS}_{j,i} = 2.21 \times 10^{-3}$	$\text{FANS}_{j,i} = 0.35$	$\text{GANS}_{j,i} = 0.63$	$\text{HANS}_{j,i} = 0.01$	$\text{KANS}_{j,i} = 4.51 \times 10^{-3}$

Example B4

$$\begin{array}{llllll} j := 4 & \text{mmMean} := 2.95 + (0.065 \cdot j) & \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}}\right) & \text{MeshMean} = 6.36 \\ & \text{mmMean} = 3.21 & & & & \\ i := 4 & \text{mmSD} := 0.05 \cdot i + 0.60 & \text{mmSD} = 0.8 & & & \\ & \text{3.5 mesh} & \text{5 mesh} & \text{8 mesh} & \text{10 mesh} & \text{14 mesh} \\ \text{EANS}_{j,i} = 1.35 \times 10^{-3} & \text{FANS}_{j,i} = 0.16 & \text{GANS}_{j,i} = 0.69 & \text{HANS}_{j,i} = 0.07 & \text{KANS}_{j,i} = 0.06 \end{array}$$

Example B5

$$\begin{array}{llllll} j := 7 & \text{mmMean} := 2.95 + (0.065 \cdot j) & \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}}\right) & \text{MeshMean} = 6.06 \\ & \text{mmMean} = 3.41 & & & & \\ i := 4 & \text{mmSD} := 0.05 \cdot i + 0.60 & \text{mmSD} = 0.8 & & & \\ & \text{3.5 mesh} & \text{5 mesh} & \text{8 mesh} & \text{10 mesh} & \text{14 mesh} \\ \text{EANS}_{j,i} = 2.46 \times 10^{-3} & \text{FANS}_{j,i} = 0.21 & \text{GANS}_{j,i} = 0.68 & \text{HANS}_{j,i} = 0.05 & \text{KANS}_{j,i} = 0.05 \end{array}$$

Example B6

$$\begin{array}{llllll} j := 11 & \text{mmMean} := 2.95 + (0.065 \cdot j) & \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}}\right) & \text{MeshMean} = 5.71 \\ & \text{mmMean} = 3.67 & & & & \\ i := 4 & \text{mmSD} := 0.05 \cdot i + 0.60 & \text{mmSD} = 0.8 & & & \\ & \text{3.5 mesh} & \text{5 mesh} & \text{8 mesh} & \text{10 mesh} & \text{14 mesh} \\ \text{EANS}_{j,i} = 5.2 \times 10^{-3} & \text{FANS}_{j,i} = 0.28 & \text{GANS}_{j,i} = 0.64 & \text{HANS}_{j,i} = 0.04 & \text{KANS}_{j,i} = 0.03 \end{array}$$

Example B7

$$\begin{array}{llllll} j := 7 & \text{mmMean} := 2.95 + (0.065 \cdot j) & \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}}\right) & \text{MeshMean} = 6.06 \\ & \text{mmMean} = 3.41 & & & & \\ i := 7 & \text{mmSD} := 0.05 \cdot i + 0.60 & \text{mmSD} = 0.95 & & & \\ & \text{3.5 mesh} & \text{5 mesh} & \text{8 mesh} & \text{10 mesh} & \text{14 mesh} \\ \text{EANS}_{j,i} = 8.93 \times 10^{-3} & \text{FANS}_{j,i} = 0.24 & \text{GANS}_{j,i} = 0.6 & \text{HANS}_{j,i} = 0.06 & \text{KANS}_{j,i} = 0.07 \end{array}$$

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APPENDIX C. LogNormally Distributed 10-14 Mesh Sodalime

Problem: A) Find a Log-Gaussian (log-normal) distribution that fulfills the requirements of the original specification for 8-12 (10-14 US) mesh size CO₂ absorbent (sodalime).

B) Find allowable % retention of absorbent for sieve sizes used in the NEDU T&E laboratory.

ORIGIN= 1

j := 1..12 i := 1..8 Iteration range

Log-Gaussian (log-normal) distribution parameters

Mean Standard deviation

$\mu_j := .32 + .03 \cdot j$ $\sigma_i := i \cdot 0.025 - .02$

Log-Normal distribution:

$$y(x, \mu, \sigma) := \left(\frac{1}{\sqrt{2 \cdot \pi \cdot \sigma}} \right) \cdot e^{-\frac{(\ln(x) - \mu)^2}{2 \cdot (\sigma)^2}}$$

$$\sigma = \begin{pmatrix} 5 \times 10^{-3} \\ 0.03 \\ 0.06 \\ 0.08 \\ 0.11 \\ 0.13 \\ 0.16 \\ 0.18 \end{pmatrix} \quad \mu = \begin{pmatrix} 0.35 \\ 0.38 \\ 0.41 \\ 0.44 \\ 0.47 \\ 0.5 \\ 0.53 \\ 0.56 \\ 0.59 \\ 0.62 \\ 0.65 \\ 0.68 \end{pmatrix}$$

Integral equations relevant to specifications for 10-14 mesh absorbent

All mesh sizes expressed in diameter of mesh screen (in mm).

<7 mesh

$$A_{j,i} := \int_{2.80}^{20} y(x, \mu_j, \sigma_i) dx$$

7-10 mesh

$$B_{j,i} := \int_{2.0}^{2.8} y(x, \mu_j, \sigma_i) dx$$

10-14 mesh

$$C_{j,i} := \int_{1.4}^{2.0} y(x, \mu_j, \sigma_i) dx$$

14-40 mesh

$$D_{j,i} := \int_{0.425}^{1.4} y(x, \mu_j, \sigma_i) dx$$

The proposed draft sodalime specification calls for no more than 1% larger than 7 mesh, no more than 30% in the 7-10 mesh size range, no less than 48% in the 10-14 range, no more than 20% between 14 and 40 mesh.

Out of 135 candidate distributions, we found 46 that met all criteria for the specification, and were therefore classified as model distributions. The manner in which these 46 distributions were identified are indicated on the following pages. Of those 46, 6 are listed on the last two pages.

Fraction of absorbent larger than #7 mesh

Matrix of absorbent fraction greater than #7 mesh (2.8 mm) in size for various distribution j's (means) and i's (standard deviations).

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 4.97 \times 10^{-10} & 5.85 \times 10^{-7} & 2.87 \times 10^{-5} & 3.13 \times 10^{-4} \\ 0 & 0 & 0 & 3.92 \times 10^{-15} & 3.04 \times 10^{-9} & 1.9 \times 10^{-6} & 6.61 \times 10^{-5} & 4.6 \times 10^{-4} \\ 0 & 0 & 0 & 7.68 \times 10^{-14} & 1.71 \times 10^{-8} & 5.88 \times 10^{-6} & 1.47 \times 10^{-4} & 8.61 \times 10^{-4} \\ 0 & 0 & 0 & 1.31 \times 10^{-12} & 8.84 \times 10^{-8} & 1.72 \times 10^{-5} & 3.14 \times 10^{-4} & 1.55 \times 10^{-3} \\ 0 & 0 & 0 & 1.93 \times 10^{-11} & 4.22 \times 10^{-7} & 4.78 \times 10^{-5} & 4.64 \times 10^{-4} & 2.77 \times 10^{-3} \\ 0 & 0 & 0 & 2.48 \times 10^{-10} & 1.86 \times 10^{-6} & 1.26 \times 10^{-4} & 9.24 \times 10^{-4} & 4.81 \times 10^{-3} \\ 0 & 0 & 0 & 2.77 \times 10^{-9} & 7.54 \times 10^{-6} & 3.15 \times 10^{-4} & 1.85 \times 10^{-3} & 8.15 \times 10^{-3} \\ 0 & 0 & 0 & 2.69 \times 10^{-8} & 2.82 \times 10^{-5} & 4.78 \times 10^{-4} & 3.58 \times 10^{-3} & 0.01 \\ 0 & 0 & 1.59 \times 10^{-14} & 2.26 \times 10^{-7} & 9.72 \times 10^{-5} & 1.05 \times 10^{-3} & 6.7 \times 10^{-3} & 0.02 \\ 0 & 0 & 1.07 \times 10^{-12} & 1.66 \times 10^{-6} & 3.09 \times 10^{-4} & 2.36 \times 10^{-3} & 0.01 & 0.03 \\ 0 & 0 & 5.37 \times 10^{-11} & 1.05 \times 10^{-5} & 5.14 \times 10^{-4} & 5.09 \times 10^{-3} & 0.02 & 0.05 \\ 0 & 0 & 2 \times 10^{-9} & 5.82 \times 10^{-5} & 1.25 \times 10^{-3} & 0.01 & 0.04 & 0.08 \end{pmatrix}$$

Binary Outcome

Specific Specification Criterion

$$xa_{j,i} := \text{if}(A_{j,i} < .01, 1, 0)$$

$$xa = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

1 = meets specification
0 = does not meet specification

Each "1" identifies a theoretical distribution that meets the portion of the specification that requires no more than 1% of the absorbent can be larger than #7 mesh.

