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This research is motivated by the problem of motion planning of autonomous vehicles in an uncertain environment. A possible approach to reduce the computational complexity of the motion planning problem for a nonlinear, high dimensional system, is based on a quantization of the system dynamics, in the sense that we restrict the feasible nominal system trajectories to the family of time-parametrized curves that can be obtained by the Inter-connection of appropriately defined primitives. These primitives will then constitute a ``maneuver library'' from which the nominal trajectory will be constructed. Instead of solving an optimal control problem over a high-dimensional, continuous space, we solve a mixed integer programming problem, over a much smaller space. At the core of the control architecture lies a hybrid automaton, the states of which represent feasible trajectory primitives for the vehicle. Each constituent subsystem of the hybrid controller will be the agent responsible for the maneuver execution. The task of the automaton will be the generation of complete, feasible and ``optimal'' trajectories, via the interconnection of the available primitives. Apart from the reduction in computational complexity, one of the objectives of this approach is the ability to provide a mathematical foundation for generating a provably stable hierarchical system, and for developing the tools to analyze robustness in the presence of uncertainty in the process as well as in the environment.

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# HIERARCHICAL NONLINEAR CONTROL FOR UNMANNED AERIAL VEHICLES

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## 1 Summary of Past Research

Below is a summary of the research conducted under NSF support, grant number ECS-9907466 (1999-2003) and grant ECS-9796099 (1997-2000).

### 1.1 Identification of Complex Systems

Here, we discuss the problem of identifying a complex system with a limited-complexity model using finite corrupted data. Complex systems are ones that cannot be uniformly approximated by a finite dimensional space. Nevertheless, our *prejudice* is represented by selecting a finitely parameterized set of models from which an estimate of the original system will ultimately be drawn. We introduced a new formulation that shows how such a model should be selected from data. We demonstrated this paradigm on the class of linear time-invariant stable systems. We also introduce a new information-theoretic criterion for model quality evaluation that is usable in the case of under modeling.

#### 1.1.1 Finite Dimensional Models of Systems with Unknown Dimension

This work addressed the fundamental question of identifying a finite-dimensional model from data generated by a possibly infinite-dimensional system using noisy input-output data. We assume that the actual process  $T_0 \in \mathcal{T}$  is  $\gamma$  away from a given finitely parametrized subspace  $\mathcal{G}$ . The main result of this work is summarized as follows. Let  $G_0$  be the best approximation of  $T_0$  in  $\mathcal{G}$ . the the following is true:

**Theorem 1** *Given the following assumption:  $\mathcal{T}$  is  $\ell_1$ ,  $\mathcal{G}$  is the set of FIR systems of order  $M$ , noise is white (or low correlated deterministic), prior  $\mathcal{I}$ :  $\text{dist}(T_0, \mathcal{G})$  is bounded by  $\gamma$ , then there exists an input  $u$  such that the least squares estimator converges to  $G_0$ :*

$$\sup_{T \in \mathcal{I}} \sup_{w \in \mathcal{W}^L} \|\hat{G}_n - G_0\| \leq (C_1 \gamma + C_2 M) \frac{\log(n)}{\sqrt{n}}$$

*In addition, the estimator is not tuned to  $\gamma$ . The above convergence rate is also polynomial.*

These results with a complete discussion on input design are in [11].

These results were also extended to classes of nonlinear systems. In addition, the paper [12] presents a consistent formulation for designing robust controllers from Data. It is shown that, if  $T$  is the standard Hardy-Sobolev space, then the resulting error bounds can be directly incorporated for synthesizing robust controllers. All resulting problems can be solved using convex optimization techniques.

### 1.1.2 Model Description Complexity (MDC)

We introduced MDC as a measure of distance between an estimate of a model and the actual process in order to evaluate the quality of a model class for estimating a Data set constructed from a complex input. In the special case where all the model classes contain the model that generated the Data, MDC coincides with the minimum description length (MDL) measure. Using MDC, we proposed a method for order estimation when the process has an unknown degree and does not have an a priori known rate of decay. These results are summarized in the recent Ph.D thesis [14, 15].

