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14. ABSTRACT

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The focus of the current project was the development of the technologies needed to support robust p-version mesh generation of general non-manifold geometric domains of interest to the NAVY.

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Final Technical Report

Contract Number: N00014-99-1-0725

Project: Automated p-Version Mesh Generation for Naval Structures

Principal Investigator: Mark S. Shephard

1. Technical Objective

This research is considering the technologies needed to realize the promise of optimum rates of convergence of adaptive high order methods based on hp-discretizations. The focus of the current project was the development of the technologies needed to support robust p-version mesh generation of general non-manifold geometric domains of interest to the Navy.

2. Technical Approach

To meet the goal of providing appropriate meshes for p-version finite element analysis for general domains, an automatic mesh generation procedure to create curved finite element meshes with sufficient order of geometric approximation based on the order of polynomial used in the finite element basis was created. Although the initial straight-sided mesh generated by automatic mesh generators has all valid elements, the process of curving the mesh edges and faces to the appropriate model boundaries often yields elements with invalid shapes. Therefore, the developed procedures perform mesh modifications to produce a set of valid curved elements.

A key ingredient is the geometric representation of the mesh entities. The standard method of Lagrangian interpolation does not lend itself to effective procedures as the order of the elements increases. Therefore, an alternative geometric form based on Bezier approximating geometry was developed.

Many of the physical domains of interest to the Navy are structures dominated by thin sections for which the desired finite element discretizations employ thin volume elements. For the most effective application of p-version finite elements it is desirable to generate elements in these thin volumes that do not have long diagonals going through the "thickness direction". This creates problems for current automatic mesh generators that use only tetrahedra. Therefore, procedures were developed to eliminate, to the greatest possible extent, such diagonals.

3. Results

3.1 Procedure for the generation of curved meshes

The procedure to curve the elements of an initially straight-sided mesh employ a set of mesh curving tools based on standard operators and a set of mesh modification operators that are capable of modifying the local mesh topology. The full set of mesh modification operators includes swap, split, and collapse operators that can be applied to various mesh entities. In addition to those applied as independent operations, there are times when they can be applied in various combinations. Finally, there is a general cavity creation and triangulation procedure that can be applied as needed.

These procedures [6,10] have been implemented in the mesh generation tool kit [7] and have been shown to provide superior results to previous more ad-hoc approaches [2,3]. Experience on the geometry correction of straight-sided meshes (that is, the snapping of refinement points [4]) indicates that careful analysis of the local situation is most effective in determining which mesh modification operations to apply to ensure there is progress in improving the mesh. The development of these procedure to curved elements is complicated by the need to do the checks against the curved element geometry which does introduce substantial computational cost. Figure 1. shows the application of this procedure where both quadratic and cubic element geometries are used, while Figure 2. shows the application of the procedure on a mechanical part.
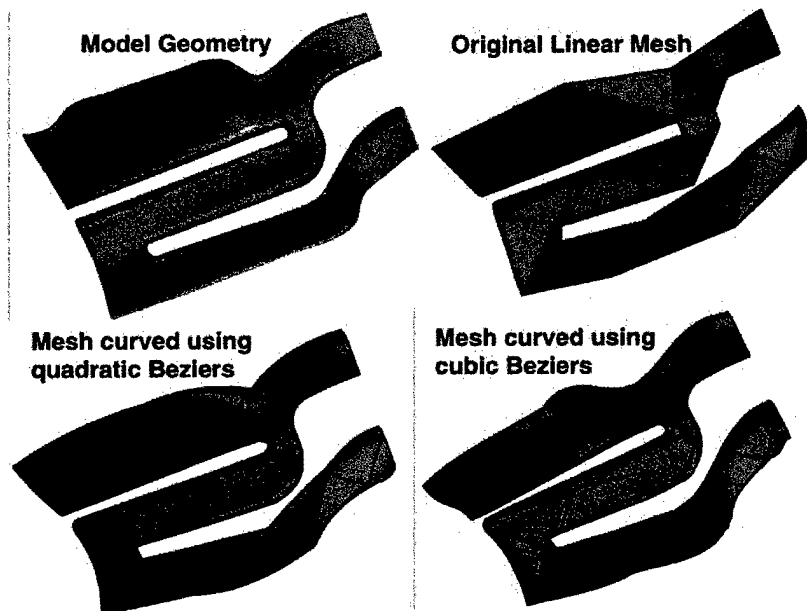

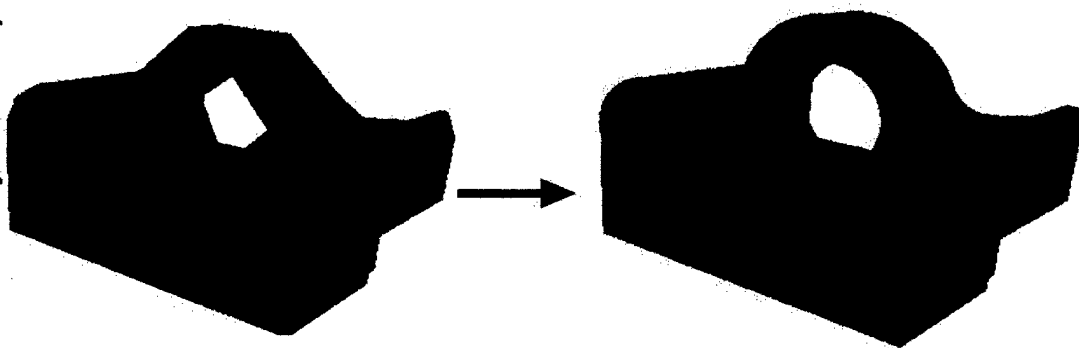Figure 1. Mesh curving example using quadratic and cubic element geometry

