Formulation of Maximized Weighted Averages in *URTURIP* Technique

Bruno Migeon, Philippe Deforge, Pierre Marché

Laboratoire Vision et Robotique
63, avenue de Lattre de Tassigny, 18020 Bourges Cedex - France
e-mail: {migeon, deforge, marche}@bourges.univ-orleans.fr

Abstract - **The existing methods developed in URTURIP Technique (Ultrasound Reflection-mode Tomography Using Radial Image Processing) can be considered as a part of a reconstruction family called Maximized Weighted Averages Technique, and it is the object of this paper. It is shown that a reconstruction can be viewed as a filter and that the three commonly used methods can be rewritten with the same formulation. Moreover, interesting new methods can be developed by using other adapted filters.**

Keywords - **URTURIP Technique, Ultrasound Tomography, 2D Reconstruction.**

I. Introduction

As a part of our project concerned with the development of an ultrasound scanner dedicated to limb study, the URTURIP Technique (Ultrasound Reflection-mode Tomography Using Radial Image Processing) has been developed [1]. It consists in using classical B-scan images instead of projections [2-5] and gives qualitative images instead of quantitative images. The final goal of this project is the 2D and 3D reconstruction of anatomical structures at limb level by using echographic image processing. The developed process consists of several successive steps like : multiple reflection removing [6], 2D reconstruction [1], segmentation [7], contour association [8], contour interpolation [9], 3D reconstruction and visualization [10]. It has been validated by in vitro experiments on anatomical pieces of limbs of new-borns using a simple acquisition system prototype [10]. Now, in vivo experiments are carried out thanks to a calibration technique [11] which allows a good positioning of the probe and an accurate determination of the rotation centre.

Most of the existing methods developed in URTURIP Technique can be considered as a part of a reconstruction family called Maximized Weighted Averages Technique, and it is the object of this paper. It is shown that a reconstruction can be viewed as a filter and that the three commonly used methods can be rewritten with the same formulation. Moreover, several new interesting methods can be developed by using other filters. It allows adapting the reconstruction to an application and choosing a good contrast between the different structures to be observed.

II. The URTURIP Technique

A. Principle of reconstruction

The Ultrasound Reflection-mode Tomography Using Radial Image Processing (URTURIP Technique) principle is to utilize radial B-scan images instead of projections as most other methods do [2-5]. In comparison, fewer radial directions are needed, it is less time-consuming, but qualitative images instead of quantitative ones are computed.

Let L_i^* i=1,..,N be N radial images obtained from N angulary equidistant directions around the rotation center where $L_i^*(k, l)$ denotes the luminance of the (k, l)-pixel on the L_i^* image.

An adjustment step consists in turning each L_i^* image around the rotation center with its own acquisition angle, to construct N adjusted images L_i in such a way that a pixel (x, y) on each L_i image corresponds to the same real point of the cross-section.

Then, a method based on the URTURIP Technique reconstructs an image L by a combination of the N adjusted images L_i , i.e. $L(x, y) = f(L_i(x,y))$, i=1..N, where f denotes the reconstruction method.

B. The commonly used methods

The three commonly used methods are the method of maxima, the method of averages and the method of maximized averages [1]. They are defined as follows.

The method of maxima is given by :

$$\forall(x,y) \qquad L(x,y) = \max_i L_i(x,y) \qquad i = 1..N$$

The method of averages is given by :

$$\forall(x,y) \qquad L(x,y) = \frac{1}{N}\sum_{i=1}^{N} L_i(x,y)$$

The maximized averages method combines the respective advantages of previous two basic method in order to obtain a good image quality even with few radial images. Its formula is as follows:

$$\forall(x,y) \qquad L(x,y) = \max_i L'_i(x,y) \qquad i = 1..N,\ N \geq 5$$

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Abstract

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with $$L'_i(x,y) = \frac{1}{2m+1} \sum_{j=n_i}^{n_{i+1}} L_{k(j)}(x,y) \qquad i = 1..N$$

$$\begin{cases} n_i &= i - m \\ n_{i+1} &= i + m \end{cases} \qquad m = \begin{cases} E\left(\frac{N}{4}\right) & \text{if } E\left(\frac{N}{4}\right) \neq \frac{N}{4} \\ \frac{N}{4} - 1 & \text{otherwise} \end{cases}$$

$$k(j) = \begin{cases} j & \text{if } j \in [1..N] \\ j - N & \text{if } j > N \\ j + N & \text{if } j < 1 \end{cases}$$

where E(z) denotes the integer part of z.