Fraction of absorbent between #7 and #10 mesh
(2.8 mm to 2.0 mm)

$$B = \begin{pmatrix} 0 & 0 & 4.46 \times 10^{-10} & 1.82 \times 10^{-5} & 1.11 \times 10^{-3} & 8.65 \times 10^{-3} & 0.03 & 0.06 \\ 0 & 0 & 1.26 \times 10^{-8} & 9.23 \times 10^{-5} & 2.95 \times 10^{-3} & 0.02 & 0.05 & 0.09 \\ 0 & 0 & 2.66 \times 10^{-7} & 4.09 \times 10^{-4} & 7.24 \times 10^{-3} & 0.03 & 0.07 & 0.12 \\ 0 & 0 & 4.22 \times 10^{-6} & 1.59 \times 10^{-3} & 0.02 & 0.05 & 0.11 & 0.17 \\ 0 & 1.18 \times 10^{-13} & 5.03 \times 10^{-5} & 5.41 \times 10^{-3} & 0.03 & 0.09 & 0.16 & 0.23 \\ 0 & 1.32 \times 10^{-10} & 4.51 \times 10^{-4} & 0.02 & 0.07 & 0.15 & 0.23 & 0.31 \\ 0 & 5.66 \times 10^{-8} & 3.06 \times 10^{-3} & 0.04 & 0.13 & 0.22 & 0.32 & 0.4 \\ 0 & 9.28 \times 10^{-6} & 0.02 & 0.1 & 0.22 & 0.33 & 0.42 & 0.5 \\ 0 & 5.92 \times 10^{-4} & 0.06 & 0.2 & 0.34 & 0.46 & 0.55 & 0.61 \\ 0 & 0.01 & 0.19 & 0.38 & 0.52 & 0.62 & 0.69 & 0.74 \\ 0 & 0.15 & 0.45 & 0.62 & 0.73 & 0.81 & 0.85 & 0.87 \\ 8.57 \times 10^{-3} & 0.67 & 0.84 & 0.92 & 0.98 & 1.01 & 1.02 & 1.01 \end{pmatrix}$$

Binary Outcome

Specific Specification Criterion

$$xb_{j,i} := \text{if}(B_{j,i} < .30, 1, 0)$$

$$xb = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Fraction of absorbent between #10 and #14 mesh
(2.0 mm to 1.4 mm)

$$C = \begin{pmatrix} 1.41 & 0.97 & 0.88 & 0.85 & 0.84 & 0.84 & 0.83 & 0.81 \\ 1.46 & 1.36 & 1.17 & 1.08 & 1.02 & 0.98 & 0.94 & 0.9 \\ 1.51 & 1.5 & 1.39 & 1.27 & 1.19 & 1.12 & 1.05 & 0.98 \\ 1.55 & 1.55 & 1.51 & 1.42 & 1.33 & 1.23 & 1.14 & 1.05 \\ 1.6 & 1.6 & 1.59 & 1.54 & 1.44 & 1.32 & 1.21 & 1.1 \\ 1.65 & 1.65 & 1.65 & 1.61 & 1.51 & 1.38 & 1.25 & 1.13 \\ 1.7 & 1.7 & 1.7 & 1.65 & 1.54 & 1.4 & 1.26 & 1.14 \\ 1.75 & 1.75 & 1.74 & 1.65 & 1.52 & 1.38 & 1.25 & 1.13 \\ 1.8 & 1.8 & 1.74 & 1.6 & 1.46 & 1.32 & 1.2 & 1.09 \\ 1.86 & 1.84 & 1.67 & 1.49 & 1.35 & 1.23 & 1.13 & 1.04 \\ 1.92 & 1.76 & 1.47 & 1.3 & 1.19 & 1.11 & 1.04 & 0.97 \\ 1.97 & 1.3 & 1.13 & 1.06 & 1.01 & 0.97 & 0.93 & 0.88 \end{pmatrix}$$

Binary Outcome

Specific Specification Criterion

$$xc = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$xc_{j,i} := \text{if}(C_{j,i} > .48, 1, 0)$$

Fraction of absorbent between #14 and #30 mesh
(1.40 mm to 0.60 mm)

$$D = \begin{pmatrix} 4.77 \times 10^{-3} & 0.45 & 0.54 & 0.57 & 0.58 & 0.58 & 0.58 & 0.58 \\ 0 & 0.1 & 0.29 & 0.39 & 0.44 & 0.47 & 0.49 & 0.5 \\ 0 & 9.88 \times 10^{-3} & 0.12 & 0.24 & 0.32 & 0.37 & 0.4 & 0.43 \\ 0 & 4.14 \times 10^{-4} & 0.04 & 0.13 & 0.21 & 0.28 & 0.32 & 0.36 \\ 0 & 7.02 \times 10^{-6} & 0.01 & 0.06 & 0.14 & 0.2 & 0.25 & 0.29 \\ 0 & 4.65 \times 10^{-8} & 2.03 \times 10^{-3} & 0.03 & 0.08 & 0.14 & 0.19 & 0.23 \\ 0 & 1.17 \times 10^{-10} & 3 \times 10^{-4} & 0.01 & 0.04 & 0.09 & 0.14 & 0.18 \\ 0 & 1.1 \times 10^{-13} & 3.36 \times 10^{-5} & 3.56 \times 10^{-3} & 0.02 & 0.06 & 0.1 & 0.14 \\ 0 & 0 & 2.84 \times 10^{-6} & 1.05 \times 10^{-3} & 0.01 & 0.03 & 0.07 & 0.1 \\ 0 & 0 & 1.81 \times 10^{-7} & 2.7 \times 10^{-4} & 4.7 \times 10^{-3} & 0.02 & 0.04 & 0.07 \\ 0 & 0 & 8.65 \times 10^{-9} & 6.11 \times 10^{-5} & 1.92 \times 10^{-3} & 0.01 & 0.03 & 0.05 \\ 0 & 0 & 3.12 \times 10^{-10} & 1.21 \times 10^{-5} & 7.28 \times 10^{-4} & 5.54 \times 10^{-3} & 0.02 & 0.04 \end{pmatrix}$$

Binary Outcome

Specific Specification Criterion

$$xd_{j,i} := \text{if}(D_{j,i} < .20, 1, 0)$$

$$xd = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Sum of binary outcomes

$$xtot_{j,i} := (xa_{j,i} + xb_{j,i} + xc_{j,i} + xd_{j,i})$$

$$xtot = \begin{pmatrix} 4 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 3 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 2 \\ 4 & 4 & 4 & 4 & 4 & 4 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 & 3 & 3 & 2 \\ 4 & 4 & 4 & 4 & 3 & 3 & 3 & 2 \\ 4 & 4 & 4 & 3 & 3 & 3 & 2 & 2 \\ 4 & 4 & 3 & 3 & 3 & 3 & 2 & 2 \\ 4 & 3 & 3 & 3 & 3 & 2 & 2 & 2 \end{pmatrix}$$

$$Final_{j,i} := \text{if}(xtot_{j,i} = 4, 1, 0)$$

1's identify distributions that simultaneously meet all four specification requirements. That is, these are good model distributions.

This matrix is used below as a mask or filter to select valid values for retained fractions with NEDU's mesh screens.

$$Final = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The following definite integrals define the fraction of the normal distribution falling within the confines of the six sieves available at NEDU.

3.5 sieve

$$E_{j,i} := \int_{5.6}^{20} y(x, \mu_j, \sigma_i) dx$$

5 sieve

$$F_{j,i} := \int_{4.0}^{5.6} y(x, \mu_j, \sigma_i) dx$$

8 sieve

$$G_{j,i} := \int_{2.36}^{4.0} y(x, \mu_j, \sigma_i) dx$$

10 sieve

$$H_{j,i} := \int_{2.06}^{2.36} y(x, \mu_j, \sigma_i) dx$$

14 sieve

$$K_{j,i} := \int_{1.4}^{2.06} y(x, \mu_j, \sigma_i) dx$$

30 sieve

$$L_{j,i} := \int_{.6}^{1.4} y(x, \mu_j, \sigma_i) dx$$

3.5 sieve

$$E = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.31 \times 10^{-14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.95 \times 10^{-13} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.02 \times 10^{-12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.44 \times 10^{-12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.48 \times 10^{-15} & 1.13 \times 10^{-11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.17 \times 10^{-14} & 3.59 \times 10^{-11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.33 \times 10^{-14} & 1.12 \times 10^{-10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.34 \times 10^{-13} & 3.37 \times 10^{-10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 9.9 \times 10^{-13} & 9.92 \times 10^{-10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.04 \times 10^{-12} & 2.84 \times 10^{-9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.59 \times 10^{-11} & 7.92 \times 10^{-9} \\ 0 & 0 & 0 & 0 & 0 & 4.65 \times 10^{-15} & 6.02 \times 10^{-11} & 2.15 \times 10^{-8} \end{pmatrix}$$

reactions retained by a #3.5 mesh sieve.

Results of binary filtering

$$eans = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.17 \times 10^{-14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$eans_{j,i} := E_{j,i} \cdot Final_{j,i}$$

Write results to a data file for use by *MeshFit* software.

