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General Theory of a Quantitative Method of Optical Patternation

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14. ABSTRACT A general theory of a method to quantitatively perform optical patternation in a particle field is developed. The general equations of optical radiation are considered, along with general methods of probing the particle field with illuminating light and general methods of performing the required measurements, in order to determine the assumptions, approximations, and conditions under which various kinds of measurements can be performed. Under conditions which exist in many practical applications, it is found that the capabilities of modern optical instrumentation allow the spatial distribution of various properties of a particle field such as particle volume or surface area concentrations to be measured, even when the measurement is complicated by attenuation of the illuminating light, attenuation of the signal light, and secondary emission. The measurements can be performed in fully three dimensional form or within a single two dimensional plane, and under certain conditions can be performed even when the particles are not spherical.					
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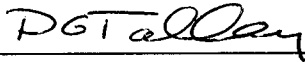
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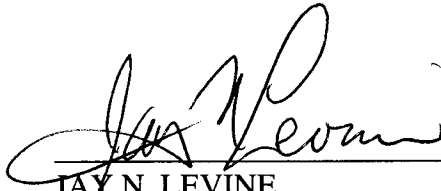
FOREWORD

This interim technical report, entitled "General Theory of a Quantitative Method of Optical Patterning," presents the results of an in-house study performed under JON 3058RF9A by AFRL/PRSA, Edwards AFB CA. The Principal Investigator/Project Manager for the Air Force Research Laboratory was Dr. Douglas G. Talley.

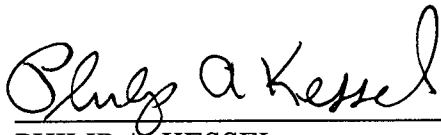
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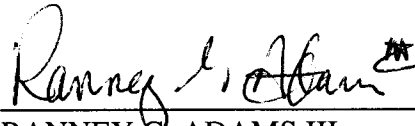
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GLOSSARY

A	area in space
a	particle surface area per unit volume in space; also magnitude of the semi-major axis of the polarization ellipse
b	magnitude of the semi-minor axis of the polarization ellipse; also characteristic transverse dimension of the beam
\tilde{c}	handedness of the polarization ellipse
\mathcal{d}	shorthand differential operator denoting a differential range of states of polarization, section A.4 of appendix A.
E	energy
\dot{E}	instantaneous time rate of energy transfer
\tilde{E}'_n	reduced detector response, section 5.3.4.1.6 of Chapter 5.
e_b	beam extinction function
e_c	signal attenuation function
g_ζ	constant of proportionality between the particle property of interest and its band coefficient
h	equation (29) of Chapter 5
I	instantaneous monochromatic polarization-specific intensity of radiation
I_0	intensity magnitude function
I_s	intensity shape function
i,j,k	indices corresponding to the x,y,z axes of Cartesian coordinates, respectively
k_n	band constant, equation (41) of Chapter 5
L	characteristic size of the particle field
\hat{n}	unit vector
N	number of band coefficients which contribute significantly to the TAMPSEC
p	probe path coordinate
P	IMPSEC component distribution function
\mathcal{P}	polarization state
\tilde{p}	polarization state variable, equals “polarized” or “unpolarized”
s	distance
t	time
x	fraction of energy removed from the beam
\vec{x}	position in space
x,y,z	Cartesian coordinates

Symbols

δ	Dirac delta function
γ	gamma function
Γ	signal attenuation integral
λ	wavelength
θ_a	angle of the semi-major axis of the polarization ellipse
ρ	particle mass per unit volume in space
σ	equation (33) of Chapter 5

Σ	effective probe volume correction factor
ω	solid angle
ξ	instantaneous monochromatic polarization-specific extinction coefficient (IMPSEC)
ξ_{abs}	instantaneous monochromatic polarization-specific absorption coefficient (IMPSAC)
ξ_c	instantaneous monochromatic polarization-specific extinction coefficient component, or IMPSEC component
ξ_f	fluorescence band coefficient
ξ_n	n th band coefficient
ξ_s	elastic scattering band coefficient
ξ_ζ	band coefficient related to the particle property ζ
ζ	particle property

Superscripts

s	pertaining to shape
'	incoming radiation, or irradiation; also, fluctuating component
–	time-averaged
0	pertaining to magnitude; also, initial

Subscripts

abs	absorption
b	beam
bf	from the beam to a surface in space
bs	band-specific
c	converted optical radiation
d	detector
f	pertaining to the fluorescence band coefficient; also, pertaining to a surface in space
fd	from a surface in space to the detector
n	pertaining to the n th band coefficient
p	probe path
pp	probe path
ps	property-specific
pv	effective probe volume
s	pertaining to the elastic scattering band coefficient; also, pertaining to shape
0	pertaining to magnitude

Acronyms

IMPSEC	instantaneous monochromatic polarization-specific extinction coefficient
IMPSAC	instantaneous monochromatic polarization-specific absorption coefficient
TAMPSEC	time-averaged instantaneous monochromatic polarization-specific extinction coefficient

PREFACE

In its original form, this report was written to support an intended patent application. As the preparation of the patent support document progressed, it became increasingly apparent that original technical information was also being generated that would be worthy of reporting in a technical document. This report is that patent support document converted with minimal changes into a technical document.

The origin of this report as a patent support document resulted in many elements of style which might not normally appear in a technical document. For example, in an effort to be as broad as possible, items tend to be referred to by their functions rather than by their identities. Thus whereas the use of a CCD camera might normally be indicated to a technically educated reader who would be familiar with what a CCD camera is, here a CCD camera is but one example of an array of single detectors, each single detector being “a device which converts the selected signal or signals received into a single response, the magnitude of which can be uniquely related to the total signal energy received over the effective collection area of the collection optics and the period of detection” (Appendix B). Another example of a technically unusual style is the redundant use of the long names of things in order to minimize any possibility of ambiguity. Thus the description of things tends to be more wordy than might usually be expected. Yet another example is that practices that might seem obvious to readers experienced in this area are broken in great and sometimes redundant detail into their fundamental components. This was again the result of an effort to be as broad as possible. The requirement to reference the fundamental components caused the document to be structured into highly nested subsections, reaching as many as eight levels in the original document. An effort has been made here to reduce the number of nested levels, but the fact remains that most of the paragraphs have their own titles if not section numbers. While at first seeming to be mainly tedious, the process of breaking “obvious” practices into their fundamental components did reveal certain important assumptions that were not otherwise realized to be necessary, and some capabilities that might not otherwise have been discovered. An important example of the latter was the realization that the method fundamentally measures band coefficients, and that it was not necessary in all cases to relate band coefficients to particle properties. This led to important insights about how the method might be applied when the particles are not spherical, a fundamental weakness of previous optical patterning methods.

By far the most significant technical contribution in this report is the theoretical analysis performed in Chapter 5, where fundamental optical theory is for the first time applied to understand the conditions under which extraordinarily complicated general relationships can be reduced to the exceedingly simplified form given in the very first equation of Chapter 1, which is a fundamental relationship required in this method as well as in many other optical patterning methods which have been reported in the past. The impatient reader may be advised to begin first with the introduction and background provided in Chapters 1 and 2, skip next to the appendices which provide the foundation of the theory, then perhaps only skim Chapters 3 and 4 but pay the most attention to sections 3.3 and 4.3, which describe the most recommended implementations of the method, and then delve in to Chapter 5. Extensive references to applicable parts of Chapters 3 and 4 are contained in Chapter 5, to which the reader may refer as necessary.

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14. ABSTRACT
A general theory of a method to quantitatively perform optical patternation in a particle field is developed. The general equations of optical radiation are considered, along with general methods of probing the particle field with illuminating light and general methods of performing the required measurements, in order to determine the assumptions, approximations, and conditions under which various kinds of measurements can be performed. Under conditions which exist in many practical applications, it is found that the capabilities of modern optical instrumentation allow the spatial distribution of various properties of a particle field such as particle volume or surface area concentrations to be measured, even when the measurement is complicated by attenuation of the illuminating light, attenuation of the signal light, and secondary emission. The measurements can be performed in fully three dimensional form or within a single two dimensional plane, and under certain conditions can be performed even when the particles are not spherical.

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1.0 INTRODUCTION

Optical patternation refers to the use of optical methods to measure the spatial distribution of some particle property of interest within a particle field. Of the various possible approaches to optical patternation, one class of methods is of particular interest because of its potential to quickly and globally measure the distribution of more than one particle property. This class of methods tends to involve an assumption that the measured signal be proportional to the product of the local illumination energy times the concentration of the property at each point measured, that is,

$$E_s \propto \zeta E, \quad (1)$$

where E_s is the total amount of signal produced, E is the total amount of illumination, and ζ is the concentration of the particle property of interest (total amount of particle property per unit volume in the particle field). An example of this class of optical patternation methods is illustrated in Figure 1. In this example, the source of illumination is a planar sheet of laser light, and the particle field is a spray. The sheet illuminates a plane within the spray, and the signals produced within the spray are detected point-by-point by a two dimensional detector. Here E would be the total continuous or pulsed laser energy delivered to the probe volume during the period of detection, and E_s would be the total signal gathered during the period of detection. If the signal is gathered at the same wavelength as the laser illumination and the particles are in the Mie scattering regime, then ζ might be the surface area concentration (total particle surface area per unit volume in the particle field). Alternatively, if the signal is gathered at some fluorescence wavelength, then ζ might be the particle volume concentration (total particle volume per unit volume in the particle field).

Application of equation (1) is often not possible, however, either because the required quantities cannot be measured, or because equation (1) simply does not apply. An example of the former would be optically thick particle fields which cause attenuation of the illumination and/or of the signal by unknown amounts, preventing the quantities E_s and E from being directly measured. An example of the latter could be attempting to perform time-averaged measurements in a statistically fluctuating particle field, depending on the nature of the statistics. Not only would ζ statistically fluctuate, random amounts of extinction of the illumination by the particle field would cause E to fluctuate as well. This gives $\overline{E_s} = \overline{\zeta} \cdot \overline{E} + \overline{\zeta E'}$, where the overbars represent time averages and the primes represent time fluctuations about the mean. Equation (1) will therefore not apply in the mean unless the fluctuations are largely uncorrelated such that $|\overline{\zeta} \cdot \overline{E}| \gg |\overline{\zeta E'}|$.

It turns out that there are in fact a rather large number of conditions which must be satisfied in order for equation (1) to be applicable and useful. Some of these are implicitly satisfied by modern detectors and methods of illumination, while others are not. One of the objectives of this report is to examine the conditions under which an equation in the form of equation (1) can be derived from a reduction of the equations of fundamental optical theory. Another objective is to develop a general implementation of the class of optical patternation methods which use equation (1) such that any known or envisioned particular implementation would be a subclass of the general implementation. This report is concerned exclusively with a theoretical examination of the method. Experimental evaluations of the method will be the subject of other reports.

Further background and literature review will first be given in Chapter 2. Then the general method of probing a particle field with illumination will be described in Chapter 3, followed by a description of the general method of detecting the required signals in Chapter 4. These general methods will then be subjected in Chapter 5 to a rigorous theoretical treatment in order to understand the conditions under which equations in the form of equation (1) can be derived. The necessary fundamental principles are developed in Appendices A-C.

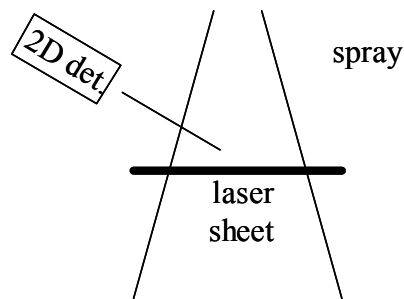


Figure 1. An optical patternator configuration.

2.0 BACKGROUND

2.1 Definition and Applications of Patternators

Particle fields occur in many aspects of everyday life. One of the more common ways in which particle fields are encountered is in the form of distributions of liquid droplets, or *sprays*. As sprays, particle fields are found in such diverse applications as water sprinklers, paint sprayers, chemical process plants, medicinal coatings for tablets and pills, aerosols for inhalation therapy, lubrication, cooling, meteorology, fabrication techniques using molten metals, and fuel and/or oxidizer sprays for heating and power generation, diesel and spark ignited reciprocating engines (e.g., automotive applications), gas turbine engines for air, marine, and ground applications, and rocket engines.

The behavior of particle fields depends in part on how particles are spatially distributed within them. The spatial distribution of particles within particle fields leads in turn to the spatial distribution of various properties of the particles within particle fields. For example, two important properties of particles the spatial distribution of which it is often important to know are the surface area of the particles, and the volume (or mass) of the particles.

A *patternator* is defined to be an instrument which measures the spatial distribution of some particle property within a particle field. The spatial distribution of a property of the particles within a particle field is sometimes referred to as the *pattern* of that property. The process of measuring the pattern of a property of the particles within a particle field is sometimes referred to as *patterning*.

Patternators can have several applications. One application is where the behavior of particle fields needs to be understood and/or predicted. Knowing the pattern of various properties of particles in a particle field often aids in understanding and predicting the behavior of those particle fields, much like knowing the distribution of cloud patterns (which are themselves a kind of particle field) can aid in predicting the weather. Another application is where quality control is desired in the production of devices used to produce particle fields. One example of a device used to produce particle fields is an atomizing nozzle used to produce liquid sprays, such as an automotive fuel injector. Ensuring that such devices produce a consistent pattern relative to some standard pattern would be one way to ensure that all such devices are manufactured with uniformly high quality.

2.2 Methods of Patterning

There are two general methods of patterning: particle-based methods, and ensemble-based methods. These methods are described more fully below

2.2.1 Particle-Based Methods. *Particle-based* methods of patterning determine the pattern of various properties from measurements of individual particles. The advantages of particle-based methods include the advantage that different particle properties can often be deduced from the measurement of only a single property of the particles. For example, if a particle-based method determines the surface area of each and every particle within a certain volume, and it is determined that all the particles are spherical, then the size, volume, and mass of each particle may also be deduced, the distributions of these properties in the volume may be deduced, and the total number of particles, and their total surface area, total volume, and total mass in the volume

may also be deduced. There are also some disadvantages of particle-based methods. One disadvantage is that complete and accurate measurements of large numbers of particles need to be obtained. Obtaining complete and accurate measurements of large numbers of particles is not only experimentally challenging, it also requires the storage and handling of large amounts of data. The resources required to perform difficult measurements and to store and handle large amounts of data must be weighed against the actual requirement of an application, which is often to know only the total amount of a single property such as the time averaged total particle mass in a given volume. Another disadvantage of particle-based methods is that only a very small fraction of the total particle field can typically be examined at a time if particle properties are to be resolved on the scale of single particles. Thus distributions over the entire extent of the particle field may be tedious to obtain.

2.2.2 Ensemble-Based Methods. *Ensemble-based* methods of patternation determine the pattern of a property of a particle field by measuring the total amounts of that property in an ensemble of particles. Individual particles are not distinguished. The ensemble of particles may be assembled over a particular volume, over a particular time, or both. The advantages of ensemble based methods include the advantage that distributions over the entire extent of the particle field are much more readily obtained, because measurements do not need to be resolved on the scale of single particles. Another advantage is that the data storage and handling requirements can be much smaller. Ensemble-based methods also have disadvantages. One disadvantage is that the number of other properties that can be deduced from a single measurement can be much smaller. This is because ensemble based methods measure only totals. For example, two different particle fields may contain the same time-averaged total surface area within a given volume, and will therefore produce the same surface area measurement, yet the time-averaged total volume of the two fields might be different. Determining the time-averaged total volume would in general require a different measurement.

