

REPORT DOCUMENTATION PAGE

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✓ Spreadsheet
✓ OTS

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MEMORANDUM FOR PRR (Contractor/In-House Publication)

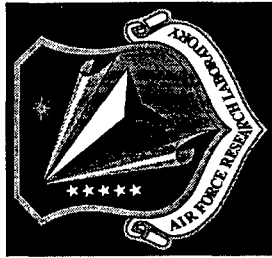
2 June 1999

FROM: PROI (TI) (STINFO)

SUBJECT: Authorization for Release of Technical Information, Control Number: AFRL-PR-ED-TP-FY99-0124
T.C. Miller, "Overdeterministic Fracture Analysis and Singular Value Decomposition"

SEM Conference slides/paper approved 25 Apr 99

(Public Release)



Overdeterministic Fracture Analysis and Singular Value Decomposition

Timothy C. Miller
Air Force Research Laboratory

Ravinder Chona
Texas A&M University

SEM Spring Conference
Cincinnati, Ohio
June 1999

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Outline of Presentation

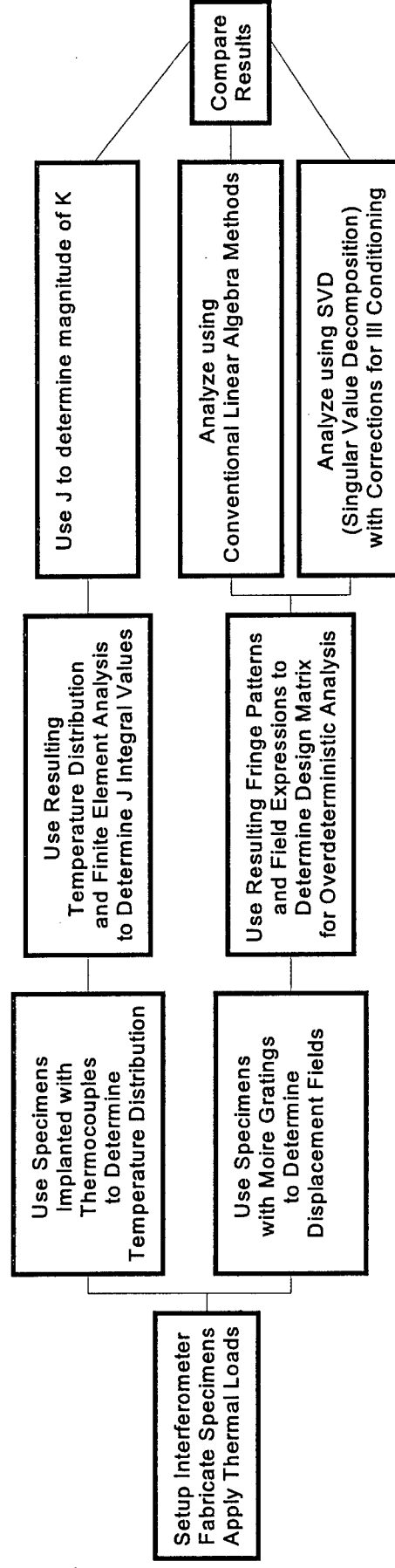
In Bimaterial Fracture Problems, Singular Value Decomposition Can Be Used to Improve Results for Stress Intensity Factor Calculations

- Experimental Procedures Used
- Causes of Ill Conditioning
- Use of Singular Value Decomposition
- Comparison of Results
- Conclusions



