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Key accomplishments were fueled by the discovery of an inherent relationship between the Multi-Stage Wiener Filter (MWF) and the iterative search method of Conjugate-Gradients (CG). This lead to a host of new algorithmic implementations of CG-MWF that offer a number of important advantages over the original implementation of MWF. The discovery that adding a"stage" to the MWF was equivalent to taking a "step" of a CG search has lead to substantial improvements to the MWF in terms of (i) computationally efficient implementations that exploit structure in the data matrices or correlation matrices, such as Toeplitz, sparseness, etc., (ii) amenability to real-time implementation, (iii) amenability to "smart" initialization (based on information learned or statistics estimated during the process of searching for a training sequence embedded in the data, for example), (iv) easy incorporation of a-priori information, constraints, and/or "past history" for accelerated convergence, and (v) amenability to complementary PC based rank reduction.

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PROJECT TITLE:

# ADVANCES IN REDUCED-RANK ADAPTIVE FILTERING: ANALYSIS OF STATISTICAL PERFORMANCE MEASURES

**PI: Michael D. Zoltowski, Purdue University**

**FINAL TECHNICAL REPORT:**
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## LONG-TERM GOALS

The primary goal of this project is to substantially reduce the computational complexity of the adaptive beamformer employed in real-time passive sonar systems for underwater surveillance. The main methodology for achieving this computational reduction is reduced-rank adaptive filtering via the method of Conjugate Gradients. Reduced-rank adaptive filtering is vitally important for underwater surveillance due to the large number of hydrophones typically employed, coupled with issues of sample support related to target mobility and the high degree of nonstationarity of the underwater environment.

## OBJECTIVES

A key objective is to develop a computationally efficient implementation of the Multi-Stage Wiener Filter (MWF) for adaptive beamforming in scan mode. The Minimum Variance (MV) Spatial Spectrum estimate for a particular bearing may be computed as the reciprocal of the inner product between the "look direction" vector for that bearing angle and the solution to the MVDR linear system of equations, with the sample covariance matrix on the left-hand-side and the "look direction" vector on the right-hand-side. Explicitly computing the inverse of the sample covariance matrix is computationally burdensome and problematic if the sample support is not adequate. The Multi-Stage Wiener Filter (MWF) provides a reduced-rank solution to the Minimum Variance Distortionless Response (MVDR) equations that is computationally efficient as well as robust to sample support. However, in scan mode, the MVDR equations need to be solved for each of a large number of "look directions." This has limited the application of MWF in MVDR based scanning. However, the recently discovered equivalence between MWF and the iterative search method of Conjugate Gradients (CG) provides the key to computationally efficient scanning via the MWF: the solution to the MVDR equations for one bearing angle is used to initialize the CG search for the solution to the MVDR equations for the next bearing angle on the grid. As a result, only a few steps of CG are needed each time the "look direction" is incremented in the process of scanning.

A second objective is to develop efficient FFT based implementations of the CG method for generating the Minimum Variance spatial spectrum that avoid the need to explicitly estimate the covariance matrix. A third objective is conduct extensive simulations to verify the efficacy of this approach in complex underwater environments. These simulations will also include an alternative to CG based on Constrained Steepest Descent. A fourth objective is to use both theoretical sensitivity analysis and the results of these simulations to develop rules for variable grid step size and the number of CG steps at each grid point.

## APPROACH

The search volume of interest is scanned by discretizing the bearing, range, and depth location variables. For each "look direction" of interest, one ostensibly has to solve the MVDR equations $\mathbf{R}_{xx}\mathbf{w}(\boldsymbol{\theta}) = \mathbf{s}(\boldsymbol{\theta})$, where $\mathbf{R}_{xx}$ is the sample covariance matrix, and $\mathbf{s}(\boldsymbol{\theta})$ and $\mathbf{w}(\boldsymbol{\theta})$ are the steering vector and adaptive beamforming weight vector for the particular look direction $\boldsymbol{\theta}$. In scan mode, the MVDR equations need to be solved for each of a large number of look directions. For each prospective look direction, MWF ostensibly starts "over again" by using the current look direction vector $\mathbf{s}(\boldsymbol{\theta})$ as the first basis vector for the forward recursion. Such a mode of operation has limited the application of MWF in MVDR based scanning.

Yet, a primary advantage of MWF deals with converging to the near-optimal adaptive filter during a period when the underlying signal statistics are relatively stationary. The larger the number of elements comprising the array, the longer it takes for the adaptive beamformer to converge to a point where it is providing a near-optimal SINR, for example. If the underlying signal statistics are varying too quickly, a high-dimensional beamformer may not converge and thus lose track. MWF based reduced-rank adaptive filtering offers substantial decreases in convergence time, leading to enhanced performance in nonstationary target scenarios.

Fortunately, the CG connection provides the key to computationally efficient scanning via the MWF. CG is an iterative search scheme; CG takes less steps to converge to the "desired solution" the closer the starting point is to the "desired solution". The solution to $\mathbf{R}_{xx}\mathbf{w}(\boldsymbol{\theta}) = \mathbf{s}(\boldsymbol{\theta})$ should be used to initialize the CG based search for the solution to $\mathbf{R}_{xx}\mathbf{w}(\boldsymbol{\theta}+\boldsymbol{\delta}) = \mathbf{s}(\boldsymbol{\theta}+\boldsymbol{\delta})$. If $\boldsymbol{\delta}$ is small enough, only two or three steps of CG are adequate for each new look direction. The first look direction will have to be solved in the "standard" MWF manner, with the first basis vector of the forward recursion equal to the steering vector for the initial "look direction."