2.3 Mechanical Patternators

The patternation method discussed in this report is an optical method of the ensemble-based kind. Henceforth, only the related art for ensemble-based methods will be reviewed. The most common type of ensemble-based patternator currently in use is referred to as a mechanical patternator. A *mechanical patternator* is a device in which tubes, not necessarily circular, are physically or “mechanically” inserted into the particle field to collect particles which enter the tubes. The tubes of a mechanical patternator are typically arranged into several sections, or bins. For instance, the tubes may have a checkerboard-like design, or they may consist of several slices that extend radially outward from the center of the device, like the slices of a pizza. By orienting the tubes so that they face the direction of flow, particles can be collected by the various sections, and after a period of time, the total mass collected in each section can be measured, for instance by weighing each section. The total mass collected in each section is typically then divided by the total time over which the sample was collected and by the area of the section, to produce a quantity known as the “mass flux,” in units of mass per unit time per unit area. The mass flux depends on both the mass of the particles entering the patternator and the velocity at which they enter. Therefore a mechanical patternator actually measures the spatial distribution of a combination of these two properties of the particles, that is, the mass of the particles, and the

velocity of the particles. Each measurement is an average over the time over which the sample was taken and the area of each collection tube.

Mechanical patternators have several disadvantages, however. One disadvantage is that the measurements are tedious and time consuming to perform. Another disadvantage is that the mechanical patternator itself disturbs the very flow it is intended to measure. Yet another disadvantage is that collection in harsh environments such as combustion environments is problematic. A further disadvantage is that collection in orientations which are not aligned with gravity is not straightforward. Thus, the collection of reliable data in complex flow fields using a mechanical patternator is difficult at best. An even further disadvantage of mechanical patternators is that mechanical patternators are not able to measure the distributions of other quantities that may be of interest, such as surface area or mass which is not weighted by velocity.

2.4 Optical Patternators

Optical patternators are another class of ensemble-based methods which offers certain advantages over mechanical patternators. In optical patternation, illuminating light interacts with the particle field to produce a signal or signals which are related to the spatial distribution of a particle property or properties. Optical patternators offer several potential advantages over mechanical patternators. One advantage is that optical patternators are relatively non-invasive to the flow. Another advantage is that optical patternators generally offer comparatively higher spatial and temporal resolution. Yet another advantage is that optical patternators can be more robust in harsh environments. A further advantage is that optical patternators can generally be applied in any physical orientation. An even further advantage is that optical patternators are potentially capable of measuring the spatial distribution of more properties than mechanical patternation can, depending on the particle field and how the light interacts with it.

One example of an optical patternator capable of measuring the spatial distribution of more than one property of a particle field would be an instrument which, first of all, detects the total light scattered at the same wavelength as the illuminating light by an ensemble of particles inside a probe volume (*see* Appendix B for a discussion of probe volumes). Under certain conditions to be explored later, the signal produced will be proportional to the product of the amount of illuminating light multiplied by the surface area concentration (total surface area per unit particle field volume) of the ensemble of particles in the probe volume. The resulting measurement can therefore provide the spatial distribution of the surface area concentration of the particles. This instrument need not be limited to measuring the surface area concentration alone, however. The same illuminating light can also at the same time be used to induce fluorescence in molecules which are either inherently present in the particles or have been added as dopants. The molecules will then fluoresce at a wavelength different than that of the illuminating light. Under certain conditions also to be explored later, the fluorescence signal produced will be proportional to the product of the amount of illuminating light multiplied by the total number of emitting molecules in the probe volume. Under certain conditions regarding the uniformity with which the emitting molecules are distributed in the particles and the magnitude of the absorption by all the molecules in each particle, the concentration of emitting molecules will in turn be proportional to the volume concentration (total particle volume per unit particle field volume) of the ensemble of particles in the probe volume. Thus information regarding the distribution of both the surface area concentration and the volume concentration in the particle field can be gathered by the same instrument.

2.5 Sources of Error in Optical Patternators

Several sources of error need to be overcome before optical patternators can be made to be quantitatively accurate. These sources of error become increasingly more severe in dense particle fields as the number of particles in the path of the illuminating light or in the path of the signal light increases. The sources of error which must be overcome before optical patterning methods can become quantitatively accurate include attenuation of the illumination light, attenuation of the signal light, and secondary emission.

2.5.1 Attenuation of the Illuminating Light. One source of error which needs to be overcome before optical patternators can be made to be quantitatively accurate consists of attenuation of the illumination light by the particle field. Particles in the path of the illuminating light will scatter and absorb a fraction of the illuminating light. As the number of particles in the path of the illuminating light increases, the amount of light scattered and absorbed increases. As a result, the amount of light remaining in the illuminating light decreases and the illuminating light is said to become attenuated. If the amount of attenuation of the illuminating light by the particle field is not known, then the amount of illumination at the measurement location will also not be known, that is, the quantity E in equation (1) will not be known. As a consequence, it will not be possible to quantitatively deduce the property of interest from the resulting signal. The reduction in amount of illumination caused by scattering or absorption by particles in the path of the illuminating light will henceforward be referred to here as *attenuation of the illuminating light* by the particle field.

2.5.2 Attenuation of the Signal Light. Another source of error which needs to be overcome before optical patternators can be made to be quantitatively accurate consists of attenuation of the signal light by the particle field. Particles in the path of the signal light will also scatter and absorb a fraction of the signal light. As the number of particles in the path of the signal light increases, the amount of light scattered and absorbed also increases. As a result, the amount of light remaining in the signal light decreases. The signal light is said to become attenuated. If the amount of attenuation of the signal light by the particle field is not known, then the amount of signal actually generated at the measurement location will also not be known, even if the amount of illumination is known there. In other words, the quantity E_S in equation (1) will not be known. As a consequence, it will not be possible to quantitatively deduce the property of interest from the resulting signal. The reduction in the amount of signal light caused by scattering and absorption by particles in the path of the signal light will henceforth be referred to here as *attenuation of the signal light* by the particle field.

2.5.3 Secondary Emission. Yet another source of error which needs to be overcome before optical patternators can be made to be quantitatively accurate consists of secondary emission. Secondary emission arises as a result of attenuation of the illuminating light and attenuation of the signal light. Light that is scattered or emitted from particles in the path of the illuminating light, light that is scattered or emitted from particles in the path of the signal, or both, can strike other particles and increase the illumination they experience to a level beyond the level that would otherwise have been produced by the original illuminating light by itself. The additional illumination of these other particles may in turn then produce additional signals. Additional sig-

nals produced by particles in response to the additional illumination striking them will henceforth be referred to as *secondary emission*. Related terms that have been used in the literature include *multiple scattering*, which is an important mechanism but not the only mechanism that can lead to secondary emission, and *secondary fluorescence*, which is one of the kinds of secondary emission but not the only kind. Measurement errors can be produced both by secondary emission from particles that are *not* in the path of the original illumination, and by secondary emission from particles that *are* in the path of the original illumination.

Secondary emission from particles which are *not* in the path of the original illumination can produce measurement errors if the secondary emission is detected by the detector. Any secondary emission which is detected by the detector will appear to have originated from the measurement location in response to the original illumination, but will in fact not be related to quantities at the measurement location. For example, consider light that is scattered away from the path of the illuminating light that strikes particles in the path between the measurement location and the detector. Any secondary emission created by these particles in the path between the measurement location and the detector could potentially also be detected by the detector, yet will not be related to quantities at the measurement location.

Secondary emission from particles which *are* in the path of the original illumination can produce measurement errors if the secondary emission becomes intermingled with and cannot be distinguished from the original illuminating light. The corrections for attenuation of the illuminating light by the particle field to be developed below correct only for attenuation of the original illuminating light. Additional illumination caused by secondary emission which becomes intermingled with and cannot be distinguished from the original illuminating light cannot be accounted for. The additional signals thereby produced will constitute a source of measurement error.

2.6 Review of Optical Patternation Techniques

Several groups have proposed optical patternator techniques. Some of these groups have failed to address any aspect of any of the sources of error described above in section 2.5. Those which have at least partially addressed any of these disadvantages are briefly reviewed next. To within the limitations of each method, all of the methods reviewed below determine the distributions of the measured quantities at least to within an undetermined constant factor. Distributions which contain an undetermined constant factor will be referred to here as *relative distributions*. On the other hand, distributions which, through calculation, measurement, or calibration, contain no undetermined quantities, will be referred to as *absolute distributions*.

2.6.1 The Work of Talley, *et. al.* Talley, *et. al.* [1,2], described a method called “Planar Liquid Laser Induced Fluorescence (PLLIF),” in which the illuminating light consisted of using two counter-propagating sheets of laser light, and the detection method consisted of using a CCD detector to visualize the two dimensional fluorescence patterns thus emitted. From the properties of fluorescence signals, they were able to determine a correction for the attenuation of the illuminating light by the particle field. They demonstrated their method by measuring the relative volume concentration distribution in a liquid spray (which they reported in the form of a mass distribution), to within an undetermined constant. The method did not require the particles to be spherical. Talley and Verdieck [3] later extended the work of Talley, *et. al.* [1,2]. They proposed a method by which the properties of fluorescence signals from the two counter-propagating

sheets of laser light could also be used to determine a correction for the attenuation of the signal light by the particle field, in addition to the correction for the attenuation of the illuminating light. The correction for the attenuation of the signal light by the particle field required a full three dimensional measurement of the particle field to be performed. The extended method would have also determined the relative volume concentration distribution, but the viability of the method was not demonstrated in reference [3]. Subsequent unpublished efforts failed to demonstrate that the method could be made to work properly.

2.6.2 The Work of Sankar, *et. al.* Sankar, *et. al.* [4] described a method in which the illumination light consisted of using a single sheet of laser light originating from only one direction, and the detection method consisted of using a CCD camera and filters to monitor the light scattered at the same wavelength as the laser light, and also a fluorescence signal emitted at a second wavelength. They proposed that taking the ratio of these two signals generated a measurement of the relative distribution of an average diameter known as the Sauter mean diameter, to within an unknown constant. In demonstrating their method in a liquid spray, they used phase Doppler interferometry to independently measure the Sauter mean diameter at single point in the spray, thus determining the value of the previously undetermined constant. The final result was therefore an absolute distribution of the Sauter mean diameter. Because the method involved taking a ratio between the fluorescence signal and the scattering signal, Sankar, *et. al.* argued that the method corrected both for attenuation of the illumination light by the particle field and for attenuation of the signal light by the particle field. Furthermore, the corrections could be performed using the information from measurements within a single plane, and did not require full three dimensional measurements of the entire particle field. Separate distributions of the surface area concentration of the particles and the volume concentration of the particles could not be obtained, however, which were not influenced by attenuation of the illuminating light and attenuation of the signal light. Furthermore, the method required the particles to be spherical.

2.6.3 The Work of Domann and Hardalupas. Domann and Hardalupas [5], in a particle based method, confirmed by calculation and careful measurements that a the ratio of the fluorescence signal to the scattering signal of a single particle could, for proper concentrations of the fluorescent molecules, indeed result in a measurement of the Sauter mean diameter, even under conditions where the lensing effect of liquid droplets produced a non-uniform illumination within the droplets. They did not consider attenuation of either the illumination light or attenuation of the signal light by the particle field, however.

2.6.4 The Work of Sellens, *et. al.* Wang, *et. al.* [6] described a method in which the illumination light consisted of using a single sheet of laser light originating from only one direction, and the detection method consisted of using a CCD camera to monitor the light scattered at the same wavelength as the laser light, and using other sensors to detect the amount of light in the laser sheet exiting a liquid spray. From the properties of the light scattered from the single sheet of laser light, they were able to determine a correction for the attenuation of the illuminating light by the particle field. Furthermore, by using theoretical results that, for spherical droplets that are large relative to the wavelength of the illuminating light, the scattering cross section at any point should be half the surface area concentration at that point, they were able to determine the absolute distribution of the surface area concentration in the absence of any unknown quantities. Later, Sellens and Deljouravesh [7] extended the work of Wang, *et. al.* [6] to using a sheet of laser light that is fan shaped rather than propagating along parallel rays, and Sellens and Wang

[8] demonstrated the method for a wider range of applications. However, this method [6-8] did not correct for attenuation of the signal light by the particle field.

2.6.5 The work of Su, *et. al.* Finally, Su, *et. al.* [9] outlined methods where the method of illumination was to use one laser sheet or two counter-propagating laser sheets, and the detection method consisted of using a CCD camera and filters to monitor the light scattered at the same wavelength as the laser light, and also a fluorescence signal emitted at a second wavelength. They used the method of Wang, *et.al.* [6] to develop a correction for the attenuation of the illuminating light by the particle field, and this information was used to also correct the fluorescence signal for the attenuation of the illuminating light by the particle field. However, their method also did not account for attenuation of the signal light by the particle field.

2.6.6 Summary of Optical Patterning Techniques. In summary, of the prior art reviewed above, only Talley and Verdieck [3] and Sankar, *et. al.* [4] have thus far proposed methods to correct for more than just attenuation of the illumination light by the particle field. Each of these methods has its drawbacks. For instance, the method proposed by Talley and Verdieck [3] required complicated optics to construct counter-propagating laser sheets, it required a measurement of the full three dimensional particle field in order to correct for the attenuation of the signal light by the particle field, and it was never demonstrated to work properly. On the other hand, the optical setup required by the method of Sankar, *et. al.* [4] was simpler than that of Talley and Verdieck [3], requiring a laser sheet from only a single direction, and measurements were required only in a single plane. However, only the distribution of the Sauter mean diameter could be measured. No measurement of either the volume concentration distribution or the surface area concentration distribution was possible. None of the methods described above addressed secondary emission.

2.7 Distinction of the Present Method

One distinction of the present method is that it is capable of measuring both the spatial distribution of surface area concentration and the spatial distribution of the volume concentration, while at the same time correcting for attenuation of the illumination light, attenuation of the signal light, and reducing the effects of secondary emission. Another distinction is that the method can perform all of these functions either in a fully three dimensional form, or within a single two-dimensional plane. Yet another distinction is that the corrections can be applied under certain conditions to the measurement of the volume concentration distribution even when the particles are not spherical. No other method is known to have been developed which has as extensive a set of capabilities.

3.0 THE METHOD OF PROBING THE PARTICLE FIELD WITH ILLUMINATING LIGHT

The method of probing the particle field with illuminating light is described in this section. The general method of probing the particle field with illuminating light is described first. Then variations of the general method will be given. Finally, the most recommended method of probing the particle field with illuminating light is summarized.

