DIPOLE LOCALIZATION METHOD BASED ON THE HIGH-RESOLUTION EEG*

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Abstract-**The dipole localization method (DLM) of the electroencephalogram (EEG) is investigated based on the high-resolution EEG. The finite volume method (FVM)-boundary element method (BEM) coupled method is used for DLM. For FVM, a novel mesh generation method is presented to overcome the geometric singularity. This method can be employed to calculate EEG forward problems and extended to calculate the high-resolution EEG (HREEG). Because of the high spatial resolution of HREEG, as well as the merits of FVM and BEM, we can expect that the FVM-BEM coupled method is more accurate than the conventional DLM.**

Keywords - **Localization, finite volume method, boundary element method, high-resolution electroencephalogram, low resolution electromagnetic tomography algorithm.**

I. INTRODUCTION

The investigation of the brain function activities is of great importance in the biomedical engineering. The brain information with both high temporal resolution and high spatial resolution (SR) is desired. Electroencephalogram (EEG) has an high temporal resolution within the millisecond range. However, the poor SR of EEG should be improved. An essential way to enhance SR is the dipole localization method (DLM) [1], which is a parameter estimation technique to account for a scalp potential distribution. In recent two decades the calculation of the cortical potentials from the measured scalp potentials is developed to overcome the blurring effect caused by the skull, which is called high-resolution EEG (HREEG).

For the realistic models, numerical methods are necessary for solving the EEG problem. The finite element method (FEM) and the boundary element method (BEM) are used successfully in biomedical electromagnetic field computation. BEM is valid to solve the models with homogeneous and isotropic media, but relatively difficult to solve the models with anisotropic conductivity, which is likely to be the reality. FEM is suitable for the field calculation, and it excels in handling complicate geometry. However, the numerical solution provided by FEM is not able to guarantee that the current continuity equation is satisfied precisely, which may result in the calculation error. The finite volume method (FVM) is an alternative approach for solving EEG problems. Similar to FEM, FVM is suitable for handling the complicated geometry with anisotropic conductivity. The numerical equations obtained from the finite volume discretization satisfy the current continuity equation. This property is necessary for the EEG calculation, where the conductivity may change greatly. Rosenfeld and Tanami [2] used FVM to solve the EEG forward problem. However, they adopted hexahedron meshes in which the geometric singularity can not be avoided for the spheroidal head model.

In this paper, FVM is used to calculate the EEG forward problem. The triangular prism meshes are employed to overcome the geometric singularity; Then a layered recursive algorithm (LRA) is developed to calculate HREEG based on FVM; After that, the FVM-BEM coupled approach is discussed for DLM. The Low Resolution Electromagnetic Tomography Algorithm (LORETA) [3] is also used for reference in DLM. Since the cortical potential distribution possesses high spatial resolution compared with the scalp potential distribution, the proposed localization method based on HREEG is expected to be more accurate than the conventional methods.

II. FORWARD PROBLEM

DLM can be solved by an iterative process of forward problems. Therefore, the investigation of efficient methods for computing forward problems is of great significance. Here the forward problem is calculated using FVM.

A. Principle of FVM

Suppose a dipole source is positioned in a conducting region, the potential distribution in the region can be expressed by the current continuity equation, i.e.

$$\oint\!\!\!\oint_S \vec{J}\cdot d\vec{S} = -\oint\!\!\!\oint_S (\sigma\nabla\varphi)\cdot d\vec{S} = I \qquad (1)$$

where σ is the conductivity of the media, I is the total current inside the closed surfaces. The boundary condition specifies that no current flows into the air, i.e. $\sigma\partial\varphi/\partial n = 0$ (n is the normal to the boundary).

To describe the discretization of the field domain, let us take a sphere for instance. The spherical surface is divided into a large number of triangles. Then it is easy to connect all the vertices of the triangles to the spherical center. Meanwhile the sphere is divided into many concentric spheres. Consequently, a large number of triangular prisms are constructed.

The potential φ at the vertices or mesh nodes is taken as the unknown to be solved. To obtain the FVM equation system, a closed surface around each node should be defined as the integration surfaces in (1). First, let us consider the two-dimensional case as shown in Fig. 1. Suppose M_{io} is the point to be considered, O_M is the barycentre of the triangle M_{io} M_{i1} M_{i2}. M_{io1} and M_{io2} are the middle points of the triangular edges. All the barycentres of triangles around M_{io} are connected with the middle points of their edges respectively, so that a closed region is shown hatched in Fig.1.