WRITEPRN("meLn1014.prn") := *eans*

5 sieve

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 3.19 \times 10^{-15} & 4.7 \times 10^{-11} & 1.76 \times 10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 2.01 \times 10^{-14} & 1.73 \times 10^{-10} & 4.67 \times 10^{-8} \\ 0 & 0 & 0 & 0 & 0 & 1.2 \times 10^{-13} & 6.15 \times 10^{-10} & 1.2 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 6.83 \times 10^{-13} & 2.11 \times 10^{-9} & 3.02 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 3.68 \times 10^{-12} & 6.95 \times 10^{-9} & 7.39 \times 10^{-7} \\ 0 & 0 & 0 & 0 & 0 & 1.89 \times 10^{-11} & 2.21 \times 10^{-8} & 1.76 \times 10^{-6} \\ 0 & 0 & 0 & 0 & 0 & 9.16 \times 10^{-11} & 6.79 \times 10^{-8} & 4.07 \times 10^{-6} \\ 0 & 0 & 0 & 0 & 7.22 \times 10^{-15} & 4.22 \times 10^{-10} & 2.01 \times 10^{-7} & 9.17 \times 10^{-6} \\ 0 & 0 & 0 & 0 & 6.81 \times 10^{-14} & 1.85 \times 10^{-9} & 5.73 \times 10^{-7} & 2.01 \times 10^{-5} \\ 0 & 0 & 0 & 0 & 5.92 \times 10^{-13} & 7.67 \times 10^{-9} & 1.58 \times 10^{-6} & 4.31 \times 10^{-5} \\ 0 & 0 & 0 & 0 & 4.76 \times 10^{-12} & 3.03 \times 10^{-8} & 4.19 \times 10^{-6} & 8.96 \times 10^{-5} \\ 0 & 0 & 0 & 0 & 3.53 \times 10^{-11} & 1.13 \times 10^{-7} & 1.07 \times 10^{-5} & 1.82 \times 10^{-4} \end{pmatrix}$$

Results of Binary filtering

$$fans = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.68 \times 10^{-12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.89 \times 10^{-11} & 2.21 \times 10^{-8} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.16 \times 10^{-11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.22 \times 10^{-15} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$fans_{j,i} := F_{j,i} \cdot Final_{j,i}$$

Maximum integral

$$\max(fans) = 2.21 \times 10^{-8}$$

8 sieve

$$G = \begin{pmatrix} 0 & 0 & 0 & 2.47 \times 10^{-10} & 1.53 \times 10^{-6} & 1.11 \times 10^{-4} & 1.27 \times 10^{-3} & 5.88 \times 10^{-3} \\ 0 & 0 & 0 & 2.64 \times 10^{-9} & 6.21 \times 10^{-6} & 2.82 \times 10^{-4} & 2.48 \times 10^{-3} & 9.78 \times 10^{-3} \\ 0 & 0 & 0 & 2.46 \times 10^{-8} & 2.33 \times 10^{-5} & 6.81 \times 10^{-4} & 4.69 \times 10^{-3} & 0.02 \\ 0 & 0 & 3.64 \times 10^{-14} & 2 \times 10^{-7} & 8.08 \times 10^{-5} & 1.56 \times 10^{-3} & 8.56 \times 10^{-3} & 0.03 \\ 0 & 0 & 2.09 \times 10^{-12} & 1.42 \times 10^{-6} & 2.59 \times 10^{-4} & 3.42 \times 10^{-3} & 0.02 & 0.04 \\ 0 & 0 & 8.95 \times 10^{-11} & 8.83 \times 10^{-6} & 7.71 \times 10^{-4} & 7.12 \times 10^{-3} & 0.03 & 0.06 \\ 0 & 0 & 2.88 \times 10^{-9} & 4.79 \times 10^{-5} & 2.12 \times 10^{-3} & 0.01 & 0.04 & 0.09 \\ 0 & 0 & 6.95 \times 10^{-8} & 2.27 \times 10^{-4} & 5.42 \times 10^{-3} & 0.03 & 0.07 & 0.12 \\ 0 & 0 & 1.26 \times 10^{-6} & 9.45 \times 10^{-4} & 0.01 & 0.05 & 0.1 & 0.17 \\ 0 & 3.8 \times 10^{-15} & 1.73 \times 10^{-5} & 3.44 \times 10^{-3} & 0.03 & 0.08 & 0.16 & 0.24 \\ 0 & 6.66 \times 10^{-12} & 1.78 \times 10^{-4} & 0.01 & 0.06 & 0.14 & 0.23 & 0.32 \\ 0 & 4.34 \times 10^{-9} & 1.39 \times 10^{-3} & 0.03 & 0.11 & 0.21 & 0.32 & 0.42 \end{pmatrix}$$

$$gans = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.64 \times 10^{-14} & 2 \times 10^{-7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.09 \times 10^{-12} & 1.42 \times 10^{-6} & 2.59 \times 10^{-4} & 3.42 \times 10^{-3} & 0 & 0 \\ 0 & 0 & 8.95 \times 10^{-11} & 8.83 \times 10^{-6} & 7.71 \times 10^{-4} & 7.12 \times 10^{-3} & 0.03 & 0 \\ 0 & 0 & 2.88 \times 10^{-9} & 4.79 \times 10^{-5} & 2.12 \times 10^{-3} & 0.01 & 0 & 0 \\ 0 & 0 & 6.95 \times 10^{-8} & 2.27 \times 10^{-4} & 5.42 \times 10^{-3} & 0 & 0 & 0 \\ 0 & 0 & 1.26 \times 10^{-6} & 9.45 \times 10^{-4} & 0 & 0 & 0 & 0 \\ 0 & 3.8 \times 10^{-15} & 1.73 \times 10^{-5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 6.66 \times 10^{-12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$gans_{j,i} := G_{j,i} \cdot Final_{j,i}$$

Maximum integral

$$\max(gans) = 0.03$$

10 sieve

$$\mathbf{H} = \begin{pmatrix} 0 & 0 & 1.28 \times 10^{-11} & 3.33 \times 10^{-6} & 4.07 \times 10^{-4} & 4.33 \times 10^{-3} & 0.02 & 0.04 \\ 0 & 0 & 4.81 \times 10^{-10} & 1.92 \times 10^{-5} & 1.16 \times 10^{-3} & 8.72 \times 10^{-3} & 0.03 & 0.05 \\ 0 & 0 & 1.36 \times 10^{-8} & 9.73 \times 10^{-5} & 3.06 \times 10^{-3} & 0.02 & 0.04 & 0.08 \\ 0 & 0 & 2.86 \times 10^{-7} & 4.3 \times 10^{-4} & 7.47 \times 10^{-3} & 0.03 & 0.07 & 0.1 \\ 0 & 0 & 4.51 \times 10^{-6} & 1.67 \times 10^{-3} & 0.02 & 0.05 & 0.1 & 0.14 \\ 0 & 1.18 \times 10^{-13} & 5.36 \times 10^{-5} & 5.66 \times 10^{-3} & 0.04 & 0.09 & 0.14 & 0.18 \\ 0 & 1.38 \times 10^{-10} & 4.79 \times 10^{-4} & 0.02 & 0.07 & 0.14 & 0.2 & 0.24 \\ 0 & 6.05 \times 10^{-8} & 3.24 \times 10^{-3} & 0.04 & 0.13 & 0.2 & 0.26 & 0.29 \\ 0 & 1.01 \times 10^{-5} & 0.02 & 0.1 & 0.21 & 0.29 & 0.34 & 0.36 \\ 0 & 6.42 \times 10^{-4} & 0.07 & 0.21 & 0.33 & 0.39 & 0.42 & 0.42 \\ 0 & 0.02 & 0.2 & 0.38 & 0.48 & 0.51 & 0.5 & 0.48 \\ 0 & 0.16 & 0.46 & 0.61 & 0.65 & 0.63 & 0.58 & 0.54 \end{pmatrix}$$

$$\mathbf{hans} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.36 \times 10^{-8} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.86 \times 10^{-7} & 4.3 \times 10^{-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.51 \times 10^{-6} & 1.67 \times 10^{-3} & 0.02 & 0.05 & 0 & 0 \\ 0 & 1.18 \times 10^{-13} & 5.36 \times 10^{-5} & 5.66 \times 10^{-3} & 0.04 & 0.09 & 0.14 & 0 \\ 0 & 1.38 \times 10^{-10} & 4.79 \times 10^{-4} & 0.02 & 0.07 & 0.14 & 0 & 0 \\ 0 & 6.05 \times 10^{-8} & 3.24 \times 10^{-3} & 0.04 & 0.13 & 0 & 0 & 0 \\ 0 & 1.01 \times 10^{-5} & 0.02 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 6.42 \times 10^{-4} & 0.07 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$\mathbf{hans}_{j,i} := \mathbf{H}_{j,i} \cdot \mathbf{Final}_{j,i}$$