3.1 The General Method of Probing the Particle Field with Illuminating Light

The general method of probing the particle field with illuminating light may be summarized as follows. The general method of probing the particle field with illuminating light is to successively probe the particle field with single beams of illuminating light, and to perform property-specific measurements along each beam. The process of successively probing the particle field with single beams of illuminating light, and performing property-specific measurements along each beam, continues repetitively until, for each particle property of interest, a representative number of property-specific measurements are performed along each property-specific probe path defined by the probing process, and a representative number of property-specific probe paths are created within an allowed contiguous sub-volume of the particle field. The new concepts and terminology introduced in the above summary are developed in detail in the sections below.

3.1.1 The Method of Probing the Particle Field Using a Single Beam of Illuminating Light. A *single beam of illuminating light* is defined in Appendix A to be light the propagation of which is limited to pass through two finite areas which are separated in space. The *effective probe volume* is defined in Appendix C to be a single volume created by the intersection of a single beam of illuminating light with the volumes of a single detector. Restricting the area of the single beams of illuminating light serves to limit the extent of the effective probe volume, and it is the key way in which the method minimizes the effects of secondary emission. The method of probing the particle field using a single beam of illuminating light may be summarized as follows. The method of probing the particle field using a single beam of illuminating light consists of using single beams of illuminating light which have been conditioned for measuring the particle property of interest according to a number of considerations governing the range of wavelengths present in the illumination, the area over which the illumination is distributed in the beam, the illumination energy delivered over the period of illumination, the period of illumination, the repeatability of the source of the illumination, and the range of polarization states present in the illumination. The considerations governing the conditioning of the single beam of illuminating light are described in detail in the sections below.

3.1.1.1 Considerations Governing the Range of Wavelengths Present in the Illumination. One set of considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest is considerations governing the range of wavelengths present in the illumination. A summary of the considerations governing the range of wavelengths present in the illumination is as follows. Considerations governing the range of wavelengths present in the illumination include the consideration of exciting a property-specific band coefficient, the consideration that signals produced within the excited property-specific band can be detected by a

property-specific single detector, the consideration that the property-specific TAMPSEC components are independent of property-specific wavelengths in the illumination, and the consideration of reducing secondary emission. The considerations governing the range of wavelengths present in the illumination are described in detail below. Variations of the considerations governing the range of wavelengths present in the illumination are also presented, and a method of partially satisfying the considerations governing the range of wavelengths present in the illumination is also given.

3.1.1.1.1 *The consideration of exciting a property-specific band coefficient.* One consideration governing the range of wavelengths present in the illumination is the consideration of exciting a property-specific band coefficient. A *property-specific band coefficient* is defined to be a band coefficient which is proportional to the quantity of the particle property of interest. The concept of a band coefficient is developed in Appendix A. At least one wavelength must be present in the illumination which excites a property-specific band coefficient, for particles that are within the range of particle parameters for measuring the particle property of interest, to within the required degree of accuracy. Wavelengths in the illumination which are used to excite selected property-specific band coefficients are referred to as *property-specific wavelengths* of the illumination. A definition of the range of particle parameters for measuring the particle property of interest is provided in the next paragraph. The *required degree of accuracy* is highly application dependent, and cannot be defined separately from the application. Further definition of the required degree of accuracy will not be attempted in this report.

The *range of particle parameters for the measurement of the particle property of interest* is defined to be the range of particle parameters within which the property-specific band coefficient will remain proportional to quantity of the particle property of interest. A *particle parameter* is defined to be a parameter which quantitatively describes some property of a particle. For example, for a spherical particle, an appropriate particle parameter might be its diameter. For a non-spherical particle, parameters describing the shape of the particle might also be required. The property-specific band coefficient for particles that are not within the range of particle parameters for the measurement of the particle property of interest may or may not be proportional to the quantity of the particle property of interest. The selection of the range of particle parameters for the measurement of the particle property of interest is governed by the requirement that failure to accurately measure the particle property of interest of particles that are not within this range should not introduce errors larger than can be tolerated by the required degree of accuracy.

3.1.1.1.2 *The consideration that signals produced within the excited property-specific band can be detected by a property-specific single detector.* Another consideration governing the range of wavelengths present in the illumination is the consideration that signals produced within the excited property-specific band can be detected by a property-specific single detector. A *signal* is defined in Appendix A to be the components of optical radiation which have been identified for detection. A *property-specific signal* is defined to be a signal produced by an excited property-specific band coefficient. This consideration arises out of the fact that it is not sufficient only that a property-specific band coefficient exist and can become excited. It is also necessary that at least some of the converted optical radiation produced can be detected as signals. Therefore at least one wavelength and state of polarization must be produced from excited property-specific band coefficients which can be detected as a signal by a property-specific single detector.

A *property-specific single detector* is defined in Chapter 4 to be a single detector which has been conditioned to detect the property-specific signal, and to filter out all other non-property-specific optical radiation that might be present, to within the required degree of accuracy, according to a number of considerations governing the response of the detector, the properties of the effective probe volume, and the location of the detector. The source of non-property specific optical radiation may include non-property-specific parts of the illuminating light itself, non-property-specific interactions between the illuminating light and the particle field, and stray illumination. *Stray illumination* is illumination which is not part of or produced as a result of interactions with the single beam of illuminating light. Stray illumination can come from sources such as room lighting. Stray illumination may itself be the source of non-property-specific optical radiation, or it may interact with the particle field to produce non-property-specific optical radiation. The capability of a single detector to filter out non-property-specific optical radiation is in part controlled by range of wavelengths present in the illumination because if a detector cannot be designed to distinguish the signals produced within the property-specific band from other optical radiation, then illumination wavelengths will need to be selected to excite another property-specific band coefficient.

3.1.1.1.3 *The consideration that the property-specific TAMPSEC components are independent of property-specific wavelengths in the illumination.* Yet another consideration governing the range of wavelengths present in the illumination is the consideration that the property-specific time-averaged monochromatic polarization-specific extinction coefficient components are independent of property-specific wavelengths in the illumination. The term “instantaneous monochromatic polarization-specific extinction coefficient” was introduced in section A.9 of Appendix A, and contracted for brevity into the acronym “IMPSEC.” Here, the term “*time-averaged monochromatic polarization-specific extinction coefficient*” will be contracted for brevity into the acronym “TAMPSEC.” In the method of analyzing the data to developed in Chapter 5, the TAMPSEC components are considered to be approximately independent of wavelength for property-specific wavelengths in the illumination. One way to approach this condition is to restrict the range of property-specific wavelengths to a narrow band. In the limit as the range of property-specific wavelengths approaches a single wavelength, this condition will become exactly true. Therefore the range of property-specific wavelengths in the single beam of illuminating light should be narrow enough that the TAMPSEC components are independent of wavelength to within the required degree of accuracy.

3.1.1.1.4 *The consideration of reducing secondary emission.* A further consideration governing the range of wavelengths present in the illumination is the consideration of reducing secondary emission. Any excess illumination not used to excite property-specific band coefficients increases the possibility of producing secondary emission. Therefore the range of wavelengths present in the illumination should be limited as much as possible to property-specific wavelengths.

3.1.1.1.5 *Variations of the considerations governing the range of wavelengths present in the illumination.* Variations of the considerations governing the range of wavelengths present in the illumination include a variation where the particle property of interest is the particle surface area concentration, a variation where the particle property of interest is the particle volume concentration, and a variation where the particle properties of interest are both the particle surface area

concentration and the particle volume concentration. Variations of the considerations governing the range of wavelengths present in the illumination are described in the next paragraphs.

A variation where the particle property of interest is the particle surface area concentration. One variation of the considerations governing the range of wavelengths present in the illumination concerns the case where the particle property of interest is the surface area concentration of the particles. Most generally, surface-area-specific illumination may be sought such that any one of the band coefficients ξ_n will be proportional to the particle surface area concentration. In this variation, the elastic scattering band coefficient ξ_s is selected. If at least one wavelength is used in the illuminating light for which the particle surface area concentration is proportional to the elastic scattering band coefficient, then $a = g_s \xi_s$, where a is the particle surface area concentration, and g_s is the constant of proportionality. In the geometric optics limit where the wavelength of the illumination is much smaller than the range of particle sizes for the measurement of the particle surface area concentration, then the scattering cross section of spherical particles will be equal to half the particle surface area concentration, giving $g_s = 2$.

A variation where the particle property of interest is the particle volume concentration. Another variation of the considerations governing the range of wavelengths present in the illumination concerns the case where particle property of interest is the particle volume concentration. Most generally, volume-concentration-specific illumination may be sought such that any one of the band coefficients will be proportional to the particle volume concentration. In this variation, the fluorescence band coefficient ξ_f is selected. If at least one wavelength is used in the illuminating light which is absorbed by molecules that are either naturally present within or are artificially doped into the particles, then depending on conditions such as the uniformity with which the molecules are distributed in the particles and the magnitude of the absorption by all the molecules in each particle, the particle volume concentration may be proportional to the fluorescence band coefficient, *i.e.* $\rho = g_f \xi_f$, where ρ is the particle volume concentration and g_f is a constant of proportionality. In general the fluorescence signal will be at a different wavelength than the wavelength of the illumination which excites it.

A variation where the particle properties of interest include both the particle surface area concentration and the particle volume concentration. Yet another variation of the considerations governing the range of wavelengths present in the illumination concerns the case where the particle properties of interest include both the particle surface area concentration and the particle volume concentration. When the particle properties of interest include both the particle surface area concentration and the particle volume concentration, both of the first two variations above are used. However, rather than use separate illumination wavelengths, in this variation the same illumination wavelength is used for both the surface area concentration measurement and the volume concentration measurement. This is possible when the fluorescence signal is at a different wavelength than the illumination, and therefore it is at a different wavelength than the elastically scattered signal. As long as a wavelength of the illumination can be found where the surface-area-concentration-specific detector and the volume-concentration-specific detector can distinguish between the scattered signal and the fluorescence signal, then the same wavelength can be used for both the surface area concentration measurement and the volume concentration measurement.

3.1.1.1.6 *A method of partially satisfying the considerations governing the range of wavelengths present in the illumination.* One method of partially satisfying the considerations governing the range of wavelengths present in the illumination is to use a specific kind of illuminating light which is a laser beam.

3.1.1.2 Considerations Governing the Area over which the Light in a Single Beam of Illuminating Light Should Be Distributed. Another set of considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest is considerations governing the area over which the light in a single beam of illuminating light should be distributed. The fact that a single beam of illuminating light is defined to be light the propagation of which is limited to pass through two finite areas which are separated in space implies that the cross sectional area of the beam will in general also be finite at any position along the beam. Considerations governing the area over which the light in a single beam of illuminating light should be distributed include the consideration of achieving adequate spatial resolution, the consideration of achieving nearly uniform detector response, the consideration of reducing bias caused by the variability of the probe volumes, the consideration of minimizing the attenuation of light across the effective probe volume (the small effective probe volume approximation of the first kind), the consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics (the small effective probe volume approximation of the second kind), the consideration of reducing secondary emission (the slender beam approximation of the first kind), the consideration of minimizing the number of directions of propagation possible within the single beam of illuminating light (the slender beam approximation of the second kind), the consideration of detecting the emitted signal to within the required degree of accuracy, and the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field. These considerations are described further in the next sections.

3.1.1.2.1 *The consideration of achieving adequate spatial resolution.* One consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of achieving adequate spatial resolution. Factors governing the degree of spatial resolution required include the necessity to relate detections to absolute locations in space from which the signals originated, and the necessity that particle properties be approximately uniform within the effective probe volume, both to within the required degree of accuracy. These necessities are described further in the next paragraphs.

The necessity to relate detections to absolute locations in space. One factor governing the consideration of achieving adequate spatial resolution is the necessity to relate detections to absolute locations in space. The absolute locations in space from which signals originated will not be known to any better accuracy than the size of the effective probe volume. Therefore the area over which the light in a single beam of illuminating light is distributed should, in combination with the design of the detector, create effective probe volumes that are small enough to relate detections to absolute locations in space from which the signals originated, to within the required degree of accuracy.

The necessity that particle properties be approximately uniform within across the single beam of illuminating light. Another factor governing the consideration of achieving adequate spatial resolution is the necessity that particle properties be approximately uniform across the single beam of illuminating light. Directions “across” a single beam of illuminating light include

all directions perpendicular to the general direction of propagation of the beam. More specifically, directions “across” a single beam of illuminating light include all directions perpendicular to the property-specific probe path, a concept which will be defined in section 3.1.2.1 below in this chapter. For the present, however, it is sufficient to understand that the property-specific probe path is roughly the general direction of propagation of the beam. The axial extent along the general direction of propagation of the beam (more specifically, along the property-specific probe path) over which particle properties are required to be approximately uniform across the single beam of illuminating light is the axial distance occupied by the effective probe volume.

Reasons why it is necessary that particle properties be considered to be approximately uniform across the single beam of illuminating light include the reason that a single detector integrates all the signal it receives from its effective probe volume into a single response, and the reason that approximately uniform particle properties across the single beam of illuminating light is one of the approximations required by the method of analyzing the data. One reason why it is necessary that particle properties be considered to be approximately uniform across the single beam of illuminating light is that a single detector integrates all the signal it receives from its effective probe volume into a single response, as discussed in Appendix C. A single detector is therefore not able to distinguish any property non-uniformities which may exist inside the effective probe volume. Ensuring that the area over which the light in a single beam of illuminating light is distributed is small enough that particle properties are approximately uniform across the single beam of illuminating light therefore ensures that particle properties will be approximately uniform within the effective probe volume, as well. Another reason why it is necessary that particle properties be considered to be approximately uniform across the single beam of illuminating light is that that approximately uniform particle properties across the single beam of illuminating light is one of the approximations required by the method of analyzing the data. This approximation is given in section 5.2.1 of Chapter 5, and used, for example, in section 5.3.3.1 of Chapter 5. Particle properties are required to be uniform across the single beam of illuminating light to within the required degree of accuracy.

3.1.1.2.2 *The consideration of achieving nearly uniform detector response.* Another consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of achieving nearly uniform detector response. As discussed in section C.8 of Appendix C, limiting illumination to regions in the response volumes of the detector to the volumes of nearly uniform detector response will improve the accuracy to which the detector response can be related to particle properties. Therefore, the area over which the light in a single beam of illuminating light is distributed should also, in combination with the design of the detector, create effective probe volumes which are limited as much as possible to the volumes of nearly uniform detector response.

3.1.1.2.3 *The consideration of reducing bias caused by the variability of the probe volumes.* Yet another consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of reducing bias caused by the variability of the probe volumes. As indicated above in section C.7 of Appendix C, in general there can be as many probe volumes as there are response volumes of the detector. However, with ensemble based methods, it is necessary to define a single effective probe volume to represent all of the probe volumes. Any variability in the probe volumes can cause bias by preferentially measuring those particles having larger probe volumes and relating them to the same effective probe vol-

ume. By further reducing the illumination to only small regions within the volumes of nearly uniform detector response, it will often be possible to even further reduce the variability of the probe volumes to a point where most if not all of the probe volumes will be nearly identical. Therefore, the area over which the light in a single beam of illuminating light is distributed should also, in combination with the design of the detector, create effective probe volumes which are small enough that the variability in the probe volumes can be minimized.