Outline of Experimental Methodology

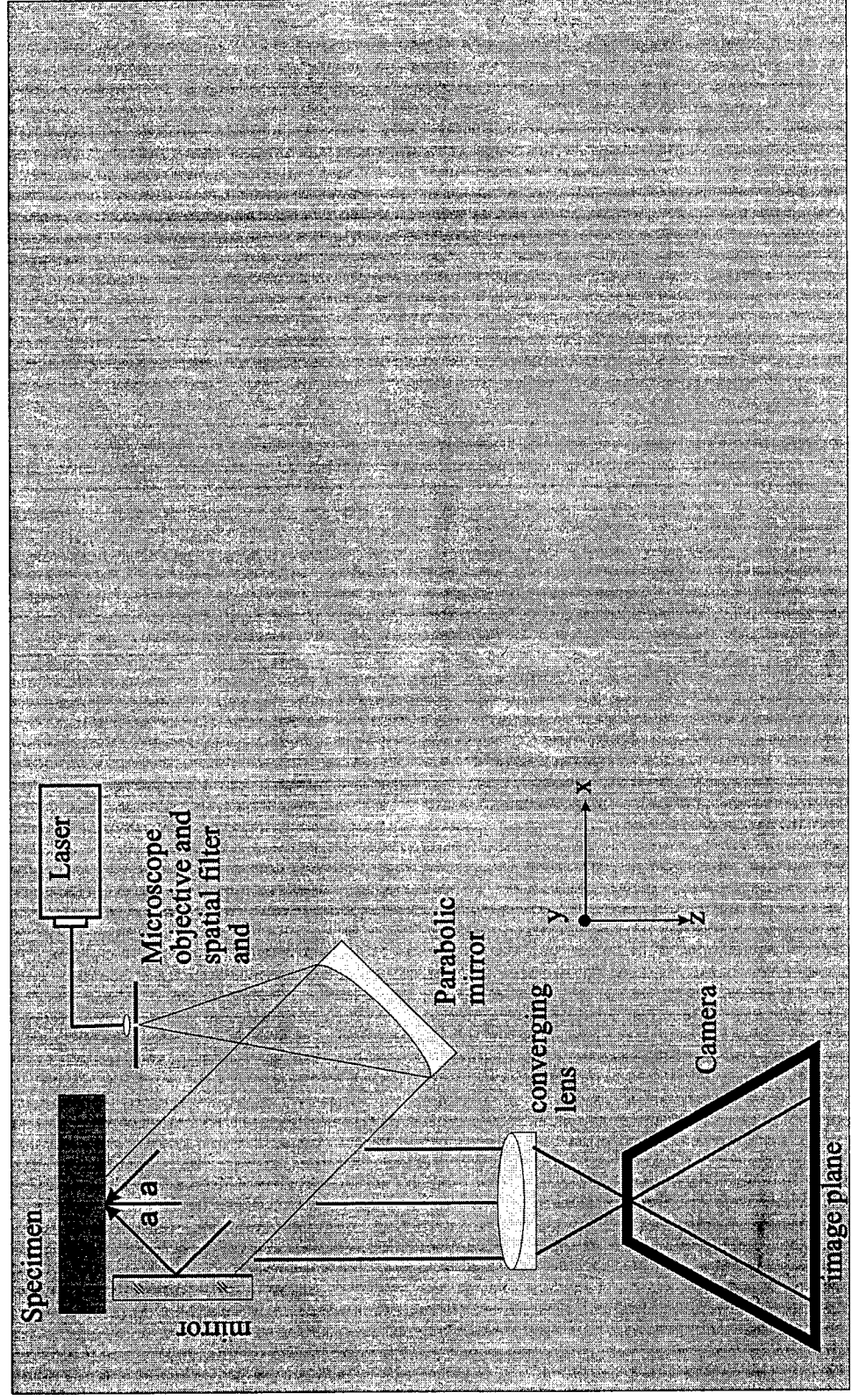
Magnitude of Complex Stress Intensity Factors Were Arrived at By Three Different Means

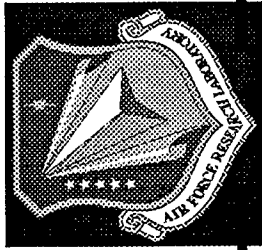




Moire Interferometer Setup

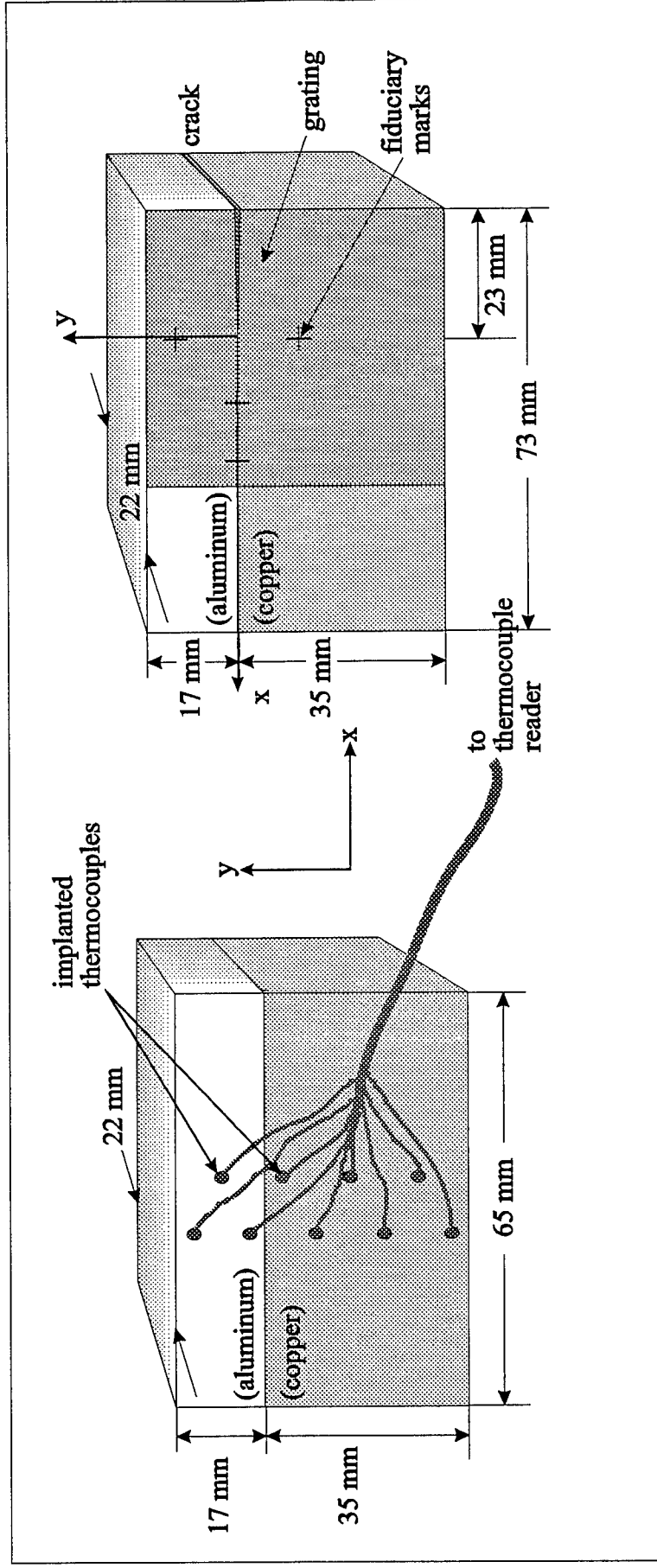
Simple Interferometer is Used to Capture U Displacement Fields





