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Statistical Treatment of Crack Propagation Data

R. A. Heller[1]

Summary

Crack propagation data are available for two different particulate composite materials. A larger set of 38 observations is used to draw statistical inference for the second set consisting of 5 data. The Paris crack propagation relation, whose two parameters are functionally related, is used.

Introduction

Two sets of data, on particulate composite materials containing hard particles in a rubbery matrix have been examined; one consisting of 38 observations and a smaller one of 5 observations. It is desired to obtain statistical inference for the smaller data set, including confidence limits, with the aid of the larger group of observations. It has been postulated that the rate of crack-propagation follows the Paris relationship and that the two parameters of this rule are functionally related. It has also been indicated that the parameters are log normally and normally distributed. The two materials designated as MM for the larger set and MX For the smaller, though different have similar characteristics. Furthermore MM was tested at a cross-head speed of 0.1 in/min and MX at 0.2 in/min.

According to the Paris rule, the rate of crack propagation

$$\frac{da}{dt} = C_1 K_1^{C_2} = \dot{a} \tag{1}$$

where C_1 and C_2 are correlated parameters and K_I is the stress intensity factor in psi $\sqrt{\text{in}}$.

Taking logarithms of both sides

$$\log \dot{a} = \log C_1 + C_2 \log K_I \tag{2}$$

$\log C_1$ as well as C_2 are assumed to be normally distributed random variables (C_1 is consequently log normally distributed).

As a result, $\log \dot{a}$, the sum of two normally distributed variables, is also normal with a mean of

$$\overline{\log \dot{a}} = \overline{\log C_1} + \overline{C_2 \log K_I} \tag{3}$$

and standard deviation

$$\sigma_{\log \dot{a}} = [\sigma^2_{\log C_1} + \log^2 K_I \sigma^2_{C_2}]^{1/2} \tag{4}$$

The relationship between $\log C_1$ and C_2 is linear

$$\log C_1 = A - BC_2 \tag{5}$$

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or in terms of standardized variables

$$(C_2 - \overline{C}_2)/\sigma_{C_2} = -(\log\ C_1 - \overline{\log\ C_1})/\sigma_{\log\ C_1} \tag{6}$$

that is

$$\log\ C_1 = -\frac{\sigma_{\log\ C_1}}{\sigma_{C_2}}(C_2 - \overline{C}_2) + \overline{\log\ C_1} \tag{7}$$

Substituting Eq. 7 into Eq. 2

$$\log\ \dot{a} = C_2\left(\log\ K_I = \frac{\sigma_{\log\ C_1}}{\sigma_{C_2}}\right) + \frac{\sigma_{\log\ C_1}}{\sigma_{C_2}}\overline{C}_2 + \overline{\log\ C_1} \tag{8}$$

is obtained. Therefore, the variance

$$\sigma^2_{\log\ \dot{a}} = [\sigma_{C_2}\ \log\ K_I - \sigma_{\log\ C_1}]^2 \tag{9}$$

Data Analysis

The two sets of data: 38 observations of log C_1 and C_2 for MM, at a cross-head speed of 0.1 in/min and 5 observations for MX, at a cross-head speed of 0.2 in/min are listed in Table I together with their sample means and sample standard deviations. The crack propagation rates, log $\dot{a}$ for $K_I = 40$ psi $\sqrt{\text{in}}$ are also listed in the table.

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Table 1 C_2, log C_1 and log $\dot{a}$