3.1.1.2.4 *The consideration of minimizing the attenuation of light across the effective probe volume (the small effective probe volume approximation of the first kind).* A further consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of minimizing the attenuation light across the effective probe volume. This consideration applies both to attenuation of the illuminating light and attenuation of the signal light. If the attenuation of light across the effective probe volume is too large, then it will not be possible to use the method of analyzing the data to be developed in Chapter 5 to relate the detected signal to the property-specific band coefficient. Note that the attenuation of light across the entire particle field might still be significant; here the attenuation of light needs to be small only over the limited path across the effective probe volume. Therefore, the area over which the light in a single beam of illuminating light is distributed should also, in combination with the design of the detector, create effective probe volumes which are significantly smaller than the dimension of the particle field when the attenuation light across the particle field is significant. Satisfying the consideration of minimizing the attenuation of the light across the effective probe volume will lead to the *small effective probe volume approximation of the first kind*, to be used in Chapter 5 to develop the method of analyzing the data.

3.1.1.2.5 *The consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics (the small effective probe volume approximation of the second kind).* An even further consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics. The *effective collection area* of the detector collection optics is defined in section B.1 of Appendix B to be the area over which signals enter the collection optics that are actually detected. From each point in the effective probe volume, the finite effective collection area of the detector collection optics will define a particular range of directions. In the limit as the size of the effective probe volume approaches a single point, the range of directions described from each point in the effective probe volume would approach the same range of directions. The consideration here is that the effective probe volume be small enough that all the ranges of directions, if not identical, will be approximately the same, to within the required degree of accuracy. Therefore, the area over which the light in a single beam of illuminating light is distributed should also, in combination with the design and location of the detector, create effective probe volumes which are small enough to allow the range of directions defined from each point in the effective probe volume to the effective collection area of the detector optics to all be approximately the same. Satisfying the consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics will lead to the *small effective probe volume approximation of the second kind*, also to be used in Chapter 5 to develop the method of analyzing the data.

3.1.1.2.6 *The consideration of reducing secondary emission (the slender beam approximation of the first kind)*. An additional consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of reducing secondary emission. Factors governing the consideration of reducing secondary emission include the necessity to reduce the amount of secondary emission from particles which are not in the path of the original illumination, and the necessity to reduce the amount of secondary emission from particles which are in the path of the original illumination. These necessities are further discussed in the next paragraphs. Satisfying the consideration of reducing secondary emission will lead to the *slender beam approximation of the first kind*, also to be used in Chapter 5 to develop the method of analyzing the data.

The necessity to reduce the amount of secondary emission from particles which are not in the path of the original illumination. One factor governing the reduction of secondary emission is the necessity to reduce the amount of secondary emission from particles which are not in the path of the original illumination. Any excess area in the beam of illuminating light could add additional light that might be scattered away from the path of the illuminating light, strike other particles that are within the response volume of the detector but are not within the effective probe volume, and produce signals that are erroneously attributed to having originated from within the effective probe volume. Therefore, the area over which the light in a single beam of illuminating light is distributed should, in combination with the illumination energy delivered over the period of illumination, also be as small as possible in order to reduce the amount of secondary emission from particles which are not in the path of the original illumination.

The necessity to reduce the amount of secondary emission from particles which are in the path of the original illumination. Another factor governing the reduction of secondary emission is the necessity to reduce the amount of secondary emission from particles which are in the path of the original illumination. Any excess area in the beam of illuminating light could add additional light that might become intermingled with and cannot be distinguished from the original illuminating light. As discussed in section 2.5 of Chapter 2, the corrections for attenuation of the illuminating light by the particle field to be developed in the method of analyzing the data in Chapter 5 correct only for attenuation of the original illuminating light. Additional illumination caused by secondary emission which becomes intermingled with and cannot be distinguished from the original illuminating light is not accounted for. Therefore, the area over which the light in a single beam of illuminating light is distributed should, in combination with the design of the detector and the illumination energy delivered over the period of illumination, also be as small as possible in order to reduce the amount of secondary emission from particles which are in the path of the original illumination.

3.1.1.2.7 *The consideration of minimizing the number of directions of propagation possible within the single beam of illuminating light (the slender beam approximation of the second kind)*. Another additional consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of minimizing the number of directions of propagation possible within the single beam of illuminating light. The area over which the light in a single beam of illuminating light is distributed should also create a progression of beam cross sections at each axial distance along the beam which are small enough to minimize the number of directions of propagation possible within the single beam of illuminating light. Satisfying the consideration of minimizing the number of directions of propagation

possible within the single beam of illumination will lead to the slender beam approximation of the second kind, also to be used in Chapter 5 to develop the method of analyzing the data when the illumination light cannot be assumed to be collimated.

3.1.1.2.8 *The consideration of detecting the signal received to within the required degree of accuracy.* A yet another additional consideration yet governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of detecting the emitted signal to within the required degree of accuracy. Factors governing the consideration of detecting the emitted signal to within the required degree of accuracy include the necessity to produce enough signal to be accurately detected by the detector, and the necessity to avoid producing so much signal that the detector becomes saturated. These factors are described further in the following paragraphs.

The necessity to produce enough signal to be accurately detected by the detector. One factor governing the consideration of detecting the emitted signal to within the required degree of accuracy is the necessity to produce enough signal to be accurately detected by the detector. The considerations described herein have thus far lead to the general guidance that the effective probe volume should be reduced to the minimum possible volume. Offsetting this general guidance is the present guidance regarding the necessity that the signal received can be detected with the required degree of accuracy. Reducing the size of the effective probe volume by reducing the area over which the light in the single beam of illuminating light is distributed can also reduce the amount signal which is produced from the effective probe volume. This signal will be further reduced by any attenuation of the signal light by the particle field which may exist. If the range of signal levels actually detected by the detector is too small for the detector to resolve small differences between the signals, then it will not be possible to measure the signals to within the required degree of accuracy. Therefore, the area over which the light in a single beam of illuminating light is distributed should, in combination with the design of the detector and the illumination energy delivered over the period of detection, also create effective probe volumes which are large enough to emit enough signal to be detected to within the required degree of accuracy at each location along the beam.

The necessity to avoid producing so much signal that the detector becomes saturated. Another factor governing the consideration of detecting the emitted signal to within the required degree of accuracy is the necessity to avoid producing so much signal that the detector becomes saturated. For most detectors, there is an upper bound to the total signal energy that can be received, beyond which the response of the detector can no longer be related to the total signal energy received. Therefore the area over which the light in a single beam of illuminating light is distributed should, in combination with the design of the detector and the illumination energy delivered over the period of illumination, also create effective probe volumes which are small enough to avoid producing so much signal that the detector becomes saturated.

3.1.1.2.9 *The consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field.* A further additional consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field. Adding too much illumination energy over too short a period of time and over too small of an area could increase the illumination intensity to such an extent that undesirable interactions with the particle field could be produced. For example, the particles could become heated, vapor-

ized, or otherwise destroyed, or the illumination could cause motions in the surrounding gas that would affect the particles which would not otherwise be present. Therefore, the area over which the light in a single beam of illuminating light is distributed should, in combination with the design of the detector, the illumination energy delivered over the period of illumination, and the period of illumination, also create effective probe volumes which are large enough to prevent perturbing the behavior of the particle field to an unacceptable degree.

3.1.1.2.10 *The consideration of maintaining a repeatable effective probe volume correction factor.* An even further additional consideration governing the area over which the light in a single beam of illuminating light should be distributed is the consideration of maintaining a repeatable effective probe volume correction factor. The effective probe volume correction factor Σ is defined in section 5.3.7.6 of Chapter 5. It is a complicated term that accounts in part for the fact that the volume of the effective probe volume will in general be different than the volume along the single beam of illuminating light which it represents, and for the fact that the magnitude of the property-specific signals produced will therefore depend on where the effective probe volume is located in the beam and on the local intensity distribution of the beam there. Maintaining a repeatable effective probe volume correction factor for each measurement performed along a given property-specific probe path allows the method of analyzing the data to be considerably simplified. (See Section 3.1.2.1 below in this chapter for the definition of a property-specific probe path). The effective probe volume correction factor is also influenced by the geometry of the effective probe volume and the geometry of the detector. Therefore, the area over which the light in a single beam of illuminating light is distributed should, in combination with the design of the detector and the repeatability of the source of the single beam of illuminating light, cause a repeatable effective probe volume correction factor to be maintained along the length of the property-specific probe path, to within the required degree of accuracy.

A method for causing the repeatability of the effective probe volume correction factor to be independent of variations in the area over which the light in a single beam of illuminating light is distributed. Although there are many ways the area over which the light in a single beam of illuminating light is distributed can interact with the design of the detector and the repeatability of the source of the single beam of illuminating light to create a repeatable effective probe volume correction factor, the simplest method is to make the repeatability of the effective probe volume correction factor independent of variations in the area over which the light in a single beam of illuminating light is distributed. One method of making the effective probe volume correction factor independent of variations in the area over which the light in a single beam of illuminating light is distributed is to minimize variations in area along the length of the beam. One method for minimizing variations in area along the length of the beam is to use a parallel-propagating laser beam.

3.1.1.3 Considerations Governing the Illumination Energy Delivered over the Period of Illumination. Yet another set of considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest is considerations governing the illumination energy delivered over the period of illumination. Considerations governing the illumination energy delivered over the period of illumination include the consideration of reducing secondary emission, the consideration of detecting the signal received with the required degree of accuracy, and the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field.

3.1.1.3.1 *The consideration of reducing secondary emission.* One consideration governing the illumination energy delivered over the period of illumination is the consideration of reducing secondary emission. Any excess illumination adds unnecessary light that could contribute to secondary emission. Therefore the illumination energy delivered over the period of detection should, in combination with the area over which the light in a single beam of illuminating light is distributed, be as small as possible in order to reduce the amount of secondary emission.

3.1.1.3.2 *The consideration of detecting the signal received with the required degree of accuracy.* Another consideration governing the illumination energy delivered over the period of illumination is the consideration of detecting the signal received with the required degree of accuracy. As discussed in section 3.1.1.2.8, factors governing the consideration of detecting the emitted signal to within the required degree of accuracy include the necessity to produce enough signal to be accurately detected by the detector, and the necessity to avoid producing so much signal that the detector becomes saturated. Therefore the illumination energy delivered over the period of illumination should be such that, in combination with the design of the detector and the area over which the light in a single beam of illuminating light is distributed, the signal or signals produced can be detected with the required degree of accuracy at each location measured along the single beam of illuminating light.

3.1.1.3.3 *The consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field.* Yet another consideration governing the illumination energy delivered over the period of illumination is the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field. Adding too much illumination over too short a period of time and over too small of an area could increase the illumination intensity to such an extent that undesirable interactions with the particle field could be produced. Therefore the illumination energy delivered over the period of illumination should, in combination with the area over which the light in a single beam of illuminating light is distributed, the period of illumination, and the design of the detector, be small enough that the behavior of the particle field will not be perturbed to an unacceptable degree.

3.1.1.4 Considerations Governing the Period of Illumination. A further set of considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest is considerations governing the period of illumination. The *period of illumination* is defined to be the total time over which the illumination energy is delivered for a single detection. The period of illumination is to be distinguished from the *period of detection*, which is defined to be the total time over which a single detector is sensitized to collect signals, which is controlled by the detector. The period of illumination may be shorter than the period of detection, but inasmuch as there will be no interest in any signals produced when the detector is not sensitized to collect signals, the period of illumination will be considered to last no longer than the period of detection. Considerations governing the period of illumination include the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field, the consideration of preventing temporal variations in the source of the single beam of illuminating light from leaving residual spatial variations in the beam, and the consideration of minimizing the detection of signals when there is no illumination.

3.1.1.4.1 *The consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field.* One consideration governing the period of illumination is the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field. For a given illumination energy, adding the illumination over too small of an area and too short of a time period may cause unacceptable perturbations of the particle field. Therefore the period of illumination should be long enough that, in combination with the illumination energy delivered over the period of illumination and the area over which the light in a single beam of illuminating light is distributed, the behavior of the particle field will not be perturbed to an unacceptable degree.

3.1.1.4.2 *The consideration of preventing temporal variations in the source of the single beam of illuminating light from leaving residual spatial variations in the beam.* Another consideration governing the period of illumination is the consideration of preventing temporal variations in the source of the single beam of illuminating light from leaving residual spatial variations in the beam. If time variations in the source of the single beam of illuminating light are rapid enough, it is conceivable that residual spatial variations in the beam could be produced, as discussed in section A.11 of Appendix A. The method of analyzing the data to be developed in section 2.4 is not capable of accounting for residual spatial variations caused by temporal variations in the source of the beam. Therefore the period of illumination should be long enough that, for a large fraction of the period of illumination, time variations in the source of the single beam of illuminating light will be so much slower than the time it takes light to traverse the particle field, that any resulting residual spatial variations will be negligible.

3.1.1.4.3 *The consideration of minimizing the detection of signals when there is no illumination.* Yet another consideration governing the period of illumination is the consideration of minimizing the detection of signals when there is no illumination. If the period of illumination is shorter than the period of detection, then any signals collected during the time over which there is no illumination can only lead to an erroneous response of the detector. Therefore the period of illumination should, in combination with the design of the detector, be at least as long as the period of detection.

3.1.1.5 Considerations Governing the Repeatability of the Source of the Single Beam of Illuminating Light. An even further set of considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest is considerations governing the repeatability of the source of the single beam of illuminating light. Repeatability of the source of the single beam of illuminating light refers to the repeatability of the intensity and directional distribution over time. Considerations governing the repeatability of the single beam of illuminating light include the consideration of maintaining a repeatable effective probe volume correction factor.

The consideration of maintaining a repeatable effective probe volume correction factor. One consideration governing the repeatability of the source of the single beam of illuminating light is the consideration of maintaining a repeatable effective probe volume correction factor. The source of the single beam of illuminating light should, in combination with the design of the detector and the area over which the light in a single beam of illuminating light is distributed, exhibit sufficient repeatability to cause a repeatable effective probe volume correction factor to be

maintained along the length of the property-specific probe path, as discussed in section 3.1.1.2.10.

A method for causing the repeatability of the effective probe volume correction factor to be independent of the repeatability of the source of the single beam of illuminating light. Although there are many ways the repeatability of the source of the single beam of illuminating light can interact with the area over which the light in a single beam of illuminating light is distributed and the design of the detector to create a repeatable effective probe volume correction factor, the simplest method is to make the repeatability of the effective probe volume correction factor independent of the repeatability of the source of the single beam of illuminating light. One method of making the repeatability of the effective probe volume correction factor independent of the repeatability of the source of the single beam of illuminating light is to use a source where the illumination intensity $I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ can be separated into the product of a magnitude function $I_0(t)$ which depends only on time, and a shape function $I_s(\lambda, \mathcal{P}, \bar{x}, \hat{n})$ which is independent of time, such that $I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) = I_0(t)I_s(\lambda, \mathcal{P}, \bar{x}, \hat{n})$.

