Effect of Cardiac Phases and Conductivity Inhomogeneities of the Thorax Models on ECG Lead Selection and Reconstruction

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abstract **- ECG lead selection and reconstruction were investigated in the present study using ECG source-to-measurement transfer matrices computed in inhomogeneous and homogeneous conductor thorax-heart models, which represent end-systolic and end-diastolic cardiac phases. ECG leads were selected from a set of 120 leads for body surface potential mapping. The transfer matrices of the 120-lead system were reconstructed from a transfer matrix of selected leads. Effects of changing the cardiac phase and the conductivities on reconstruction performance are reported here. The number of selected leads well matched to the results of earlier works conducted by other researchers.**
Keywords - Body surface potential mapping, ECG lead, cardiac phase, volume conductor model

I. Introduction

Body surface potential mapping (BSPM or BSM) is a method to display electric field potential distribution on the torso surface generated by the cardiac activity. From generated maps, features related to diagnostic categories are observed [1]. There are BSM measurement configurations mostly varied by the number of ECG leads and electrode positions, for example [2, 3, 4] and more summarized by [5]. While there is not a standard configuration for BSM, several computational studies have been reported aiming at specifying necessary electrode positions or at reducing the number of positions. Some of them are, say, dataset approach, e.g., Barr et. al. [6], Lux et. al. [7], and Kornreich et. al. [8], which use real ECG data, and the others are model approach, e.g., Dössel et. al. [9], which use ECG source-to-measurement transfer system computed in the thorax-heart models. The present study is a model approach and applies a lead selection algorithm, which is mathematically the same as that in Lux et. al.'s dataset approach, to a matrix of source-to-measurement transfer coefficients. The ECG source in Dössel et. al.'s model was the epicardial potentials, while in the present study's are dipole current sources distributed in the entire heart.

Since the potentials on BSM are detected by unipolar leads, selection of electrode positions and that of ECG leads can be considered as questions equivalent to each other if we are allowed not to be concerned with the reference electrode configuration.

A uniqueness of the present study may be that lead selections and reconstruction coefficients are computed in models of different cardiac phases and with different conductivity modelings, and those obtained in each model are also applied to the other models.

II. Study Material

Four thorax models with abbreviations used in the present study are listed below.

- i-sys, Inhomogeneous conductor model, end-systolic cardiac phase,
- i-dia, Inhomogeneous, end-diastolic,
- h-sys, Homogeneous, end-systolic,
- h-dia, Homogeneous, end-diastolic.

They were inhomogeneous models and infinite homogeneous models. With each conductivity modeling, two thorax-heart models were constructed; one had the heart of the end-systolic cardiac phase, and the other had the heart replaced with that of the end-diastolic phase. In each of the inhomogeneous models, ECG source-to-measurement transfer matrix, $\mathbf{C}$, was computed using the finite difference method (FDM). $\mathbf{C}$ transforms dipole current source vectors at all FDM node points forming the heart model. The numbers of such nodes were 12032 and 13120 in the end-systolic heart model and in the end-diastolic model, respectively. The full-BSM lead system had 120 unipolar leads with the Wilson central terminal as the reference level including 117 body surface leads and three limb leads [2]. Therefore, the matrix size of $\mathbf{C}$ was 120-by-M where M was 12032×3 in the systolic phase model and 13120×3 in the diastolic phase model. A vector $\mathbf{u}$ consisting of source coefficients of all dipole sources is transformed to a 120-length vector $\mathbf{v}$ consisting of 120 lead voltages:

$$\mathbf{v} = \mathbf{C}\mathbf{u} \tag{1}$$

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Abstract

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Each row in $\mathbf{C}$ transforms $\mathbf{u}$ into an ECG voltage of one of the BSM leads.

The transfer matrices in the homogeneous models were computed using the dipole field equation [10], but the source points and the electrode locations were the same as in the inhomogeneous models.

