Proceedings – 23rd Annual Conference – IEEE/EMBS Oct.25-28, 2001, Istanbul, TURKEY

An Analytical Method for Rolling Contact of Articular Cartilages in Diarthrodial Joint

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Abstract- **An analytical method is presented to investigate the stresses and strains in the cartilage layers of diarthrodial joint under rolling contact. Each cartilage layer in contact is assumed to be biphasic, composed of a linear elastic solid phase and a Newtonian viscous fluid, whereas the subchondral bone is simplified as a rigid body. The contact range of two cartilage layers is discretized, where the surface tractions are piecewise given. A Galerkin-penalty method is applied to form the finite element formulation for the cartilage layers. The surface tractions, stresses and strains in the cartilage layers are then obtained for each time step with a numerical procedure of rolling contact. Results show that the interstitial fluid plays a fundamental role in the distributions of the stresses and strains in the cartilage. The normal solid stress reaches its maximum on the cartilage-subchondral bone interface. The coefficient of friction at the contact surface has a great effect on the tangential traction while it has little effect on the normal traction. The difference of rolling velocity between the two cartilage layers has an increasing effect on the tangential traction as the coefficient of friction increases.**
***Keywords*- Rolling contact, Articular cartilage, Diarthrodial joint.**

I. INTRODUCTION

The function of diarthrodial joint is to transmit loads and allow motion between the bones of the musculoskeletal system. It is considered that the mechanical failure of articular cartilage in diarthrodial joint, such as keen joint, is the direct cause of osteoarthritis. A variety of theoretical models have been developed for estimating the contact pressure or the stress distribution in loaded articular cartilage [1]~[3]. However, it is necessary to make further research considering various factors, including the friction of two cartilage layers under rolling contact, for overall understanding of the mechanism of degeneration.

In this paper an analytical model of two cartilage layers under rolling contact, which is based on the KLM linear biphasic theory, is developed. A piecewise discretization method together with a FEM procedure is adopted to obtain the mechanical response of articular cartilage.

II. METHODS

A. Contact model

Figure 1 shows the contact configuration of the two cylindrical biphasic cartilage layers of radii R_i, thickness b_i, and surface velocities V_i (i=1, 2) with a contact width $2a$ and a normal approach d under a normal load per unit length P along the longitudinal axis of the cylinder. The subchondral bone of each layer is assumed impermeable and rigid in this analysis. The inertial force in each layer is ignored as it is sufficiently small compared to the contact pressure. Each cylindrical layer can be approximated by a flat layer bonded to a rigid half-plane if the contact width is small compared to the cylinders' radii.

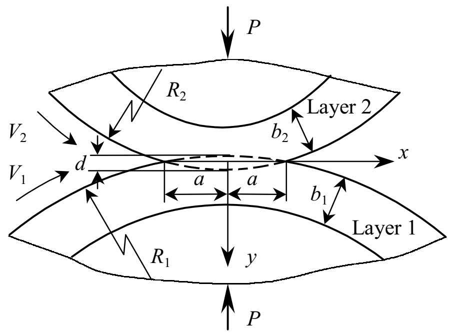

Fig. 1. Contact model of two cartilage layers

B. Governing equations

According to the linear biphasic theory [4], the solid and fluid stresses, $\boldsymbol{\sigma}^s$ and $\boldsymbol{\sigma}^f$, are related to the interstitial fluid pressure p and the effective solid stress $\boldsymbol{\sigma}^e$ by

$$\boldsymbol{\sigma}^s = -\varphi^s p\mathbf{I} + \boldsymbol{\sigma}^e \quad (1a)$$

$$\boldsymbol{\sigma}^f = -\varphi^f p\mathbf{I} \quad (1b)$$

where φ is the volume fraction ($\varphi^s + \varphi^f = 1$), λ and μ are the Lame constant of the solid phase, which together define the aggregate modulus $H_A = \lambda + 2\mu$. The effective stress $\boldsymbol{\sigma}^e$ is relate to the solid strain $\boldsymbol{\varepsilon}$ in the form:

$$\boldsymbol{\sigma}^e = \lambda\,\mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon} \quad (2)$$

The solid and fluid quasistatic momentum equations, and the continuity equation are given by

$$\nabla\boldsymbol{\sigma}^s + K(\mathbf{v}^f - \mathbf{v}^s) = 0 \quad (3a)$$

$$\nabla\boldsymbol{\sigma}^f - K(\mathbf{v}^f - \mathbf{v}^s) = 0 \quad (3b)$$

$$\nabla(\varphi^s\mathbf{v}^s + \varphi^f\mathbf{v}^f) = 0 \quad (3c)$$

where $\mathbf{v}$ is the phase velocity, and the drag constant K is related to the tissure permeability k by $K = (\varphi^f)^2/k$.