Test No.	C_2	log C_1	log $\dot{a}$	Test No.	C_2	log C_1	log $\dot{a}$
103	2.42161	-8.91898	-5.03956	132	1.527031	-5.86331	-3.41701
105	1.76926	-6.7678	-3.93344	133	1.827629	-6.91117	-3.98331
106	1.507719	-5.85985	-3.4449	134	1.608571	-6.17429	-3.59736
107	1.563306	-6.09666	-3.59224	136	2.094406	-7.88458	-4.52935
108	1.996521	-7.48854	-4.29011	137	1.169022	-4.7417	-2.87893
109	2.326644	-8.55145	-4.82417	138	1.381992	-5.43623	-3.22227
110	1.718226	-6.53345	-3.78085	139	1.273468	-5.06151	-3.02142
111	1.882954	-7.08362	-4.06712	140	2.019887	-7.53751	-4.30165
112	2.212894	-8.21514	-4.67008	141	1.944194	-7.29269	-4.17809
113	1.885258	-7.15843	-4.13824	142	2.516193	-9.19896	-5.16802
114	2.286922	-8.46155	-4.7979	143	1.994757	-7.45045	-4.25485
115	1.961836	-7.40744	-5.80544	144	1.735048	-6.99552	-4.21597
117	1.758835	-6.6976	-3.87995	145	2.349725	-8.59174	-4.82748
118	2.358755	-8.72234	-4.94362	Mean	1.845786	-6.99665	-4.07457
119	1.403963	-5.53812	-3.28897	Std	0.365129	1.214061	0.70221
120	2.205351	-8.19052	-4.65755				
121	1.981772	-7.43898	-4.26416				
122	1.645804	-6.34225	-3.70568				
123	2.259029	-8.3731	-4.75414	G13L	2.857	-7.554	-2.97709
124	1.22662	-4.96987	-2.78909	G13R	4.255	-10.287	-3.47049
125	1.867377	-7.09606	-4.10452	G23	2.775	-7.359	-2.91345
128	1.787407	-6.82663	-3.96321	G19L	3.881	-9.581	-3.57748
129	1.156759	-4.6236	-2.77048	G19R	4.461	-10.724	-3.36364
130	1.961972	-7.344	-4.20092	Mean	3.6458	-9.101	-3.26043
131	1.551135	-6.02706	-3.54124	Std	0.786049	1.557153	0.29832

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The data C_2 vs log C_1, are plotted for the two materials, MM and MX, in Fig. 1 and indicate the validity of linear relationships with correlation coefficients of $\rho = -0.99836$ and -0.99994 respectively.

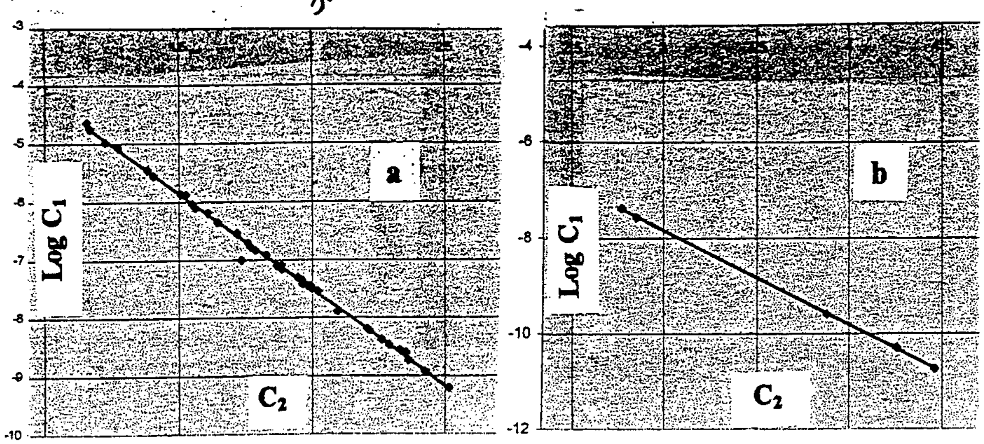

Fig. 1. Log C_1 vs. C_2 for a) MM and b) MX

It is seen that the range of values are quite different. For MM, C_2 varies between 1.16 and 2.52 with corresponding log C_1 values of -4.6 and -9.2. For MX, C_2 varies from 2.8 to 4.46 with log C_1 from -7.55 to -10.72.

The differences are attributable to the two different materials rather than to the changed cross-head speeds. Though the cross-head speed for MX is double that for MM, the mode of cracking is similar. It is expected that only at much greater cross-heads speeds would cracking change to a brittle fracture.

Utilizing Eqs. 3 and 4, log $\dot{a}$ has been calculated for each datum for a stress-intensity factor of $K_I = 40$ psi$\sqrt{\text{in}}$ (Table 1).

The data were plotted on normal probability paper in Fig. 2. Though all datum points fit the normal distribution reasonably well, all MX values are higher than log $\dot{a}$ for MM.