III. Method

The lead selection and reconstruction method here is based on the Gram-Schmidt Orthogonalization (GSO) [11], and it is an iterative algorithm applied to I vector entries $\mathbf{a}_{i=1:I}$ in a matrix $\mathbf{A} = \begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_I\end{bmatrix}$ which is the transpose of an ECG source-to-measurement transfer matrix, i.e., $\mathbf{A} \leftarrow \mathbf{C}^{\mathrm{t}}$. I is the maximum number of ECG leads; namely, $I = 120$. $\mathbf{A}$ can be given as a matrix product

$$\mathbf{A} = \mathbf{QR} \tag{2}$$

where the columns, $\mathbf{q}_{i=1:I}$, of matrix $\mathbf{Q}$ are mutually orthogonal basis vectors of $\mathbf{A}$, and $\mathbf{R}$ is an I-by-I matrix. Let $\mathbf{O}$ denote the inverse matrix of $\mathbf{R}$ so that $\mathbf{Q}$ is given as

$$\mathbf{AO} = \mathbf{Q} \tag{3}$$

$\mathbf{A}$ is reconstructed (or approximated) from N columns selected from $\mathbf{A}$. The reconstructed matrix is expressed as $\mathbf{A}^{(N)}$ where the superscript (N) indicates the number of selected columns in $\mathbf{A}$. The reconstruction error matrix $\mathbf{E}^{(N)}$ is defined as $\mathbf{E}^{(N)} = \mathbf{A} - \mathbf{A}^{(N)}$ and it consists of error vectors $\mathbf{e}_{i=1:I}$:

$$\mathbf{E}^{(N)} = \begin{bmatrix}\mathbf{e}_1^{(N)} & \mathbf{e}_2^{(N)} & \cdots & \mathbf{e}_I^{(N)}\end{bmatrix} \tag{4}$$

N also denotes the iteration number. The iterative process selects one column in $\mathbf{A}$ in each iteration. As the initialization (when $N = 0$), $\mathbf{E}^{(0)}$ is set equal to $\mathbf{A}$.

In each iteration, $N = 1 : I$, the index of error vector whose 2-norm is the largest of $\mathbf{e}_{i=1:I}^{(N-1)}$ is detected and stored as Nth element, s_N , in the lead selection sequence list $\mathcal{S}$:

$$s_N = \arg\max_{i=1}^{I} \|\mathbf{e}_i^{(N-1)}\|_2 , \ N = 1 : I \tag{5}$$

$$\mathcal{S} = \begin{pmatrix} s_1, & s_2, & \ldots , & s_I \end{pmatrix} \tag{6}$$

$\mathbf{q}_N$ is set equal to $\mathbf{e}_{s_N}^{(N-1)}$, detected by (5):

$$\mathbf{q}_N = \mathbf{e}_{s_N}^{(N-1)} , \ N = 1 : I \tag{7}$$

and row N in $\mathbf{R}$ are set by

$$r_{N,i} = \frac{{\mathbf{q}_N}^{\mathrm{t}} \mathbf{e}_i^{(N-1)}}{{\mathbf{e}_i^{(N-1)}}^{\mathrm{t}} \mathbf{e}_i^{(N-1)}} , \ i = 1 : I \tag{8}$$

where $r_{N,i}$ is the element on Nth row in ith column of $\mathbf{R}$, and ${\mathbf{q}_N}^{\mathrm{t}} \mathbf{e}_i^{(N)}$ is the inner product of $\mathbf{q}_N$ and $\mathbf{e}_i^{(N)}$.

As seen in (7), $\mathbf{q}_{n=1:N}$ are copies of $\mathbf{e}_{s_n}^{(n-1)}$ which are transformations of N columns in $\mathbf{A}$ whose indices are first N entries in the selection sequence $\mathcal{S}$.