3.1.1.6 Considerations Governing the Range of Polarization States Present in the Illumination. An additional set of considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest is considerations governing the range of polarization states present in the illumination. One consideration governing the range of polarization states present in the illumination is the consideration of simplifying the method of analyzing the data when the small detector approximation of the first kind cannot be applied. Maintaining a constant effective probe volume correction factor for each state of polarization as discussed in section 3.1.1.5.1 above is not by itself sufficient to considerably simplify the method of analyzing the data. Restriction of the range of polarization states may also be necessary. The small detector approximation of the first kind will be introduced in section 4.1.1.3.1 of Chapter 4 in which the number of directions from the effective probe volume to the detector is minimized. If this approximation cannot be applied, then considerably simplifying the method of analyzing the data will not be possible unless the range of polarization states in the illumination can be restricted to a single polarization state. If the small detector approximation of the first kind can be applied, the range of polarization states is not strictly required to be limited. However, having more than one polarization state contributes no additional capability to the invention. Therefore, the range of polarization states in the illumination should whenever possible be limited to only a single polarization state.

3.1.2 The Method of Successively Probing the Particle Field Using Single Beams of Illuminating Light. The method of probing the particle field with a single beam of illuminating light having now been described, the method of successively probing the particle field using single beams of illuminating light can now be described. The method of successively probing the particle field using single beams of illuminating light consists of the following series of steps. First, the particle field is probed somewhere with a single beam of illuminating light. Secondly, property-specific single measurements are performed along the beam according to the method of performing the required measurements to be developed in Chapter 4. Thirdly, the particle field is probed again with a second single beam of illuminating light. This second beam of illuminating light may either probe the same part of the particle field as the first beam, or it may probe a different part of the particle field. To accomplish the latter, either the beam projecting optics, or the particle field, or both, can be moved. In any case, property-specific single measurements are

again performed along the second beam of illuminating light according to the method of performing the required measurements to be developed in Chapter 4. The process of successively probing the particle field with single beams of illuminating light, and performing property-specific single measurements along the beams, continues repetitively until, for each particle property of interest, a representative number of property-specific single measurements are performed along each property-specific probe path defined by the probing process, and a representative number of property-specific probe paths are created within an allowed contiguous sub-volume of the particle field. Definitions of the above terms are provided next.

3.1.2.1 Definition of a Property-Specific Probe Path. A *property-specific probe path* may be defined as follows. Within a single beam of illuminating light, there will be a large number of straight lines (mathematical constructs having length but no volume) which are entirely contained within the part of the beam that is within the particle field. One of these lines will be selected to be the property-specific probe path. In the course of successively probing the particle field using single beams of illuminating light, it will be necessary that every property-specific probe path be illuminated with single beams of illuminating light that entirely contain the property-specific probe path as many times as are necessary in order to perform a representative number of property-specific single measurements along each path. If the duration of a single beam of illuminating light is long enough, as with a continuous source of light, it may be possible that only one beam of illuminating light will be required to perform a representative number of property-specific single measurements along a probe path. In that case, any of the straight lines entirely contained within the beam of illuminating light may be arbitrarily selected to be the property-specific probe path. If the duration of a beam is too short, as with a pulsed source of light, then the probe path might need to be illuminated more than once with single beams of illuminating light in order to perform a representative number of property-specific single measurements along the probe path. In the latter case, the property-specific probe path can be selected arbitrarily only from the subset of straight lines which is common to all of the beams required to perform a representative number of property-specific single measurements. Each beam of illuminating light used to probe the particle field which does not entirely contain a previously defined property-specific probe path will need to be assigned a new property-specific probe path, and a representative number of property-specific single measurements will need to be performed along the new probe path, before any of the measurements taken along the new probe path can be used. There is no required order in the manner of successively probing the particle field with single beams of illuminating light, as long as a representative number of property-specific single measurements are performed along each property-specific probe path. However, in most cases, the most practical method will be to complete all of the required measurements along a single probe path before moving on to the next probe path. In general, a different set of property-specific probe paths will need to be defined by the probing process for each property of interest. However, if property-specific single measurements can be performed for more than one property of interest along a single probe path, the same probe path can be considered to be specific to each property.

3.1.2.2 Definition of a Representative Number of Property-Specific Single Measurements. A *representative number of property-specific single measurements* along a property-specific probe path is defined to be a representative number of property-specific single measurements at each

distance measured along the property-specific probe path, for a representative number of distances along the property-specific probe path.

A *representative number of property-specific single measurements at each distance measured along the property-specific probe path* is defined to be the number of measurements necessary to determine the time-averaged property-specific band coefficient, to within the required degree of accuracy. One required property of the particle field to be developed in Chapter 5 is the property of statistical stationarity. The property of statistical stationarity essentially means that the average particle properties do not vary with time during the period of time all required measurements are being performed. However, fluctuations around the averages can still occur with time. If the time over which a property-specific single measurement is made is too short, then a measurement might only be made of a fluctuation, not the average. Therefore, if the time over which a single measurement is performed is not long enough for the measurement to reflect the average, then more than one single measurement will need to be made at the same distance along the property-specific probe path so that the sum of the separate measurements can be combined to form the average.

A *representative number of distances along the property-specific probe path* may be defined as follows. In practice, it will only be possible to perform a representative number of property-specific single measurements only for a finite number of distances along a property-specific probe path. The finite number of distances will populate the property-specific probe path like pearls along a string of pearls. Gaps will exist between the finite number of distances where measurements will not have been performed. The number of distances along the property-specific probe path at which a representative number of property-specific single measurements are made will be considered to be *representative* of that property-specific probe path if, for all the gaps along the property-specific probe path which were not measured, it must be possible in the end to estimate by some means what the property-specific band coefficient would have been measured to be, had measurements been performed in these gaps, to within the required degree of accuracy. Normally the method of estimating what the property-specific band coefficient would have been measured to be will require interpolations to be performed between locations where effective probe volumes in fact existed. However, any method of estimation can be used. If interpolation is used, the method of interpolation may be linear interpolation, or it may be any higher order method of interpolation.

3.1.2.3 Definition of a Representative Number of Property-Specific Probe Paths. A *representative number of property-specific probe paths* within an allowed contiguous sub-volume of the particle field may be defined as follows. In practice, it will only be possible to probe the allowed contiguous sub-volume of the particle field with a finite number of property-specific probe paths. Gaps will exist between the finite number of property-specific probe paths where probe paths will not have been defined, and along which measurements will not have been performed. For example, if the single beams of illuminating light came from random directions, then the sum total volume of all the property-specific probe paths might resemble a random stack of hay or a random pile of toothpicks which fills much of the allowed contiguous sub-volume of the particle field, but not the entire sub-volume due to the spaces between the hay stalks or the toothpicks. As another example, if the beams of illuminating light all came from the same direction but were displaced from each other by differing distances, then the sum total volume of all the property-specific probe paths might resemble a forest of trees or poles extending from one direction through the sub-volume, but with some spaces between them. The sum of all the property-

specific probe paths will be considered to be *representative* of the allowed contiguous sub-volume of the particle field if, for all the gaps between the property-specific probe paths in the sub-volume, it must be possible in the end to estimate by some means what the property-specific band coefficient would have been measured to be, had these gaps been probed, to within the required degree of accuracy. Normally the method of estimating what the property-specific band coefficient would have been measured to be will require interpolations to be performed between locations which were in fact probed. However, any method of estimation can be used. If interpolation is used, the method of interpolation may be linear interpolation, or it may be any higher order method of interpolation.

3.1.2.4 Definition of an Allowed Contiguous Sub-Volume of the Particle Field. An *allowed contiguous sub-volume of the particle field* is defined to be any contiguous volume in the particle field, including possibly the volume of the entire particle field itself, for which the following is true for each and every property-specific single measurement performed within that sub-volume during the probing process: for each and every property-specific single measurement, all parts of the particle field which might exist between the effective probe volume and the detector must also be probed by a representative number of property-specific probe paths.

As an example of an allowed contiguous sub-volume of a particle field, suppose the particle field is contained within and completely fills a sphere. Further suppose all that the detectors are located at a short distance from the front side of the sphere. Now consider the contiguous sub-volume of the particle field contained by the back hemisphere of this sphere. Then the contiguous sub-volume defined by the back hemisphere of this sphere would not be considered to be an allowed contiguous sub-volume of the particle field because most if not all of the measurements from within the back hemisphere must pass through the front hemisphere, which is not probed (if the front hemisphere were probed in addition to the back hemisphere to remedy this problem, then the contiguous sub-volume would no longer be the back hemisphere by itself. It would be the entire sphere). On the other hand, if the contiguous sub-volume is defined to be only the front hemisphere, then all measurements in the front hemisphere would pass through the front hemisphere, the entire volume of which is probed. Therefore the front hemisphere would be an allowed contiguous sub-volume of the particle field.

3.2 Variations of the General Method of Probing the Particle Field with Illuminating Light

The general method of probing the particle field with illuminating light now having been described, variations of the general embodiment of the method of probing the particle field with illuminating light may now be introduced. A large number of variations are possible. A partial list of variations of the general method of probing the particle field with illuminating light includes reducing the allowed contiguous sub-volume of the particle field to a single plane of interest, introducing more order into the probing process, and probing the particle field simultaneously using more than one beam of illuminating light. These variations are described next below.

3.2.1 The Variation of Reducing the Allowed Contiguous Sub-Volume of the Particle Field to a Single Plane of Interest. One variation of the general method of probing the particle field with illuminating light is the variation of reducing the allowed contiguous sub-volume of the particle field to a single plane of interest. In many applications, the distribution of particle properties is often sought only over one or perhaps a few planes in the particle field. The general

method of probing the particle field with illuminating light, however, requires that the particle field be probed over an allowed contiguous sub-volume of the particle field. Information about the desired plane could be obtained by containing the plane within the allowed contiguous sub-volume. However, probing an entire volume to gain information about a single plane can be tedious. Under the variation of reducing the allowed contiguous sub-volume of the particle field to a single plane of interest, the particle field is swept with single beams of illuminating light only within the plane of interest. The combination of the planar area with the thickness of the beam of illuminating light perpendicular to the plane then forms a contiguous planar-shaped volume like a table top. If all detections are made through the end of the planar volume, like looking into the edge of the table top, then all parts of the particle field which exist between each probe volume and each detection will also have been probed for each and every measurement. The planar contiguous sub-volume of the particle field would therefore be an allowed contiguous sub-volume of the particle field. Thus the particle field can be probed only within the plane of interest.

3.2.2 The Variation of Introducing More Order Into the Probing Process. Another variation of the general method of probing the particle field with illuminating light is the variation of introducing more order into the probing process. In the general method of probing the particle field with illuminating light, it is allowed that successive probes by single beams of illuminating light might occur from random directions. This possibility led to the “random stack of hay” and “random pile of toothpicks” analogies discussed in section 3.1.2.3 above. In this variation, any method which introduces more order or less randomness into the probing process is included. Introducing more order into the probing process might be done in order to simplify the probing process, or the measurement process, or the analysis process, or any combination of these. A large number of variations are possible. A partial list of variations included in introducing more order into the probing process includes the variation of introducing more order into the probing process in a two dimensional plane of interest, and the variation of introducing more order into the probing process in a three dimensional volume of interest. These variations are described next.

3.2.2.1 The Variation of Introducing More Order into the Probing Process in a Two Dimensional Plane of Interest. One variation of introducing more order into the probing process is the variation of introducing more order into the probing process in a two dimensional plane of interest. A large number of variations are possible. A partial list of variations of introducing more order into the probing process in a two dimensional plane of interest includes probing with beams that all propagate in the same direction, and probing with beams which successively fan out from a single apex.

3.2.2.1.1 *The variation of probing with beams that all propagate in the same direction.* One variation of introducing more order into the probing process in a two dimensional plane of interest is the variation of probing with beams that all propagate in the same direction. Under this variation, the plane of interest is probed with single beams of illuminating light that all come from the same direction, but which are displaced from one another by various distances in the plane of interest perpendicular to the common direction of propagation. A large number of variations are possible. A partial list of variations of probing with beams that all propagate in the same direction includes arranging the probe paths with equal displacements, fixing the particle field

and traversing the illumination optics, and fixing the illumination optics and traversing the particle field.

The variation of arranging the probe paths with equal displacements. One variation of probing with beams that all propagate in the same direction is the variation of arranging the probe paths with equal displacements. Most generally, there is no requirement that the displacements between the probe paths within the plane of interest be equal. Arranging the probe paths with equal displacements can simplify both the probing process and the analysis process.

The variation of fixing the particle field and traversing the illumination optics. Another variation of probing with beams that all propagate in the same direction is the variation of fixing the particle field and traversing the illumination optics. This can simplify the optics to the optics necessary to create only a single continuous beam, for example, where the optics are traversed such that the beam traverses the particle field within the plane of interest.

The variation of fixing the illumination optics and traversing the particle field. Yet another variation of probing with beams that all propagate in the same direction is the variation of fixing the illumination optics and traversing the particle field. An example of fixing the illumination optics and traversing the particle field would be traversing a spray nozzle while the optics are not moved. Fixing the illumination optics and traversing the particle has the same advantage as in the above paragraph, but may offer the additional advantage that the detection optics need not be traversed.

3.2.2.1.2 *The variation of probing with beams which successively fan out from a single apex.* Another variation of introducing more order into the probing process in a two dimensional plane of interest is the variation of probing with beams which successively fan out from a single apex. This variation might correspond to a beam created by a single set of optics which is directed into the particle field by reflecting it off of a mirror. Successive probings could then be produced by simply rotating the mirror, which would create the fan effect.

3.2.2.2 The Variation of Introducing More Order into the Probing Process in a Three Dimensional Volume of Interest. Another variation of introducing more order into the probing process is the variation of introducing more order into the probing process in a three dimensional volume of interest. A large number of variations are possible. A partial list of variations of introducing more order into the probing process in a three dimensional volume of interest includes probing with beams that all propagate in the same direction and probing within multiple planes of interest.

3.2.2.2.1 *The variation of probing with beams that all propagate in the same direction.* One variation of introducing more order into the probing process in a three dimensional volume of interest is the variation of probing with beams that all propagate in the same direction. This variation is similar to the variation in section 3.2.2.1.1 where a single plane of interest is probed with beams that all propagate in the same direction. Here, the beams are not required to stay within a single plane. A large number of variations are possible. A partial list of variations of probing with beams that all propagate in the same direction includes arranging the probe paths in a regular rectangular array, fixing the particle field and traversing the illumination optics, and fixing the illumination optics and traversing the particle field.

The variation of arranging the probe paths in a regular rectangular array. One variation of probing with beams that all propagate in the same direction is the variation of arranging the

probe paths in a regular rectangular array. Most generally, there is no requirement that the displacements between the probe paths be arranged in a regular rectangular array. Arranging the probe paths in a regular rectangular array can simplify both the probing process and the analysis process. This variation is analogous to the variation of arranging the probe paths with equal displacements in a single plane of interest, as discussed in section 3.2.2.1.1.

The variation of fixing the particle field and traversing the illumination optics. Another variation of probing with beams that all propagate in the same direction is the variation of fixing the particle field and traversing the illumination optics. This can simplify the optics to the optics necessary to create only a single continuous beam, for example, where the optics are traversed such that the beam traverses the particle field within the plane of interest. This variation is analogous to the variation in section 3.2.2.1.1 for a single plane of interest.

