NEW DOMAIN BLOCK PARTITIONING BASED ON COMPLEXITY MEASURE OF ECG

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Abstract-**This paper introduces a new domain block partitioning scheme for a nonlinear iterated function systems (NIFS) compression of electrocardiogram (ECG) signals, based on their complexity measure. The idea behind the scheme is based on the multifractal characteristics of the ECG signal. The partitioning is intended to reduce the time-consuming inverse problem in fractal compression. The proposed technique gives computational complexity of *O*(*N*) for a time series with length *N*. The segmented NIFS achieves a compression ratio of 8.8:1 under a distortion error of 5.8%, as compared to that of 6.0:1 obtained by ∅ien and Nårstad's orthogonal transform.**

Keywords-**ECG, IFS, affine transform, multifractality, complexity measure.**

I. INTRODUCTION

A fractal object may be expressed in terms of a series of mapping transforms of itself. In data compression of the traditional linear iterated function systems (IFS) approach, a time-consuming search needs to be performed on the entire signal to obtain an optimal match between a small range and some other part of the signal because the linear affine transform is not flexible to model complicated signals locally. For example, The computational complexity of Barnsley's collage coding for an image of $N \times N$ size is $O(N^6)$ [1]. In order to reduce the computational complexity, Jacqin proposed a fractal block coding (FBC) technique $O(N^4)$ [2]. The FBC splits the image into two kinds of small blocks: domain blocks and range blocks. Each range block is compared to affine-transformed versions of the domain blocks. The most similar pair gives the optimal affine transform for that range block. Kinsner *et al.* suggested a reduced FBC $O(N^3)$ in which a neural-network based classification technique is used to find an optimal match between the range and domain blocks [3]. We have applied this approach first to images [3], and later to speech through the residual in the code excited linear prediction (CELP) technique [4]. A more efficient search approach is necessary to compress electrocardiogram (ECG) signals by fractal technique in real time. We will make use of fractal properties to develop a fast IFS approach in ECG data compression. Before the discussion of fractal compression, it is necessary to know what a fractal signal and its properties are.

A random discrete process $x(n)$ defined for all integers n, $-\infty < n < \infty$, is said to be statistically self-similar if its statistics are invariant to dilations and contractions of the waveform in time. More specifically, a random discrete process $x(n)$ is statistically self-similar with a parameter D if for any real $\alpha > 0$, it obeys the scaling relation [5]

$$x(n) \overset{p}{=} \alpha^{-D} x(\alpha n) \tag{1}$$

where $\overset{p}{=}$ denotes equality in a statistical sense, and D is related to a fractal dimension (or complexity) of the fractal object.

Equation (1) reveals two important properties in the fractal object: (i) self-similarity which signifies scale invariance and (ii) fractal dimension which signifies the structural and informational (compositional) complexity. The real α in (1) signifies the invariance to dilations and contractions, which is the foundation of the IFS. The IFS uses a set of contractive mappings from the signal to itself to represent the fractal object. However, the complexity of the fractal object is not employed by the IFS to find the mappings.

We conjecture that a strong self-similarity of the fractal object exists in the area where the signal has the same compositional complexity, and the self-similarity is much weaker between regions with different complexities. If that is true, then it is not necessary to search domain blocks which have different complexities with the range block for the multifractal object. Therefore, if we can measure the complexity of the signal and only search domain blocks which have the same complexity with the range block, the search can be very efficient.

It has been shown that the ECG signal is multifractal, with its singularity varying with time [6]. Thus, we could segment the signal into ranges with similar fractal dimensions. Based on this idea, a new scheme is proposed to segment the ECG signal for the construction of an optimal domain pool. We use the variance fractal dimension trajectory (VFDT) technique [7] to measure the complexity of the signal, and then partition it into segments. Instead of taking

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Abstract

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the domain blocks from almost every point of the signal, our pool is only composed of the partitioned segments, which reduces the search problem to $O(N)$ if we take the search size to be N. The application of a nonlinear affine transform [8] on such a domain pool can reduce the reconstruction error, while maintaining fast search. Experimental results show that our approach can achieve lower reconstruction error and faster implementation speed under the same compression ratio than the traditional IFS.