Maximum integral

$$\max(\mathbf{hans}) = 0.14$$

14 sieve

$$K = \begin{pmatrix} 1.41 & 0.97 & 0.88 & 0.85 & 0.84 & 0.84 & 0.84 & 0.82 \\ 1.46 & 1.36 & 1.17 & 1.08 & 1.03 & 0.99 & 0.96 & 0.92 \\ 1.51 & 1.5 & 1.39 & 1.27 & 1.19 & 1.13 & 1.07 & 1.01 \\ 1.55 & 1.55 & 1.51 & 1.43 & 1.34 & 1.26 & 1.17 & 1.09 \\ 1.6 & 1.6 & 1.59 & 1.54 & 1.46 & 1.36 & 1.26 & 1.16 \\ 1.65 & 1.65 & 1.65 & 1.62 & 1.54 & 1.43 & 1.31 & 1.2 \\ 1.7 & 1.7 & 1.7 & 1.68 & 1.59 & 1.47 & 1.34 & 1.22 \\ 1.75 & 1.75 & 1.75 & 1.71 & 1.61 & 1.48 & 1.35 & 1.22 \\ 1.8 & 1.8 & 1.79 & 1.71 & 1.58 & 1.45 & 1.32 & 1.2 \\ 1.86 & 1.86 & 1.8 & 1.65 & 1.51 & 1.38 & 1.26 & 1.16 \\ 1.92 & 1.9 & 1.72 & 1.53 & 1.39 & 1.28 & 1.18 & 1.09 \\ 1.97 & 1.81 & 1.51 & 1.34 & 1.23 & 1.15 & 1.08 & 1.01 \end{pmatrix}$$

$$kans = \begin{pmatrix} 1.41 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.46 & 1.36 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.51 & 1.5 & 1.39 & 0 & 0 & 0 & 0 & 0 \\ 1.55 & 1.55 & 1.51 & 1.43 & 0 & 0 & 0 & 0 \\ 1.6 & 1.6 & 1.59 & 1.54 & 1.46 & 1.36 & 0 & 0 \\ 1.65 & 1.65 & 1.65 & 1.62 & 1.54 & 1.43 & 1.31 & 0 \\ 1.7 & 1.7 & 1.7 & 1.68 & 1.59 & 1.47 & 0 & 0 \\ 1.75 & 1.75 & 1.75 & 1.71 & 1.61 & 0 & 0 & 0 \\ 1.8 & 1.8 & 1.79 & 1.71 & 0 & 0 & 0 & 0 \\ 1.86 & 1.86 & 1.8 & 0 & 0 & 0 & 0 & 0 \\ 1.92 & 1.9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.97 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$kans_{j,i} := K_{j,i} \cdot Final_{j,i}$$

Maximum integral

$$\max(kans) = 1.97$$

30 sieve

$$L = \begin{pmatrix} 4.77 \times 10^{-3} & 0.45 & 0.54 & 0.57 & 0.58 & 0.58 & 0.58 & 0.58 \\ 0 & 0.1 & 0.29 & 0.39 & 0.44 & 0.47 & 0.49 & 0.5 \\ 0 & 9.88 \times 10^{-3} & 0.12 & 0.24 & 0.32 & 0.37 & 0.4 & 0.43 \\ 0 & 3.98 \times 10^{-4} & 0.04 & 0.13 & 0.21 & 0.28 & 0.32 & 0.36 \\ 0 & 6.46 \times 10^{-6} & 0.01 & 0.06 & 0.14 & 0.2 & 0.25 & 0.29 \\ 0 & 4.12 \times 10^{-8} & 2.03 \times 10^{-3} & 0.03 & 0.08 & 0.14 & 0.19 & 0.23 \\ 0 & 1.01 \times 10^{-10} & 3 \times 10^{-4} & 0.01 & 0.04 & 0.09 & 0.14 & 0.18 \\ 0 & 9.32 \times 10^{-14} & 3.34 \times 10^{-5} & 3.56 \times 10^{-3} & 0.02 & 0.06 & 0.1 & 0.14 \\ 0 & 0 & 2.81 \times 10^{-6} & 1.05 \times 10^{-3} & 0.01 & 0.03 & 0.07 & 0.1 \\ 0 & 0 & 1.77 \times 10^{-7} & 2.7 \times 10^{-4} & 4.7 \times 10^{-3} & 0.02 & 0.04 & 0.07 \\ 0 & 0 & 8.43 \times 10^{-9} & 6.11 \times 10^{-5} & 1.92 \times 10^{-3} & 0.01 & 0.03 & 0.05 \\ 0 & 0 & 3 \times 10^{-10} & 1.21 \times 10^{-5} & 7.28 \times 10^{-4} & 5.54 \times 10^{-3} & 0.02 & 0.04 \end{pmatrix}$$

$$lans = \begin{pmatrix} 4.77 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9.88 \times 10^{-3} & 0.12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.98 \times 10^{-4} & 0.04 & 0.13 & 0 & 0 & 0 & 0 \\ 0 & 6.46 \times 10^{-6} & 0.01 & 0.06 & 0.14 & 0.2 & 0 & 0 \\ 0 & 4.12 \times 10^{-8} & 2.03 \times 10^{-3} & 0.03 & 0.08 & 0.14 & 0.19 & 0 \\ 0 & 1.01 \times 10^{-10} & 3 \times 10^{-4} & 0.01 & 0.04 & 0.09 & 0 & 0 \\ 0 & 9.32 \times 10^{-14} & 3.34 \times 10^{-5} & 3.56 \times 10^{-3} & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 2.81 \times 10^{-6} & 1.05 \times 10^{-3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.77 \times 10^{-7} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Apply binary filter

$$lans_{j,i} := L_{j,i} \cdot Final_{j,i}$$

Maximum integral

$$\max(lans) = 0.2$$

Examples of expected retained fractions across NEDU mesh sizes for distributions meeting the overall specifications. The output file of this MathCad program contains 46 such distributions. The Visual Basic program "MeshFit" fits NEDU's data from the sieve analysis to these distributions. The model distribution representing the best fit to the test sample is then compared to the sample distribution. That comparison is quantified by assessments of "Goodness of Fit" using either a simple summed error, or the Chi Square statistical test (see MeshFit documentation).

Definition of one of the 46 model distributions - down selected from 135 candidate distributions.

$$\begin{aligned} j &:= 3 & \text{mmMean} &:= 1.3 + (0.05 \cdot j) & \text{MeshMean} &:= 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) & \text{MeshMean} &= 12.75 \\ & & \text{mmMean} &= 1.45 \\ i &:= 1 & \text{mmSD} &:= (.05 \cdot i) & \text{mmSD} &= 0.05 \end{aligned}$$

Fractions retained in the NEDU sieves

<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>	<u>30 mesh</u>
$\text{gans}_{j,i} = 0$	$\text{hans}_{j,i} = 0$	$\text{kans}_{j,i} = 1.51$	$\text{lans}_{j,i} = 0$

Another model distribution

$$\begin{aligned} j &:= 12 & \text{mmMean} &:= 1.3 + (0.05 \cdot j) & \text{MeshMean} &:= 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) & \text{MeshMean} &= 9.99 \\ & & \text{mmMean} &= 1.9 \\ i &:= 1 & \text{mmSD} &:= (.05 \cdot i) & \text{mmSD} &= 0.05 \end{aligned}$$

<u>8 mesh</u>	<u>10 mesh</u>	<u>14 mesh</u>	<u>30 mesh</u>
$\text{gans}_{j,i} = 0$	$\text{hans}_{j,i} = 0$	$\text{kans}_{j,i} = 1.97$	$\text{lans}_{j,i} = 0$

Yet another model distribution

$$\begin{aligned} j &:= 9 & \text{mmMean} &:= 1.3 + (0.05 \cdot j) & \text{MeshMean} &:= 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) & \text{MeshMean} &= 10.75 \\ & & \text{mmMean} &= 1.75 \\ i &:= 8 & \text{mmSD} &:= (.05 \cdot i) & \text{mmSD} &= 0.4 \end{aligned}$$

$\text{gans}_{j,i} = 0$	$\text{hans}_{j,i} = 0$	$\text{kans}_{j,i} = 0$	$\text{lans}_{j,i} = 0$
-------------------------	-------------------------	-------------------------	-------------------------

$$\begin{aligned} j &:= 6 & \text{mmMean} &:= 1.3 + (0.05 \cdot j) & \text{MeshMean} &:= 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) & \text{MeshMean} &= 11.66 \\ & & \text{mmMean} &= 1.6 \\ i &:= 4 & \text{mmSD} &:= (.05 \cdot i) & \text{mmSD} &= 0.2 \end{aligned}$$

$\text{gans}_{j,i} = 8.83 \times 10^{-6}$	$\text{hans}_{j,i} = 5.66 \times 10^{-3}$	$\text{kans}_{j,i} = 1.62$	$\text{lans}_{j,i} = 0.03$
---	---	----------------------------	----------------------------

$$j := 12 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 9.99$$

$$\text{mmMean} = 1.9$$

$$i := 4 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.2$$

$$\text{gans}_{j,i} = 0$$

$$\text{hans}_{j,i} = 0$$

$$\text{kans}_{j,i} = 0$$

$$\text{lans}_{j,i} = 0$$

$$j := 9 \quad \text{mmMean} := 1.3 + (0.05 \cdot j) \quad \text{MeshMean} := 1.096 + \left(16.9 \cdot \frac{1}{\text{mmMean}} \right) \quad \text{MeshMean} = 10.75$$

$$\text{mmMean} = 1.75$$

$$i := 4 \quad \text{mmSD} := (.05 \cdot i) \quad \text{mmSD} = 0.2$$

$$\text{gans}_{j,i} = 9.45 \times 10^{-4}$$

$$\text{hans}_{j,i} = 0.1$$

$$\text{kans}_{j,i} = 1.71$$

$$\text{lans}_{j,i} = 1.05 \times 10^{-3}$$

APPENDIX D. Details listing for Figure 10.

