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13. ABSTRACT *(Maximum 200 words)*

Crossed-field devices: The Item A.(i) has beet. done and a manuscript was published in the Journal of Plasma Physics. This is a summary of the current understanding of the physics behind the operation of the non-relativistic devices. On Item A.(ii), a manuscript was published in the proceedings of the SPIE Orlando meeting in April, 2001. This treatment was for a planar device, operating in the same general parameter regime as the A6 magnetron. Work has been, completed on the relativistic cylindrical model of the A6 magnetron. A manuscript is being prepared to detail these results. On Item A. (iii), in an unpublished effort, we have also reviewed further the effect that the relativistic corrections would have on the non-relativistic T266 of CPI. We found that these corrections are rather small, as one would expect. A result from the relativistic electromagnetic model is that, strictly speaking, a two-dimension model of a crossed field device is not valid, unless such a device is either infinitesimally thin or operating in a stationary mode. How thin is "thin'' for computational purposes, has not been determined, Further comments will be made below. Work has continued on the saturated operating stage. A relativistic cylindrical version, has been obtained with the aid of MACSYMA. However, the expressions are sufficiently complex that it was determined that we should first, study the simplier non-relativistic planar model of the T266 , as an initial numerical study of this stage of operation. What we have observed is that our model of the relativistic cylindrical AG magnetron indicates the A6 to be potentially more unstable in the saturated operating stage, than the T266 was, This instability would show up as a breakdown in the coherent transfer of electrons from the cathode to the anode.

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Theoretical Studies in Plasmas: Crossed-Field Devices & Ionospheric Plasmas
AFOSR Contract #F49620-00-C-0001

FINAL REPORT: 14 Nov. '99 - 31 Dec. '02

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OBJECTIVES

A. For crossed–field devices, we will extend our previous classical results into the relativistic regime. The specific objectives are:
  (i) improved predictions for the classical current flow and the density profile near the anode;
  (ii) operational density profiles for relativistic devices, including studies of how the relativistic and electromagnetic corrections affect the density profiles;
  (iii) comparison of how (ii) differs from the classical results; and
  (iv) study of the parametric interactions in the fully relativistic limit.

B. Second, we are to initiate further studies of plasma interactions, linear and nonlinear, in the ionosphere. The specific objectives are:
  (i) description of the global linear solution for an RF electromagnetic wave injected into the ionosphere;
  (ii) description of parametric interactions at the UH resonance layer and how they relate to ionospheric modification experiments,
  (iii) theoretical studies of RF emissions from the bottomside of auroral discharges, and
  (iv) theoretical studies of possible explanations of polar region radar backscatter.

STATUS OF EFFORT

Crossed–field devices: The Item A.(i) has been done and a manuscript was published in the Journal of Plasma Physics. This is a summary of the current understanding of the physics behind the operation of the nonrelativistic devices.

On Item A.(ii), a mauscript was published in the proceedings of the SPIE Orlando meeting in April, 2001. This treatment was for a planar device, operating in the same general parameter regime as the A6 magnetron. Work has been completed on the relativistic cylindrical model of the A6 magnetron. A manuscript is being prepared to detail these results.

On Item A.(iii), in an unpublished effort, we have also reviewed further the effect that the relativistic corrections would have on the nonrelativistic T266 of CPI. We found that these corrections are rather small, as one would expect.

A result from the relativistic electromagnetic model is that, strictly speaking, a two-dimension model of a crossed field device is not valid, unless such a device is either infinitesimally thin or operating in a stationary mode. How thin is "thin" for computational purposes, has not been determined. Further comments will be made below.

Work has continued on the saturated operating stage. A relativistic cylindrical version has been obtained with the aid of MACSYMA. However, the expressions are sufficiently complex that it was determined that we should first study the simplier nonrelativistic planar model of the T266, as an initial numerical study of this stage of operation. What we have observed, is that our model of the relativistic cylindrical A6 magnetron indicates the A6 to be potentially more unstable in the saturated operating stage, than the T266 was. This instability would show up as a breakdown in the coherent transfer of electrons from the cathode to the anode. Due to the nature of this breakdown, it is not unreasonable to expect this to show up experimentally as a pulse shortening.

Ionospheric Plasmas: In this section, Item B.(i) has essentially been accomplished. Work is continuing on the descriptions of the modal transition points. We have explored the possibility of collaboration with Dr. William Burke of Hanscom Geophysical Laboratory on ionospheric experimental results, for which our theoretical results may be useful. This will be continuing work.