For clarity and completeness, a continuous ECG signal is first obtained from multi-channel sensors, then amplified and filtered to satisfy the Nyquist sampling theorem, then sampled to form a discrete signal, and finally quantized to form a digital ECG signal. This paper is concerned with the digital ECG form only.

II. Variance Fractal Dimension and Partitioning

Since we know that the ECG signal has multifractal characteristics [6], it is possible to segment the signal based on a local fractal dimension estimate. Fractal dimensions such as Rényi dimension, Mandelbrot dimension, correlation dimension, and Haussdorf-Besicovitch dimension, require a large number of sample points to obtain a statistical measure [9]. Usually, such a measure is a global estimate for the fractal object. Instead of a single-scale statistical measure, the variance fractal dimension (VFD) is estimated by computing the spread of the increments in the signal amplitude, based on a much smaller number of points within a window of interest, and at several scales. A time series representing a chaotic or nonchaotic process can be analyzed directly in time through its VFD [7].

Now we illustrate how to calculate the variance dimension from a given time series. Let us assume that the signal $x(n)$ is discrete in time n. The variance, σ^2, of its amplitude increments over a time increment is proportional to the time increment according to the following power law

$$Var[x(n_2) - x(n_1)] \sim |n_2 - n_1|^{2H^*} \tag{2}$$

where H^* is the Hurst exponent. By setting $\Delta n = |n_2 - n_1|$ and $(\Delta x)_{\Delta n} = x(n_2) - x(n_1)$, the exponent H^* can be calculated from

$$H^* = \lim_{\Delta n \to 0} \frac{1}{2} \frac{\log[Var(\Delta x)_{\Delta n}]}{\log(\Delta n)} \tag{3}$$

For the embedding Euclidean dimension D_E (i.e., the number of independent variables in the observed signal), the variance dimension can be computed from

$$D_\sigma = D_E - H^* \tag{4}$$

For measured data, it is not practical to set $\Delta n \to 0$. Instead of using (3), H^* can be obtained from a log-log plot in which it is related to the slope. To spread the point on the log-log plot equally, a finite sequence of time increments, $\{\Delta n_1, \Delta n_2, ..., \Delta n_m\}$, should follow a b-adic sequence (such as dyadic), as shown in Fig. 1. First, we calculate the following two log values for the plot

$$X_m = \log_b \Delta n_m \tag{5}$$

$$Y_m = \log_b(Var(\Delta x)_m) \tag{6}$$

where Δn_m is the discrete time increment at the mth order scale. Then the slope s is determined from these points by a polynomial line fitting method, with careful attention given to outliers [9]. The Hurst exponent is obtained from

$$H^* = s/2 \tag{7}$$

If the time series is stationary, this analysis produces a single VFD.

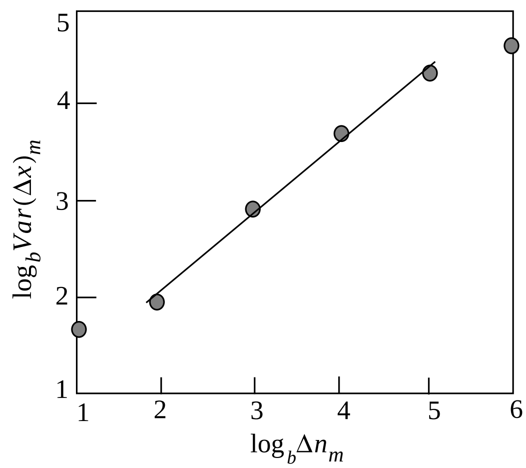

Fig. 1 The log-log plot and its line fitting.