$\mathbf{O}^{(N)}$ consisting of N columns left-most in $\mathbf{O}$ gives $\mathbf{q}_{n=1:N}$ (see (3)):

$$\mathbf{AO}^{(N)} = \mathbf{Q}^{(N)} \tag{9}$$

Using the corresponding N rows in $\mathbf{R}$ as the entries in a matrix $\mathbf{R}^{(N)}$, the reconstruction matrix, denoted by $\mathbf{H}^{(N)}$, is defined as $\mathbf{H}^{(N)} = \mathbf{O}^{(N)}\mathbf{R}^{(N)}$. $\mathbf{H}^{(N)}$ transforms $\mathbf{a}_i$, whose indices i are first N elements in $\mathcal{S}$, to $\mathbf{A}^{(N)}$: $\mathbf{AH}^{(N)} = \mathbf{A}^{(N)}$. Defining the error generator matrix, $\mathbf{D}^{(N)}$, such as

$$\mathbf{D}^{(N)} = \mathbf{I} - \mathbf{H}^{(N)} \tag{10}$$

$\mathbf{E}^{(N)}$ is directly obtained from $\mathbf{A}$: $\mathbf{AD}^{(N)} = \mathbf{E}^{(N)}$.

(2) is presented as the QR factorization, e.g., [12], in which (5) does not generally take place, either in the GSO, but $\mathcal{S} = \begin{pmatrix}1, 2, \ldots , I\end{pmatrix}$.

As the evaluation function, a normalized version of the squared Euclidean norm of $\mathbf{E}^{(N)}$, is employed and denoted by $\epsilon^{(N)}$:

$$\epsilon^{(N)} = \frac{\|\mathbf{E}^{(N)}\|_E}{\|\mathbf{A}\|_E} \tag{11}$$

$\epsilon^{(N)}$ is the "overall" reconstruction error ratio; i.e., it does not indicate an error level of particular lead.

$\mathbf{H}^{(N)}$ and $\mathbf{D}^{(N)}$ can be applied to any matrix if its matrix size is the same as $\mathbf{A}$ from which the $\mathbf{H}^{(N)}$ and the $\mathbf{D}^{(N)}$ are computed. The process to compute them is referred to as *training*, and evaluation by $\epsilon^{(N)}$ applying $\mathbf{D}^{(N)}$ to a matrix, which may or may not be the $\mathbf{A}$, is referred to as *test* in the following.

Four tests were executed in each model such that $\epsilon^{(N)}, N = 1 : I$, were computed applying the $\mathbf{D}^{(N)}, N = 1 : I$, obtained from all the four models.

IV. Result

$\epsilon^{(N)}$ in the tests with different models are compared in Figure 1. The number of leads, N, was incremented from 1 to 120 where 120, which is the number of full-set BSM leads. The minimum N to obtain $\epsilon^{(N)}$ less than or equal to certain values are briefly summarized in Table 1. They are not significantly different from the numbers previously reported by other researchers [6, 7, 9]. The cardiac phase of the heart did not strongly affect the test performance; i.e., the same electrode set and reconstruction coefficients may be used in models of different cardiac phases if all the other factors in the models are same. The inhomogeneities made significant differences, especially when

$\min N$ for $\epsilon^{(N)} \leq 0.1$

	Test Model			
Training Model	i-sys	i-dia	h-sys	h-dia
i-sys	16	18	15	18
i-dia	17	17	15	16
h-sys	18	20	12	14
h-dia	20	21	13	13

$\min N$ for $\epsilon^{(N)} \leq 0.01$

	Test Model			
Training Model	i-sys	i-dia	h-sys	h-dia
i-sys	30	35	37	37
i-dia	31	32	35	38
h-sys	53	48	29	31
h-dia	45	45	29	30

Table 1: The minimum numbers of leads, N, to obtain reconstruction error ratio $\epsilon^{(N)}$ less than or equal to 0.1 and 0.01 are tabulated. Lead selection sequences $\mathcal{S}$ and lead reconstruction error generator matrices $\mathbf{D}^{(N)}$ were obtianed in traninigs. In the tests $\epsilon^{(N)}$ were computed using the $\mathbf{D}^{(N)}$ for $N = 1, 3, \ldots$ as simulations of incrementing the leads according to $\mathcal{S}$.