The other key implication of the CG discovery is that MWF may be used in conjunction with Principal Components (PC) based rank reduction. Being an iterative search method, CG can be applied in a lower-dimensional space, the signal subspace, for example. The beamformer for each target being tracked needs to adapt with time. Dr. Norm Owlsey and others have shown that the use of Principal Components lowers the sample support needed for estimating the covariance matrix from roughly twice the number of sonar sensors to twice the number of sources. Coupled with computationally efficient algorithms for tracking the signal subspace, PC can effect a substantial reduction in the computational load by effectively compressing the high-dimensional "element space" covariance matrix to a covariance matrix of dimension equal to the number of sources, denoted $\mathbf{R}_{cc}$. CG may be applied as follows: the solution to $\mathbf{R}_{cc}(n)\mathbf{w}(\boldsymbol{\theta}(n)) = \mathbf{s}(\boldsymbol{\theta}(n))$ is used to initialize the CG search for the solution to $\mathbf{R}_{cc}(n+D)\mathbf{w}(\boldsymbol{\theta}(n+D)) = \mathbf{s}(\boldsymbol{\theta}(n+D))$, where $\mathbf{R}_{cc}(n)$ and $\mathbf{R}_{cc}(n+D)$ are the compressed sample covariance matrices at time $n$ and $n+D$, respectively, and $\boldsymbol{\theta}(n)$ and $\boldsymbol{\theta}(n+D)$ are the look directions for the target of interest at time $n$ and $n+D$, respectively. The value of $D$ depends on the update rate.

## WORK COMPLETED

Key accomplishments during the first year effort were fueled by our recent discovery of an inherent relationship between the MWF and the Iterative Search Method of Conjugate-Gradients (CG). This lead to a host of new algorithmic implementations of CG-MWF that offer a number of important advantages over the original implementation of MWF. The discovery that adding a"stage" to the MWF was equivalent to taking a "step" of a CG search has lead to substantial improvements to the MWF in terms of (i) computationally efficient implementations that exploit structure in the data matrices or correlation matrices, such as Toeplitz, sparseness, etc., (ii) amenability to real-time implementation, (iii) amenability to "smart" initialization (based on information learned or statistics estimated during the process of searching for a training sequence embedded in the data, for example), (iv) easy incorporation of a-priori information, constraints, and/or "past history" for accelerated convergence, and (v) amenability to complementary PC based rank reduction.

The basic implementation of CG-MWF for solving the $N \times N$ system of Wiener-Hopf equations $\mathbf{R}_{xx}\mathbf{w} = \mathbf{r}_{dx}$ is summarized in Table 1, where $\mathbf{r}_{dx}$ is the cross-correlation vector between the desired signal and the observed data. For the beamforming application, $\mathbf{r}_{dx}$ is the steering vector for the current look direction. We have formulated new algorithmic implementations of CG-MWF that offer a number of important advantages over the original implementation of MWF. In addition, CG-MWF offers a number of VIP advantages over both LMS and RLS based adaptive filtering.

The basic CG-MWF algorithm is delineated on the left hand side of Table 1. Each step/iteration of CG-MWF involves two vector-vector multiplies, three simple vector updates, and one matrix-vector product. The latter is the most computationally burdensome step of CG-MWF. $\mathbf{R}_{xx}$ may be estimated from $M$ blocks of data as $\mathbf{R}_{xx} = \mathbf{X}\mathbf{X}^H/M$, where $\mathbf{X}$ is an $N \times M$ Toeplitz data matrix. Through the circulant extension of a Toeplitz matrix, the code in the right hand side of Table 1 shows that the product $\mathbf{R}_{xx}\mathbf{u}_i$ can be computed via FFT's leading to two VIP advantages: (1) substantially reduced computational complexity by choosing the FFT length to be a power of two and (2) substantially reduced memory requirements as $\mathbf{R}_{xx} = \mathbf{X}\mathbf{X}^H/M$ need not be formed and thus not stored. Relative to the latter point, all that is required is a one-time FFT of the data stream from which the $M$ successive data blocks are implicitly extracted.

Recent algorithmic implementations of MWF by Ricks and Goldstein also avoid having to form (or store) a covariance matrix. In contrast to the FFT based CG-MWF algorithm described above, the innovative lattice, modular implementation of MWF of Ricks and Goldstein requires a backwards recursion as well as a forward recursion and it does not directly yield the Wiener-Hopf weights. The relationship between the standard version of MWF and their innovative lattice/modular implementation of MWF is analogous to the relationship between the standard version of RLS and fast RLS algorithms.

As part of the second year effort, we are developing further refinements to CG-MWF that advantageously exploit the fact that it works directly on the covariance matrix. In contrast, RLS either explicitly or implicitly works with the inverse of the covariance matrix. As a result of the nonlinear relationship between the covariance matrix and its inverse, the innovations we propose have no counterpart in the realm of RLS based algorithms. As a result, in addition to the huge advantage of CG-MWF over RLS under conditions of low sample support, due to the inherent reduced-rank processing of CG-MWF, there are also major advantages in terms of initialization and computational complexity. The computational complexity reduction of CG-MWF relative to RLS arises primarily from the former's ability to exploit the Toeplitz structure of the covariance matrix via FFT's. Since the inverse of a Toeplitz matrix is not Toeplitz (in general), RLS does not offer this feature.

Another key advantageous feature of CG-MWF relative to RLS is "smart initialization." Equalization of underwater digital communication signals – equalizing distortion arising from propagation effects – is used in the illustrative simulation example. In this application, one may obtain an initial estimate of the impulse response of the propagation channel through semi-blind means (limited training plus exploitation of signal properties.) We say "initial", since the channel is time-varying, in general. The initial channel estimate may be used to initialize the covariance matrix, $\mathbf{R}_{xx}$, in the CG-MWF algorithm, as well as the cross-correlation vector, $\mathbf{r}_{dx}$. The former is the outer product of the convolution matrix of the channel with itself, while the latter is is the channel itself with some zero padding. Both quantities are then updated on a per snapshot basis as time evolves to implicitly track the time-variations of the channel. Preliminary results indicate that this channel based initialization of CG-MWF, which is possible due to the fact that CG-MWF works directly with $\mathbf{R}_{xx}$, has a very dramatic effect on the convergence speed of CG-MWF. In contrast, even if an estimate of the channel is available, there are no means through which RLS can utilize it since that would require inverting the outer product of the convolution matrix of the channel with itself; this totally defeats the purpose of RLS in addition to representing a huge attendant computational burden.

Although the target application is underwater surveillance, the CG-MWF algorithm is as universally applicable as the classical adaptive filtering schemes of LMS and RLS. In fact, I believe that CG represents the next generation of adaptive filtering algorithms.