no.	Error	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
1	.995	0	0	0	0	0	0
2	.995	0	0	0	0	0	0
3	.995	0	0	0	0	0	0
4	.995	0	0	0	0	0	0
5	.995	0	0	0	0	0	0
6	.995	0	0	0	0	0	0
7	.995	0	0	0	0	0	0
8	.995	0	0	0	0	0	0
9	.3332	5.771E-05	0.07842	0.7875	0.0757	0.05342	0.004806
10	.995	0	0	0	0	0	0
11	.995	0	0	0	0	0	0
12	.995	0	0	0	0	0	0
13	.995	0	0	0	0	0	0
14	.995	0	0	0	0	0	0
15	.995	0	0	0	0	0	0
16	.995	0	0	0	0	0	0
17	.30305	8.588E-05	0.09411	0.7922	0.06605	0.04391	0.003586
18	.26701	0.0002359	0.1107	0.758	0.07048	0.05423	0.006197
19	.23123	0.0005418	0.1266	0.7252	0.07363	0.064	0.009646
20	.995	0	0	0	0	0	0
21	.995	0	0	0	0	0	0
22	.995	0	0	0	0	0	0
23	.995	0	0	0	0	0	0
24	.995	0	0	0	0	0	0
25	.26821	0.0001266	0.112	0.7924	0.05706	0.03575	0.00265
26	.23136	0.0003313	0.1292	0.7581	0.06211	0.04535	0.004763
27	.19563	0.0007314	0.1454	0.7254	0.06594	0.05469	0.007653
28	.16117	0.001406	0.1603	0.6943	0.06872	0.06345	0.01128
29	.995	0	0	0	0	0	0
30	.995	0	0	0	0	0	0
31	.995	0	0	0	0	0	0
32	.995	0	0	0	0	0	0
33	.23082	0.0001851	0.1322	0.788	0.04882	0.02884	0.00194
34	.19148	0.0004617	0.1497	0.7543	0.05427	0.03761	0.003631
35	.15603	0.0009675	0.1659	0.7219	0.05862	0.04641	0.006029
36	.12242	0.001829	0.1806	0.6912	0.06195	0.05487	0.009132
37	.13617	0.003116	0.1937	0.6623	0.06442	0.06275	0.01287
38	.995	0	0	0	0	0	0

39	.995	0	0	0	0	0	0
40	.995	0	0	0	0	0	0
41	.20119	0.000268	0.1547	0.7792	0.04135	0.02304	0.001407
no.	Error	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
42	.15302	0.0006385	0.1723	0.7464	0.04702	0.03094	0.002745
43	.11262	0.001292	0.1881	0.7149	0.05172	0.0391	0.004716
44	.08034	0.002364	0.2023	0.685	0.05549	0.04715	0.007347
45	.10663	0.00392	0.2148	0.6568	0.05842	0.05481	0.0106
46	.13164	0.006017	0.2257	0.6302	0.06062	0.06192	0.01439
47	.15536	0.008681	0.2349	0.6053	0.0622	0.06836	0.01861
48	.995	0	0	0	0	0	0
49	.16505	0.0003845	0.1796	0.7661	0.03469	0.01824	0.001011
50	.11765	0.0008571	0.1968	0.7346	0.0404	0.02525	0.002059
51	.0747	0.001713	0.2121	0.7044	0.0453	0.0327	0.003663
52	.05458	0.003037	0.2255	0.6758	0.04938	0.04026	0.005874
53	.07959	0.004906	0.2371	0.6486	0.05267	0.04762	0.008683
54	.10885	0.007366	0.2469	0.6229	0.05527	0.05458	0.01203
55	.995	0	0	0	0	0	0
56	.995	0	0	0	0	0	0
57	.12368	0.0005469	0.2069	0.7487	0.02881	0.01431	
0.0007197							
58	.07683	0.001172	0.2233	0.7191	0.03441	0.02044	0.001532
59	.03588	0.002255	0.2376	0.6907	0.03938	0.02716	0.002825
60	.05191	0.003878	0.2499	0.6635	0.04365	0.03416	0.004667
61	.09162	0.006107	0.2604	0.6377	0.04722	0.04114	0.007073
62	.995	0	0	0	0	0	0
63	.995	0	0	0	0	0	0
64	.995	0	0	0	0	0	0
65	.08128	0.0007443	0.2364	0.7275	0.0237	0.01112	
0.0005074							
66	.04414	0.001589	0.2517	0.7001	0.02907	0.01641	0.00113
67	.05431	0.002949	0.2647	0.6738	0.03399	0.0224	0.002163
68	.09256	0.004922	0.2756	0.6485	0.03834	0.0288	0.003685
69	.995	0	0	0	0	0	0
70	.995	0	0	0	0	0	0
71	.995	0	0	0	0	0	0
72	.995	0	0	0	0	0	0
73	.07784	0.001046	0.2681	0.7026	0.0193	0.008556	
0.0003544							
74	.07139	0.002137	0.2817	0.6779	0.02434	0.01307	
0.0008274							

75	.10333	0.00383	0.2931	0.654	0.02911	0.01834	0.001645
76	.995	0	0	0	0	0	0
77	.995	0	0	0	0	0	0
78	.995	0	0	0	0	0	0
79	.995	0	0	0	0	0	0
80	.995	0	0	0	0	0	0
81	.11011	0.001456	0.3017	0.6745	0.01557	0.006524	
0.0002452							
82	.13507	0.002852	0.3133	0.6527	0.02021	0.01032	
0.0006007	83	.995	0	0	0	0	0
0							
84	.995	0	0	0	0	0	0
no.	Error	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
85	.995	0	0	0	0	0	0
86	.995	0	0	0	0	0	0
87	.995	0	0	0	0	0	0
88	.995	0	0	0	0	0	0
89	.18104	0.002007	0.3369	0.6435	0.01243	0.004927	
0.0001681							
90	.995	0	0	0	0	0	0
91	.995	0	0	0	0	0	0
92	.995	0	0	0	0	0	0
93	.995	0	0	0	0	0	0
94	.995	0	0	0	0	0	0
95	.995	0	0	0	0	0	0
96	.995	0	0	0	0	0	0

Best model = 59

	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
obs:	0	24.5	68.5	7	0	0
modl:	0	23.9855	69.07	6.9365	0	0

Chi Square =.0163
degrees of freedom = 2

APPENDIX E. Details listing for Figure 11.

no.	Error	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
1	.9916	0	0	0	0	0	0
2	.9916	0	0	0	0	0	0
3	.9916	0	0	0	0	0	0
4	.9916	0	0	0	0	0	0
5	.9916	0	0	0	0	0	0
6	.9916	0	0	0	0	0	0
7	.9916	0	0	0	0	0	0
8	.9916	0	0	0	0	0	0
9	.9482	5.771E-05	0.07842	0.7875	0.0757	0.05342	0.004806
10	.9916	0	0	0	0	0	0
11	.9916	0	0	0	0	0	0
12	.9916	0	0	0	0	0	0
13	.9916	0	0	0	0	0	0
14	.9916	0	0	0	0	0	0
15	.9916	0	0	0	0	0	0
16	.9916	0	0	0	0	0	0
17	.91805	8.588E-05	0.09411	0.7922	0.06605	0.04391	0.003586
18	.88201	0.0002359	0.1107	0.758	0.07048	0.05423	0.006197
19	.84623	0.0005418	0.1266	0.7252	0.07363	0.064	0.009646
20	.9916	0	0	0	0	0	0
21	.9916	0	0	0	0	0	0
22	.9916	0	0	0	0	0	0
23	.9916	0	0	0	0	0	0
24	.9916	0	0	0	0	0	0
25	.88321	0.0001266	0.112	0.7924	0.05706	0.03575	0.00265
26	.84636	0.0003313	0.1292	0.7581	0.06211	0.04535	0.004763
27	.81063	0.0007314	0.1454	0.7254	0.06594	0.05469	0.007653
28	.77617	0.001406	0.1603	0.6943	0.06872	0.06345	0.01128
29	.9916	0	0	0	0	0	0
30	.9916	0	0	0	0	0	0
31	.9916	0	0	0	0	0	0
32	.9916	0	0	0	0	0	0
33	.84346	0.0001851	0.1322	0.788	0.04882	0.02884	0.00194
34	.80648	0.0004617	0.1497	0.7543	0.05427	0.03761	0.003631
35	.77103	0.0009675	0.1659	0.7219	0.05862	0.04641	0.006029
36	.73742	0.001829	0.1806	0.6912	0.06195	0.05487	0.009132
37	.70577	0.003116	0.1937	0.6623	0.06442	0.06275	0.01287
38	.9916	0	0	0	0	0	0
39	.9916	0	0	0	0	0	0