Specimen Fabrication for Aluminum- Copper Specimen

Shown are Specimens used with Thermal and
Displacement Field Analyses





Mechanical and Thermal Properties for the Aluminum-Copper Specimen

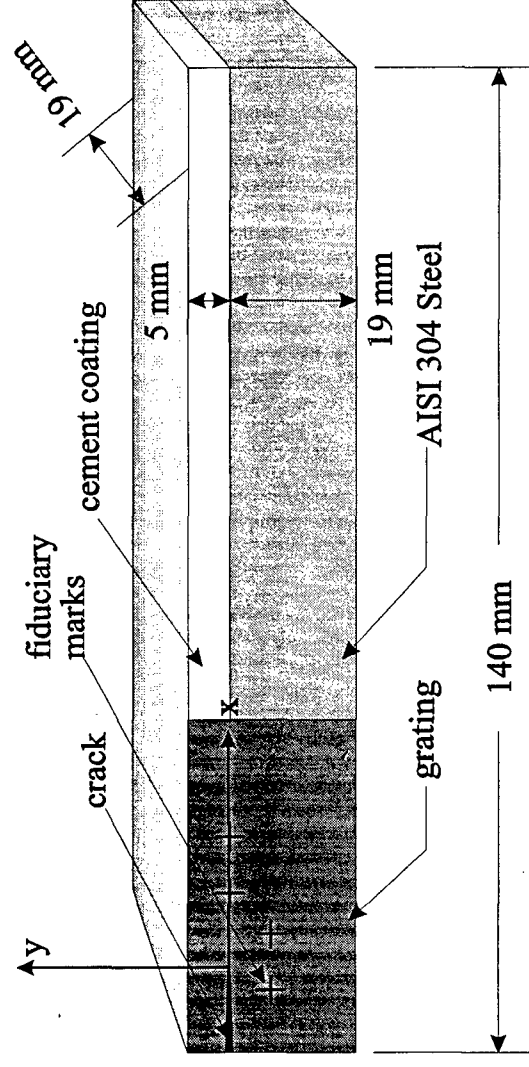
Property	Aluminum	Copper
Young's modulus [GPa]	71.7	120.0
Poisson's ratio	0.34	0.33
density [kg/m ³]	2770.0	8800.0
coefficient of thermal expansion [10 ⁻⁶ /K]	24.0	17.0
thermal conductivity [W/m ² K]	270.0	441.6
specific heat [J/kg K]	875.0	420.0



Specimen Fabrication for Steel-Thermocouple Cement Specimen

Highly Dissimilar Materials and Nearby Free Surface
Effects are Incorporated

(A Similar Specimen
was Constructed for
the Specimen with
Implanted
Thermocouples)