## RESULTS

A simple illustrative simulation is presented to demonstrate the efficacy of the efficient FFT based CG-MWF algorithm. The application is linear equalization of a multipath distorted QPSK signal. In addition to a direct path, there is a 70% ghost at a half-symbol delay with a phase of 165°. Factoring in the pulse shaping, the effective channel length was roughly $L = 13$. An equalizer length of $N_g = 20$ was selected; the taps were symbol-spaced. $N = 32$ is the FFT length.

The multipath channel is frequency selective, thereby causing the intersymbol interference shown in the received constellation plotted in the upper-left corner of Figure 1. Applying the respective equalizing weight vectors obtained after 3 steps and 5 steps of FFT based CG-MWF yields the constellations plotted in the lower-left and lower-right corners of Figure 1, respectively. This demonstrates that near-optimal performance may be obtained with a few steps of CG-MWF, and that CG-MWF may be implemented efficiently via FFT processing. In addition to the savings associated with the FFT based computation of the elements of the matrix-vector product $\mathbf{R}_{xx}\mathbf{u}_i$ at each step of CG, we also avoid the computation associated with explicitly estimating $\mathbf{R}_{xx}$.

## IMPACT/APPLICATIONS

This work will yield substantial reductions in the computational complexity associated with implementing adaptive beamformers in real-time passive sonar systems for underwater surveillance.

## TRANSITIONS

An all-day tutorial entitled "TUTORIAL ON REDUCED-RANK ADAPTIVE FILTERING BASED ON THE MULTI-STAGE WIENER FILTER" was presented on 18 June 2002 to the CITE Group at Rome Labs headed by Dr. Bruce Suter. Dr. Bruce Suter and CITE will implement and assess the performance of the Conjugate-Gradient based reduced rank filtering schemes developed as part of this project. We will continue to report of this ongoing transition as it progresses.

## RELATED PROJECTS

There is a synergism between this work and research being conducted in parallel for a National Science Foundation project entitled "Reduced-Dimension Decision Feedback Equalizers (DFE's) for 4G High-Speed Wireless Digital Communications." This is a single Principal Investigator grant funded by the Computing and Communications Research Division of the CISE Directorate at NSF. (Grant Number: CCR-0118842. Duration: 1 Sept. 2001 - 31 Aug. 2004.) Equalization of Digital TV is an ideal application for CG based adaptive filtering due to the high dimensionality of the DFE; current receiver chips have 576 DFE taps that need to be adapted with time. In addition to the synergism relative to theoretical developments, there is a synergism emerging with respect to the underwater application following from a lecture recently delivered at the Second Sensor Array and Multichannel Workshop held 4-6 August 2002 in Rosslyn, VA. The lecture was by Dr. Johann F. Bhme of the Ruhr-University in Bochum, Germany, and was entitled "Multichannel Blind Equalization in Digital Underwater Acoustic Communications: Experimental Results."

## PUBLICATIONS


- Michael Zoltowski, "Conjugate Gradient Based Adaptive Filtering with Application to Space-Time Processing for Wireless Communications," *Second IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM 2002)*, Rosslyn, VA, 5-6 August 2002.
- Matthew Weippert, John Hiemstra, J. Scott Goldstein, and Michael Zoltowski, "Insights from the Relationship Between the Multistage Wiener Filter and the Method of Conjugate Gradients," *Second IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM 2002)*, Rosslyn, VA, 5-6 August 2002.

| |
|---|
| $\mathbf{w}_0 = \mathbf{0}$ |
| $\mathbf{u}_1 = \hat{\mathbf{r}}_{dx}$ |
| $\mathbf{t}_1 = -\mathbf{u}_1$ |
| $\ell_1 = \mathbf{t}_1^H \mathbf{t}_1$ |
| for $i = 1, \ldots, D$ |
| $\mathbf{v} = \mathbf{X}\mathbf{X}^H \mathbf{u}_i$ |
| $\eta_i = \ell_i / \mathbf{u}_i^H \mathbf{v}$ |
| $\mathbf{w}_i = \mathbf{w}_{i-1} + \eta_i \mathbf{u}_i$ |
| $\mathbf{t}_{i+1} = \mathbf{t}_i + \eta_i \mathbf{v}$ |
| $\ell_{i+1} = \mathbf{t}_{i+1}^H \mathbf{t}_{i+1}$ |
| $\Psi_i = \ell_{i+1} / \ell_i$ |
| $\mathbf{u}_{i+1} = -\mathbf{t}_{i+1} + \Psi_i \mathbf{u}_i$ |

Direct Block CG-MWF.

```
L=block length; N=FFT length;
M=weight vector length; N=M+L-1
X=fft(xd,N);
F=(exp(-j*2*pi/N).^[0:N-1]').*conj(X);
w=zeros(M,1); u=rdx; g=-u;
l=g'*g;
for i=1:Nstop,
d=ifft(X.*fft(u,N),N); y=d(end-L+1:end,1);
z=ifft(F.*fft(y,N),N); v=z(end-M+1:end,1);
eta=l/(u'*v);
w_old=w;
w=w_old+eta*u;
g_old=g;
g=g_old+eta*v;
l_old=l;
l=g'*g;
psi=l/l_old;
uold=u;
u=-g+psi*u_old;
end
```