In order to eliminate the influence of the material differences, the log $\dot{a}$ values were standardized using the appropriate means and standard deviations for each material (Table 1). The data were then arranged in increasing order and were plotted on normal probability paper (Fig. 3).

It is apparent that, in this standardized form both sets are essentially normally distributed and that they belong to the same population, the MX data are interspersed with MM values.

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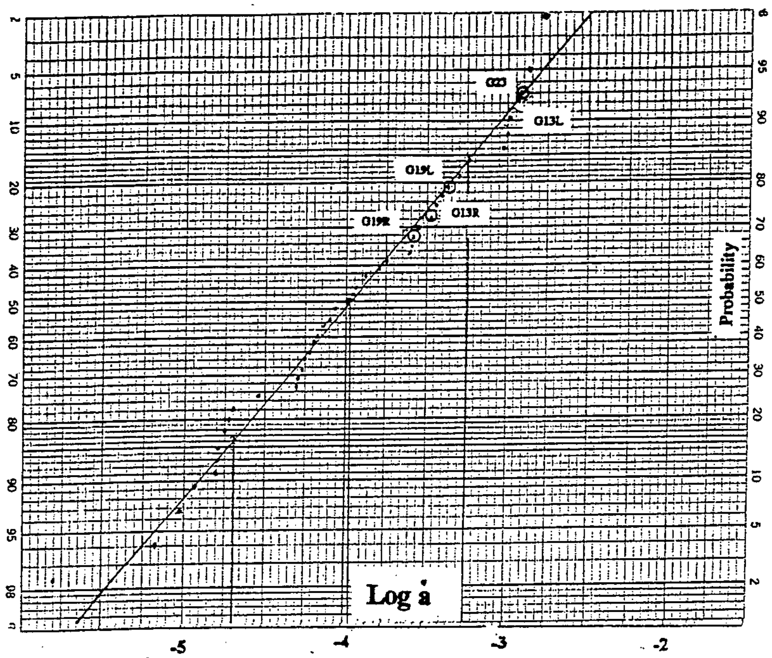

Fig. 2. Normal probability plot of log $\dot{a}$ for the two materials

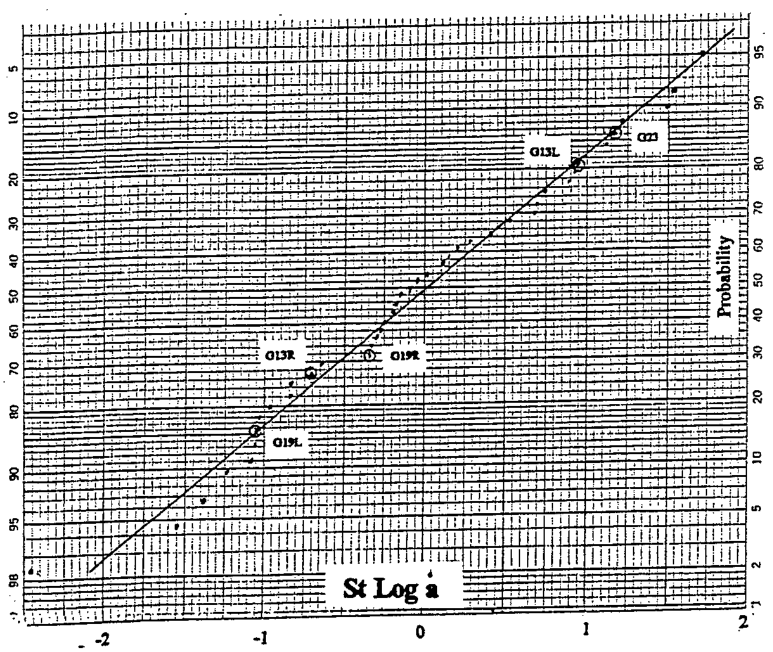

Fig. 3. Normal probability plot of standardized log $\dot{a}$ for the two materials

Confidence Limits

Confidence limits may be established for the normally distributed data. The meaning of such limits is that the true population mean will lie between these limits with a given probability $(1-\alpha)$. The width of the confidence interval is a function of the number of observations, n. The greater the value of n, the narrower is the confidence band.