Fractal analysis of nonstationary signals is usually conducted in terms of fractal dimensions as a function of time, thus resulting in the VFD trajectory (VFDT) which reflects the change of the compositional complexity of the signal with time. In fact, the VFDT represents the multifractal characteristics of signals with time. The VFDT is generated by calculating local VFDs for a rectangular sliding window that is displaced along the entire signal recording of length N. For each position of the window, the VFD is calculated with Δn_m set to an integer, ranging from 1 to the length M of the sliding window. The b-adic rule is not followed to accommodate the small M.

To partition the ECG signal optimally, two parameters need to be considered in computing the VFDT, including the rectangular sliding window size M for the local VFD calculation, as well as its displacement. The displacement is set to 1 sample point, which leads to the highest accuracy to locate partitioning points for the ECG signal at hand. The N should be in the range where the signal can be considered as stationary over the length of the window. If the window size is too large, it causes the dimensions of locally distinct fractals to be buried in the dimensions of their most significant adjacent fractals, and at the same time is computationally expensive. On the other hand, a relatively small window cannot provide sufficient data to cover the stationary components of the signal for the analysis. Therefore, the correct selection of the window is important. The window size is obtained experimentally, as described in Section IV. Generally, the parameter setting should be dependent on the nature of the signal being analyzed and can be studied experimentally [10][11].

III. The Nonlinear IFS (NIFS)

In an IFS-based data compression, the optimal domain block is searched point-by-point in the signal. It is very time-consuming. Now with the ECG signal which is segmented according to the complexity, we propose a new search scheme. According to (1), the self-similarity of a fractal signal exists in segments with the same complexity. Thus, since the point-by-point search of the entire signal is not necessary, we can reduce the search to the blocks segmented by the VFDT technique. In other word, the domain pool is composed of the blocks segmented according to different compositional complexities in the signal. The number of domain blocks in the domain pool is now much smaller than that in the traditional IFS approach.

After the new domain pool is ready, we apply the IFS technique to compress the ECG signal. The main effort in the IFS compression is to find a set of affine transform W between the range blocks and domain blocks as following

$$W\begin{pmatrix} n \\ x \end{pmatrix} = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix} \begin{bmatrix} n \\ x \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \tag{8}$$

where a, c, d, e, and f are the transform coefficients. Then, the compression ratio and the reconstruction error are calculated based on the set of transform to evaluate the performance of the algorithm.

Figure 2 gives an original ECG signal and its reconstruction by the IFS. Although the search speed of finding the transform set is fast, the compression performance suffers from large reconstruction errors, especially in the QRS area. One of the reasons is that the affine transform defined by (8) is not a proper transform for the ECG signal.

Since (8) defines a linear transform, it should not be applied to signals with nonlinear characteristics. The ECG signal, like speech and images in nature, comes from a nonlinear system and is very complicated. Although it is self-similar in the area with the same compositional complexity in the ECG signal, the affine transform cannot represent the abrupt change in the QRS complex.

The self-similar characteristics can be represented by a nonlinear IFS. Instead of the affine transform, we have used an extended nonlinear IFS (NIFS) transform to compress the ECG signal in another paper [8]. For a one-dimensional signal $x(n)$, a generalized transform is defined as

$$W\begin{pmatrix} n \\ x \end{pmatrix} = \begin{bmatrix} an + e \\ g(x, n) \end{bmatrix} \tag{9}$$

where the coefficient a is limited to the range $[0, 1)$ to guarantee contraction of signals in time, $an + e$ carries out a mapping from a domain block to a range block in time, and $g(x, n)$ is a nonlinear function. If $g(x, n)$ is also a contractive transform, then (9) satisfies the convergence condition of the IFS.