**Table 1.** Standard CG-MWF (left) and FFT Implementation of CG-MWF (right).

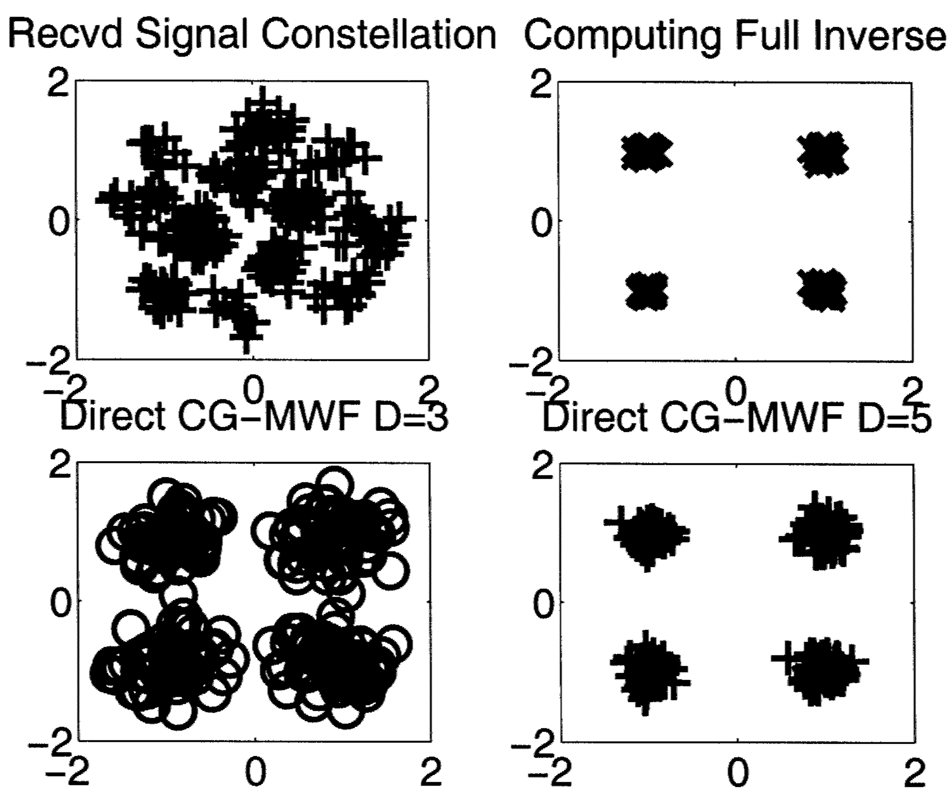
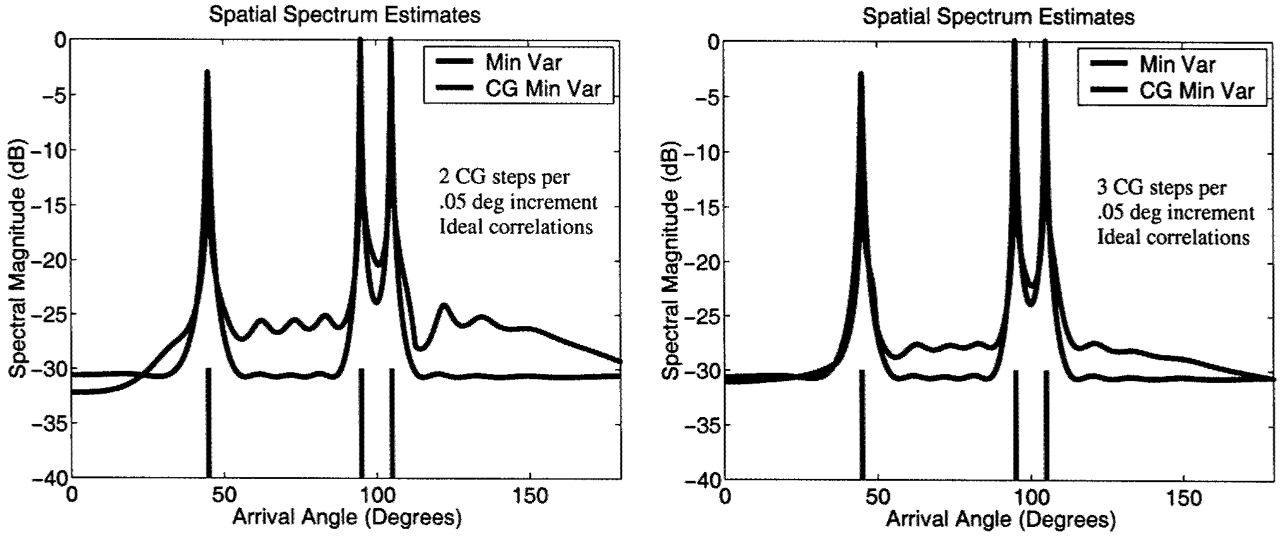