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Now generalize the two-dimensional case to the three-dimension case, as in Fig. 2. Triangle M_{io} M_{i1} M_{i2} is the interface of two adjacent triangular prisms. Other five triangles surrounding point M_{io} in Fig. 1 can be obtained by other triangular prism interfaces. The hatched region of Fig. 1 is moved to the middle section of the upper and lower triangular prisms respectively, which compose the top and the bottom face of the closed surface surrounding point M_{io}. Other parts of the closed surfaces are rectangular surfaces as shown in Fig. 2.

Suppose the number of the surface segments of the polygonal prism is *N*. By taking the surface of the prism as the integral surface in (1), we can get an equation as

$$\sum_{i=1}^{N}\iint_{S_i}\sigma\nabla\varphi\cdot d\vec{S}_i=\sum_{i=1}^{N}\iint_{S_i}\sigma\nabla_{n_i}\varphi dS_i=-I \qquad (2)$$

here n_i is the normal to the surface segment *i*. The integral on each surface segment can be evaluated by the Gaussian integration method.

The potential in a mesh element is expressed by the potentials at its nodes and the shape function of the triangular prism as follows

$$\varphi_G=\sum_{i=1}^{6}F_i\varphi_i \qquad (3)$$

where F_i is the shape function. The gradient of the potential at each Gaussian integration point on the surface is given by

$$\nabla_{n_i}\varphi_G=\frac{\sum\limits_{i=1}^{N}\varphi_i\dfrac{\partial F_i}{\partial u}}{\sqrt{(\sum\limits_{i=1}^{N}x_i\dfrac{\partial F_i}{\partial u})^2+(\sum\limits_{i=1}^{N}y_i\dfrac{\partial F_i}{\partial u})^2+(\sum\limits_{i=1}^{N}z_i\dfrac{\partial F_i}{\partial u})^2}} \qquad (4)$$

By substituting (4) to (2), the discrete equation is achieved, whose unknowns are the potentials at all nodes of all the triangular prisms possessing the considered node. Suppose the total number of nodes is *m*, then a system of *m* equations can be obtained by the approach above, which is just the equation of FVM. By solving the equation system, the potential at each node can be obtained.

B. Numerical Simulation of the Forward Problem

A four-layer sphere model is used to represent the brain, the cerebrospinal fluid (CSF), the skull and the scalp. The radius of the interfaces and the spherical surface are 6.3, 6.5, 7.1, and 7.5 cm respectively. A unit radial dipole is positioned at (5.8, 0, 0) cm and its positive direction is along the *x*-axis. The conductivity of the cortex, CSF and scalp are isotropic, which are 0.33 S/m, 1.0 S/m, and 0.33 S/m respectively. While the conductivity of the skull is anisotropic with the radial conductivity of 0.0042 S/m and the tangential conductivity of 0.042 S/m. The potential distribution on the circular boundary formed by cutting the sphere along xoz plane is shown in Fig. 3. The error of the numerical solution compared with the analytical solution introduced in [4] is within 1.8%.

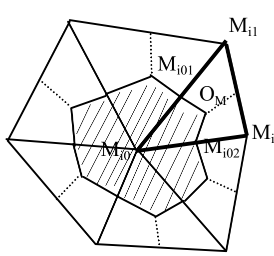


Fig. 1. Closed surface in the two dimensional case

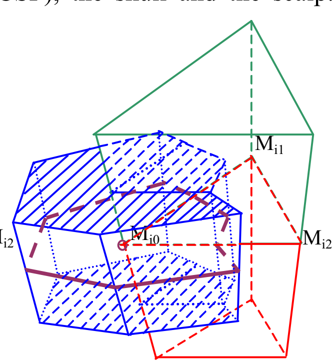


Fig. 2. Definition of the polygonal prism around node M_{io}.

III. High-Resolution EEG

The methods for obtaining HREEG are based on the fact that the region from the cortex to the scalp, including the CSF, the skull and the scalp, is passive electrically. Therefore, the electromagnetic process in this region is depicted by the Laplace equation. With the boundary conditions, the cortical potential distribution can be obtained.