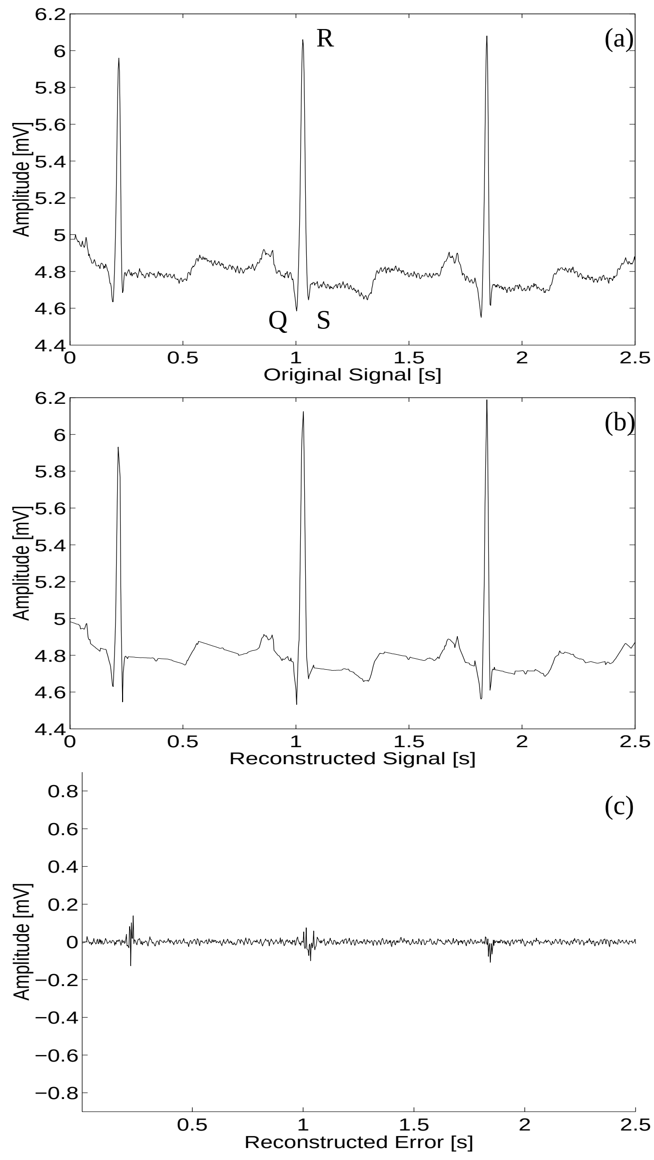

Fig. 2 The ECG signal compression with the IFS approach. (a) Original ECG signal. (b) Reconstructed ECG signal. (c) Reconstruction error.

Taylor's remainder formula and the least square error are employed to find the nonlinear function, $g(x, n)$. The performance of the nonlinear approach may compete with the traditional one, as the nonlinear transform gives more concentrated distribution of the mapped domain address around the first point in a segment than the affine transform [8], which is useful to develop a fast IFS algorithm.

In principle, Equation (9) may fit an arbitrary function. Therefore, we combine the nonlinear IFS and the VFDT technique to develop a fast IFS algorithm for multifractal signal compression. Such scheme must be validated by experiment results.

IV. Experimental Results and Analysis

In this section, experiments are performed to compress the ECG signal by combining the VFDT and the NIFS together. The ECG signal sampled at 360 Hz with 11 bit resolution was obtained from the MIT-BIH ECG database [12]. We compress the ECG data contained in the file, x_100.txt, which is a 10-minute recording.