C. Boundary conditions

Define the relative fluid flux $\mathbf{w} = \varphi^f(\mathbf{v}^f - \mathbf{v}^s)$ and the solid displacement $\mathbf{u}$. Since the subchondral bone of each layer is assumed impermeable and rigid in this study, the cartilage-bone boundary conditions are as follows:

$$\text{At } y = b_i,\ (u_x)_i = (u_y)_i = (w_y)_i = 0,\ -\infty < x < \infty \quad (4)$$

At the contact interface, the boundary conditions stipulate continuity of the fluid pressure, effective traction normal displacements and normal fluid flux within the contact

Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)
25 Oct 2001	N/A	-

Title and Subtitle An Analytical Method for Rolling Contact of Articular Cartilages in Diarthrodial Joint	**Contract Number**
	Grant Number
	Program Element Number
Author(s)	**Project Number**
	Task Number
	Work Unit Number
Performing Organization Name(s) and Address(es) Tongji University School of Life Science and Biomedical Engineering Shanghai, China	**Performing Organization Report Number**
Sponsoring/Monitoring Agency Name(s) and Address(es) US Army Research, Development & Standardization Group (UK) PSC 802 Box 15 FPO AE 09499-1500	**Sponsor/Monitor's Acronym(s)**
	Sponsor/Monitor's Report Number(s)
Distribution/Availability Statement Approved for public release, distribution unlimited	
Supplementary Notes Papers from 23rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society, October 25-28, 2001, held in Istanbul, Turkey. See also ADM001351 for entire conference on cd-rom.	
Abstract	
Subject Terms	
Report Classification unclassified	**Classification of this page** unclassified
Classification of Abstract unclassified	**Limitation of Abstract** UU
Number of Pages 3	

region. Thus:

At $y=0,\ -p_i+(\sigma_{yy}^e)_i=t_y(x),\ (\sigma_{xy}^e)_i=t_x(x)$ (5a)

$$\sum_{i=1}^{2}(u_y)_i=d+ex/R-x^2/2R,\ |x|\le a \quad (5b)$$

$$\sum_{i=1}^{2}(w_y)_i=0 \quad (5c)$$

where $t_x(x)$, $t_y(x)$ are the normal and tangential traction at the contact interface respectively, e is an offset between the contact center and the line which connects the centers of the two cylinders , and R is the equivalent radius of curvature, $R_1R_2/(R_1+R_2)$. Outside of the contact region the surface tractions reduce to zero, so that:

At $y=0,\ p_i=0,\ (\sigma_{yy}^e)_i=0,\ (\sigma_{xy}^e)_i=0,\ |x|>a$ (6)

Finally, the total load per unit length P applied across the surface must be balanced by the normal traction at the contact interface,

$$P=\int_{-a}^{a}t_y(x)dx \quad (7)$$

In a steady-state of rolling contact, the surface velocities f_i (i=1, 2) of the layers 1 and 2 in the contact region can be written as follows:

$$f_i=V_i[1+(\varepsilon_x)_i],\ |x|\le a \quad (8)$$

The contact region is divided into two parts: one is called the stick zone where two contacting layers have the same surface velocity, and another is called the micro-slip zone where the two layer surfaces slid each other. At the stick zone, we have from (8):

$$\frac{V_1}{V_2}=\frac{1+(\varepsilon_x)_2}{1+(\varepsilon_x)_1}\approx 1+(\varepsilon_x)_2-(\varepsilon_x)_1 \quad (9)$$

In the micro-slip zone, the Coulomb's law is assumed to satisfy and then

$$t_x(x)=\mathrm{sgn}(f_2-f_1)\upsilon t_y(x) \quad (10)$$

where υ is the coefficient of friction between two layer surfaces.