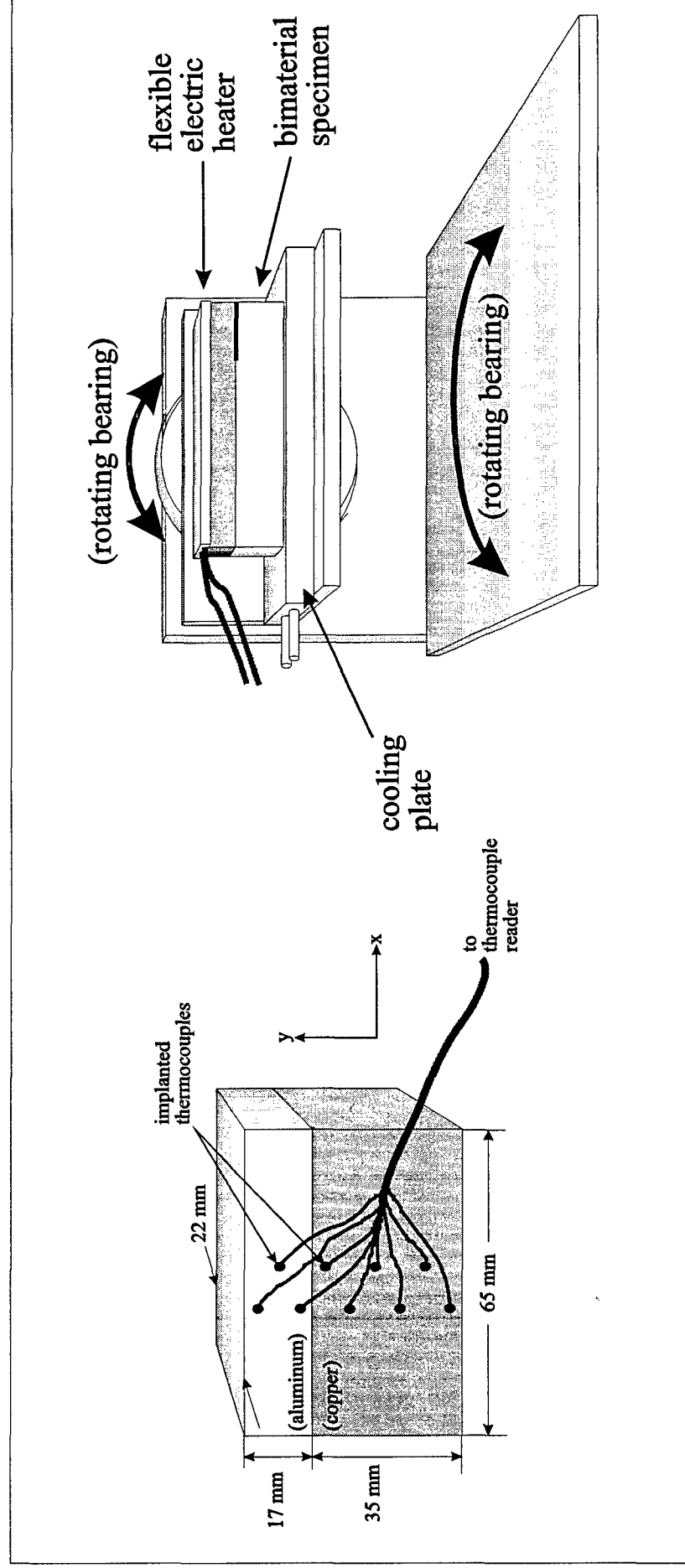
Mechanical and Thermal Properties for the Steel-TCC Cement Specimen

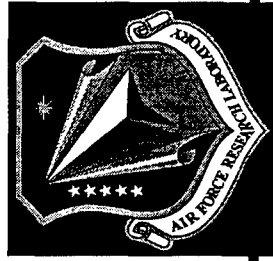
Property	AISI 304 Steel	TC cement
Young's Modulus [GPa]	218	3.24
Poisson's ratio	0.29	0.30
density [kg/m ³]	7834	3173
coefficient of thermal expansion [10 ⁻⁶ /K]	17.2	19.7
thermal conductivity [W/m ² K]	16.2	1.15
specific heat [J/kg K]	500	100



Application of Thermal Loads to Specimens

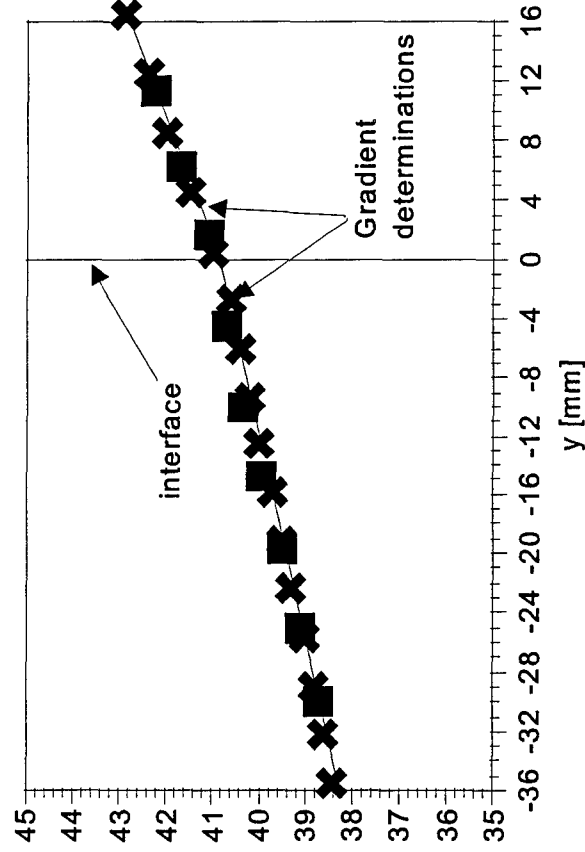
Applying Heating to Top of Specimen and Cooling to Bottom Provides One-Dimensional Temperature Fields
(Except Near the Crack Tip)