ACCOMPLISHMENTS/NEW FINDINGS

1. We have completed a working computer code for the relativistic cylindrical model of a magnetron operating in the initiation stage. Currently, the parameters are set for the A6, but the code is written in such a way that one may adjust the parameters for any cylindical device, relativistic and nonrelativistic. We have obtained a comprehensive collection of numerical data at the operating voltages of 300, 325, and 350 Kvs. The operating ranges found agree quite well with the known experimental values.
2. We have shown that there are two distinct stages of operation in a magnetron, as a function of time. In a crossed-field amplifier, there are the same two stages, but they are separated in space, and not in time. The first stage in a magnetron occurs as one is initiating it, starting from a cold device. This stage is called the "initiation mode stage". In this stage, the RF wave is growing due to the linear instability of the shear flow, which then initates a nonlinear diffusion process, whereby the Brillouin flow becomes reshaped such that the background (DC) fields will be in equilibrium with the ponderomotive forces arising due to the RF fields. This is the stage that we have been, and are currently studying, theoretically and numerically. The second stage is the final stage in the operation of the device. (Actually, there is an intermediate transistion stage between these two, but that stage could only be addressed by numerically solving the time-dependent equations.) This final stage must occur because eventually, the initiation stage will saturate. This is because with a linear growth rate, eventually the density oscillations will become so large, that they will begin to drive the total density zero at some point. If they would become any larger, then they would create a negative density of electrons, which is physically impossible. Thus, when this RF amplitude is achieved, then the RF wave must cease its growth. From this point on, the device must enter into what we will term as the "saturated operating stage". Here, the growth rate must be zero. However, power will still be produced and will flow out of the vacuum (plasma) region and into the slow-wave structure. The characteristics and nature of this stage are mostly unknown and still to be explored. We have theoretically investigated some of these characteristics, mainly showing that at sufficiently high DC vertical currents (the DC current drawn through the device), a stable, nonsingular and coherent DC background cannot exist. When this happens, one would expect the general coherency to be lost, and the DC background to break up into chaotic motion. If so, then the power generation by the device would abruptly crease.
3. The results of previous studies of the planar models had indicated that the parameter regime for a relativistic device would be more limited than for a nonrelativistic device. However, more recent numerical results have indicated that the cylindrical geometry mitigates some, if not many, of the problems and restrictions found in the relativistic planar model. In any planar model, there can be only one diocotron (wave-particle) resonance. However, in cylindrical geometry, provide that the cylindrical aspect ratio (the ratio of the outer and inner radii) is sufficiently large, there can be two diocotron resonances. In fact, several of our cylindrical solutions for the A6 model have demonstated this feature. The consequence of this is that in the cylindrical geometry, with the cathode on the inside and the anode on the outside, and with a reasonably large aspect ratio, the electrons in the outer sheath can be held in resonance with the RF wave for a much longer period, than they could in any planar model. Thus, these electrons could be expected to deliver more energy to the RF wave than in the planar case. The main reason for this is the following. First, ignoring the relativistic factors for simplicity, in a cylindical device, the diocotron resonance occurs at the values of r where $\omega_d = m\omega$, where ω_d is the angular drift rate of the drifting electrons, given by $|\mathbf{E}_r \times \mathbf{B}_z|/r$, m is the mode number, and ω is the RF frequency. Now, as one moves out along the radius from the cathode, the magnitude of $\mathbf{E}_r$ increases due to the high density, driving ω_d to higher values. But as one moves further out and away from the core of the sheath, the electron density soon becomes more rarified, and $\mathbf{E}_r$ starts to decrease in magnitude as $1/r$. As an net result, $|\omega_d|$ reaches a maximum and then decreases, thus allowing the diocotron resonance to occur at two locations between the cathode and anode.