D. Solution

The contact region (-a, a) shown in Fig.1 is equally divided into 2S elements and the distributions of the normal and tangential traction, $t_x(x)$, $t_y(x)$, in the contact region are approximated by a series of straight lines defined by $t_x^{(n)}$ and $t_y^{(n)}$ at the points of intersection, n, as shown in Fig.2. These approximations are equivalent to that $t_x(x)$ and $t_y(x)$ are each represented by 2S-1 overlapping triangle elements with equal base of width 2a/S. The tractions represented by the nth triangle element in the local coordinate system (x*, y*) originated at the point n are given in the form:

$$\tilde{t}_x(x^*)=t_x^{(n)}(1-S/a|x^*|) \quad (11a)$$

$$\tilde{t}_y(x^*)=t_y^{(n)}(1-S/a|x^*|) \quad (11b)$$

A Galerkin-penalty finite element method [5] is then employed to solve the displacements and stresses of each layer under the tractions shown in Fig. 2. In this approach, a weighted residual statement is constructed for a solid phase and for the fluid phase, and the penalty method is used to introduce the continuity equation. The governing first order

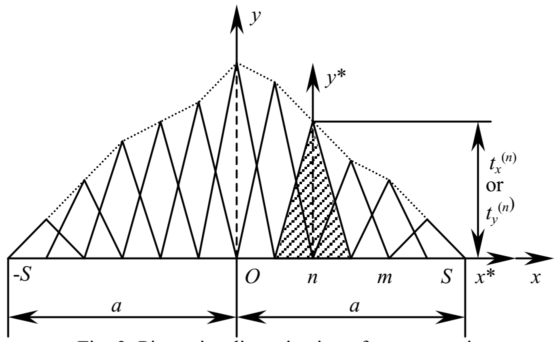

Fig. 2. Piecewise discretization of contact region

coupled differential equations for the finite element assembled system are in the form:

$$\mathbf{Cv}+\mathbf{Ku}=\mathbf{F} \quad (12)$$

where **C, K** and **F** are the conductivity matrix, stiffness matrix and force vector, respectively. We solve (11) using finite difference techniques. Considering times t_m and t_{m+1} separated by a time increment $\Delta t = t_{m+1} - t_m$ and using the Generalized Trapezoidal Family of first order difference rules, We obtain:

$$[\mathbf{C}+\omega\Delta t\mathbf{K}]\mathbf{v}_{m+1}=\mathbf{F}_{m+1}-\mathbf{K}[\mathbf{u}_m+\Delta t(1-\omega)\mathbf{v}_m],\quad 0\leqslant\omega\leqslant 1 \quad (13)$$

Equation (12) is a velocity based formulation and can be solved recursively for $\mathbf{v}_{m+1}$ once a set of boundary conditions and initial conditions have been given. Then the nodal displacements $\mathbf{u}_{m+1}$ can be computed by

$$\mathbf{u}_{m+1}=\mathbf{u}_m+\Delta t[(1-\omega)\mathbf{v}_m+\omega\mathbf{v}_{m+1}] \quad (14)$$

We can obtain the displacements of the contact surface in each layer under the nth traction element in which $t_x^{(n)}$ and $t_y^{(n)}$ (n=-S+1, ⋯, S-1) are put to unit in (11a) and (11b) by using (13) and (14). According to (9), (10) and the above results, we can solve the unknowns $t_x^{(n)}$ and $t_y^{(n)}$ (n=-S+1, ⋯, S-1) when the contact half-width a or normal approach d, surface velocity V_i (i=1, 2) and so on are given. Then solving (13) and (14) again, we can calculate the displacements **u** and phase velocities **v** due to the whole surface tractions shown in Fig.2. The interstitial fluid pressure in the cartilage layers can be obtained by

$$p=-\beta\nabla(\phi^s\mathbf{v}^s+\phi^f\mathbf{v}^f) \quad (15)$$

where β is the penalty parameter which is selected large enough to enforce the continuity constraint condition, (3c), but not so large that the governing matrix equations become ill conditioned. Finally, we obtain $\boldsymbol{\sigma}^s$, $\boldsymbol{\sigma}^f$ and $\boldsymbol{\sigma}^e$ according to (1a), (1b) and (2).

III. RESULTS AND CONCLUSIONS

In the present results, a=2mm, b_1=b_2=2mm, R_1=8mm, R_2=6mm, λ=0.05MPa, μ=0.25MPa, φ^s=0.2, φ^f=0.8, k=4.0 × 10^{-15}m^4/Ns.

Figure 3 shows the contact tractions for two values of the coefficient of friction υ (0.001 and 0.3). It is seen that υ has a great effect on the tangential traction while it has little effect on the normal traction.