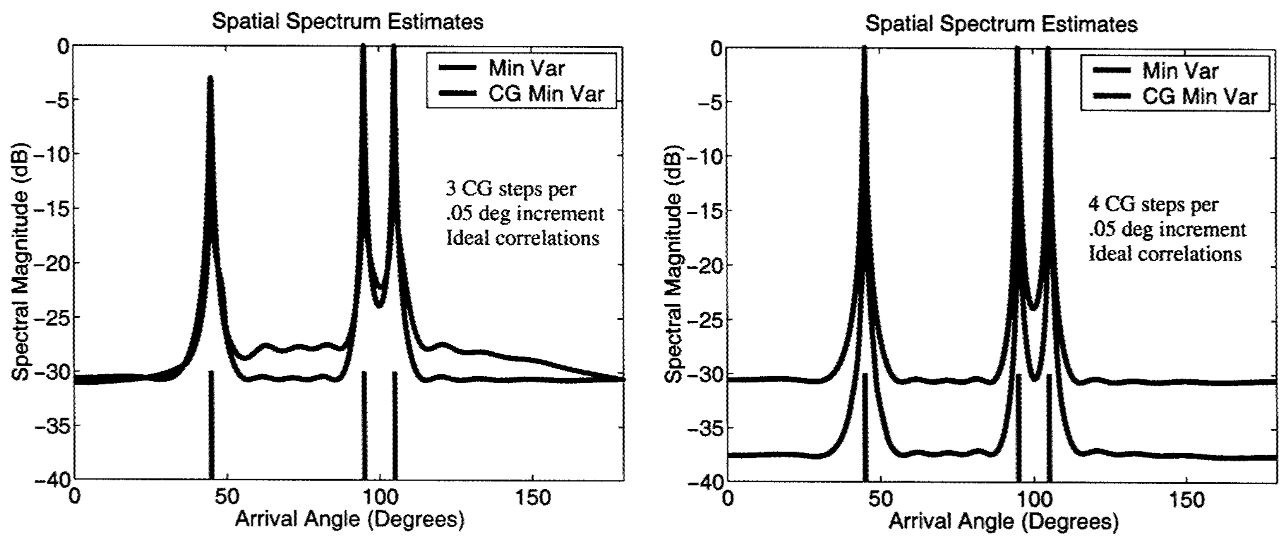

**Figure 1.** FFT Direct CG-MWF Applied to Equalization of QPSK

**CG Spectral Estimation – Ideal Correlations**
**Varying Number of CG Steps per Angle Increment**



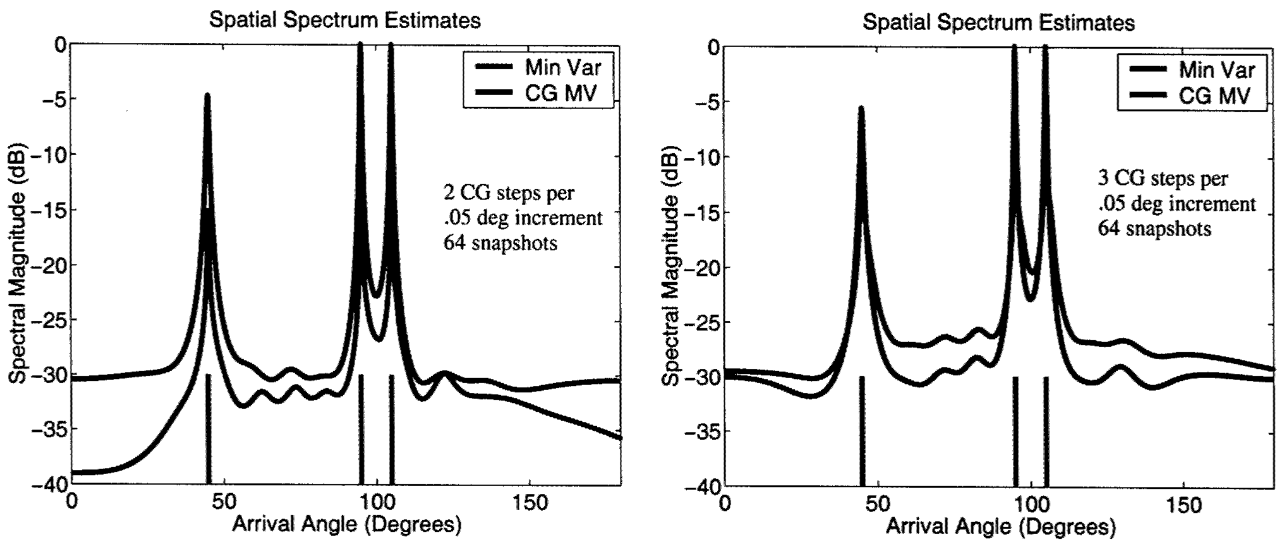

12 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG Spectral Estimation – Ideal Correlations**
**Varying Number of CG Steps per Angle Increment**



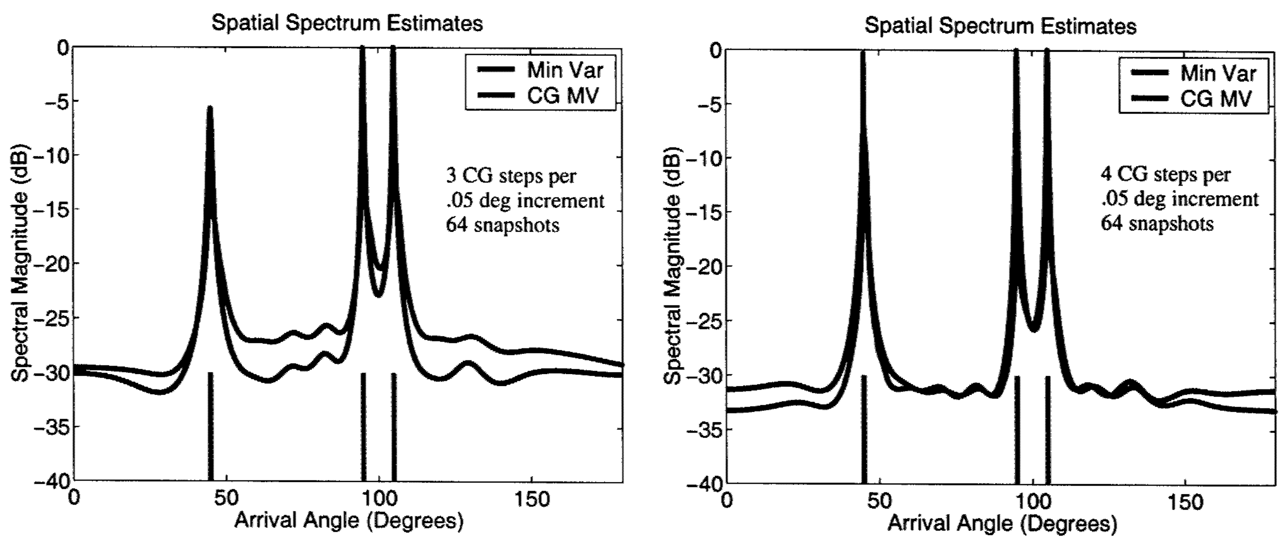

12 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG Spectral Estimation**
**Varying Number of CG Steps per Angle Increment**



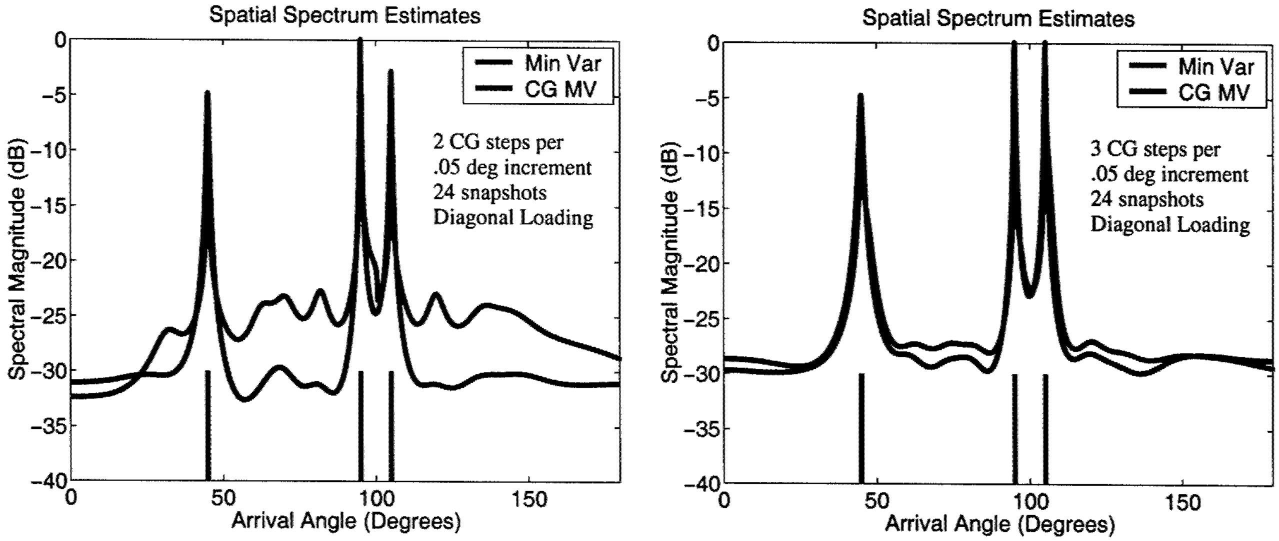

12 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG Spectral Estimation**
**Varying Number of CG Steps per Angle Increment**