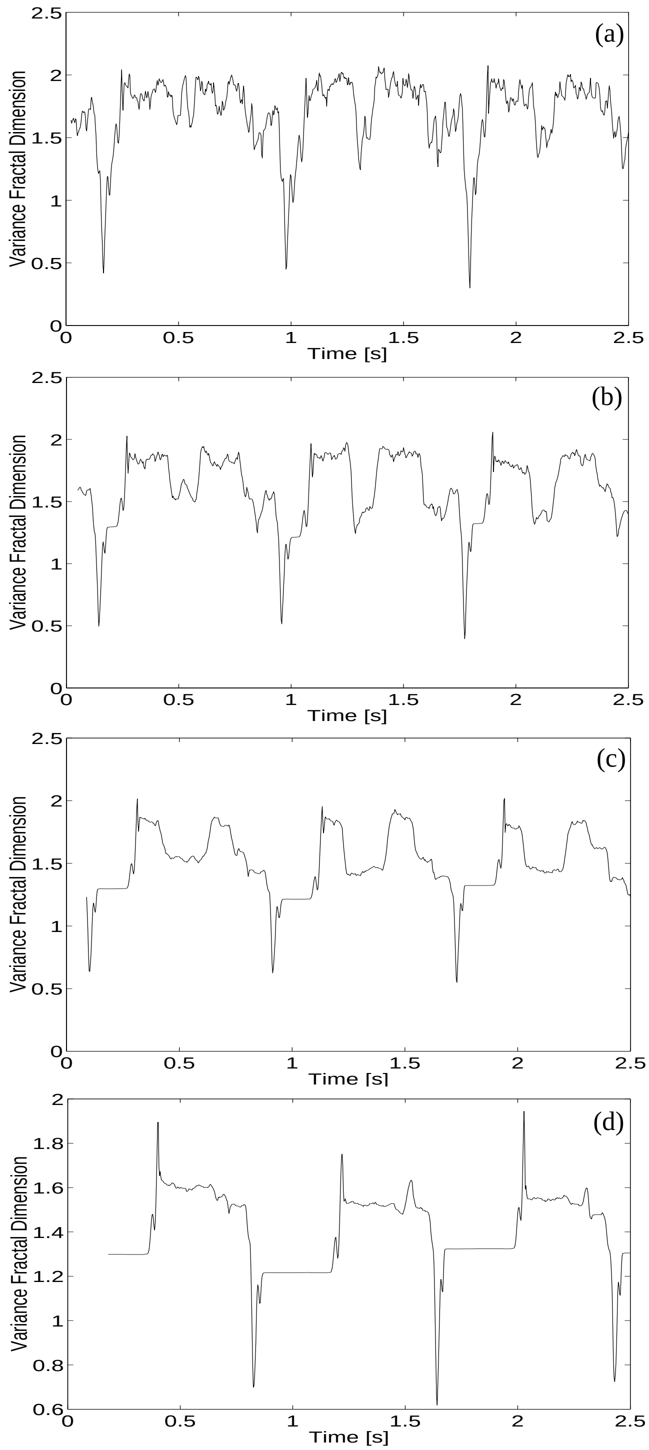

Fig. 3 The variance fractal dimension trajectory of an ECG signal with window size of (a) 16, (b) 32, (c) 64, and (d) 128 sample points.