The variation of fixing the illumination optics and traversing the particle field. Yet another variation of probing with beams that all propagate in the same direction is the variation of fixing the illumination optics and traversing the particle field. An example of fixing the illumination optics and traversing the particle field would be traversing a spray nozzle while the optics are not moved. This variation is analogous to the variation of section 3.2.2.1.1 for a single plane of interest.

3.2.2.2.2 *The variation of probing within multiple planes of interest.* Another variation of introducing more order into the probing process in a three dimensional volume of interest is the variation of probing within multiple planes of interest. Under the variation of probing within multiple planes of interest, any of the methods given in section 3.2.2.1 of introducing more order into the probing process in a two dimensional plane of interest are used within multiple planes of

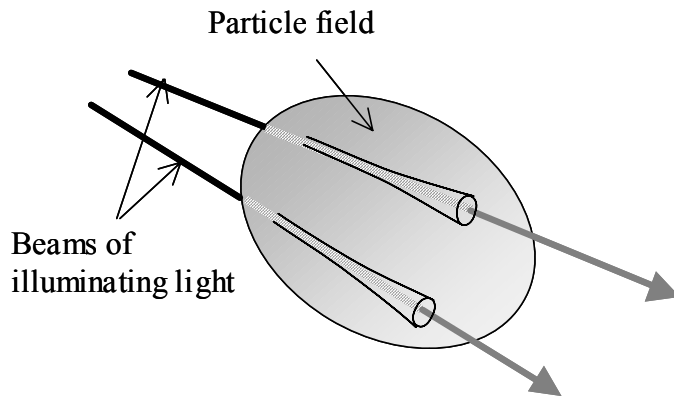


Figure 2. Two beams of illuminating light propagating through a particle field

interest. A large number of variations are possible. A partial list of variations included in probing within multiple planes of interest includes probing within selected planes constituting a disconnected volume, and probing within planes constituting a connected volume.

The variation of probing within selected planes constituting a disconnected volume. One variation of probing within multiple planes of interest is the variation of probing within selected planes constituting a disconnected volume. In this variation, probing would occur only within selected planes which have particular interest. No attempt is made to ensure the planes are spaced closely enough together to make interpolation between the planes reliable.

The variation of probing within planes constituting a connected volume. Another variation of probing within multiple planes of interest is the variation of probing within planes constituting a connected volume. In this variation, a volume of interest is probed with planes spaced suitably close together so that interpolation between the planes is reliable. A large number of variations are possible. A partial list of variations of probing within planes constituting a connected volume includes maintaining an equal spacing between the planes.

3.2.3 The Variation of Probing the Particle Field Simultaneously with More than One Beam of Illuminating Light. Yet another variation of the general method of probing the particle field with illuminating light is the variation of probing the particle field simultaneously with more than one beam of illuminating light. Two such beams are illustrated in Figure 2, although the number of beams is not necessarily limited to two. Provided that the detectors and the optics are available, this variation could, for example, speed up the rate at which measurements can be made. Considerations exist which govern the number and orientation of the beams of illuminating light when more than one beam of illuminating light is used to simultaneously probe the particle field. Variations of probing the particle field simultaneously with more than one beam of illuminating light also exist.

3.2.3.1 Considerations Governing the Number and Orientation of the Beams of Illuminating Light When More Than One Beam of Illuminating Light is used to Simultaneously Probe the Particle Field. Considerations governing the number and orientation of the beams of illuminating light when more than one beam of illuminating light is used to simultaneously probe the particle field include the consideration that more than one beam of illuminating light does not contribute significantly to the signal produced within the response volume of any detector, and the consideration of minimizing secondary emission.

3.2.3.1.1 The consideration that more than one beam of illuminating light does not contribute significantly to the signal produced within the response volume of any detector. One consideration governing the number and orientation of the beams of illuminating light when more than one beam of illuminating light is used to simultaneously probe the particle field is the consideration that more than one beam of illuminating light does not contribute significantly to the signal produced within the response volume of any detector. An example of several beam configurations is given in Figure 3. The hatched lines in Figure 3 represent the beams of illuminating light. The circles represent detector response volumes. A measurement at position 1 would be an acceptable measurement because only one beam is within the response volume of the detector. Position 2 represents one beam crossing behind another beam. A measurement at position 2 might be acceptable if the beams are separated by enough distance that the signal produced by only one of the beams contributes significantly to the signal in the response volume of the detector. If both beams produce a significant contribution to the

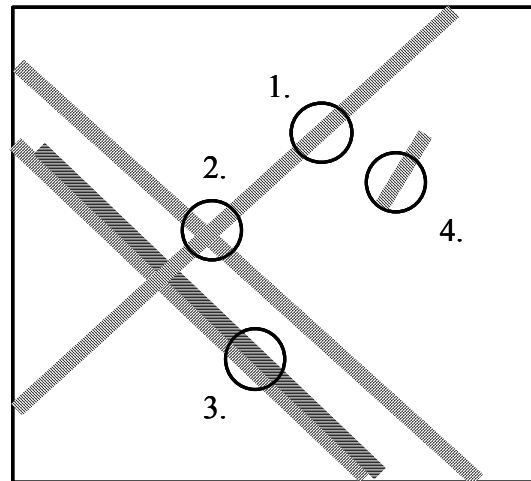


Figure 3. Randomly oriented multiple beams. The circles represent detector response volumes.

signal within the response volume of the detector located at position 2, the measurement would not be acceptable. Similarly, a measurement at position 3, representing parallel beams behind each other, may or may not be acceptable depending on their separation with respect to the size of the detector response volume. Finally, position 4, representing the case of a beam pointing almost directly at the detector, would probably be unacceptable by the criteria of sections 3.1.1.2.1 and 3.1.1.2.2, due to inadequate spatial resolution in the axial direction, and due to signal being produced from parts of the response volume where the detector response is non-uniform.

3.2.3.1.2 *The consideration of minimizing secondary emission.* Another consideration governing the number and orientation of the beams of illuminating light when more than one beam of illuminating light is used to simultaneously probe the particle field is the consideration of minimizing secondary emission. Whenever more than one beam of illuminating light probes the particle field simultaneously, each additional beam of illuminating light increases the total amount of illumination which could cause secondary emission. The consideration of minimizing secondary emission generally requires restricting the number of simultaneous beams of illuminating light to the fewest number possible.

3.2.3.2 Variations of Probing the Particle Field Simultaneously using More than One Beam of Illuminating Light. A large number of variations of simultaneously probing the particle field with more than one beam of illuminating light are possible. A partial list of variations of simultaneously probing the particle field with more than one beam of illuminating light includes arranging the beams to propagate within a continuous planar sheet of light, and arranging selected beams of illuminating light in a planar comb of light.

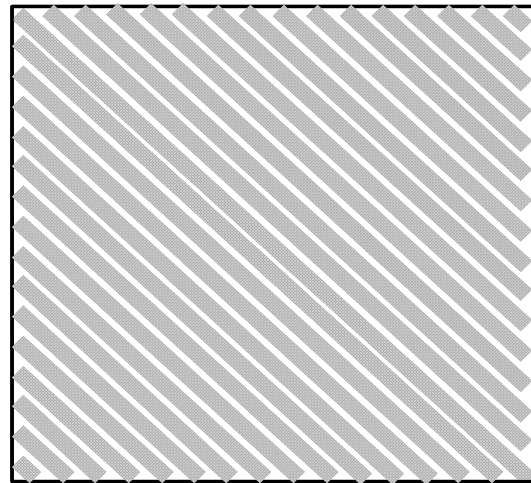


Figure 4. Multiple beams making up a sheet of illuminating light.

3.2.3.2.1 *The variation of arranging the beams to propagate within a continuous planar sheet of light.* One variation of probing the particle field simultaneously using more than one beam of illuminating

light is the variation of arranging the beams to propagate within a continuous planar sheet of light. One variation of arranging the beams to propagate within a continuous planar sheet of light is arranging the beams of illuminating light to propagate in parallel as a continuous planar sheet of light. Arranging the beams of illuminating light to propagate in parallel as a continuous planar sheet of light is illustrated in Figure 4. Planar sheets of illuminating light have often been used in optical patterning methods, as discussed in Chapter 2.

Another variation of arranging the beams to propagate within a continuous planar sheet of light is the variation of arranging the beams of illuminating light to propagate from a single common point in a continuous planar fan of light. Such an effect can be produced by cylindrical lenses, for example. This method has also often been used in optical patterning methods.

Two disadvantages of arranging the beams to propagate within a continuous planar sheet of light are that secondary emission is increased, and that probing only within a single plane of in-

terest is no longer possible. The latter disadvantage is due to the fact that more than one beam would contribute significantly to the signal produced within the response volume of the detector.

3.2.3.2.2 *The variation of arranging selected beams of illuminating light in a planar comb of light.* Another variation of probing the particle field simultaneously using more than one beam of illuminating light is the variation of arranging selected beams of illuminating light in a planar comb of light. As discussed above in section 3.2.3.2.1, one of the disadvantages of arranging the beams to propagate within a continuous planar sheet of light are that secondary emission is increased. One way to reduce the secondary emission and still retain the ability to probe the particle field with multiple beams in a planar geometry would be reduce the number of beams to a select few, in a kind of “comb” geometry. As intended here, the comb geometry would apply both to the rectilinear plane case, as illustrated in Figure 5, and to the planar fan case, as in the spokes of a wheel.

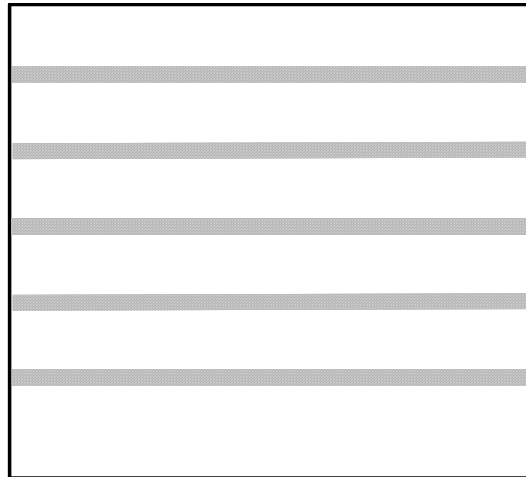


Figure 5. Multiple beams making up planar comb of illuminating light

3.3 The Most Recommended Method of Probing the Particle Field with Illuminating Light

With the general method and variations of the general method of probing the particle field with illuminating light now having been described, the most recommended method of probing the particle field with illuminating light may now be introduced. The most recommended method of probing the particle field with illuminating light contains the following elements:

3.3.1 Probing the particle field with a single beam of illuminating light.

3.3.1.1 Conditioning the single beam of illuminating light according to the general considerations given in section 3.1.1.

3.3.1.1.1 Conditioning the single beam of illuminating light where the particle property of interest is the particle surface area per unit volume and the property-

specific band coefficient is the elastic scattering coefficient, wherein wavelengths are selected to be much smaller than the range of particle sizes for the measurement of particle surface area.

- 3.3.1.1.2 Conditioning the single beam of illuminating light where the particle property of interest is also the particle mass per unit volume and the property-specific band coefficient is the fluorescence band coefficient, wherein the same wavelength selected in section 3.3.1.1.1 also excites fluorescence in the particles.
 - 3.3.1.1.2.1 Exciting fluorescence in molecules that are naturally present in the particles.
 - 3.3.1.1.2.2 Exciting fluorescence in molecules which are artificially doped into the particles.
- 3.3.1.1.3 Having only the wavelength used in sections 3.3.1.1.1 and 3.3.1.1.2 be present in the beam
- 3.3.1.1.4 Limiting the range of polarization states to only a single polarization state.
- 3.3.1.2 Using a single beam of illuminating light which is a monochromatic parallel-propagating laser beam.
 - 3.3.1.2.1 Using a continuous laser beam.
 - 3.3.1.2.2 Using a pulsed laser beam.
- 3.3.2 Successively probing the particle field with the monochromatic parallel-propagating laser beam, according to the requirements given in section 3.1.2.
 - 3.3.2.1 Probing the particle field only within a single plane of interest.
 - 3.3.2.1.1 Holding the optics of the monochromatic parallel-propagating laser beam fixed, and probing the plane of interest is by translating the particle field.

4.0 THE METHOD OF PERFORMING THE REQUIRED MEASUREMENTS

The method of performing the required measurements is described below. The general method of performing the required measurements is described first. Then variations of the general embodiment will be given. Finally, the most recommended method of performing the required measurements is presented.

4.1 The General Method of Performing the Required Measurements

The general method of performing the required measurements is to successively perform property-specific single measurements along property-specific probe paths, until a representative number of property-specific single measurements are performed along each property-specific probe path defined by the method of probing the particle field with illuminating light. A *property-specific probe path* was defined in section 3.1.2.1 of Chapter 3, and a *representative number of property-specific single measurements* was defined in section 3.1.2.2 of Chapter 3.

The method of performing a property-specific single measurement consists of the following. A *detection* is defined in Appendix B to be the single response of a single detector. A *measurement*, as distinguished from a detection, is defined herein to be a detection, plus a determination of all the other quantities required to make use of the detection. The method of performing a property-specific single measurement consists of gathering sufficient information so that the below list of quantities will be known, measured, or calculable.

4.1.1 The Magnitude of the Response of a Property-Specific Single Detector. The first item in the list of quantities that must be known, measured, or calculable is the magnitude of the response of a property-specific single detector. A *property-specific single detector* is defined to be a single detector which has been conditioned to detect the property-specific signal, and to filter out all other non-property-specific optical radiation that might be present, to within the required degree of accuracy. The property-specific single detector is designed according to a number of considerations governing the response of the detector, the properties of the effective probe volume, and the location of the detector. These considerations are described in greater detail next.

4.1.1.1 Considerations Governing the Response of a Property-Specific Single Detector. One set of considerations involved in designing a property-specific single detector is the set of considerations governing the response of the detector. Considerations governing the response of a property-specific single detector include the consideration of detecting property-specific signals, the consideration of detecting property-specific signals only at wavelengths where the TAMP-SEC is approximately the same at each wavelength, the consideration of filtering out all other non-property-specific optical radiation that might be present, the consideration of detecting the emitted signal to within the required degree of accuracy, the consideration of minimizing the detection of signals when there is no illumination, and the consideration of minimizing undetected drift in the response of the detector. Variations of the considerations governing the response of a property-specific single detector include a variation where the particle property of interest is the particle surface area, and a variation where the particle property of interest is the particle mass. These considerations and the variations of them are described in greater detail next.

4.1.1.1.1 *The consideration of detecting property-specific signals.* One consideration governing the response of a property-specific single detector is the consideration of detecting property-specific signals. Recall that the single beam of illuminating light was conditioned in section 3.1.1.1.1 of Chapter 3 to contain at least one wavelength which excites a property-specific band coefficient, and further conditioned in section 3.1.1.1.2 of Chapter 3 so that signals produced within the excited property-specific band can be detected by a property-specific single detector. The corresponding consideration here is that the property-specific single detector must be able to detect these property-specific signals.