III. THE MAXIMIZED WEIGHTED AVERAGES

A. Formulation

The three previous method can be described by a same formulation defined as follows :

$$\forall (x,y) \in L \qquad L(x,y) = \underset{i}{\mathrm{Max}}\ L'_i(x,y) \qquad i = 1..N$$

with $$L'_i(x,y) = \frac{1}{\sum_{j=-N/2+1}^{N/2} C(j)} \sum_{j=1}^{N} C(n(j)) L_j(x,y)$$

where

$$n(j) = \begin{cases} j - i + N & \text{if } j < \mathrm{Sup}\left(i - \frac{N}{2},\ 1\right) \\ j - i & \text{if } \mathrm{Sup}\left(i - \frac{N}{2},\ 1\right) \leq j \leq \mathrm{Inf}\left(i + \frac{N}{2},\ N\right) \\ j - i - N & \text{if } j > \mathrm{Inf}\left(i + \frac{N}{2},\ N\right) \end{cases}$$

C is a filter defining each method. Moreover, C being a symmetric filter, its restriction C^* on [0, N/2] can be considered and the formulation becomes :

$$\forall (x,y) \in L \qquad L(x,y) = \underset{i}{\mathrm{Max}}\ L'_i(x,y) \qquad i = 1..N$$

with $$L'_i(x,y) = \frac{1}{C^*(0) + 2\sum_{j=1}^{N/2} C^*(j)} \sum_{j=1}^{N} C^*(n(j)) L_j(x,y)$$

where $$n(j) = \begin{cases} |j - i| & \text{if } |j - i| \leq \frac{N}{2} \\ N - |j - i| & \text{if } |j - i| > \frac{N}{2} \end{cases}$$

B. Rewritten of the existing methods

It clearly appears that the three commonly used methods correspond to three common filters (uniform, impulsive and square window). They can be rewritten with the same formulation and the corresponding filter C or C^* which defines the method used.

The method of *averages* (fig. 1) is defined by :

$$C(k) = 1 \quad \forall\ k = -N/2+1, .., N/2 \qquad \text{or}$$
$$C^*(k) = 1 \quad \forall\ k = 0, .., N/2$$

The method of *maxima* (fig. 2) is defined by :

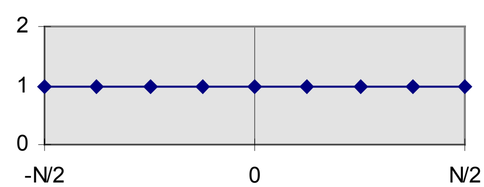
$$C(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & k \in [-N/2+1, N/2] - \{0\} \end{cases} \qquad \text{or}$$

$$C^*(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } 1 \leq k \leq N/2 \end{cases}$$

The method of *maximized averages* (fig. 3) is defined by :

$$C(k) = \begin{cases} 1 & \text{if } -m < k < +m \\ 0 & k \in [-N/2+1, N/2] - \{0\} \end{cases} \qquad \text{or}$$

$$C^*(k) = \begin{cases} 1 & \text{if } -m < k < +m \\ 0 & \text{if } 1 \leq k \leq N/2 \end{cases}$$