40	.9916	0	0	0	0	0	0
41	.79889	0.000268	0.1547	0.7792	0.04135	0.02304	0.001407
42	.76206	0.0006385	0.1723	0.7464	0.04702	0.03094	0.002745
no.	Error	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
43	.72762	0.001292	0.1881	0.7149	0.05172	0.0391	0.004716
44	.69534	0.002364	0.2023	0.685	0.05549	0.04715	0.007347
45	.66523	0.00392	0.2148	0.6568	0.05842	0.05481	0.0106
46	.63704	0.006017	0.2257	0.6302	0.06062	0.06192	0.01439
47	.61096	0.008681	0.2349	0.6053	0.0622	0.06836	0.01861
48	.9916	0	0	0	0	0	0
49	.74943	0.0003845	0.1796	0.7661	0.03469	0.01824	0.001011
50	.71345	0.0008571	0.1968	0.7346	0.0404	0.02525	0.002059
51	.6803	0.001713	0.2121	0.7044	0.0453	0.0327	0.003663
52	.64994	0.003037	0.2255	0.6758	0.04938	0.04026	0.005874
53	.62179	0.004906	0.2371	0.6486	0.05267	0.04762	0.008683
54	.59585	0.007366	0.2469	0.6229	0.05527	0.05458	0.01203
55	.9916	0	0	0	0	0	0
56	.9916	0	0	0	0	0	0
57	.69492	0.0005469	0.2069	0.7487	0.02881	0.01431	
0.0007197							
58	.66065	0.001172	0.2233	0.7191	0.03441	0.02044	0.001532
59	.62964	0.002255	0.2376	0.6907	0.03938	0.02716	0.002825
60	.60141	0.003878	0.2499	0.6635	0.04365	0.03416	0.004667
61	.57566	0.006107	0.2604	0.6377	0.04722	0.04114	0.007073
62	.9916	0	0	0	0	0	0
63	.9916	0	0	0	0	0	0
64	.9916	0	0	0	0	0	0
65	.63592	0.0007443	0.2364	0.7275	0.0237	0.01112	
0.0005074							
66	.60388	0.001589	0.2517	0.7001	0.02907	0.01641	0.00113
67	.57549	0.002949	0.2647	0.6738	0.03399	0.0224	0.002163
68	.55004	0.004922	0.2756	0.6485	0.03834	0.0288	0.003685
69	.9916	0	0	0	0	0	0
70	.9916	0	0	0	0	0	0
71	.9916	0	0	0	0	0	0
72	.9916	0	0	0	0	0	0
73	.57236	0.001046	0.2681	0.7026	0.0193	0.008556	
0.0003544							
74	.54361	0.002137	0.2817	0.6779	0.02434	0.01307	
0.0008274							
75	.51835	0.00383	0.2931	0.654	0.02911	0.01834	0.001645

76	.9916	0	0	0	0	0	0
77	.9916	0	0	0	0	0	0
78	.9916	0	0	0	0	0	0
79	.9916	0	0	0	0	0	0
80	.9916	0	0	0	0	0	0
81	.50489	0.001456	0.3017	0.6745	0.01557	0.006524	
0.0002452							
82	.47993	0.002852	0.3133	0.6527	0.02021	0.01032	
0.0006007							
83	.9916	0	0	0	0	0	0
84	.9916	0	0	0	0	0	0
85	.9916	0	0	0	0	0	0
no.	Error	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
86	.9916	0	0	0	0	0	0
87	.9916	0	0	0	0	0	0
88	.9916	0	0	0	0	0	0
89	.4367	0.002007	0.3369	0.6435	0.01243	0.004927	
0.0001681							
90	.9916	0	0	0	0	0	0
91	.9916	0	0	0	0	0	0
92	.9916	0	0	0	0	0	0
93	.9916	0	0	0	0	0	0
94	.9916	0	0	0	0	0	0
95	.9916	0	0	0	0	0	0
96	.9916	0	0	0	0	0	0

Best model = 89

	3.5 mesh	5 mesh	8 mesh	10 mesh	14 mesh	30 mesh
obs:	0	55.1	44.12	0	0	0
modl:	0	33.8907	66.10251	0	0	0

Chi Square = 20.5834
degrees of freedom = 1

APPENDIX F. MeshFit Code

```
VERSION 4.00
Dim fracRetain(8, 50, 50) As Single
Dim Frac3 As Single
Dim Frac5 As Single
Dim Frac8 As Single
Dim Frac10 As Single
Dim Frac14 As Single
Dim Frac30 As Single
Dim mesh35fit As Single
Dim mesh5fit As Single
Dim mesh8fit As Single
Dim mesh10fit As Single
Dim mesh14fit As Single
Dim mesh30fit As Single

Dim ErrorFrac As Single
Dim Fractotal As Single

Dim i As Integer
Dim imax As Integer
Dim j As Integer
Dim jmax As Integer
Dim s As Integer
Dim sieveNumb As Integer
Dim bestn As Integer

Private Sub Command1_Click()
If Fractotal > 1# Then
    Beep
    Beep
    Beep
    Label(15).Visible = True
    Exit Sub
End If

Call TotalFraction

Description$ = Text7.Text

If Choice.Option1.Value = True Then
    Open "e:\meshfit\mesh408.prn" For Input As #1
    jmax = 12
    imax = 8
End If

If Choice.Option2.Value = True Then
    Open "e:\meshfit\mesh1014.prn" For Input As #1
    jmax = 11
    imax = 9
End If

If Choice.Option3.Value = True Then
    Open "e:\meshfit\mesLn408.prn" For Input As #1
    jmax = 12
    imax = 8
End If

j = 1
i = 1
s = 1
sieveNumb = 6
```

```

Do Until s > sieveNumb
    j = 1
    Do Until j > jmax
        i = 1
        Do Until i > imax
            Input #1, fracRetain(s, j, i) ' Get data
            i = i + 1
        Loop
        j = j + 1
    Loop
    s = s + 1
Loop

Close #1 ' Close file.
minerror = 50

n = 1
If Check1.Value = 1 Then
    Printer.Print 'create blank line
    Printer.Font.Name = "Arial"
    Printer.Font.Size = 10
    Printer.Print Description$ 'print test description
    Printer.Print
    Printer.Font.Underline = True
    Printer.Print "no.", "Error", "3.5 mesh", "5 mesh", "8 mesh", "10 mesh", "14
mesh", "30 mesh"
    Printer.Font.Underline = False
End If

For j = 1 To jmax
    For i = 1 To imax
        If Choice.Option1.Value = True Then 'for 408 mesh
            distMean = 3.1 + (0.05 * j) 'mean in mm
            distSD = 0.6 + (0.05 * i) 'SD in mm
        Else 'for 812 mesh
            distMean = 1.3 + (0.05 * j) 'mean in mm
            distSD = 0.05 * i 'SD in mm
        End If

        ErrorFrac35 = Abs(Frac35 - fracRetain(1, j, i))
        ErrorFrac5 = Abs(Frac5 - fracRetain(2, j, i))
        ErrorFrac8 = Abs(Frac8 - fracRetain(3, j, i))
        ErrorFrac10 = Abs(Frac10 - fracRetain(4, j, i))
        ErrorFrac14 = Abs(Frac14 - fracRetain(5, j, i))
        ErrorFrac30 = Abs(Frac30 - fracRetain(6, j, i))

        sumerror = ErrorFrac5 + ErrorFrac8 + ErrorFrac10 + ErrorFrac14

        sumerrorp = Format(sumerror, "#.#####")

        If Check1.Value = 1 Then Printer.Print n, sumerrorp, fracRetain(1,
j, i), fracRetain(2, j, i), fracRetain(3, j, i), fracRetain(4, j, i), fracRetain(5, j,
i), fracRetain(6, j, i)
        End If

        If sumerror < minerror Then
            bestn = n
            bestj = j
            besti = i
            minerror = sumerror
            bestdistMean = distMean

```

```

        bestdistSD = distSD
        meshMean = 1.096 + (16.91 * 1 / bestdistMean) 'mean in mesh size
        meshSDup = 1.096 + (16.91 * 1 / (bestdistMean - bestdistSD)) 'mean - SD in
mesh size
        meshSDdn = 1.096 + (16.91 * 1 / (bestdistMean + bestdistSD)) 'mean + SD in
mesh size
        meshMean = Format(meshMean, "##.##")
        meshSDup = Format(meshSDup, "##.##")
        meshSDdn = Format(meshSDdn, "##.##")
    End If
    n = n + 1 'increment counter
Next i
Next j
Label2.Caption = Format$(bestdistSD, "##.##")

mesh35fit = fracRetain(1, bestj, besti)
mesh5fit = fracRetain(2, bestj, besti)
mesh8fit = fracRetain(3, bestj, besti)
mesh10fit = fracRetain(4, bestj, besti)
mesh14fit = fracRetain(5, bestj, besti)
mesh30fit = fracRetain(6, bestj, besti)

Label(20).Caption = Format$(fracRetain(1, bestj, besti) * 100, "###.###")
Label(21).Caption = Format$(fracRetain(2, bestj, besti) * 100, "###.###")
Label(22).Caption = Format$(fracRetain(3, bestj, besti) * 100, "###.###")
Label(23).Caption = Format$(fracRetain(4, bestj, besti) * 100, "###.###")
Label(24).Caption = Format$(fracRetain(5, bestj, besti) * 100, "###.###")
Label(25).Caption = Format$(fracRetain(6, bestj, besti) * 100, "###.###")
For idx = 20 To 25
    If Label(idx).Caption = "." Then Label(idx).Caption = "0"
Next idx

Call ChiSquare
Call BarGraph

Label(37).Caption = Format$(minerror, "###.###")
Label(39).Caption = Str$(bestn)
Label(17).Caption = Str$(meshMean)
Label(18).Caption = Str$(meshSDup)
Label(40).Caption = Str$(meshSDdn)
End Sub

Private Sub Command2_Click()
    End
End Sub

Private Sub Command3_Click()
    Mesh.BackColor = &HFFFFFF
    Check1.BackColor = &HFFFFFF
    For idx = 0 To 39
        Label(idx).BackColor = &HFFFFFF 'control array
    Next idx

    Printer.DrawMode = 13
    Printer.ScaleLeft = -((Printer.Width - Mesh.Width) / 2)
    Printer.ScaleTop = -((Printer.Height - Mesh.Height) / 2)
    PrintForm 'print graphic screen
    Printer.EndDoc

    Mesh.BackColor = &H8000000F
    Check1.BackColor = &H8000000F