For small numbers of observations, confidence intervals are based on Student's t distribution [1]. For a variable X, with sample mean, $\overline{X}$ and standard deviation, S, the population mean $\mu_{1-\alpha}$ will lie between the following confidence limits

Are you referring to an actual person or to a student?

$$\left[\left(\overline{X} - t_{\alpha/2}, n-1\ S/\sqrt{n}\right) < \mu_{\alpha-1} < \left(\overline{X} + T_{\alpha/2}, n-1\ S/\sqrt{n}\right)\right] \tag{10}$$

where n is the sample size and $t_{\alpha/2,n-1}$, $n-1$ is the tabulated value of the t distribution with $\alpha/2$ and number of degrees of freedom $f = n-1$.

In the case of log $\dot{a}$, for MM, with a confidence $1-\alpha$ of .99 $\overline{\log \dot{a}} = 4.07457$, $S_{\log\ \dot{a}} = .702212$ and $n = 38$, $\alpha = .01$, $\alpha/2 = .005$, $f = 37$, $p = 1-\alpha/2 = .995$.

$t_{.995,37} = 2.72$ (by interpolation)

$$\left[-4.07457 - 2.72\left(\frac{.70221}{\sqrt{38}}\right)\right] < \mu_{\log\ \dot{a}(1-\alpha)} < \left[-4.07457 + 2.72\left(\frac{.702212}{\sqrt{38}}\right)\right] \tag{11}$$

Therefore

$$-4.38442 < \mu_{\log\ \dot{a}(1-\alpha)} < -3.76472 \tag{12}$$

a difference of 0.62.

The true mean lies between -4.38442 and -3.76472 with a confidence of 0.99.

In contrast, for MX with $\overline{\log \dot{a}} = -3.26042$, $S_{\log\ \dot{a}} = .298318$, $n = 5$, for a confidence of $1-\alpha = .99$, $f = n-1 = 4$, $t_{\alpha/2,n-1} = t_{.995,4} = 4.604$

$$\left(-3.26043 - 4.604\frac{.298318}{\sqrt{5}}\right) < \mu_{\log\ \dot{a}(1-\alpha)} < \left(-3.26043 + 4.604\frac{.298318}{\sqrt{5}}\right) \tag{13}$$

$$(-3.87466 < \mu_{\log\ \dot{a}(1-\alpha)} < -2.64620) \tag{14}$$

with a difference of 1.228.

The confidence interval for MX is much greater than for MM, because the former set has a small sample size.

To improve on the confidence interval for the smaller sample, the standardized normal distribution, Fig. 4, may be used. With a mean of zero and standard deviation of unity, Eq. 11 is modified as

Where is figure 4?

$$\left[\left(0 - t_{\alpha/2,n-1}1/\sqrt{n}\right) < \mu_{\alpha-1} < \left(0 + t_{\alpha/2,n-1}1/\sqrt{n}\right)\right] \tag{15}$$

Again for $1 - \alpha = .99$, with $n = 43$, $t_{.995,42} = 2.6948$.

For the standardized variables

$$\left[\left(0 - 2.6948\ 1/\sqrt{44}\right) < \mu_{.99} < \left(0 + 2.6948\ 1/\sqrt{44}\right)\right] \tag{16}$$

or

$$-0.4063 < \mu_{.99} < .4063$$

$$X_\ell < \mu_{.99} < Xu$$

Performing the standardization backwards

$$(x_\ell - \overline{x})/S_x = X_\ell,\ (x_u - \overline{x})/S_x = X_u$$

Where ℓ and u indicate the lower and upper confidence levels, $\overline{x}$ and S_x are the mean and standard deviat ion of log $\dot{a}$ for MX

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$$X_\ell = \log \dot{a}_\ell = -.29832 \times .4063 - 3.26043 = -3.3816$$

$$X_u = \log \dot{a}_u = .29832 \times .4063 - 3.26043 = -3.1392$$

Therefore the mean log $\dot{a}$ for MX with a confidence of .99 lies between

$$3.3816 < \mu_{.99} < -3.1392$$

with an interval width of .242 instead of 1.228 based on the original data.

The validity of the suggested calculation hinges on the assumption of normality for both sets of data and that crack propagation is not significantly different in the two materials.

Reference

1. Beyer, W. H., (1966) "CRC Handbook of Tables for Probability and Statistics" The Chemical Rubber Co. pp. 225-232.