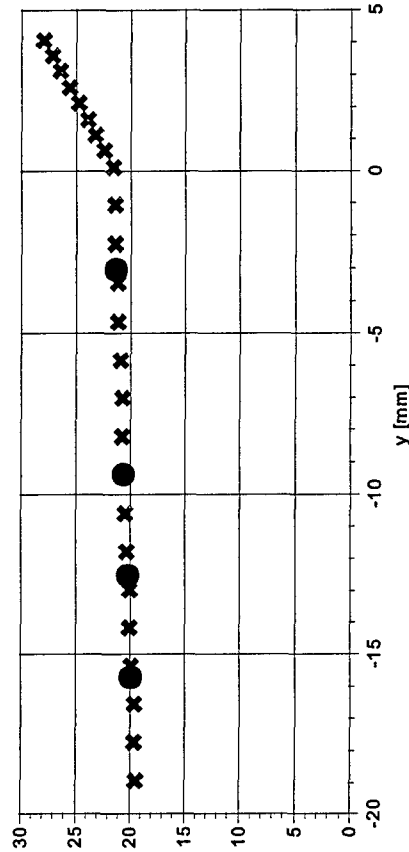
Determination of Temperature Distributions from Thermocouple Data

Thermal Boundary Conditions Can be Simulated Using Finite Element Methods



■ experimental ✕ finite element results

Aluminum-Copper Specimen



✕ Computational Results ● Experimental Results

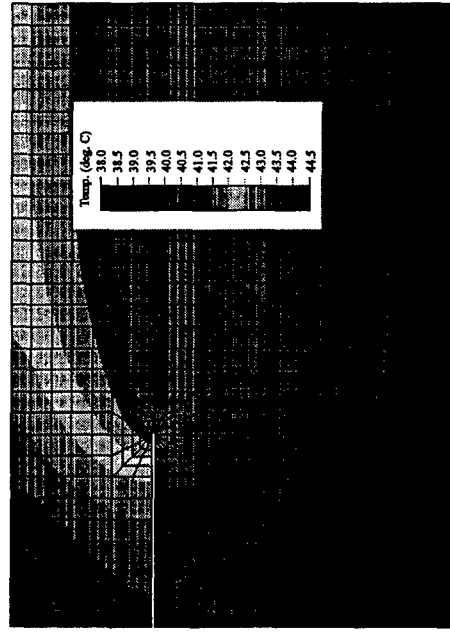
Steel-TCC Specimen



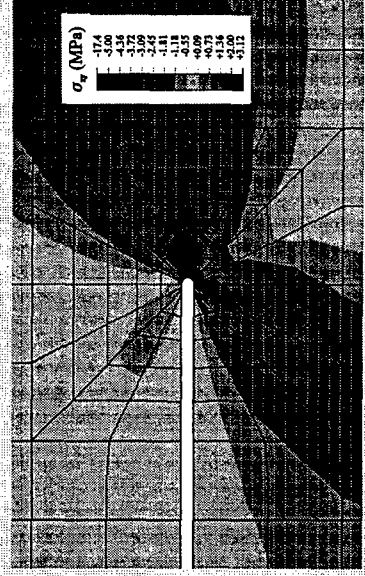
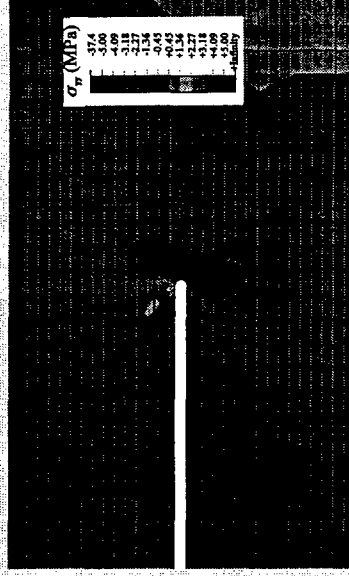
Thermal Loads and Numerical Modeling Gives Magnitude of $|K|$