A. Layered Recursive Algorithm for HREEG

A layered recursive algorithm (LRA) is presented to realize HREEG. This algorithm is to calculate the cortical potentials from the known scalp potentials, so that it can be considered as the inverse application of FVM.

According to the passive property of the skull, the right term of (2) is 0. Thus, the current continuity equation for node M_0 is expressed by

$$\sum a_i^{(n-1)}\varphi_i^{(n-1)}+\sum a_i^{(n)}\varphi_i^{(n)}+\sum a_i^{(n+1)}\varphi_i^{(n+1)}=0 \quad (5)$$

here *n* refers to the index of layers named in ascending order from inside to outside, a_i is the coefficient of the matrix.

Suppose the node potentials in layers *n*+1 and *n* are known, by moving these terms to the right side of the equation, (5) is changed into

$$\sum a_i^{(n-1)}\varphi_i^{(n-1)}=-\sum a_i^{(n)}\varphi_i^{(n)}-\sum a_i^{(n+1)}\varphi_i^{(n+1)} \quad (6)$$

The unknowns in equation (6) are the node potentials in the layer *n*-1. (6) is also expressed as a matrix equation

$$A^{(n-1)}\Phi^{(n-1)}=-A^{(n)}\Phi^{(n)}-A^{(n+1)}\Phi^{(n+1)} \qquad (7)$$

According to the boundary condition on the outmost layer

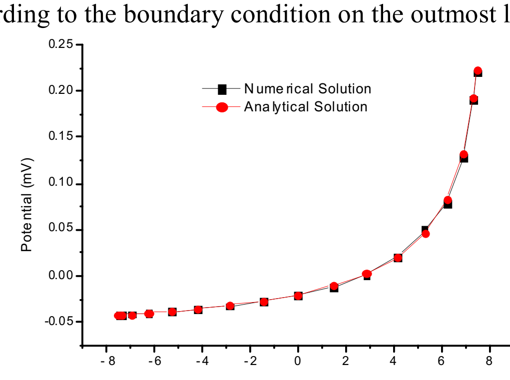


Fig. 3. Potential distribution on the surface of a four-layer sphere with anisotropic conductivity.

$$\frac{\partial \Phi}{\partial n} = 0 \tag{8}$$

we can get the relationship of the potential on the outmost layer denoted by n and that on the second layer denoted by n-1 as follows

$$\Phi^{(n-1)} - \Phi^{(n)} = 0 \tag{9}$$

It is because that the side edges of triangular prisms are along the normal direction of the surface. Since potential on layer n is known, which is obtained by the measurement, the potential distribution of the layer n-1 is obtained. From the known potential distribution of layer n and layer n-1, the potential distribution of layer n-2 is obtained by (7). Consequently, the potential distribution of each layer is deduced recursively. It is thus called the "layered recursive algorithm".

B. Simulation of LRA

A four-layer sphere model with anisotropic conductivity is simulated. The radius of the interfaces and the spherical surface are defined as 6.0, 6.5, 7.0, and 7.5 cm respectively. A unit radial dipole is located at (5.0, 0, 0) cm, and its positive direction is along the x-axis. The conductivity of the cortex, CSF and scalp are isotropic, with the conductivity of 0.33 S/m, 1.0 S/m, 0.33 S/m respectively. The conductivity of the skull is anisotropic with radial conductivity of 0.0042 S/m and the tangential conductivity of 0.042 S/m. Fig. 4 shows the potential distribution on each layer. Since the conductivity in the skull is much lower than the scalp and CSF, the potential magnitude decreases rapidly. It means the cortical potential is much greater than the scalp potential.

IV. Dipole Localization

HREEG is obtained as described above, which is of high spatial resolution compared with the potential distribution on the scalp. Thus, the dipole localization based on HREEG can achieve higher localization accuracy. The FVM-BEM coupled approach is developed for the new DLM based on HREEG.