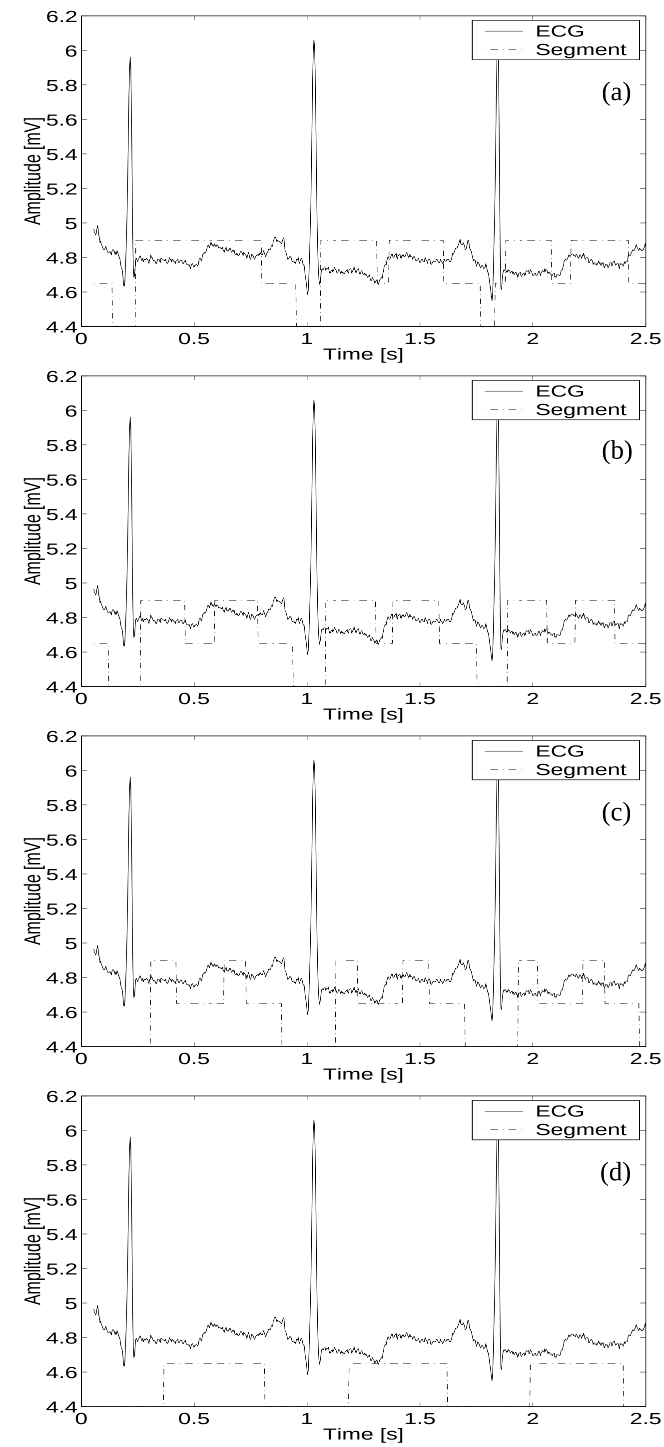

Fig. 4 Partitioning of an ECG signal based on complexity estimate with window size of (a) 16, (b) 32, (c) 64, and (d) 128 sample points.

To achieve good compression performance, the sliding window size for the VFDT, as well as the expansion order, and the quantization resolution should be chosen properly for the algorithm. We can see that the complexity segments change with the window size. An optimal sliding-window size is chosen experimentally by inspecting the VFDT, computed for different numbers of samples varying from *Nmin* to *Nmax*, as shown in Fig. 3. The histogram of the VFDT of an ECG signal shown in Fig. 5 has three peaks. For the VFDT shown in Fig. 3, two thresholds are selected experimentally to separate the trajectory into three levels, thus segmenting the ECG into three areas with different complexity. Figures 4(a) to 4(d) illustrate the corresponding segmentations of the ECG signal. Figure 4(d) shows that the sliding window with 128 sample points give too wide areas for the low complexity parts. Figure 4(c) with 64 sample point window is still too wide. On the other hand, Fig. 4(a) shows that the wide segmentation in the high complexity areas is caused by the window size of 16 sample points being too short. Consequently, Fig. 4(b) with window size of 32 sample points gives a proper segmentation for the ECG signal because it is partitioned into intervals with (i) high but uniform complexity, (ii) middle but varying complexity and (iii) low complexity representing the QRS complex. In this paper, the sliding window with 32 sample point width is chosen as an optimal parameter for the complexity measure of the ECG signal.

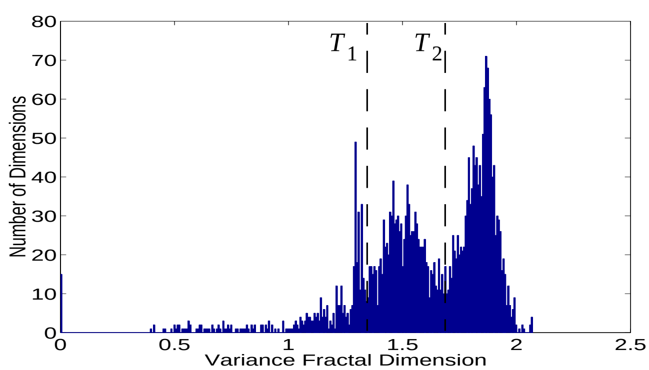

Fig. 5 Histogram of the VFDT of an ECG signal with sliding window of 32 sample points.