```

```

        For idx = 0 To 39
            Label(idx).BackColor = &H8000000F      'control array
        Next idx
    End Sub

    Private Sub Command4_Click()
        Command1.SetFocus
        Choice.Show 0
    End Sub

    Private Sub Command5_Click()
        Text1.Text = ""
        Text2.Text = ""
        Text3.Text = ""
        Text4.Text = ""
        Text5.Text = ""
        Text6.Text = ""
    End Sub

    Private Sub Form_Load()
        Label(16).Visible = False
        Graph1.RandomData = 0
        Graph1.NumPoints = 6
        Graph1.LeftTitle = "Fraction Retained"
        Graph1.BottomTitle = "Mesh Size"
        Graph1.NumSets = 2
    End Sub

    Private Sub VScroll1_Change()
        Text7.Text = Format$(VScroll1.Value, "##")
    End Sub

    Private Sub Option1_Click()
    End Sub

    Private Sub Option2_Click()
    End Sub

    Private Sub Text1_Change()
        Label(16).Visible = False
        Call TotalFraction
    End Sub

    Private Sub Text2_Change()
        Label(16).Visible = False
        Call TotalFraction
    End Sub

    Private Sub Text3_Change()
        Label(16).Visible = False
        Call TotalFraction
    End Sub

```

```

Private Sub Text4_Change()
    Label(16).Visible = False
    Call TotalFraction
End Sub

Private Sub Text5_Change()
    Label(16).Visible = False
    Call TotalFraction
End Sub

Private Sub Text6_Change()
    Label(16).Visible = False
    Call TotalFraction
End Sub

Public Sub TotalFraction()
    Frac3 = Val(Text1.Text) / 100
    Frac5 = Val(Text2.Text) / 100
    Frac8 = Val(Text3.Text) / 100
    Frac10 = Val(Text4.Text) / 100
    Frac14 = Val(Text5.Text) / 100
    Frac30 = Val(Text6.Text) / 100
    Fractotal = Frac3 + Frac5 + Frac8 + Frac10 + Frac14 + Frac30
    If Fractotal <= 1 Then
        Gauge1.Value = 100 * Fractotal
    Else: Gauge1.Value = 0
    End If
    Label(15).Caption = Format$(100 * Fractotal, "#.##")
End Sub

Public Sub BarGraph()
    Graph1.DrawMode = 0
    Graph1.PatternData = 5
    Graph1.YAxisMax = 1
    Graph1.AutoInc = 1
    Graph1.LabelText = "    3.5"
    Graph1.LabelText = "    5"
    Graph1.LabelText = "    8"
    Graph1.LabelText = "   10"
    Graph1.LabelText = "   14"
    Graph1.LabelText = "   30"
    Graph1.GraphData = Frac3
    Graph1.GraphData = Frac5
    Graph1.GraphData = Frac8
    Graph1.GraphData = Frac10
    Graph1.GraphData = Frac14
    Graph1.GraphData = Frac30
    Graph1.GraphData = mesh35fit
    Graph1.GraphData = mesh5fit
    Graph1.GraphData = mesh8fit
    Graph1.GraphData = mesh10fit
    Graph1.GraphData = mesh14fit
    Graph1.GraphData = mesh30fit
    Graph1.LegendText = "measured"
    Graph1.LegendText = "best fit"
    Graph1.DrawMode = 2
End Sub

```

```

Public Sub ChiSquare()
combined = 0
mult = 100
Frac3mult = mult * Frac3
Frac5mult = mult * Frac5
Frac8mult = mult * Frac8
Frac10mult = mult * Frac10
Frac14mult = mult * Frac14
Frac30mult = mult * Frac30

If Frac30mult < 5 Then
    combine30 = 1
    Frac14mult = Frac14mult + Frac30mult
    mesh14fit = mesh14fit + mesh30fit
    Frac30mult = 0
    mesh30fit = 0
End If

If Frac14mult < 5 Then
    combine14 = 1
    Frac10mult = Frac10mult + Frac14mult
    mesh10fit = mesh10fit + mesh14fit
    Frac14mult = 0
    mesh14fit = 0
End If

If Frac10mult < 5 Then
    combine10 = 1
    Frac8mult = Frac8mult + Frac10mult
    mesh8fit = mesh8fit + mesh10fit
    Frac10mult = 0
    mesh10fit = 0
End If

If Frac8mult < 5 Then
    combine8 = 1
    Frac10mult = Frac10mult + Frac8mult
    mesh10fit = mesh10fit + mesh8fit
    Frac8mult = 0
    mesh8fit = 0
End If

If Frac3mult < 5 Then
    combine35 = 1
    Frac5mult = Frac5mult + Frac3mult
    mesh5fit = mesh5fit + mesh35fit
    Frac3mult = 0
    mesh35fit = 0
End If

If Frac5mult < 5 Then
    combine5 = 1
    Frac8mult = Frac8mult + Frac5mult
    mesh8fit = mesh8fit + mesh5fit
    Frac5mult = 0
    mesh5fit = 0
End If

```

```

mesh35mult = mult * mesh35fit
mesh5mult = mult * mesh5fit
mesh8mult = mult * mesh8fit
mesh10mult = mult * mesh10fit
mesh14mult = mult * mesh14fit
mesh30mult = mult * mesh30fit

If Check1.Value = 1 Then
    Printer.Print
    Printer.Print "Best model ="; bestn
    Printer.Print
    Printer.Font.Underline = True
    Printer.Print " 3.5 mesh", "5 mesh", "8 mesh", "10 mesh", "14 mesh", "30 mesh"
    Printer.Font.Underline = False
    Printer.Print "obs: "; Frac3mult, Frac5mult, Frac8mult, Frac10mult, Frac14mult,
Frac30mult
    Printer.Print "modl: "; mesh35mult, mesh5mult, mesh8mult, mesh10mult, mesh14mult,
mesh30mult
End If

combined = combine35 + combine5 + combine8 + combine10 + combine14 + combine30
degfreedom = 6 - combined - 1      '6 sieves - sieves combined - 1

If mesh35fit > 0.00001 Then
    chi35 = (Frac3mult - mesh35mult) ^ 2 / mesh35mult
Else: chi35 = 0
End If

If mesh5fit > 0.00001 Then
    chi5 = (Frac5mult - mesh5mult) ^ 2 / mesh5mult
Else: chi5 = 0
End If

If mesh8fit > 0.00001 Then
    chi8 = (Frac8mult - mesh8mult) ^ 2 / mesh8mult
Else: chi8 = 0
End If

If mesh10fit > 0.00001 Then
    chi10 = (Frac10mult - mesh10mult) ^ 2 / mesh10mult
Else: chi10 = 0
End If

If mesh14fit > 0.00001 Then
    chi14 = (Frac14mult - mesh14mult) ^ 2 / mesh14mult
Else: chi14 = 0
End If

If mesh30fit > 0.00001 Then
    chi30 = (Frac30mult - mesh30mult) ^ 2 / mesh30mult
Else: chi30 = 0
End If

ChiStatistic = chi35 + chi5 + chi8 + chi10 + chi14 + chi30
Label(26).Caption = Format$(ChiStatistic, "###.####")
Label(29).Caption = degfreedom

If Check1.Value = 1 Then
    Printer.Print
    Printer.Print "Chi Square ="; Format$(ChiStatistic, "###.####")
    Printer.Print "degrees of freedom ="; degfreedom

```

```

End If

significance = 0
If degfreedom = 1 And ChiStatistic > 3.84146 Then significance = 1

If degfreedom = 2 And ChiStatistic > 5.99147 Then significance = 1

If degfreedom = 3 And ChiStatistic > 7.81473 Then significance = 1

If degfreedom = 4 And ChiStatistic > 9.48773 Then significance = 1

If degfreedom = 5 And ChiStatistic > 11.0705 Then significance = 1

If significance = 1 Then
    Label(30).ForeColor = &H0&
    Label(30).Caption = "Significantly different from best model !"
End If

If significance = 0 Then
    Label(30).ForeColor = &H0
    Label(30).Caption = "Same as model"
End If

End Sub

```

Appendix G *MeshFit* SUMMARY

When NEDU evaluates sodalime quality, we determine, among other things, whether or not the absorbent meets specifications for granule size distributions. However, because of disparities between mesh sizes called out in specifications and sieve sizes used in testing, a simple comparison is not always possible. *MeshFit* software makes those comparisons simple for the operator, and gives a clear answer as to whether a sodalime sample meets specifications.