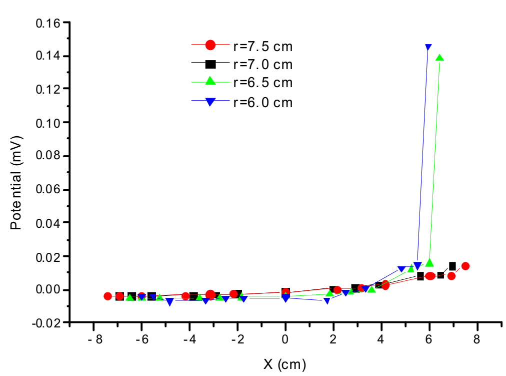


Fig. 4. Potential distribution on the layers of a four-layer sphere with anisotropic conductivity by LRA.

A. FVM-BEM Coupled Approach

For the discussed model, FVM is employed from the scalp surface to the cortical interface, and BEM is employed inside the cortical interface.

The BEM equation is expressed by [5]

$$\varphi = \gamma \iiint_V \frac{\rho}{4\pi\varepsilon r} dV + \gamma \iint_{S-S_\delta} \varphi \frac{1}{4\pi r^2} \cos\theta dS + \gamma \oiint_S \frac{1}{4\pi r} \frac{\partial \varphi}{\partial n} dS$$

$$\gamma = \begin{cases} 0, & M_0 \notin V \\ 1, & M_0 \in V \\ 2, & M_0 \in S \end{cases} \tag{10}$$

here φ is the potential, ε is the dielectric coefficient, $\vec{r}$ is a vector directed from the field point M_0 to $d\vec{S}$, θ is the angle included between $\vec{r}$ and the normal direction of $d\vec{S}$, V is the field domain, S is the boundary of V, S_δ is the boundary where the point M_0 is excluded from V. Since M_0 is on the interface, therefore $\gamma = 2$. According to the electrostatic analogy theory, i.e. ε is substituted by σ, and $\iiint_V \frac{\rho}{4\pi\varepsilon r} dV$ is substituted by $\iiint_V \frac{\vec{J} \cdot \vec{r}_s{}^0}{4\pi\sigma r_s{}^2} dV$, we can get

$$\varphi(Y) = 2G(Y) + 2 \iint_{S-S_\delta} \varphi(X) \frac{1}{4\pi r^2} \cos\theta dS + 2 \oiint_S \frac{1}{4\pi r} \frac{\partial \varphi(Y)}{\partial n} dS \tag{11}$$

In this formulation,

$$G(Y) = \iiint_V \frac{\vec{J} \cdot \vec{r}_s{}^0}{4\pi\sigma r_s{}^2} dV \tag{12}$$

which is the potential at the field point Y if the medium were homogeneous and of infinite extent, $\vec{r}_s = \vec{r}_s(X, Y)$ is a vector directed from the source point X to the field point Y, and $d\vec{S} = d\vec{S}(X)$ is an element of the bounding surface S directed along the local normal.

The cortical surface is divided into a large number of small triangles the same as the discretization of FVM. Hence, the integral in (11) can be approximated by

$$\iint_{S-S_\delta} \varphi \frac{\cos\theta dS}{r^2} = \sum_{j=1}^{N} \varphi_j \iint_{\Delta S} \frac{\cos\theta dS}{r^2} \tag{13}$$

$$\oiint_S \frac{1}{r} \frac{\partial \varphi}{\partial n} dS = \sum_{j=1}^{N} \frac{\partial \varphi_j}{\partial n} \iint_{\Delta S} \frac{1}{r} dS \tag{14}$$

here N is the total number of triangles on the cortical surface.

The boundary condition on the cortical interface is

$$\varphi_{ci} = \varphi_{co} = \varphi \tag{15}$$

$$\sigma_i \frac{\partial \varphi_{ci}}{\partial n} = \sigma_o \frac{\partial \varphi_{co}}{\partial n} = J_n \tag{16}$$

here σ_i and σ_o are the conductivity of the inside and outside of the interface respectively, φ_{ci} represents the cortical potential of the inside interface, φ_{co} represents the cortical potential of the outside interface. $\frac{\partial \varphi_{co}}{\partial n}$ is approximated by

$$\frac{\partial \varphi_{co}}{\partial n} = \frac{\varphi^{(n+1)} - \varphi^{(n)}}{d} \tag{17}$$

here $\varphi^{(n)}$ is the potential on the cortical interface, $\varphi^{(n+1)}$ is the potential just outside the cortical interface in the mesh generation layer, d is the distance between these two layers.