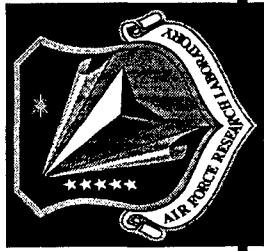
Finite Element Results Give Stress Fields, J Integral, and Magnitude
of Complex Stress Intensity Factor

Temperature Contours



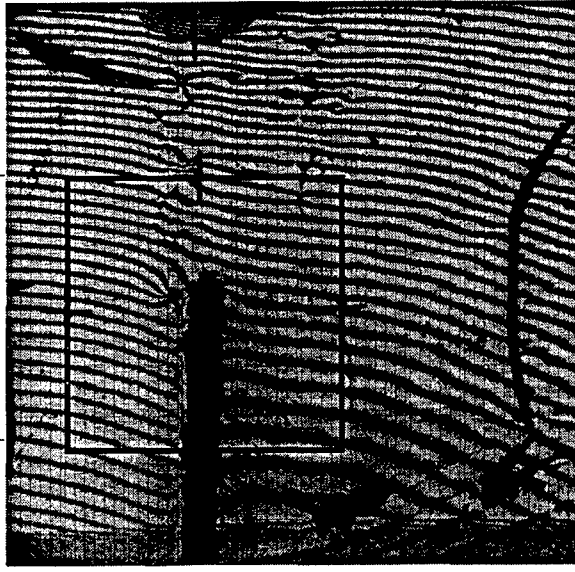
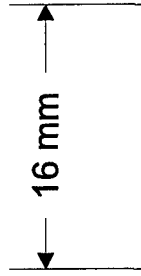
Stress Contours





Moiré Interferometry is Used to Obtain Fringe Patterns for Subsequent Analysis

Fringe Patterns Are Digitized and Data is Used in Overdeterministic Analysis



Aluminum-Copper Specimen



Steel-TCC Specimen



Causes of Unstable Solutions

In some cases the normal equations work adequately, but in other cases a very ill-conditioned matrix occurs, and the result is fitted parameters with very large magnitudes that are unstably balanced to cancel out almost precisely when the fitted function is evaluated.

This occurs most often because the data do not clearly distinguish between two or more of the basis functions provided. The result is a large set of ambiguous solutions exist.

In difficult problems, the ambiguities may be hard to discern.



Displacement Field Equations for Homogeneous Materials

Overall Field Equation is a Combination of Arbitrary
Translation/Rotation and Near Tip Deformations

$$u_y = P x + Q y + R$$

$$u_x = \frac{1}{E} \left[\sum_{j=0}^{\infty} C_{2j} \frac{r^{j+1/2}}{j+1/2} ((1-v)\cos(j+1/2)\theta - (1+v)(j+1/2) \sin\theta \sin(j-1/2)\theta) \right. \\ \left. + \sum_{j=0}^{\infty} C_{2j+1} \frac{r^{j+1}}{j+1} (2\cos(j+1)\theta - (1+v)(j+1) \sin\theta \sin(j\theta)) \right]$$

$$N(r,\theta) = L(r,\theta) + M(r,\theta)$$



Displacement Field Equations for Bimaterial Problems

$$L(x,y) = P_1x + R \quad y \geq 0$$

$$P_2x + R \quad y < 0$$

$$M(x,y) = \frac{1}{2\mu_1} [a_{0r}r^{1/2}(f_{0r})_1 - a_{0j}r^{1/2}(f_{0j})_1 + b_{0r}r(g_{0r})_1 - b_{0j}r(g_{0j})_1] \quad y \geq 0$$

$$= \frac{1}{2\mu_2} [a_{0r}r^{1/2}(f_{0r})_2 - a_{0j}r^{1/2}(f_{0j})_2 + b_{0r}r(g_{0r})_2 - b_{0j}r(g_{0j})_2] \quad y < 0$$

$$N(r,\theta) = L(r,\theta) + M(r,\theta)$$



Solution of Linear Algebra Problems Using Conventional Methods

Obtain
Displacement
Data from
Moire
Interferometer
Setup

Formulate
Linear
Algebra
Problem
Using
Displacement
Data and
Combined
Field
Expressions

Use Linear
Algebra to
Transform
Rectangular
Design Matrix
into a Square
Matrix

Solve Linear
Algebra
Problem to
Determine
Unknown
Coefficients in
Filed
Expression (and
Complex Stress
Intensity
Factor)