$\epsilon^{(N)}$ became low. The difference of conductivities examined here is an extreme case — inhomogeneous or infinite extent of homogeneous conductivity. The results suggest that the inhomogeneity/homogeneity setting should not be changed for accurate lead reconstructions in model-based studies.

The reconstruction matrix, $\mathbf{H}$, does not have an effect of amplification or attenuation. Its effect is a rotation in a vector space. Thus, only relative conductivities in each model concerns when models are different. The effects by changing models are most likely due to distribution patterns of conductivities.

V. Conclusion

The numbers of leads necessary for certain levels of full-BSM reconstruction were not significantly different from those reported by other researchers. Difference of cardiac phases of the models did not affect the reconstruction performance while difference of inhomogeneous/homogeneous conductivities clearly affected. The lead selection method in the preset study is mathematically the same as one applied to real ECG dataset [7]. The next interest will be exchangeability of lead selection and reconstruction coefficients obtained by model approach and by dataset approach.

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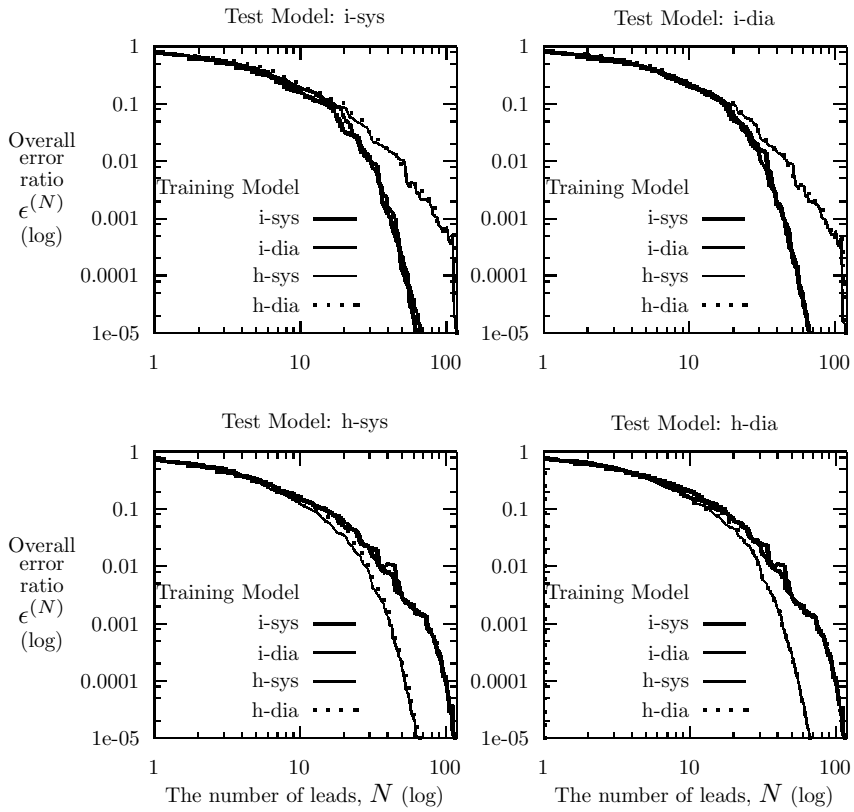

Figure 1: The error generator matrices, $\mathbf{D}^{(N)}$, were obtained from four thorax models (as training models) and applied to the four models (as test models). The overall error ratio is an evaluation function of lead reconstruction performance. N is the number of leads to be selected, i.e., it is the number of vectors selected from the transfer matrix of test model from which the entire transfer matrix itself is reconstructed using the reconstruction matrix. The lead selections and incorporating reconstruction matrices were obtained from training models. The error generator matrix generates the error matrix in reconstruction tests. The plots above show changes in the reconstruction error by incrementing the number of selected leads. The abbreviations of the models are as follows: i-sys: inhomogeneous end-systolic model, i-dia: inhomogeneous end-diastolic model, h-sys: homogeneous end-systolic model, h-dia: homogeneous end-diastolic model.