Figure 2. Mesh curving for a mechanical part

3.2 Geometric Representation of Curved Mesh Entities

The two basic approaches to represent the geometry of mesh edges and faces on the model boundary are:
- Exact model geometry through geometric model interrogation. This method has been previously implemented and found to be computationally expensive [1].

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- A geometric approximation of sufficient order to ensure proper convergence of the finite element solution.

For mesh edges on the interior of the model that need to be curved to preserve the shape of attached already curved elements, any polynomial fit of an order consistent with the finite element basis of the element is satisfactory.

Two options for the geometric approximation of curved boundary and interior mesh edges and faces considered are Lagrange interpolation and Bezier approximating polynomials. Although the definition of Lagrange polynomials is straight forward, the determination of interference and control of these polynomials for higher than quadratic geometry becomes difficult. Therefore, efforts on this project focused on the development of the Bezier geometric approximation [5,6,9]. The key Bezier geometry properties of interest are:

- Can be as high a degree as desired
- Convex hull provides smoother and more controllable approximation
- Better properties to allow more efficient intersection checks
- Derivatives and products of Beziers are also Beziers
- Efficient algorithms for degree elevation and subdivision

The key technical issues associated with the implementation of Bezier element geometry are:

- Defining mesh entity shapes on the model boundary based on interactions with the geometric modeling system. This includes determining the points on the model entity that the mesh entity should pass through, the corresponding parametric locations on the mesh entity for each interpolation point, and if the resulting shape self-intersects and how to correct the shape if needed
- Setting mesh entity order based on entity closure order requirement
- Ensuring the validity of the elements
- Geometric properties and operations to use in the process of correcting mesh entity shapes
- Providing an interface for the mesh curving routine

The mesh entity Bezier approximations must be constructed from pointwise interrogations of the geometric model. The process must account for issues associated with geometric modeling systems face parametric coordinates including periodic faces and edges, degenerate points on faces and parametric system distortion. Mesh edges classified on model face are parameterized based on a cord length procedure to control the parametric space distortion. Procedures to deal with parametric space periodicity and typical degeneracies have been defined.

The next key task was determination and elimination of any self-intersections that arise. Self-intersections can occur in the interpolation geometry when the model geometry is of higher degree than the degree of the mesh entity or is a piece-wise shape (such as a NURB). Inexpensive methods for determining potential self-intersections are being devised. In the case of mesh edges, the method makes use of the variation diminishing property of Bezier curves, which states that an infinite plane can not intersect a Bezier curve more times than it intersects its control polyline. This allows the development of a procedure to find obvious edge intersection problems.

Detecting self-intersections in mesh faces is more difficult since Bezier Surfaces do not exhibit the variation diminishing property. The process begins with an analogous test

defined for Bezier Surfaces based on the projection of the control polygon onto the planar face of the linear mesh face. If the projection of the control polygon's points form a non-intersecting tessellation the surface is less likely to self-intersect. This builds on the fact that the control polygon resulting from continually degree elevating a surface will in the limit become the Bezier Surface.

The previous implementations forced all mesh entities to be raised to the highest degree requested. Since many models have planar faces and linear edges, this restriction results in mesh entity shapes that are more complex than needed. By allowing mesh entities to have different polynomial degrees, mesh entity shapes are as complex as required. In order to allow mesh entities to have different polynomial degrees, a mesh entity may need to provide higher order representations in order for another mesh entity that has the entity as part of its boundary to be of higher order.

Determining the validity of region elements requires that the determinate of the Jacobian be positive. Properties of Bezier regions useful in this process are:

- Partial derivatives of the region are themselves Bezier functions
- The Jacobian determinant is defined by box-product of partial derivatives
- Since the product of 2 Bezier functions is also a Bezier function, the Jacobian determinant is also a Bezier function
- In the case of a tetrahedron the function is of order $3(p-1)$, where p is the order of the original shape
- Since a Bezier function is bounded by its convex hull, the Jacobian determinant function inside the region is bounded by the convex hull of its control points (which in this case are scalars)
- A region is valid globally if the minimum control point of the Jacobian determinant function is positive

The box product terms that compose the Jacobian determinant function can be used to determine how a region should be corrected. An outline of the procedure is