Solution of Linear Algebra Problems Using Conventional Methods

$$\{N\} = [f] \{C\}$$

$m \times 1 \quad m \times n \quad n \times 1$

$$[f]^T \{N\} = [f]^T [f] \{C\}$$

$$\text{Let } \{d\} = [f]^T \{N\}, \quad [a] = [f]^T [f]$$

$$\{d\} = [a] \{C\}$$

$n \times 1 \quad n \times n \quad n \times 1$



The Use of the SVD Method to Solve Overdeterministic Problems

Condition of Matrix is Determined and Adjusted for, if Necessary

Obtain
Displacement
Data from
Moire
Interferometer
Setup

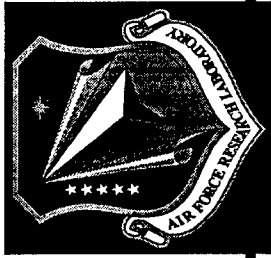
Formulate
Linear
Algebra
Problem
Using
Displacement
Data and
Combined
Field
Expressions

Decompose
Rectangular
Design Matrix
into $[U]$, $[V]$,
and $[W]$
Matrices

Check $[W]$
Matrix for
Presence of
Ill-
Conditioning

Correct for
Ill-
Conditioning,
if Necessary

Solve Linear
Algebra
Problem to
Determine
Unknown
Coefficients in
Filed
Expression (and
Complex Stress
Intensity
Factor)



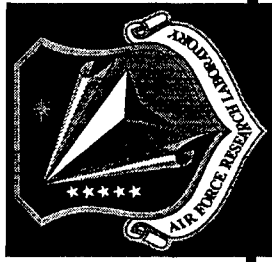
How the SVD Method Corrects for Ill-Conditioning

$$\{N\} = [f]\{C\} = [U][w][V]^T \{C\}$$

$m \times n \quad n \times n \quad n \times n$

Large condition numbers indicate ill-conditioning. When the reciprocal of the condition number approaches the computational precision, the presence of ill-conditioning will significantly affect solution accuracy. If this occurs, the adverse effects can be minimized by setting the terms $1/w_j$ in equal to zero for all sufficiently small w_j .

The effect of ill-conditioning is to produce an infinite set of solutions that all approximately solve the linear equation $\{N\} = [f]\{C\}$. Zeroing these diagonal elements selects from this set the solution the one that minimizes the residual $R = |\{N\} - [f]\{C\}|$. The result is that the SVD method with correction is often better than both direct methods and uncorrected SVD methods, as is shown by the experimental results below.



Use of the SVD Method to Solve Linear Algebra Problems

$$\{N\} = [f] \{C\}$$

$$m \times 1 \quad m \times n \quad n \times 1$$

$$\{N\} = [f]\{C\} = [U][w][V]^T \{C\}$$

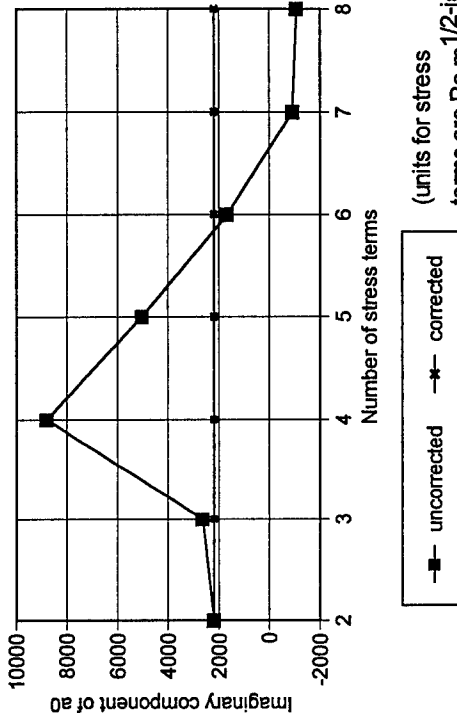
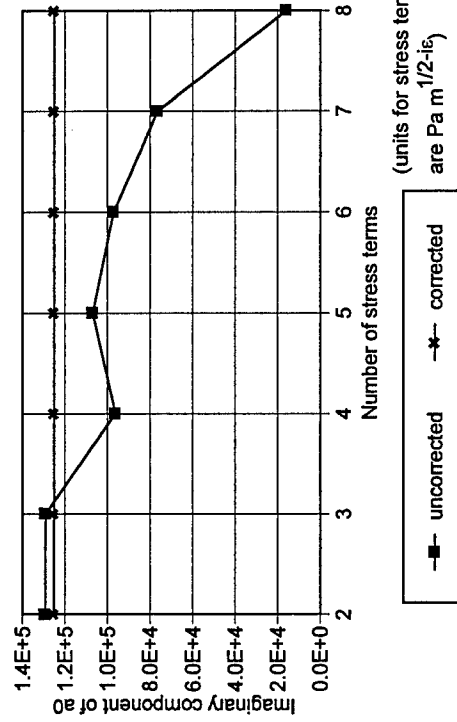
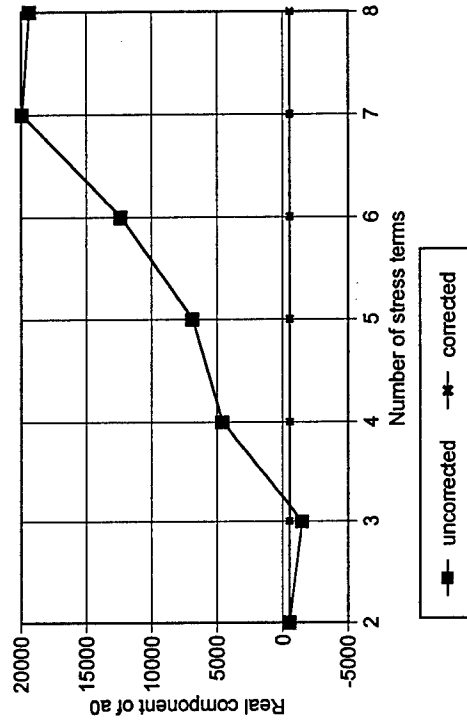
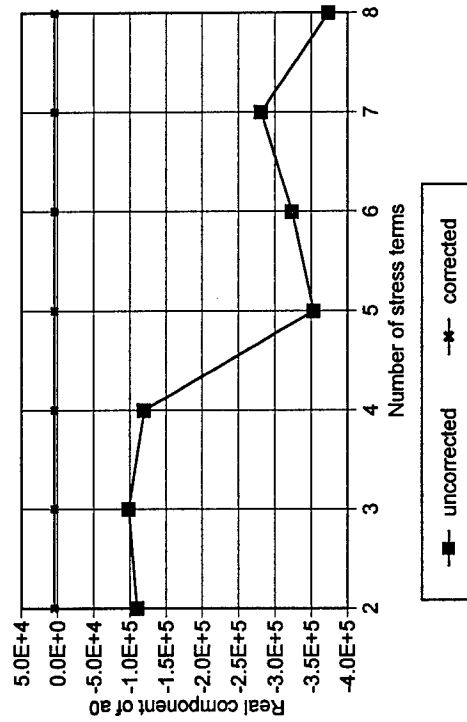
$$m \times n \quad n \times n \quad n \times n$$