### 1.1.3 Surface Lyapunov Functions

Many systems of interest are dynamic systems whose behavior is determined by the interaction of continuous and discrete dynamics. These systems typically contain variables or signals that take values from a continuous set and also variables that take values from a discrete, typically finite set. These continuous or discrete-valued variables or signals depend on independent variables such as time, which may also be continuous or discrete. Such systems are known as Hybrid Systems. Although widely used, not much is known about analysis of hybrid systems. This work attempts to take a step forward in understanding and developing tools to systematically analyze certain classes of hybrid systems. In particular, it focuses on a class of hybrid systems known as Piecewise Linear Systems (PLS). These are characterized by a finite number of affine linear dynamical models together with a set of rules for switching among these models. Even for simple classes of PLS, very little theoretical results are known. More precisely, one typically cannot assess a priori the guaranteed stability, robustness, and performance properties of PLS designs. Rather, any such properties are inferred from extensive computer simulations. In other words, complete and systematic analysis and design methodologies have yet to emerge. In this work, we develop an entirely new constructive global analysis methodology for PLS. This methodology consists in inferring global properties of PLS solely by studying their behavior at switching surfaces associated with PLS. The main idea is to analyze impact maps, i.e., maps from one switching surface to the next switching surface. These maps are proven globally stable by constructing quadratic Lyapunov functions on switching surfaces. Impact maps are known to be "unfriendly" maps in the sense that they are highly nonlinear, multivalued, and not continuous. We found, however, that an impact map induced by an LTI flow between two switching surfaces can be represented as a linear transformation analytically parametrized by a scalar function of the state. Moreover, level sets of this function are convex subsets of linear manifolds. This representation of impact maps allows the search for quadratic Lyapunov functions on switching surfaces to be done by simply solving a set of LMIs. Global asymptotic stability of limit cycles and equilibrium points of PLS can this way be efficiently checked. The classes of PLS analyzed in this work are LTI systems in feedback with an hysteresis, an on/off controller, or a saturation. Although this analysis methodology yields only sufficient criteria of stability, it has shown to be very successful in globally analyzing a large

number of examples with a locally stable limit cycle or equilibrium point. In fact, it is still an open problem whether there exists an example with a globally stable limit cycle or equilibrium point that could not be successfully analyzed with this new methodology. Examples analyzed include systems of relative degree larger than one and of high dimension, for which no other analysis methodology could be applied. We have shown that this methodology can be efficiently applied to not only globally analyze stability of limit cycles and equilibrium points, but also robustness, and performance of PLS. Using similar ideas, performance of on/off systems in the sense that bounded inputs generate bounded outputs, can also be checked. Among those on/off and saturation systems analyzed are systems with unstable nonlinearity sectors for which classical methods like Popov criterion, Zames-Falb criterion, IQCs, fail to analyze. This success in globally analyzing stability, robustness, and performance of certain classes of PLS has shown the power of this new methodology, and suggests its potential towards the analysis of larger and more complex PLS. Results are reported in [17, 16, 18, 19]

## 1.2 Hierarchical Nonlinear Control: Robust Hybrid Automaton

This research is motivated by the problem of motion planning of autonomous vehicles in an uncertain environment. A possible approach to reduce the computational complexity of the motion planning problem for a nonlinear, high dimensional system, is based on a quantization of the system dynamics, in the sense that we restrict the feasible nominal system trajectories to the family of time-parametrized curves that can be obtained by the interconnection of appropriately defined primitives. These primitives will then constitute a “maneuver library” from which the nominal trajectory will be constructed. Instead of solving an optimal control problem over a high-dimensional, continuous space, we solve a mixed integer programming problem, over a much smaller space.