4. The theoretical results for our model of the A6 are qualitatively close to the known experimental results for the A6. However, one must keep in mind that these results are only for the initiation stage, and the experimental results are generally for the saturation stage, where one has power being steadily delivered. Nevertheless, the fact that our parameters match closely with the experimental results on the saturation stage, is what one would expect, simply because, in order for a device to reach the saturation stage, it has to first pass through the initiation stage. Thus our results outline the doorway through which the device must be initiated in order to reach a delivery of steady power.
5. A through study of the structure of the electromagnetic fields in the initiation stage has revealed an interesting theoretical result. We found that the usual assumption that there is a solution which is translationally invariant in the direction parallel to the ambient magnetic field, is not valid for any operating device. In other words, the real problem in the initiation stage is three-dimensional, and cannot be reduced to two dimensions, unless the device is sufficiently "thin". This result is probably well known to experimentalists, but what is interesting, is that it does not show up in the theoretical modeling until one carefully considers the structure of the electromagnetic fields in the initation stage, which must be time dependent and exponentially growing. What this means that there is a limit to the thickness of any such device, if it is to operate as we would expect it to. The reason for this is not too surprising. When operating, a device has a radial DC current from the anode to the cathode. This current must create a second-order asmuthial magnetic field, $B_{\phi,2}$, which by Ampere's Law, must linearly grow in the direction parallel to the external, ambient magnetic field, $B_{z,0}$. In other words, as the RF fields grow in the device and it delivers more and more power, then as one moves away from the central plane of the device, $B_{\phi,2}$ will grow linearly as z, as well as exponential in time. As a consequence of this, strictly speaking, the entire problem becomes three-dimensional, in that a z-dependence would have to be included. However, provided that we restrict our attention to a sufficiently thin device, then one can retain the two-dimensional theory. This problem does not occur in electrostatic models, since such models ignore the effects of the non-ambient magnetic fields.
6. Work has continued on new forms of solitons, such as embedded solitons and "virtual" solitons. In collaboration with Prof. Malomed of Tel Aviv University, we have recently studied a system of Type II second harmonic generation, which contained dispersion and Kerr nonlinearities. This is not an integrable system, but it is a system which is physically relevant to selected experiments. In this system, one can find embedded solitons, as well as regular solitary waves, or solitons. Currently we are plotting out the parameter regime for existence of the solitary waves. This is a 6th order system of ODE's, and finding even one soliton solution is not a simple process, since one must locate that exact single set of initial values, in a space of three parameters. We have solved this problem by developing a general Fortran code for finding such soliton solutions in a high order system.

   Related to this, we have collaborated with Dr. V. Gerdjikov in studying how the soliton stability of these solitons could be determined numerically. There are two ways to do this. First, one could solve the full PDE's upon just slightly perturbing the soliton solution. If the solution is stable, it would relax back to the initial solution. If it is unstable, one would see it breaking up. The other way is to simply perturb the full PDE's analytically, and reduce the problem to an eigenvalue problem. It this case, one would have a 12th order, linear eigenvalue problem, consisting of 12 coupled linear, ordinary differential equations. We are studying this problem to see if an analogy of Levison's Theorem can be formulated for such systems, which could be executed numerically.

PERSONNEL SUPPORTED

- Faculty: D.J. Kaup
- Other:
  - Prof. Subash Antani (Edgewood College, Madison, Wisc., consultant, nonlinear interactions in the ionosphere.)
  - Dr. N.N. Rao (Theoretical Physics Division, Physical Research Laboratory, Ahmedabad, INDIA; consultant and employee; plasma interactions in the ionosphere.)
  - Prof. Roy Choudhury (University of Central Florida, consultant, nonlinear interactions)
  - Dr. J.P. Gu (Institute for Simulation and Training, University of Central Florida, Orlando, FL, Subcontract to UCF for code writing and computer computations)
  - Ms. Irina Polandova (Employee, code writing and computer computations)
  - Dr. V Gerjikov (Sofia, Bulgaria; consultant)

PUBLICATIONS

* SUBMITTED

* Books/Book Chapters

* Journals

- *Embedded Solitons in Lagrangian and Semi-Lagrangian Systems* D.J. Kaup and Boris A. Malomed (To appear in Physica D).

* Conferences

- *Recent Results on Second Harmonic Generation*, D. J. Kaup and H. Steudel (Submitted to the Proceedings of the Nov. 2000 AMS sectional meeting, Birmingham, Alabama. To appear as an AMS Comtemporary Mathematics Series.)

* ACCEPTED

* Books/Book Chapters

- *The Legacy of the Inverse Scattering Transform* Contemporary Mathematics (Published by the AMS) **301**, (2002) [Eds. Jerry Bona, Roy Chouldhury, D. J. Kaup].