Furthermore, (11) is transformed into

$$\iiint_V \frac{\vec{J} \cdot \vec{r}_s^{\,0}}{4\pi\sigma r_s^{\,2}} dV = \frac{1}{2}\varphi(Y) - \iint_{S-S_\delta} \varphi(X) \frac{1}{4\pi r^2} \cos\theta dS - \oiint_S \frac{1}{4\pi r} \frac{\partial \varphi(Y)}{\partial n} dS \tag{18}$$

Since the potential outside the cortical interface is known, the right terms of the equation can be calculated, the only unknown is $\vec{J}$.

To estimate $\vec{J}$, the Low Resolution Electromagnetic Tomography Algorithm (LORETA) is used for reference. The principle of LORETA is equivalence of the cortical parameter distribution as the linear superposition of the cortical parameter distribution generated by each current dipole source.

Suppose the dipoles in the brain are all along the normal direction, the model of the EEG signal is

$$\boldsymbol{GJ} + \boldsymbol{N} = \boldsymbol{S} \tag{19}$$

here $\boldsymbol{S} \in m \times 1$ is the right terms in (18), m is the number of the vertex on the cortical interface; $\boldsymbol{J} \in n \times 1$ is the amplitude of the unknown dipole sources, n is the total number of the discretized points around the dipole sources; $\boldsymbol{N}$ is the noise; $\boldsymbol{G} \in m \times n$ is the coefficient matrix of the left term in (18). Because it is only an analytical calculation, the speed is high.

The destination function of the inverse problem is the residual sum of squares (RSS) between the measurement of $\boldsymbol{S}$ and the left terms in (19). It is solved by the linear optimization.

To increase the stability of the inverse problem, the singular value decomposition (SVD) is used as following

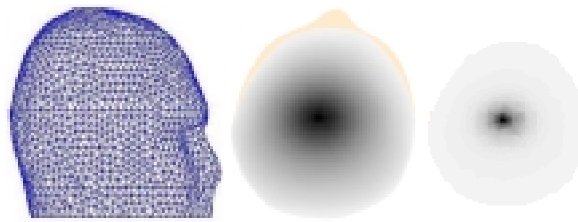
$$\boldsymbol{G} = U \Sigma V^T \tag{20}$$

here $\Sigma = diag[\lambda_1, \cdots, \lambda_m]$ is the singular value, $\lambda_i > 0$, i = 1,…,l; $\lambda_i \approx 0, i = l+1, \cdots, m$. λ_i (i = l+1,…,m) approximated to 0 is neglected.

In general, the amplitude of the dipole on the true source position is greater than the amplitude on the other positions. Suppose there are totally n discrete dipole amplitudes to be estimated, the dipole with relatively small amplitude can be neglected from the calculation. Therefore, we can remove a dipole from the dipole distribution in each iterative process. The final dipole position left is just the true dipole position.

B. Dipole Localization for the Realistic Head Model

The approaches above are extended to the realistic head model. The mesh generation on the head surface is shown in Fig. 5(a). The head model consists of four compartments, which are the scalp, the skull, CSF and the cortex from outside to inside. The conductivity are 0.33 S/m, 0.0042 S/m, 1.0 S/m, 0.33 S/m respectively. One unit dipole is placed inside the cortical interface at (0.00, 0.00, 6.40) cm. The potential distribution on the scalp surface and the cortical interface are shown in Fig. 5(b) and 5(c) respectively. Based on the HREEG, the localization result is (-0.29, -0.10, 6.44) cm. The error is approximately 0.32 cm.



(a) (b) (c)

Fig. 5. Realistic head model. (a) mesh generation on the surface of the realistic head model; (b) potential distribution on the scalp; (c) potential distribution on the cortex.

V. CONCLUSIONS

In this paper, the FVM-BEM coupled method is introduced for DLM. First, a novel FVM is developed to solve the forward problem and the HREEG. The geometrical singularity is overcome by using the triangular prism meshes. Then based on HREEG, the FVM-BEM coupled method is developed to localize dipole source. The simulation results show that the numerical solution is well accordant with the analytical solution in the forward problem. The localization result is close to the true source position in the realistic head model. As described above, we can conclude that FVM is of high accuracy in the realization of the forward problem and HREEG. Because of the high spatial resolution of HREEG, as well as the merits of FVM and BEM, the proposed method can achieve high precision in the dipole localization.

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