At the core of the control architecture lies a hybrid automaton, the states of which represent feasible trajectory primitives for the vehicle. Each constituent subsystem of the hybrid controller will be the agent responsible for the maneuver execution. The task of the automaton will be the generation of complete, feasible and “optimal” trajectories, via the interconnection of the available primitives. Apart from the reduction in computational complexity, one of the objectives of this approach is the ability to provide a mathematical foundation for generating a provably stable hierarchical system, and for developing the tools to analyze robustness in the presence of uncertainty in the process as well as in the environment.

The proposed architecture builds upon, and exploits the symmetries of autonomous vehicle dynamics, and is based on the definition of two classes of primitives: trim trajectories and “maneuvers” (seen as transition between two trim trajectories). Fundamental properties of the proposed architecture, such as well-posedness, reachability, and consistency have been characterized.

The hybrid automaton can be seen as a new *modeling tool*, in which the ODE describing the system’s continuous dynamics is replaced by the transition rules on the directed graph representing the automaton, and by the associated hybrid system evolution. Results on this development are reported in [4, 5, 6].

### 1.2.1 Application: Real-Time Motion Planning

A very important problem, especially when controlling autonomous vehicles, is represented by motion planning in the presence of fixed or moving obstacles. We are using the expression “motion planning” as opposed to the traditional “path planning” because we want to emphasize the

role of the dynamics of the system, or of non-holonomicity constraints, on the allowable feasible trajectories; this problem is also known in the literature as kinodynamic planning.

One of the fundamental conceptual steps in addressing a problem is the selection of the appropriate representation of the system state, constraints and of the decision variables. The approach that we propose, through the introduction of the hybrid control architecture described in the previous section, can be seen as a “maneuver space”. Using this representation, the hybrid automaton already encodes all the relevant information about the system dynamics and dynamic constraints (such as non-holonomicity).

The hybrid control architecture lends itself in a very straightforward manner to the implementation of a new class of motion planning algorithms, based on a randomized road map approach. By relaxing the requirement for completeness to probabilistic completeness, this class of algorithms can ensure successful termination with arbitrarily high probability, while maintaining computational tractability. Results on motion planning are reported in [3, 7, 8].

We introduced a version of randomized roadmap planner, which is based on the use of Lyapunov functions for local motion planning, and hence provides a very efficient and natural way of dealing with the system dynamics, with the associated stability guarantees. Moreover the algorithm is very general, and can be used with both continuous state-space and hybrid models of a vehicle’s dynamics.

From a theoretical point of view, it was shown how to perform uniform sampling in the reachable space of the vehicle, as opposed to sampling in the input space. Real-time issues were directly addressed: in the case in which finite computation time and available resources do not allow the computation of a feasible solution before a decision has to be made, it was shown how to ensure safety and how to choose likely candidates for further exploration [3, 4].

### **Actor-critic learning algorithms**

We have initiated a new research direction, on actor-critic algorithms for simulation-based optimization in stochastic control. These methods combine value function approximation (as in Neuro-Dynamic Programming or Reinforcement Learning), with gradient descent within a parametrized family of policies or control laws. In particular, learning of the value function is used to construct gradient estimates with improved statistical properties. While variants of actor-critic algorithms have often been proposed within the past 20 years, it was always on a heuristic basis. In our work, we obtain for the first time a sound family of actor-critic methods with rigorous convergence guarantees. Of course, much analysis remains to be done, e.g., in demonstrating some form of statistical efficiency and in identifying promising applications [9].

### **Computational Complexity**

On this subject, we have settled a long-standing open problem. We have shown that the boundedness of all products of a given finite set of matrices cannot be decided algorithmically. This problem relates to the stability of time-varying as well as hybrid systems. In addition, we have completed a systematic exposition and major survey of the literature on the computational complexity aspects of systems and control problems [1, 2].

## Personnel Supported

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### 1.3 Transitions to Technology

**Draper Laboratory:** The work on autonomous systems was transported to Draper Laboratory as part of an integrated simulation and control environment for autonomous Helicopter. With a follow up DARPA support two small Helicopter at MIT and Draper have been built to demonstrate this theory.

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