4.1.1.1.2 *The consideration of detecting property-specific signals only at wavelengths where the TAMPSEC is approximately the same at each wavelength.* Another consideration governing the response of a property-specific single detector is the consideration of detecting property-specific signals only at wavelengths where the TAMPSEC is approximately the same at each wavelength. Recall that “TAMPSEC” is an acronym which was defined in section 3.1.1.1.3 of Chapter 3 to stand for “time-averaged instantaneous monochromatic polarization-specific extinction coefficient.” The meaning of the terms in this acronym is discussed in **Appendix A**. In the method of analyzing the data to developed in Chapter 5, the TAMPSEC is considered to be approximately the same for each property-specific wavelength in the signal. One way to approach this condition is to restrict the range of property-specific signal wavelengths which the detector can detect to increasingly narrow bands. In the limit as the range of property-specific signal wavelengths approaches a single wavelength, this condition will become exactly true. Therefore the range of wavelengths which the property-specific single detector can detect should be narrow enough for the TAMPSEC to be approximately the same at each wavelength, to within the required degree of accuracy.

4.1.1.1.3 *The consideration of filtering out all other non-property-specific optical radiation that might be present.* Yet another consideration governing the response of the property-specific single detector is the consideration of filtering out all other non-property-specific optical radiation that might be present. A property-specific single detector must by definition be able to filter out non-property-specific optical radiation. The property-specific band coefficient will have been selected in sections 3.1.1.1.1 and 3.1.1.1.2 of Chapter 3 so that it will be possible to do this.

4.1.1.1.4 *The consideration of detecting the signal received to within the required degree of accuracy.* A further consideration governing the response of the property-specific single detector is the consideration of detecting the emitted signal to within the required degree of accuracy. Factors governing detecting the emitted signal to within the required degree of accuracy include the necessity to be sensitive enough to accurately respond to the signal, and the necessity to avoid being so sensitive that the detector becomes saturated.

The necessity to be sensitive enough to accurately respond to the signal. One factor governing detecting the emitted signal to within the required degree of accuracy is the necessity to be sensitive enough to accurately respond to the signal. The response of the detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, the illumination energy delivered over the period of detection, and the size of the effective probe volume, be sensitive enough to accurately respond to the signal. A further discussion of this necessity may be found in section 3.1.1.2.8 of Chapter 3.

The necessity to avoid being so sensitive that the detector becomes saturated. Another factor governing detecting the emitted signal to within the required degree of accuracy is the necessity to avoid being so sensitive that the detector becomes saturated. The response of the detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, the illumination energy delivered over the period of detection, and the size of the effective probe volume, not be so sensitive that the detector becomes saturated. A further discussion of this necessity may be found in section 3.1.1.2.8 of Chapter 3.

4.1.1.1.5 *The consideration of minimizing the detection of signals when there is no illumination.* An even further consideration governing the response of the detector is the consideration of minimizing the detection of signals when there is no illumination. The period of detection should, in conjunction with the design of the single beam of illuminating light, not exceed the period of illumination. A further discussion of this consideration may be found in section 3.1.1.4.3 of Chapter 3.

4.1.1.1.6 *The consideration of minimizing undetected drift in the response of the detector.* An even further yet consideration governing the response of the detector is the consideration of minimizing undetected drift in the response of the detector. Any undetected change in sensitivity of the detector during the course of a measurement or during the course of making many measurements which would change the magnitude of the response of the detector to a given signal will introduce errors in relating the magnitude of the detector response to the total signal energy received. Therefore the response of the detector should minimize any drift in the response of the detector. Methods for minimizing undetected drift in the response of the detector include using stable electronics, periodic calibration, and preventing particles in the particle field from contaminating the collection optics.

4.1.1.1.7 *Variations of the considerations governing the response of a property-specific single detector.* Variations of the considerations governing the response of a property-specific single detector include a variation where the particle property of interest is the particle surface area, and a variation where the particle property of interest is the particle mass.

A variation where the particle property of interest is the particle surface area. One variation of the considerations governing the response of a property-specific single detector is a variation where the particle property of interest is the particle surface area. It will be recalled from section 3.1.1.1.5 of Chapter 3 that when the particle property of interest is the particle surface area, at least one wavelength will be present in the illumination for which the elastic scattering band coefficient is proportional to the particle surface area density. The corresponding variation of the considerations governing the response of a property-specific single detector would be to use wavelength filters to allow the scattered signal at the scattering wavelengths to pass, while filtering out all other wavelengths.

A variation where the particle property of interest is the particle mass. Another variation of the considerations governing the response of a property-specific single detector is a variation where the particle property of interest is the particle mass. It will be recalled from section 3.1.1.1.5 of Chapter 3 that when the particle property of interest is the particle mass, at least one wavelength will be present in the illumination for which the fluorescence band coefficient will be proportional to the particle mass density. The corresponding variation of the considerations governing the response of a property-specific single detector would be to use wavelength filters to

allow the fluorescence signal to pass, while filtering out all other wavelengths, including the wavelength of the scattered light.

4.1.1.2 Considerations Governing the Properties of the Effective Probe Volume. Another set of considerations involved in designing a property-specific single detector is the set of considerations governing the properties of the effective probe volume. Considerations governing the properties of the effective probe volume include the consideration of achieving adequate spatial resolution, the consideration of achieving nearly uniform detector response, the consideration of reducing bias caused by the variability of the probe volumes, the consideration of minimizing attenuation of the illuminating light and attenuation of the signal light across the effective probe volume, the consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics, the consideration of reducing secondary emission (the slender beam approximation of the first kind), the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field, and the consideration of maintaining a repeatable effective probe volume correction factor. These considerations are described in greater detail next.

4.1.1.2.1 *The consideration of achieving adequate spatial resolution.* One consideration governing the properties of the effective probe volume is the consideration of achieving adequate spatial resolution. Factors governing the degree of spatial resolution required include the necessity to relate detections to absolute locations in space from which the signals originated, and the necessity that particle properties be approximately uniform within the effective probe volume, both to within the required degree of accuracy.

The necessity to relate detections to absolute locations in space. One factor governing the consideration of achieving adequate spatial resolution is the necessity to relate detections to absolute locations in space. The design of the property-specific single detector should, in combination with area over which the light in a single beam of illuminating light is distributed, create an effective probe volume which is as small as possible in order to relate detections to absolute locations in space from which the signals originated, to within the required degree of accuracy. A further discussion of this necessity may be found in section 3.1.1.2.1 of Chapter 3.

The necessity that particle properties be approximately uniform within the effective probe volume. Another factor governing the consideration of achieving adequate spatial resolution is the necessity that particle properties be approximately uniform within the effective probe volume. The design of the property-specific single detector should, in combination with area over which the light in a single beam of illuminating light is distributed, create an effective probe volume which is as small as possible so that particle properties are approximately uniform within the effective probe volume, to within the required degree of accuracy. A further discussion of this necessity may be found in section 3.1.1.2.1 of Chapter 3.

4.1.1.2.2 *The consideration of achieving nearly uniform detector response.* Another consideration governing the properties of the effective probe volume is the consideration of achieving nearly uniform detector response. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, create an effective probe volume which is limited as much as possible to the volumes of nearly uniform detector response. A further discussion of this consideration may be found in section 3.1.1.2.2 of Chapter 3.

4.1.1.2.3 *The consideration of reducing bias caused by the variability of the probe volumes.* Yet another consideration governing the properties of the effective probe volume is the consideration of reducing bias caused by the variability of the probe volumes. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, create an effective probe volume which is as small as possible so that the variability in the probe volumes can be minimized. A further discussion of this consideration may be found in section 3.1.1.2.3 of Chapter 3.

4.1.1.2.4 *The consideration of minimizing attenuation of the illuminating light and attenuation of the signal light across the effective probe volume (the small effective probe volume approximation of the first kind).* A further consideration governing the properties of the effective probe volume is the consideration of minimizing attenuation of the illuminating light and attenuation of the signal light across the effective probe volume. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, create an effective probe volume which is significantly smaller than the dimension of the particle field when attenuation of the illuminating light and attenuation of the signal light across the particle field is significant. A further discussion of this consideration may be found in section 3.1.1.2.4 of Chapter 3.

4.1.1.2.5 *The consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics (the effective small probe volume approximation of the second kind).* An even further consideration governing the properties of the effective probe volume is the consideration of defining approximately the same range of directions from each point in the effective probe volume to the effective collection area of the detector optics. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, create an effective probe volume which is small enough to allow the range of directions defined from each point in the effective probe volume to the effective collection area of the detector optics to all be approximately the same. A further discussion of this consideration may be found in section 3.1.1.2.5 of Chapter 3.

4.1.1.2.6 *The consideration of reducing secondary emission (the slender beam approximation of the first kind).* An additional consideration governing the properties of the effective probe volume is the consideration of reducing secondary emission. Factors governing the reduction of secondary emission include the necessity to reduce the amount of secondary emission from particles which are not in the path of the original illumination, and the necessity to reduce the amount of secondary emission from particles which are in the path of the original illumination.

The necessity to reduce the amount of secondary emission from particles which are not in the path of the original illumination. One factor governing the reduction of secondary emission is the necessity to reduce the amount of secondary emission from particles which are not in the path of the original illumination. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed and the illumination energy delivered over the period of illumination, create an effective probe volume which is as small as possible in order to reduce the amount of secondary emission from par-

ticles which are not in the path of the original illumination. A further discussion of this necessity may be found in section 3.1.1.2.6 of Chapter 3.

The necessity to reduce the amount of secondary emission from particles which are in the path of the original illumination. Another factor governing the reduction of secondary emission is the necessity to reduce the amount of secondary emission from particles which are in the path of the original illumination. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed and the illumination energy delivered over the period of illumination, create an effective probe volume which is as small as possible in order to reduce the amount of secondary emission from particles which are in the path of the original illumination. A further discussion of this necessity may be found in section 3.1.1.2.6 of Chapter 3.

4.1.1.2.7 *The consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field.* Another additional consideration governing the properties of the effective probe volume is the consideration of preventing the single beam of illuminating light from perturbing the behavior of the particle field. The design of the property-specific single detector should, in combination with the area over which the light in a single beam of illuminating light is distributed, the illumination energy delivered over the period of illumination, and the period of illumination, create an effective probe volume which is large enough to prevent perturbing the behavior of the particle field to an unacceptable degree. A further discussion of this consideration may be found in section 3.1.1.2.9 of Chapter 3.

4.1.1.2.8 *The consideration of maintaining a repeatable effective probe volume correction factor.* An even further additional consideration governing the properties of the effective probe volume is the consideration of maintaining a repeatable effective probe volume correction factor. The design of the property-specific single detector should, when combined with the repeatability of the source of the single beam of illuminating light, the area over which the light in a single beam of illuminating light is distributed, and the location of the detector, create effective probe volumes which exhibit sufficient repeatability to cause a repeatable effective probe volume correction factor to be maintained along the length of the property-specific probe path. A further discussion of this consideration may be found in section 3.1.1.2.10 of Chapter 3.

A method for causing the repeatability of the effective probe volume correction factor to be independent of the repeatability of the of the effective probe volumes. Although there are many ways the repeatability of the effective probe volume can interact with the repeatability of the source of the single beam of illuminating light, the area over which the light in a single beam of illuminating light is distributed, and the location of the detector to create a repeatable effective probe volume correction factor, the simplest method is to make the repeatability of the effective probe volume correction factor independent of the repeatability of the effective probe volume. One method of making the repeatability of the effective probe volume correction factor independent of the repeatability of the effective probe volume is to use effective probe volumes having geometries which are as repeatable as possible.

4.1.1.3 Considerations Governing the Location of the Detector and the Size of the Effective Collection Area of the Collection Optics. Yet another set of considerations involved in designing a property-specific single detector is considerations governing the location of the detector and the size of the effective collection area of the collection optics. Considerations governing the lo-

cation of the detector and the size of the effective collection area of the collection optics include the consideration of minimizing the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics, the consideration of minimizing the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field, and the consideration of maintaining a repeatable effective probe volume correction factor.

4.1.1.3.1 *The consideration of minimizing the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics (the small detector approximation of the first kind).* One consideration governing the location of the detector and the size of the effective collection area of the collection optics is the consideration of minimizing the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics. In the small probe volume approximation of the second kind discussed in section 3.1.1.2.5 of Chapter 3, the small size of the effective probe volume allowed the solid angles defined from each point in the effective probe volume to the effective collection area of the collection optics to all be approximately the same. Under this consideration, either the effective collection area of the collection optics is also considered to be small, or the distance from the effective probe volume to the effective collection area of the collection optics is considered to be large, such that the solid angles defined by any point in the effective probe volume to the effective collection area of the collection optics reduces to very narrow solid angles all oriented in approximately the same direction. Satisfying the consideration of minimizing the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics will lead to the *small detector approximation of the first kind*, which can simplify the method of analyzing the data to developed in Chapter 5. In the limit as the number of directions from the property-specific probe path to the effective collection area of the collection optics approaches a single direction, and the solid angle approaches zero, the small detector approximation of the first kind will become identically true.

A method for minimizing the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics. One method for minimizing the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics is to locate the detector at a suitably large distance from the effective probe volume, or to make the effective collection area of the collection optics suitably small, or both.

4.1.1.3.2 *The consideration of minimizing the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field.* Another consideration governing the location of the detector and the size of the effective collection area of the collection optics is the consideration of minimizing the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field. This effect is caused by the finite size of the effective collection area of the collection optics, and can become a problem near the boundaries of the allowed contiguous sub-volume of the particle field. Suppose for example that the upper boundary of the allowed contiguous sub-volume of the particle field is a plane. Then if some portion of

the effective collection area should exist above that plane, then the signal reaching that portion of the collection optics could be attenuated by particles above the plane which are not probed by single beams of illuminating light. Corrections for attenuation of the signal light according to the method of analyzing the data to be developed in Chapter 5 cannot be accurately applied to signals which are partly or wholly attenuated by particles not contained within the allowed contiguous sub-volume of the particle field. Therefore this effect will constitute a source of error. Raising the boundary of the allowed contiguous sub-volume of the particle field so that these particles can be included will merely change the location of the boundary where the problem occurs, but will not correct the problem unless the new boundary is located above all parts of the particle field. Therefore, the location of the detector and the size of the effective collection area of the collection optics should minimize the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field.

A method for minimizing the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field. One method for minimizing the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field is to mask off the light coming from those paths so that this light does not reach the sensitive element. In the most recommended method of probing the particle field with illuminating light, the particle field is probed by single beams of illuminating light only within the plane of interest. The combination of the plane of interest with the thickness of the beam will constitute an allowed contiguous sub-volume of the particle field only if the all detections are made through the end of the planar volume, like looking into the edge of the table top. In this case the effective collection area of practical collection optics might easily extend significantly beyond the upper and lower boundaries of this table top-like volume, and the error caused could potentially be significant. The mask in this case could be a linear slit to allow only the light originating within the table top-like plane to reach the sensitive element. The linear slit would also tend to reduce the amount of secondary emission which is detected by the detector.