Table 1: Compression performance investigation of the combined VFDT and NIFS approach with quantization resolution and polynomial expansion change on an ECG signal.

Resolution (bit)	k=1		k=2		k=3		k=4	
	PRD	Rcr	PRD	Rcr	PRD	Rcr	PRD	Rcr
7	27%	5.1	9.7%	4.5	9.3%	2.2	14%	1.6
8	7.1%	5.3	6.8%	4.3	5.5%	3.4	8.1%	2.7
9	6.7%	5.4	6.3%	6.9	5.8%	5.3	5.9%	4.4
10	6.2%	6.2	5.9%	7.7	5.2%	5.3	5.6%	5.1
11	5.8%	6.1	5.9%	8.8	5.7%	7.4	5.8%	6.3
12	5.9%	6.7	5.9%	8.6	6.1%	7.7	5.7%	6.5
13	5.9%	6.4	5.6%	8.5	5.8%	8.1	5.9%	7.7
14	6.0%	7.0	6.2%	9.4	5.6%	7.9	5.7%	8.0

Before the NIFS approach is applied to the ECG signal, the domain pool is prepared based on the signal partitioning according to the complexity measure. Instead of constructing the traditional domain pool by shifting a block point-by-point in the time series, the blocks in the new domain pool are bounded by the partitioning. Although an exact reduced number of domain blocks cannot be given, the partitioning may reduce the domain-block number to about $T/5$ times in our scheme, where T is the average period of the ECG signal. We use a personal computer with Pentium(R) III of 600 MHz to run Matlab programs of the ECG compression algorithm. The linear IFS requires 3540 s to compress an ECG signal of length of 1024, while the segmented NIFS requires 106 s only, which is over 33 times faster.

Now the NIFS is applied on the ECG signal using the reduced domain pool. Table 1 shows the influence of the order k of the Taylor series expansion and quantization resolution on the compression performance of an ECG signal. By changing the expansion order from 1st to 4th, as well as the resolution of the quantizer from 7 bits to 14 bits, various percent root-mean-square difference (*PRD*) and compression ratio (*Rcr*) are obtained [8]. By monitoring the change among them, optimal parameters for the NIFS on the ECG signal can be found. Table 1 shows that when the resolution increases, the *PRD* decreases and the *Rcr* increases. A high-resolution quantizer will benefit the *PRD* but not the *Rcr.* An optimal quantization resolution may be found based on the trade-off between the *RMS* and the *Rcr.* When the quantizer takes more than 10 bits, the *PRD* and the *Rcr* change little with the resolution increasing further. One can choose an 11-bit nonuniform quantizer with a Laplacian distribution for each coefficient. In the future, we will use different quantizer for different coefficient.

One may predict that for a high expansion order the *PRD* should decrease because of the more powerful modelling ability of the NIFS. However, a higher order results in more coefficients, thus leading to a lower compression ratio. Consequently, there must be a trade-off between the *RMS* and the *Rcr* for choosing the optimal order k. Although Table 1 demonstrates that the change in k has almost no influence on the *PRD* after resolution of 9 bits, it results in different *Rcr.* The expansion order of 2 is taken as the optimal parameter in this paper because it can achieve the highest compression ratio under about the same *PRD* in the experiments.

It was found that the NIFS achieves higher compression ratio than the traditional IFS under the same reconstruction error in the ECG signal compression with the segmentation. When $PRD = 5.8\%$, ∅ien and Nårstad achieved 6.0:1 with their IFS ECG compression by orthogonal transform [13]. With the optimal choice of order 2 and 11 bit nonuniform quantizer, we get the compression ratio about 6.1:1 for the linear affine transform and about 8.8:1 for the NIFS. Thus, the NIFS outperforms the traditional IFS approach under the fast domain block search.