$$\{C\} = [V] \cdot [\text{diag}(1/w_i)] \cdot (U^T \cdot \{N\})$$



Correcting for Ill-Conditioning Using SVD Gives Improved and More Stable Results

Shown Are Results for Aluminum-Copper Specimen (Left) and Steel-TCC Specimen (Right)



(units for stress terms are $\text{Pa m}^{1/2}$)

(units for stress terms are $\text{Pa m}^{1/2}$)



Results for Aluminum-Copper Bimaterial Specimen

Parameter	Trial 1	Trial 2	Average
P_1	375.8×10^6	395.6×10^6	385.7×10^6
P_2	338.6×10^6	346.5×10^6	342.6×10^6
$a_{or} [Pa m^{1/2-\epsilon}]$	4523	4920	4721
$a_{oj} [Pa m^{1/2+\epsilon}]$	125200	118300	121700
$ K' [Mpa m^{1/2}]$ (defined at 1 mm)	0.630	0.596	0.613
phase of K' [degrees] (defined at 1 mm)	-87.9	-87.6	-87.8
Related measurements			
Measurement	Source	Value	Related parameter
$(\epsilon_{xx})_1$	Fringe pattern plots (see Fig. 37)	394.0×10^6	P_1
$(\epsilon_{xx})_2$	Fringe pattern plots (see Fig. 37)	279.0×10^6	P_2
$ K' [Mpa m^{1/2}]$ (defined at 1 mm)	Finite element J integral calculation	0.601	$ K' $ or $ a_0 $ (above)



Results for Steel-TCC Bimaterial Specimen

Parameter	Experimental Value		
a_{or} [Pa m ^{1/2-ε}]	-526		
a_{oj} [Pa m ^{1/2-ε}]	2203		
$ K' $ [Mpa m ^{1/2}] (defined at 1 mm)	12.1		
phase of K' [degrees] (defined at 1 mm)	31.8		
Related measurements			
Measurement	Source	Value	Related parameter
$ K' $ [Mpa m ^{1/2}] (defined at 1 mm)	Finite element J integral calculation	11.1	$ K' $ or $ a_0 $ (above)



Comparison of Experimental and Numerical Results

Use of Local Collocation Method with SVD Gives Good Agreement with Numerical Computations

Material Pair	SVD-Local Collocation Method	Finite Element Results
aluminum-copper	0.613	0.601
steel-thermocouple cement	12.1	11.1

(Units are MPa m^{1/2})



Conclusions

- The Presence of a Nonzero Bimaterial Parameter in Interfacial Fracture Problems Can Cause Problems When Overdeterministic Methods Are Employed.
- One Means to Remedy the Problem of Ill-Conditioning is to use Singular Value Decomposition to Solve the Linear Algebra Problem.