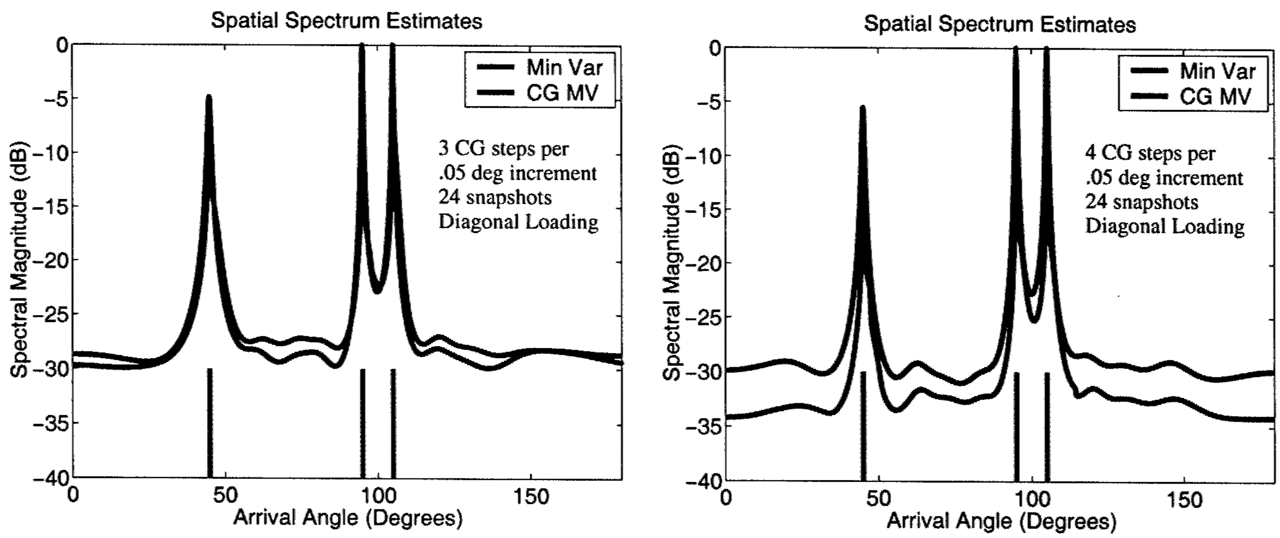

12 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG-FFT Spectral Estimation**
**Small Number of Snapshots with Diagonal Loading**



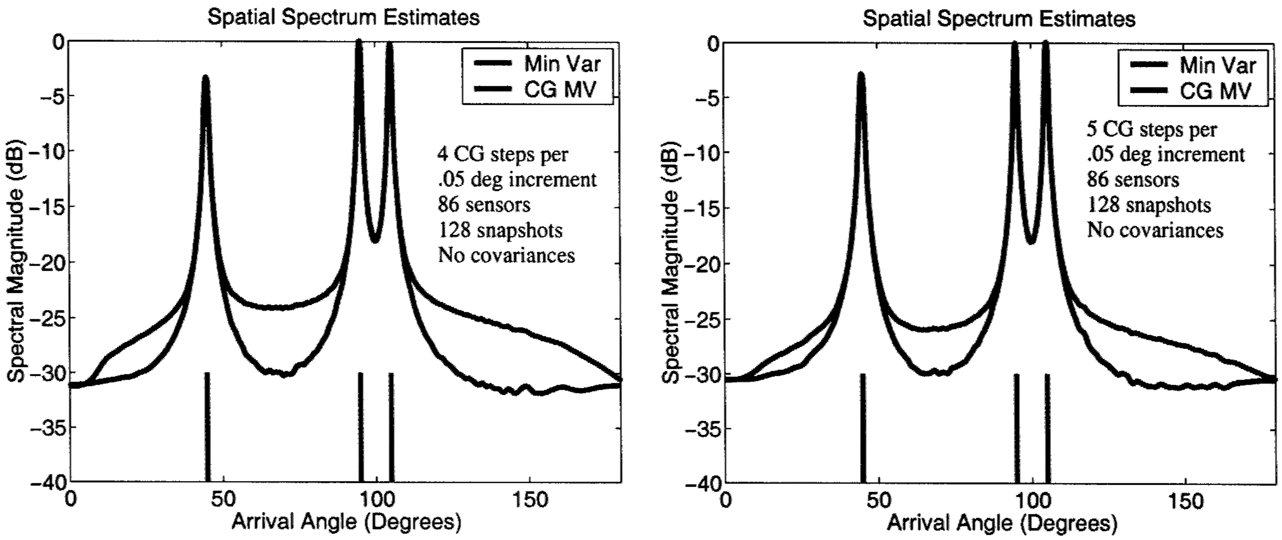

12 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG-FFT Spectral Estimation**
**Small Number of Snapshots with Diagonal Loading**