The software relies on the mathematical translation of sodalime specifications into specific targets for sieve-based sodalime testing. First, mathematical distributions of granule sizes are determined that meet the sodalime specification. Those distributions, called model distributions, are then subjected to a mathematical sieving operation. After the test bench operator enters the results of the actual physical sieving operation, *MeshFit* compares those results to the “sieved” model distributions. If the physical sample matches one of the model distributions, then the sample meets the granule size specification.

It is possible that a test sample will match none of the model distributions particularly well. Fortunately, the degree of match or mismatch can be measured and used as an evaluation factor for the test sample. *MeshFit* finds the one model distribution that best agrees with the test sample data. A statistical test then determines whether or not the best fit is adequate; that is, whether the actual sample and the model are in close enough agreement. The goodness of fit between the test data and the best model distribution is found through the classical chi square statistical test for goodness of fit.

When using *MeshFit*, the user selects whether the test sample is from a 4-8 mesh sample or 10-14 U.S. mesh (8-12 British mesh) sample. The user then enters the mean retained percentages for all sieve screens obtained from the sieving of five or more sodalime samples. The software next identifies the best fit to a model distribution, and plots in bar graph form the mesh distribution of both the sample and the best model. Finally, the chi square goodness of fit statistic is computed, with significance being determined from the chi square and the degrees of freedom for the test (Figures G-1 and G-2).

The chi square test enables the testing of observed versus expected values for multiple data groupings. In this case, we compare the observed fraction retained by a particular sieve screen against the expected fraction for the closest normal distribution meeting the specifications.

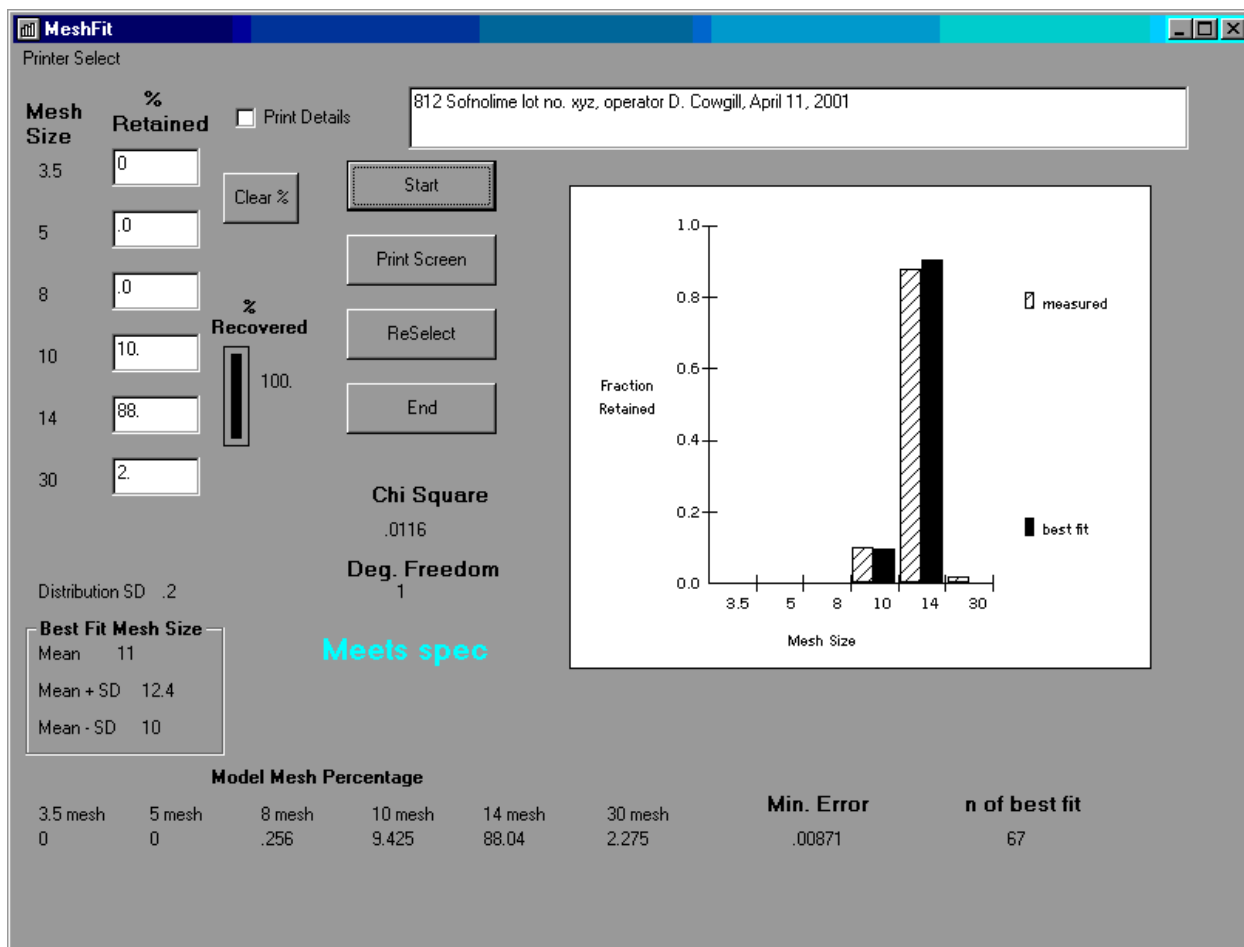


Figure G-1. MeshFit printout for a sample of 812 Sofnolime that meets the British specification for the granule size distribution.

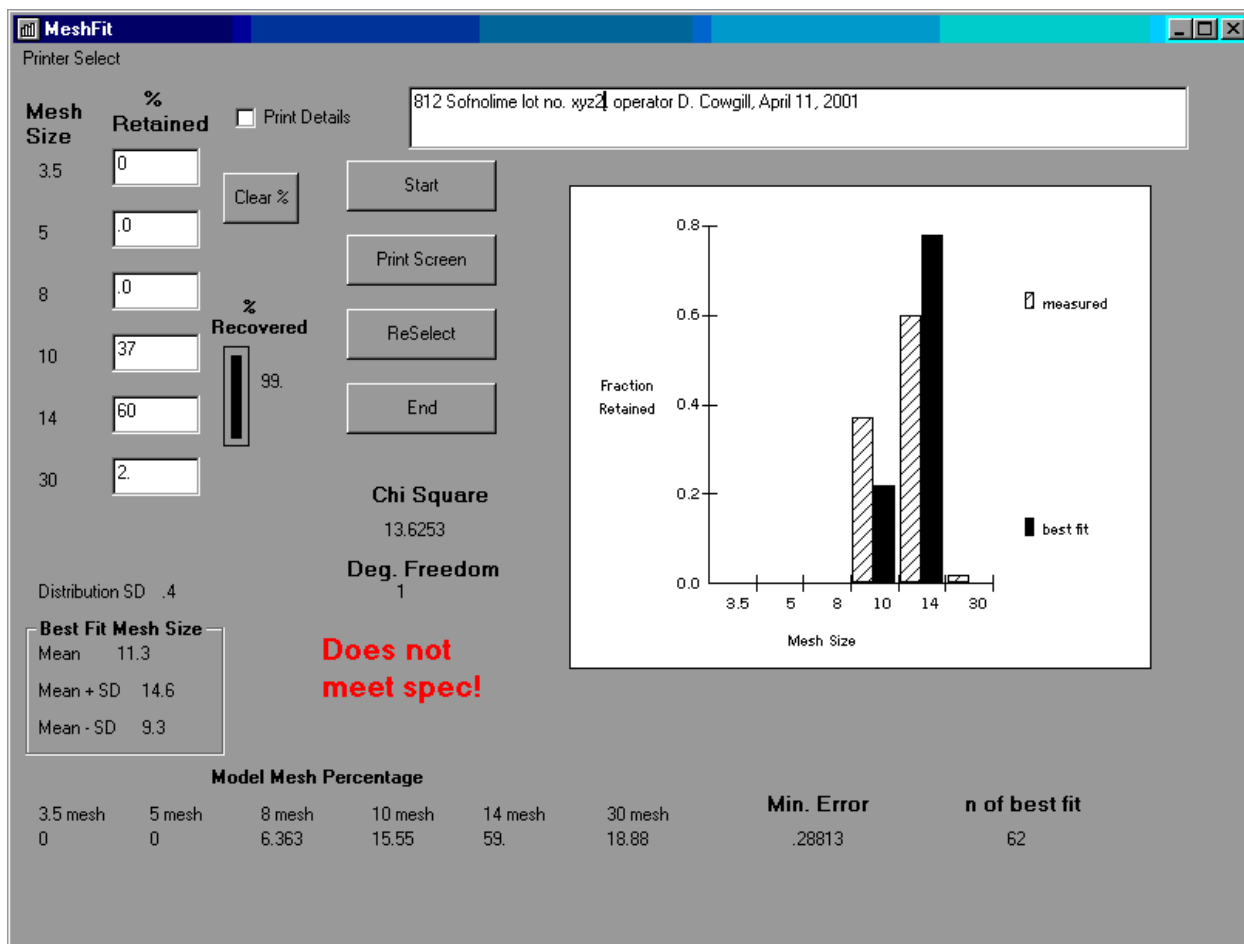


Figure G-2. MeshFit printout for a sample of 812 Sofnolime that does not meet the British specification for granule size distribution.