- For each Jacobian that is negative:
- Identify the minimum Box Product term contributing to the Jacobian
- Identify the region control points involved in the box product vectors that can be moved. Control Points may be constrained due to being associated with mesh entities on the model boundary or constrained to prevent other mesh regions from becoming invalid
- Identify the minimum angle to make the Box Product Term positive
- Determine which control points of region that defines the vector should be displaced in order to rotate the vector
- If this change would result in invalidating a neighboring region it must be modified to accommodate the shape change
- If the region is still invalid then perform one of the following:
  - Degree elevate the region's shape if possible in order to increase the degrees of freedom available
  - Sub-divide the region in order to refine the mesh and introduce more degrees of freedom

After the surface mesh has been properly curved and all the mesh regions have been made valid, the last step in the mesh curving process is to detect global mesh intersections. This

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procedure uses the convex hull property of a Bezier curve, surface, or volume which states it is contained in the convex hull formed by its control points. Therefore, if the convex hulls of two mesh entities do not intersect then the entities do not intersect. If the convex hulls do intersect then the convex hulls can be refined by subdivision. Refinement continues until either it is determined that the hulls do not intersect or that an intersection has occurred (within an acceptable tolerance). Once an intersection is detected, the control points of the interfering geometry are displaced in order to remove the intersection.

To isolate the details of the Bezier mesh geometry representation from the mesh curving and modification procedures, all interactions with the shape information is through general classes. The use of the topologically specific sub-classes allows for the effective accounting for the local parametric spaces of the mesh entities. Every subclass shares the same interface with degree, mesh entity and a set of control points location. The information in the element entity shape subclasses is operated on by operators which are used in the program logic that curves the mesh entities and performs the appropriate mesh modification operations to ensure the final curved mesh is valid.

In addition to the ability to create curved meshes, a number of issues related to the details of the geometric approximation used, ranging from the basic order used to the selection of interpolation points for fitting the Bezier geometry, must be considered. Although theoretical studies clearly show a loss of rate of convergence when the geometric approximation is too low, many argue that for purposes of "engineering accuracy" only quadratic element geometry is needed. The simple studies presented in references [5,6] demonstrate this is not true and that very poor results are obtained when the order of the basis exceeds that of the element geometric approximation for coarse meshes. These studies also indicate the importance of accounting for the curvature for selecting the distribution of interpolation points for fitting the Bezier geometry.

3.3 Specialized Mesh Generation for Thin Sections

Many Naval structures are constructed as an assembly of stiffened shells where the thickness of the individual components can vary through the structure. One approach to the p-version acoustics analysis of these structures is to represent the thickness as a volume component in the model and use volume elements where the p-value in the thickness direction is low. This allows the direct generation of a mesh of volume elements in both the portion of the water domain represented and in the thin volumes of the shell structures. The problem with this approach is the meshing procedures use tetrahedra elements so there would be long through the thickness diagonals created that degrade the ability to apply effectively p-version representations through the thickness. An alternative approach is to represent the thin sections as faces in the geometric model with thickness attributes associated with them. The needed meshes in the thin sections would then be generated by first getting a curved surface mesh for all those faces (see an example in Figure 3.). The volume meshes would then be created through the application of specialized meshing operations that avoid long through the thickness diagonals. The key to the development of this approach is the effective treatment of the face intersections where more than two faces share the edge.

A study of typical stiffened shell intersection was undertaken to determine the most appropriate mesh configuration which:
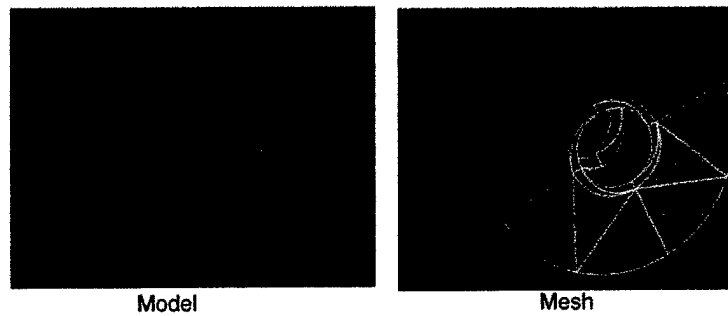- Do not alter the desired surface mesh

| Model | Mesh |

Figure 3. Curved non-manifold mesh of domain with thin sections.

- Create the minimal number of volume elements
- Do not have long diagonal mesh edges in the through thickness direction

The alternative approaches to perform such meshing procedures indicated the need for the following mesh topologies:

- hexahedron
- wedge
- pyramid
- five-noded created by collapsing one of the triangular faces of a wedge to a edge
- seven-noded created by collapsing one edge causing two of the faces to become of the triangular

The construction of the element shape functions for all these element topologies is to employ the shape function decomposition approach of reference [8]. Note that the pyramid, five-noded degenerate wedge and 7-noded-degenerate hexahedra do not have an appropriate natural coordinate system available. In each case the element must be constructed by the appropriate degeneration of the parametric system for the hexahedron, wedge and hexahedron respectively. This introduces a second mapping that must be accounted for in the construction of the region blend functions and in performing numerical integration. The region blends are constructed by mapping from the original parent element to the degenerate form, defining the inverse mapping and substituting this as appropriate into the original region functions for the non-degenerate form. Note the integration process must integrate over the degenerate form. Therefore, the Jacobian of the mapping from the non-degenerate parametric form to the degenerate form appears in the integrand evaluation.

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