* Journals

- *Solitary Waves in Perturbed Generalized Nonlinear Schrödinger Equations*, J. Yang and D.J. Kaup, SIAM J. Appl. Math. **60**, 967-89 (2000).
- *Embedded solitons in second-harmonic-generating systems*, J. Yang, B.A.Malomed and D.J. Kaup, Phys. Rev. Letters **83**, 1958-61 (1999).
- *Collision-induced pulse timing jitter in a WDM system with strong dispersion management* , D.J. Kaup, B.A. Malomed and J. Yang, JOSA B **16**, 1628-35 (1999).
- *The Inverse Scattering Transform on a Finite Interval*, H. Steudel and D.J. Kaup, J. Physics A: Math. Gen. **32**, 6219-31 (1999).
- *Parametric Interactions Inside a Magnetron* with J. O. El-Reedy and Gary E. Thomas, J. Plasma Phys. **64**, 489-506 (2000).
- *Embedded solitons: a new type of solitary wave*, J. Yang, B.A.Malomed, D.J. Kaup, and A.R. Champneys, Math. & Computers in Simulation **5**, 585-600 (2001).
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- + *Theoretical modeling of crossed-field electron vacuum devices*, D.J. Kaup, Phys. of Plasmas **8**, 2473-80 (2001).
- + *Inverse scattering method applied to degenerate two–photon propagation in the low excitation limit*, H. Steudel and D.J. Kaup, J. Phys. A: Math. Gen. **33**, 1445-1457 (2000).
- + *Virtual Solitons and the Asymptotics of the Second Harmonic Generation*, D.J. Kaup and H. Steudel, Inverse Problems **17**, 959-970 (2001).
- + *Parametric Amplification of Chirped Pulses in the Presence of a Large Phase Mismatch*, E. Ibragimov, Allan Struthers, and D.J. Kaup, JOSA B **18**, 1872-6 (2001).
- + *The Legacy of the IST* D. J. Kaup, Contemporary Mathematics (Published by the AMS) **301**, 1-14 (2002).

* Conferences

- + *Density Profiles and Current Flow in a Crossed-Field, Electron Vacuum Device*, D.J. Kaup and G. E. Thomas, Proceedings of the 2nd International Conference on Crossed–Field Vacuum Devices, Boston, Mass., 17-18 June, 1998.
- + *Chaotic Instabilities and Density Profiles in a Crossed-Field Electron Vacuum Device* D.J. Kaup and G.E. Thomas, Proceedings of SPIE, **4031**, 54-64 (2000).
- + *Theoretical Studies of a Planar Model of a Relativistic Magnetron*, D. J. KAUP, Proceedings of SPIE, **4371**, 49-56 (2001).
- + *Initiation and Stationary Operating States in a Crossed-Field Vacuum Electron Device* D. J. Kaup, Proc. of SPIE, **4720** 67-74, (2002).

* Participation/Presentations At Meetings, Conferences, Seminars, Etc

- + *Effects of InterChannel Collisions on Dispersion Managed Solitons*, (with B. Malome and J. Yang), "SOLITONS, COLLAPSES, AND TURBULENCE: Achievements, Developments and Perspectives", Landau Institute for Theoretical Physics, Chernogolovka, Russia, 8 August, 1999.
- + *Inverse Scattering on a Finite Interval*, (with H. Steudel), "SOLITONS, COLLAPSES, AND TURBULENCE: Achievements, Developments and Perspectives", Landau Institute for Theoretical Physics, Chernogolovka, Russia, 8 August, 1999.
- + *The Raman Effect in the Presence of Soliton Collisions in Multichannel DM Systems*, (with T. Lakoba), "SOLITONS, COLLAPSES, AND TURBULENCE: Achievements, Developments and Perspectives", Landau Institute for Theoretical Physics, Chernogolovka, Russia, 8 August, 1999.
- + *Nonlinear Theory of Magnetrons and CFAs*, Nonlinear Optics Workshop, Univ. of Ariz., Tucson, Ariz., 17 Sept. 1999.
- + *Effect of Phase Mismatch on the Process of Optical Parametric Amplification*, (with Allan Struthers and Edem Ibragimov), OSA, Santa Clara, Califorina, 28 Sept., 1999.
- + *Conversion of Ordinary Mode into Z-Mode During Ionospheric Heating*, (with S. Antani and N. Rao), APS Plasma Physics meeting, Seattle, WA, 16 Nov., 1999.
- + *Linear Stability of Stationary Operating Density Profiles in a Crossed-Field, Electron Vacuum Device*, (with Gary E. Thomas), APS Plasma Physics meeting, Seattle, WA, 18 Nov., 1999.
- + *Chaotic Instabilities and Density Profiles in a Crossed-Field Electron Vacuum Device* D.J. Kaup and G.E. Thomas, SPIE AeroSense 2000 Symposium, Orlando, Florida, 24 April, 2000.
- + *Optical Fibers and Solitons* , Dept. of Mathematics, University of Central Florida, Orlando, Florida, 26 April, 2000.
- + *Modeling ans Simulation: A Case Study with Magnetrons*, Institute for Simulation and Training, University of Central Florida, Orlando, Florida, 27 April, 2000.
- + *Virtual Solitons*, Friedrich-Schiller University, Jena, Germany, June 15, 2000.
- + *Virtual Solitons*, RCP 264: Inverse Problems and Nonlinearity, Montpellier University, Montpellier, France, June 21, 2000.
- + *Asymptotic Solution of the Second Harmonic Generation Equations*, AMS Sectional Meeting, Birmingham, AL, Nov. 10, 2000.