Figure 6 shows a compression and reconstruction of an

ECG signal by the proposed scheme with optimal parameters. Unlike the reconstruction error obtained by the IFS, here the error is controlled within a certain range. The QRS complex in the ECG signal is reconstructed almost perfectly.

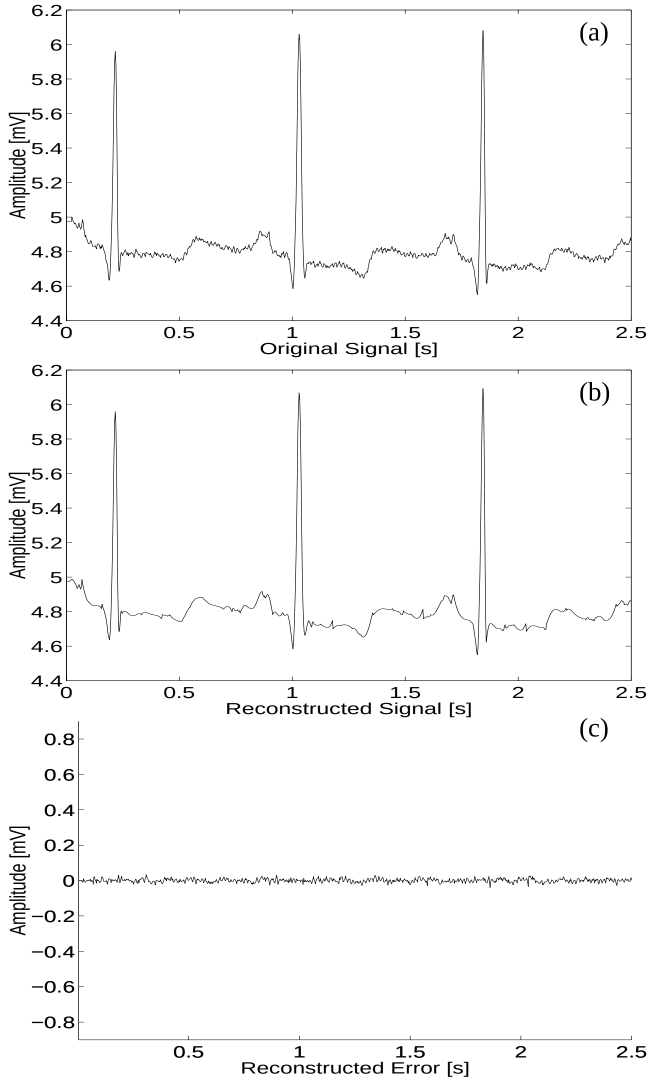

Fig. 6 The ECG signal compression with the combined VFDT and NIFS approach under expansion order 2 and 11 bit nonuniform quantizer. (a) Original signal. (b) Reconstructed signal. (c) Reconstruction error.

V. Conclusion

In this paper, we apply the NIFS and signal partitioning approach based on the compositional complexity measure by the VFDT to compress a one-dimensional ECG signal fast. The domain search speed is reduced to $O(N)$ for a time series with length N, compared with the computational complexity of $O(N^2)$ of the IFS.

With the domain pool prepared by the VFDT, the compression ratio obtained by the IFS decreases from 11.1:1 to 6.1:1 sharply. Opposite to that, the *Rcr* decreases from 10.2:1 to 8.8:1 for the NIFS, which outperforms the traditional IFS technique for the ECG signal compression [8]. The compression ratio given by the segmented NIFS is also higher than that of 6.0:1 obtained by ∅ien and Nårstad under the same *PRD* of 5.8%. We conclude that the NIFS has more flexible modelling ability than the IFS. Compared with a much weaker self-similarity between regions with different complexities, a strong self-similarity of fractal object exists in the area with the same complexity.

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