Figure 4 shows the distribution of the normal stress for y/b_1=0.05, 0.55 and 0.95 in layer 1. It is seen that the normal stress reaches its maximum on the cartilage-subchondral bone interface.

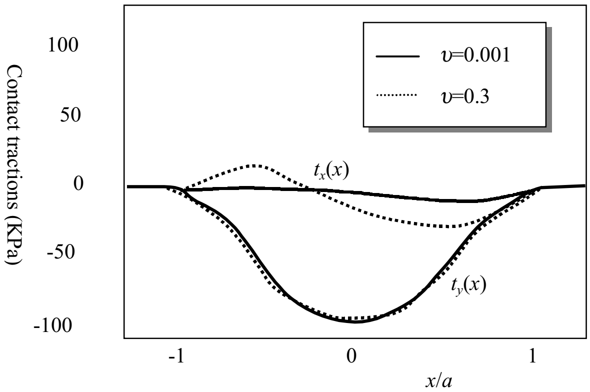

Fig.3 Contact tractions $t_x(x)$ and $t_y(x)$ for υ=0.001 and 0.3 (V_1/V_2=0.95)

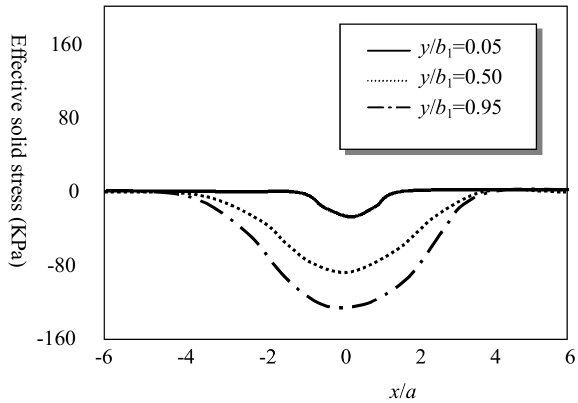

Fig. 4. Effective solid Stress σ^e_{yy} for y/b_1=0.05, 0.55 and 0.95 in layer 1(V_1/V_2=0.95)

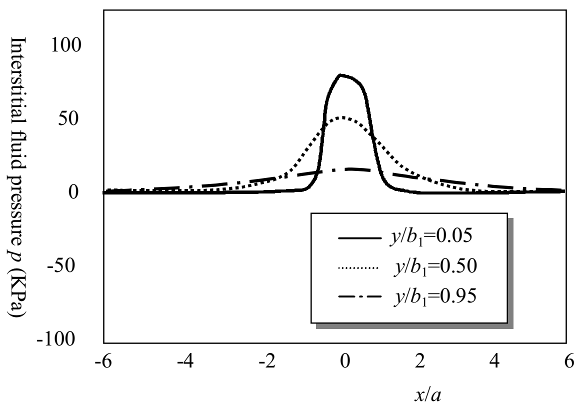

Fig. 5. Interstitial fluid pressure p for y/b_1=0.05, 0.55 and 0.95 in layer 1(V_1/V_2=0.95)

Figure 5 shows the interstitial fluid pressure p for y/b_1=0.05, 0.55 and 0.95 in layer 1. It is observed that the interstitial fluid pressurization could contribute more than 80% of the total applied load at the contact surface.

Figure 6 displays the tangential traction $t_x(x)$ for V_1/V_2=0.95 and 0.90 in layer 1. It is observed that the magnitude of the tangential traction increases as the difference of rolling velocity between the two cartilage layers enlarges. Besides, the effect of the difference on the tangential traction becomes larger as the coefficient of friction increases.

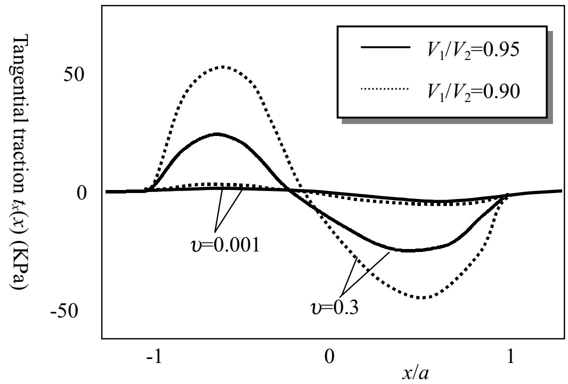

Fig. 6. Tangential traction $t_x(x)$ for V_1/V_2=0.95 and 0.90 in layer 1

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