- \+ *Theoretical Modeling of Crossed-Field Electron Vacuum Devices*, APS Plasma Physics meeting, Quebec City, Quebec, Canada, 23-27 Oct., 2000.
- \+ *Introduction to Solitons*, Median Seminar Series, Institute for Simulation and Training, University of Central Florida, Orlando FL, March 16, 2001.
- \+ *False Instabilities in Variational Methods*, Second IMACS International Conference on "NONLINEAR EVOLUTION EQUATIONS AND WAVE PHENOMENA: COMPUTATION AND THEORY", Athens, GA, April 9, 2001.
- \+ *Theoretical Studies of a Planar Model of a Relativistic Magnetron*, SPIE AeroSense 2001, Orlando, FL, April 17, 2001.
- \+ *Second Harmonic Generation and an Unsolved IST*, The Legacy of the Inverse Scattering Transform, AMS Summer Research Conference, Mount Holyoke, MA, June 17, 2001.
- \+ *The Inverse Scattering Transform of Three Level Coherent Pulse Propagation*, "SOLITON EQUATIONS: APPLICATIONS AND THEORY", Univ. of Colorado at Colorado Springs, 10 Aug. 2001.
- \+ *Novel Solitons*, "Arizona Applied Math Fest", University of Arizona, Tucson, Ariz., Nov. 3, 2001.
- \+ *Stationary Solutions of the Magnetron Equations*, APS Plasma Physics meeting, Long Beach, Calif., 27 Nov., 2001.
- \+ *Initiation and Stationary Operating States in a Crossed-Field Vacuum Electron Device* D. J. Kaup, SPIE Aerosense meeting, 2 April, 2002.
- \+ *Modeling of a Relativistic Magnetron*, APS Plasma Physics Meeting, Orlando FL, Nov. 13, 2002.
- \+ *Virtual Solitons*, Nonlinear Waves Seminar, Duke University, Raleigh, NC, April 18, 2002.
- \+ *Second Harmonic Generation as an Inverse Problem*, " SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives", Landau Institute for Theoretical Physics, Chernogolovka, Russia, August 20, 2002.

* Consultative And Advisory Functions To Other Laboratories And Agencies

- * Consultant to CPII (formerly Varian Beverly). Involved in the application of comparison of theory with experimental data for CFA and magnetrons.
- \+ Advisor to the Directed Energy Laboratory, Kirkland AFB.
- \+ Advisor to Hanscom Geophysical Laboratory, Hanscom AFB.

* Transitions

NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES

It is still too early to be definite, but the potential instability of the background noted in Proc. of SPIE, **4720** 67-74, (2002), could well be responsible for the typical pulse shortening noted in HPMs. As we have discussed above, our current results on the initiation stage shows that such is indeed within the realm of possibility. It will be for the research on the new contract to confirm or deny this possibility. However, this instability does have some very interesting features. 1)In the initiation stage, for the A6, at maximum growth, the second-order DC radial velocity can be on the order of one-half of the magnitude of the RF radial velocity. (For the T266, there was always a factor of at least 4.) 2)Thus by the time the device reaches the saturation stage, this difference could become even less. 3)With a small difference between these quantities, the instability could easily be initiated by some random fluctuation. 4)With a random fluctuation, there would be an irregularity in when the pulse shortening occured. 5)When this instability turns on, it would be expected to create a chaotic background, destroying coherence and thereby terminating power production.

HONORS/AWARDS

Appointed Provost Distinguished Research Professor, University of Central Florida, Orlando, FL. in 2001.