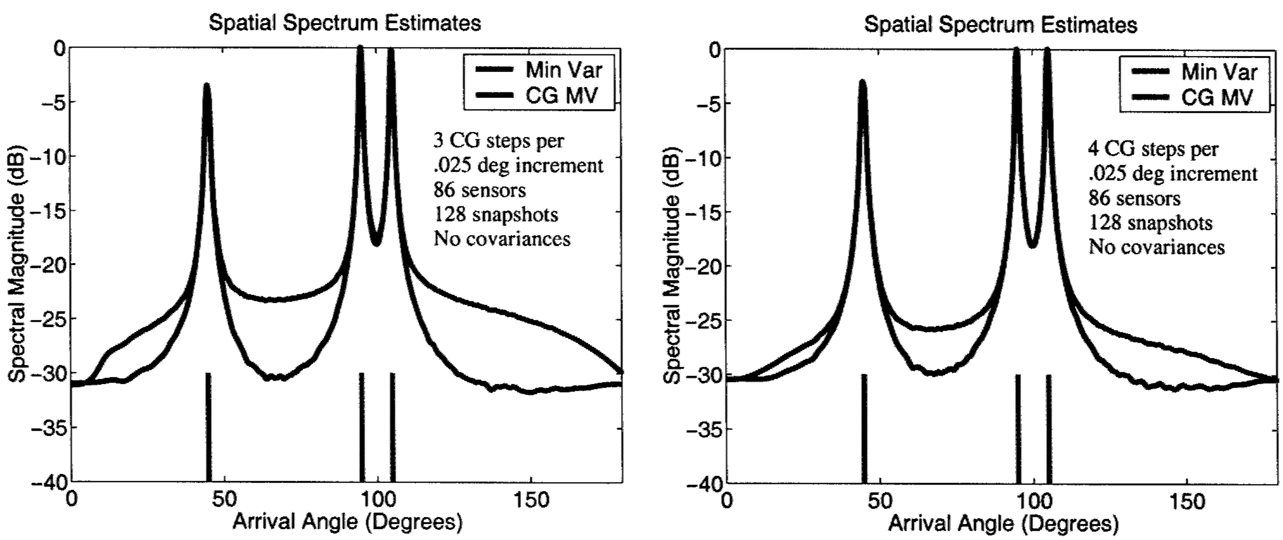

12 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG-FFT Spectral Estimation: No Covariance Matrix**
**Number of CG Steps per Angle Increment**



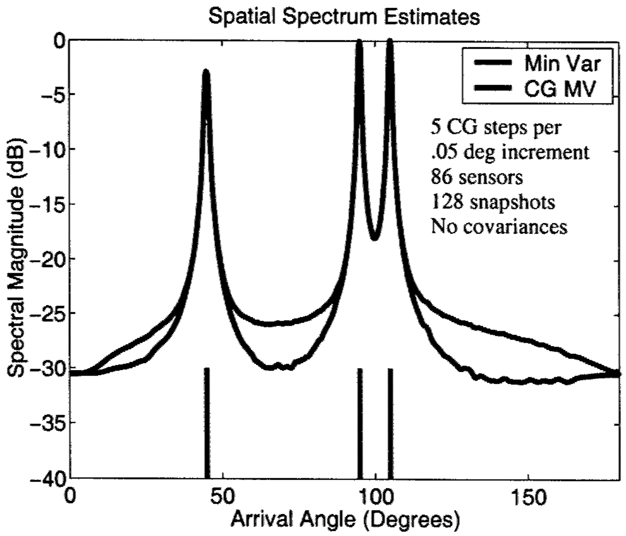

86 sensor ULA; $\Delta = 0.05°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG-FFT Spectral Estimation: No Covariance Matrix**
**Number of CG Steps per Angle Increment**