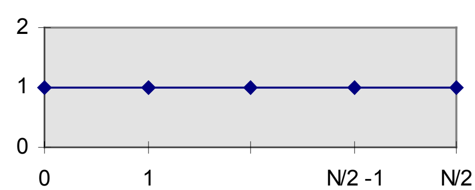


Fig. 1 : C and C* for the method of *averages*

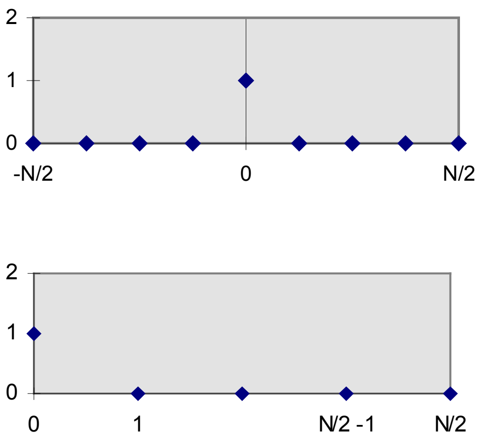


Fig. 2 : C and C* for the method of *maxima*

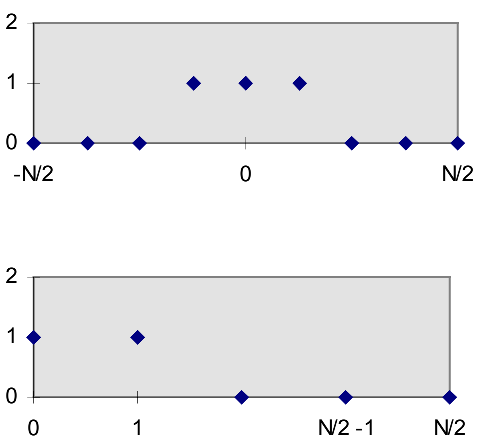


Fig. 3 : C and C* for the method of *maximized averages*

C. New methods

Of course, in this family of *Maximized Weighted Averages Methods*, other methods can be developed. In the context of a comparative study, different methods have been tested using different filters [12] like Haming, Hanning, median, ...

The *Maximized Weighted Averages Methods* using an exponential filter (fig. 4) or a gaussian filter (fig. 5) give quite interesting and good results and are currently used in our project. They are defined by :

$C(k) = e^{-\sigma|k|}$ for $k \in [-N/2+1, N/2]$ or

$C^*(k) = e^{-\sigma k}$ for $k \in [0, N/2]$ and

$C(k) = e^{-k^2/2\sigma^2}$ for $k \in [-N/2+1, N/2]$ or

$C^*(k) = e^{-k^2/2\sigma^2}$ for $k \in [0, N/2]$, where $\sigma > 0$ is an interesting coefficient to weight the influence of radial images. It can be adapted to different applications and depth field configurations.

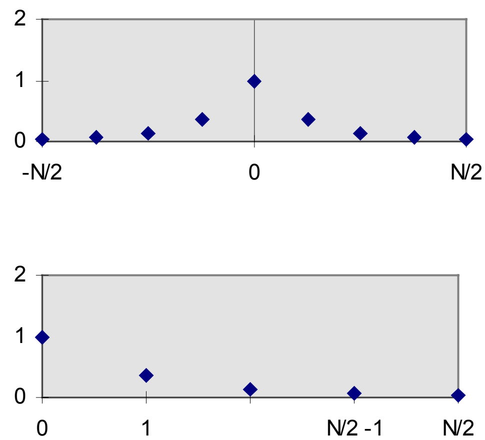


Fig. 4 : C and C* for a exponential filter

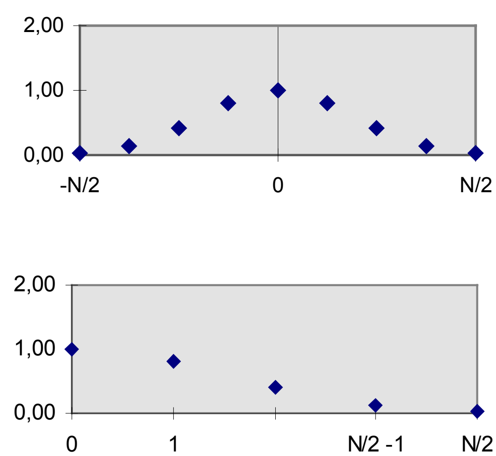
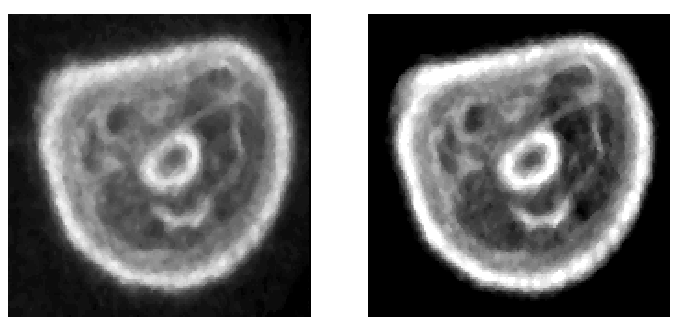


Fig. 5 : C and C* for a gaussian filter



(a) gaussian filter (b) exponential filter

Fig. 6 : 2D reconstruction at thigh level

Fig. 6 shows a reconstructed cross-section at thigh level with maximized weighted averages methods respectively based on a gaussian filter and an exponential filter. It gives good contrast to distinguish the different anatomical structures.

IV. Conclusion

In the context of a project of the development of an ultrasound scanner dedicated to limb study, the URTURIP Technique has been developed.

In this paper, Maximized Weighted Averages methods are described. Thanks to a unique formulation all the three commonly used methods can be rewritten. It is shown that they correspond to different classical filters. Moreover, in the same category of Maximized Weighted Averages, new interesting methods can be developed by using other filters.

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