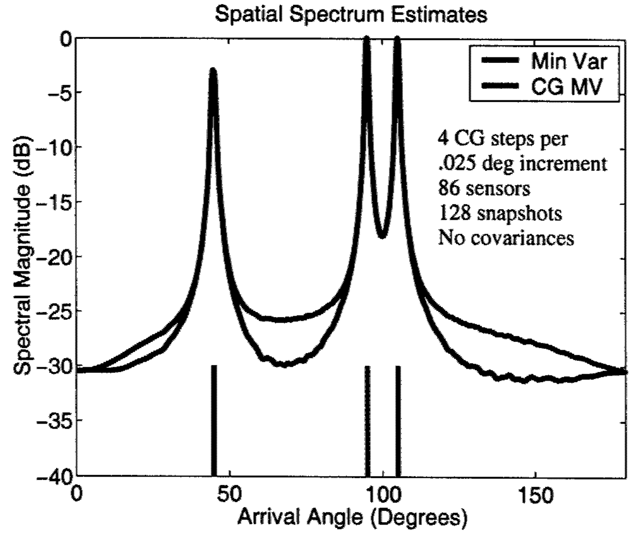



86 sensors ULA; $\Delta = 0.025°$ SNR1=14 dB; SNR2=SNR3=20 dB

**CG-FFT Spectral Estimation: No Covariance Matrix**
**Number of CG Steps versus Angle Increment**

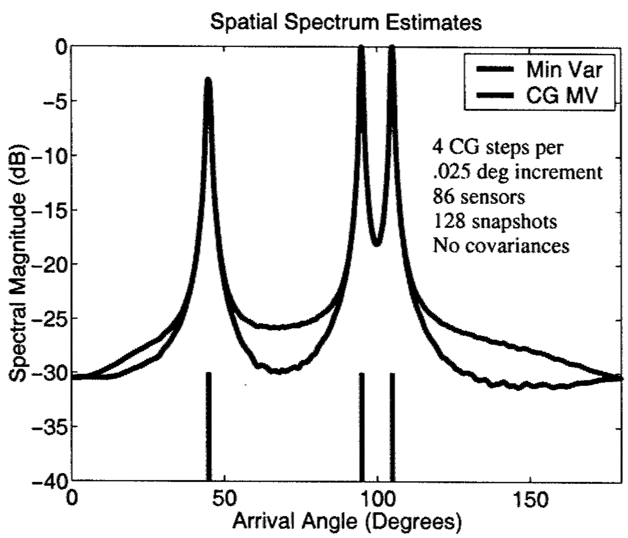



86 sensor ULA

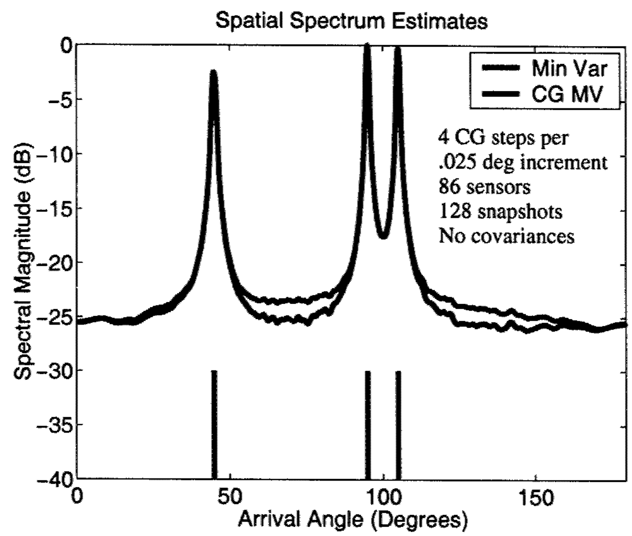



SNR1=14 dB; SNR2=SNR3=20 dB

**CG-FFT Spectral Estimation: No Covariance Matrix**
**Number of CG Steps versus SNR**

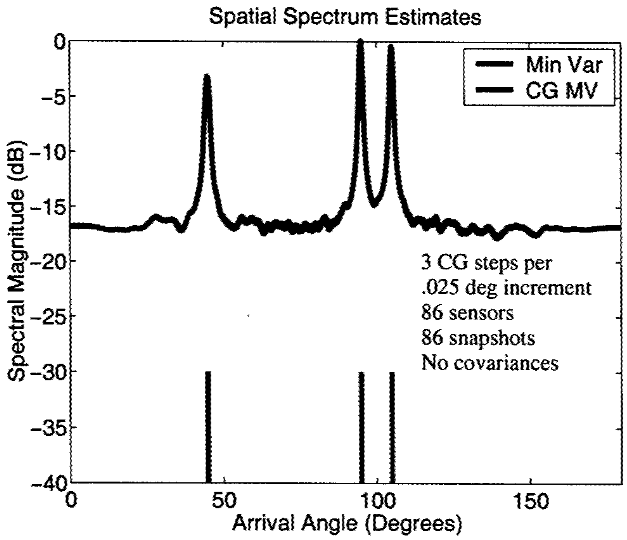



SNR1=14 dB; SNR2=SNR3=20 dB

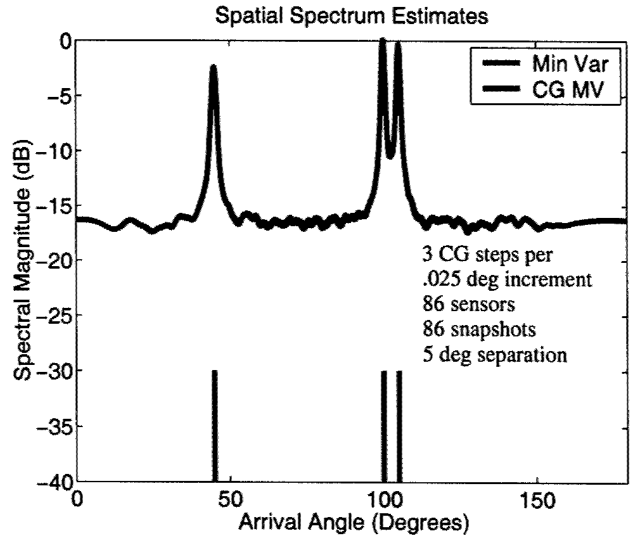



SNR1=4 dB; SNR2=SNR3=10 dB

**CG-FFT Spectral Estimation: No Covariance Matrix**
**SNR, Angular Separation, and Number of Snapshots Decreased**

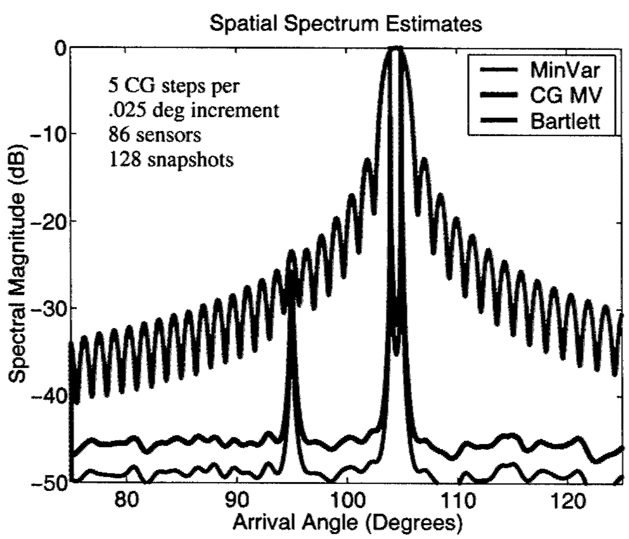



SNR1=-6 dB; SNR2=SNR3=0 dB
angle between two strongest
sources equals 10 degrees
(diagonal loading)

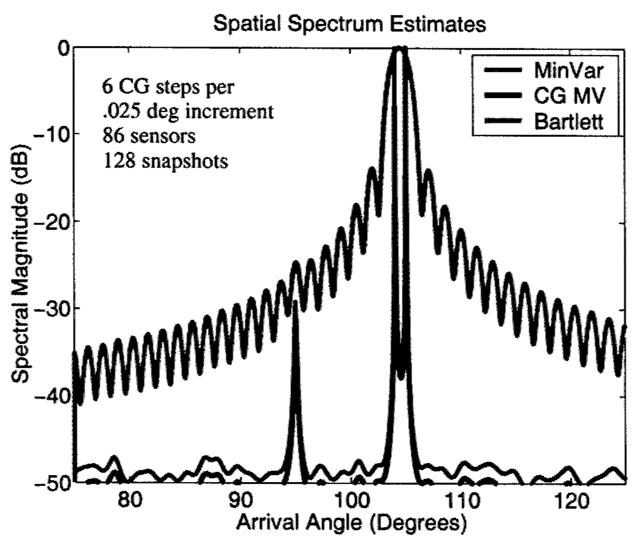



SNR1=-6 dB; SNR2=SNR3=0 dB
angle between two strongest
sources equals 5 degrees
(diagonal loading)

**CG Minimum Variance Spectral Estimation**
**True MV versus CG MV versus Bartlett**




SNR1=0 dB; SNR2=SNR3=30 dB




AOA1=95°; AOA2=104°; AOA3=105°

# 1 PERSONNEL SUPPORTED

| | |
|---|---|
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| Graduate Students: | Samina Chowdhury |