4.1.1.3.3 *The consideration of maintaining a repeatable effective probe volume correction factor.* Yet another consideration governing the location of the detector and the size of the effective collection area of the collection optics is the consideration of maintaining a repeatable effective probe volume correction factor. The location of the detector and the size of the effective collection area of the collection optics should, when combined with the area over which the light in a single beam of illuminating light is distributed, the repeatability of the source of the single beam of illuminating light, and the properties of the effective probe volume, exhibit sufficient repeatability to cause a repeatable effective probe volume correction factor to be maintained along the length of the property-specific probe path. A further discussion of this consideration may be found in section 3.1.1.2.10 of Chapter 3.

A method for causing the repeatability of the effective probe volume correction factor to be independent of the repeatability of the location of the detector and the size of the effective collection area. Although there are many ways the repeatability of the location of the detector and the size of the effective collection area can interact with the repeatability of the effective probe volume, the repeatability of the source of the single beam of illuminating light, and the area over which the light in a single beam of illuminating light is distributed, the simplest method is to

make the repeatability of the effective probe volume correction factor independent of the repeatability of the location of the detector and the size of the effective collection area. One method of making the repeatability of the effective probe volume correction factor independent of the repeatability of the location of the detector and the size of the effective collection area is to maintain a fixed relationship between the detector and the single beam of illumination light. Maintaining a fixed relationship between the detector and the single beam of illumination light means maintaining a fixed distance from the detector to the beam, and maintaining fixed angles between the detector and the beam. The detector is allowed to traverse along the beam as long as the distance to the beam is fixed and all angles relative to the beam are fixed.

4.1.1.3.4 *The consideration of minimizing the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor (the small detector approximation of the second kind).* A further consideration governing the location of the detector and the size of the effective collection area of the collection optics is the consideration of minimizing the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor. A variation of the method of performing the required measurements will be described below in section 4.2.1.2 of this chapter where more than one property-specific single measurement is performed simultaneously by using more than one sensitive element behind a common set of collection optics. When this is done, it will in general no longer be possible to maintain a precisely fixed relationship between each detector and the single beam of illumination light, as required in section 4.1.1.3.3 to cause the repeatability of the effective probe volume correction factor to be independent of the repeatability of the location of the detector and the size of the effective collection area. Small changes in angles will be involved between each sensitive element and the position along the property-specific probe path of the corresponding effective probe volume, and there will be small changes in the geometry of the effective probe volumes. The location of the detector and the size of the effective collection area of the collection optics must therefore be such that the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor is minimized. Satisfying the consideration of minimizing the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor will lead to the *small detector approximation of the second kind*. Satisfying the small detector approximation of the first kind involved having small solid angles from anywhere in the effective probe volume to the effective collection area of the collection optics. Here, satisfying the small detector approximation of the second kind involves small angles subtended from anywhere on the effective collection area of the collection optics to anywhere along the property-specific probe path which the detector views. In the limit as the angle subtended from anywhere on the effective collection area of the collection optics to anywhere on the property-specific probe path which the detector views approaches zero, the small detector approximation of the second kind becomes identically true.

A method for minimizing the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor. One method of minimizing the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor is to locate the detector a suitably large distance away from the property-specific probe path.

4.1.2 The Identity of the Property-Specific Band Coefficient to which the Response of the Property-Specific Single Detector Corresponds. The second item in the list of quantities that must be known, measured, or calculable is the identity of the property-specific band coefficient to which the response of the property-specific single detector corresponds.

4.1.3 The Identity of the Property-Specific Probe Path to which the Response of the Property-Specific Single Detector Corresponds. The third item in the list of quantities that must be known, measured, or calculable is the identity of the property-specific probe path to which the response of the property-specific single detector corresponds. The identity of the property-specific probe path to which the response of the property-specific single detector corresponds consists of specifying any point \vec{x}_{pp} through which the property-specific probe path passes and also specifying a unit vector \hat{n}_{pp} which describes the direction of the property-specific probe path.

4.1.4 The Distance along the Property-Specific Probe Path to which the Response of the Property-Specific Single Detector is Assigned to Correspond. The fourth item in the list of quantities that must be known, measured, or calculable is the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond. The distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond is given by a rational number S ; $-\infty \leq s \leq \infty$, such that $\vec{x}_{pp} + s\hat{n}_{pp}$ specifies the location of the point having the nearest distance from any point in the effective probe volume of the detection to the property-specific probe path. It is possible that more than one point in the effective probe volume of the detection corresponds to the nearest distance to the property-specific probe path. For example, if the property-specific probe path passes through the effective probe volume for the detection, there might be any number of points within the effective probe volume of the detection for which the nearest distance to the property-specific probe path is zero. In such cases, any of the points along the property-specific probe path which correspond to the nearest distance to the effective probe volume of the detection may be selected.

4.1.5 The Total Property-Specific Illumination Energy Entering the Allowed Contiguous Sub-Volume of the Particle Field. The fifth item in the list of quantities that must be known, measured, or calculable is the total property-specific illumination energy entering the allowed contiguous volume of the particle field. The total property-specific illumination energy entering the allowed contiguous volume of the particle field is the total illumination energy entering the allowed contiguous volume of the particle field integrated over all property-specific wavelengths, over all property-specific states of polarization, over the total period of illumination, and over the entire area of the beam at some point before the beam enters the allowed contiguous sub-volume of the particle field. If this quantity is not otherwise known, then it must be measured by a detector which is separate from the property-specific single detector. The detector for measuring the total property-specific illumination energy entering the allowed contiguous sub-volume of the particle field must produce a response the magnitude of which can be uniquely related to the total property-specific illumination energy delivered over the period of illumination. It must be capable filtering out any non-property-specific illumination which might be present. It must minimize the detection of signals when there is no property-specific illumina-

tion. Its response must exhibit minimum drift in the course of a detection and in the course of many detections. Finally, it must perform all of these functions to within the required degree of accuracy. No attenuation of the property-specific illumination must occur between the point of measurement and the point where the illumination enters the allowed contiguous sub-volume of the particle field. Any means of accomplishing the above requirements is acceptable. One common way to perform the required measurement would be to deflect a small fraction of the light from the single beam of illuminating light towards a detector using a partially reflecting mirror. Alternately, if the total signal-specific illumination energy of the single beam of illuminating light does not vary from beam to beam, the measurement can be made before the process of probing the particle field commences.

4.1.6 The Total Property-Specific Illumination Energy Exiting the Allowed Contiguous Sub-Volume of the Particle Field. The sixth item in the list of quantities that must be known, measured, or calculable is the total property-specific illumination energy exiting the allowed contiguous sub-volume of the particle field. The total property-specific illumination energy exiting the allowed contiguous sub-volume of the particle field is the total illumination energy exiting the allowed contiguous sub-volume of the particle field integrated over all property-specific wavelengths, over all property-specific states of polarization, over the total period of illumination, and over the entire area of the beam at some point after the beam exits the particle field. No attenuation of the property-specific illumination must occur between the point where the illumination exits the allowed contiguous sub-volume of the particle field and the point of measurement. The requirements governing the detector which detects it are the same as discussed above in section 4.1.5. However, the detector which measures the total property-specific illumination energy exiting the allowed contiguous sub-volume of the particle field will have to deal with the effects of secondary emission to a much greater extent than the detector which measures the total property-specific illumination energy entering the allowed contiguous sub-volume of the particle field. Additional wavelength filters may serve to eliminate or significantly attenuate wavelengths in the secondary emission which are different than the signal-specific wavelengths. Secondary emission at the same wavelength or wavelengths as the property-specific illumination will be harder to deal with. One method of minimizing errors caused by secondary emission at the same wavelengths as the property-specific illumination is to use spatial filters in the detector.

The method of using spatial filters to minimize errors caused by secondary emission at the same wavelengths as the property-specific illumination. One method of minimizing errors caused by secondary emission at the same wavelengths as the property-specific illumination is to use spatial filters in the detector. Illumination in the beam of illuminating light will exit only through a specific area and will tend to be directed over a relatively narrow range of directions, especially under a slender beam approximation. Secondary emission at the same wavelengths as the property-specific illumination, however, exits over a much larger area and over a much larger range of directions. Masking off light which does not pass through the exit area of the beam may therefore do much to minimize the detection of secondary emission at the same wavelengths as the property-specific wavelengths. Using more than one mask in series at different axial locations along the exit beam may succeed in further blocking secondary emission at the same wavelengths as the property-specific wavelengths which succeeds in passing through the first mask but is not traveling in the same narrow range of directions as the exit beam. Such illumination may be blocked by the second or subsequent masks.

4.1.7 The Size, Shape, Extent, and Location of the Effective Collection Area of the Collection Optics of the Property-Specific Single Detector. The seventh item in the list of quantities that must be known, measured, or calculable is mathematical coordinates describing the size, shape, extent, and location of the effective collection area of the collection optics of the property-specific single detector.

4.1.8 The Size, Shape, Extent, and Location of the Allowed Contiguous Sub-Volume of the Particle Field. The eighth item in the list of quantities that must be known, measured, or calculable is mathematical coordinates describing the size, shape, extent, and location of the allowed contiguous sub-volume of the particle field. Errors in describing the size, shape, extent, and location of the allowed contiguous sub-volume of the particle field that include volumes that are not part of the allowed contiguous sub-volume of the particle field but where no attenuation of the illumination light and no attenuation of the signal light occurs are preferable to omitting parts of the volume of the allowed contiguous sub-volume of the particle field where attenuation of the illumination light and attenuation of the signal light does occur.

4.1.9 The Value of the TAMPSEC at the Point Along the Property-Specific Probe Path to Which the Response of the Property-Specific Single Detector is Assigned to Correspond. The ninth item in the list of quantities that must be known, measured, or calculable is the value of the time-averaged monochromatic polarization-specific extinction coefficient (TAMPSEC) at the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond. If the property-specific band coefficient dominates the TAMPSEC such that $\xi_n \approx \xi$, then no further measurements are required by this section. For example, if the property-specific band coefficient is the elastic scattering band coefficient ξ_s , and if extinction is almost entirely due to elastic scattering, then no further measurements are required by this section. However, if other band coefficients are also significant, and if the TAMPSEC is not otherwise known, then the requirement under this section is that measurements specific to the other band coefficients must also be performed according to section 4.1 of this chapter, probing the particle field with additional single beams of illuminating light according to section 3.1 of Chapter 3 as necessary, until it is possible to calculate the TAMPSEC, *i.e.*, $\xi = \sum_n \xi_n$, where the summation is over the significant band coefficients.

4.1.10 Optional Items. The final items in the list of quantities constituting a measurement are quantities that may optionally be known, measurable, or calculable. Quantities that may optionally be known, measurable, or calculable include the identity of the particle property to which the response of the property-specific detector corresponds, the drift in the relationship between the property-specific band coefficient and the particle property of interest, the size, shape, extent, and location of the applicable effective probe volume, the distribution in intensity of the single beam of illuminating light entering the allowed contiguous sub-volume of the particle field, and the distribution in intensity of the single beam of illuminating light exiting the allowed contiguous sub-volume of the particle field.

4.1.10.1 The Identity of the Particle Property to which the Response of the Property-Specific Single Detector Corresponds. One item in the list of quantities that may optionally be known, measured, or calculable is the identity of the particle property to which the response of a prop-

erty-specific single detector corresponds. As will be fully discussed in Chapter 5, the quantities directly measured by the method described in this report are actually the property-specific band coefficients. Additional assumptions are then required to relate the property-specific band coefficients to the particle property of interest. If only knowing the property-specific band coefficients is sufficient, there is no need to relate the band coefficients to any particular particle property. However, if it is desired to relate the band coefficients to particular particle properties, then it will be necessary to know the identity of the particle property to which the response of a property-specific single detector corresponds.

4.1.10.2 The Drift in the Relationship between the Property-Specific Band Coefficient and the Particle Property of Interest. Another item in the list of quantities that may optionally be known, measured, or calculable is the drift in the relationship between the property-specific band coefficient and the particle property of interest. As was briefly introduced in section 4.1.10.1 above and will be fully discussed in Chapter 5, the quantities directly measured by the method described in this report are actually the property-specific band coefficients. If only knowing the property-specific band coefficients is sufficient, there is no need to relate the band coefficients to any particular particle property. However, if it is desired to relate the band coefficients to particular particle properties, then it will be necessary to know the relationship between the property-specific band coefficient and the particle property of interest. This will not be possible if the relationship drifts, changes, or is otherwise not known. Therefore the drift, if any, in the relationship between the property-specific band coefficient and the particle property of interest must be known, measured, or calculable.

A method for measuring the drift in the relationship between the property-specific band coefficient and the particle property of interest when the particle property of interest the particle volume concentration and the property-specific band coefficient is the fluorescence coefficient. A variation of the considerations governing the range of wavelengths present in the illumination when the particle property of interest is the particle volume concentration will be recalled from section 3.1.1.1.5 of Chapter 3 to be selecting the fluorescence band coefficient. It has been noted that the response of the fluorescence molecules in the particles can sometimes drift over time. This has been noted to occur, for example, when the particle field is a spray, the fluorescence molecules have been artificially doped into the liquid, and the spray liquid is recirculated into a tank. In this case, drift in the response of the fluorescence molecules can be caused by repetitive exposure of the fluorescence molecules to the illuminating light as a result of the recirculation. One method of measuring the drift in the response of the fluorescence is to place an optical cell in the tubing or pipe supplying liquid to the spray nozzle. Illuminating the cell with illuminating light having the same properties as the illuminating light used to probe the particle field, and measuring the fluorescence response of the cell, allows corrections for any drift in the response of the fluorescence to be developed as a function of time.

4.1.10.3 The Size, Shape, Extent, and Location of the Applicable Effective Probe Volume. Yet another item in the list of quantities that may optionally be known, measurable, or calculable is mathematical coordinates describing the size, shape, extent, and location of the applicable effective probe volume. The corresponding required item was given in section 4.1.4 of this chapter to be the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond. This distance was specified there to be the distance to a point along the probe path having the nearest distance to any point in the effective

probe volume. Here, the optional item is a complete mathematical description of the size, shape, extent, and location of the applicable effective probe volume.

4.1.10.4 The Distribution in Intensity of the Single Beam of Illuminating Light Entering the Allowed Contiguous Sub-Volume of the Particle Field. A further item in the list of quantities that may optionally be known, measurable, or calculable is the distribution in intensity of the single beam of illuminating light entering the allowed contiguous sub-volume of the particle field. The corresponding required item was given in section 4.1.5 of this chapter to be the total property-specific illumination energy entering the allowed contiguous volume of the particle field. The total property-specific illumination energy entering the allowed contiguous volume of the particle field is an integrated quantity over the intensity distribution, which does not require the intensity distribution to be known as long as the integral is known. Here the optional item is the distribution of intensity itself, as a function of wavelength, state of polarization, time, direction, and location in the beam.

4.1.10.5 The Distribution in Intensity of the Single Beam of Illuminating Light Exiting the Allowed Contiguous Sub-Volume of the Particle Field. Yet another item in the list of quantities that may optionally be known, measurable, or calculable is the distribution in intensity of the single beam of illuminating light exiting the allowed contiguous sub-volume of the particle field. The corresponding required item was given in section 4.1.6 of this chapter to be the total property-specific illumination energy exiting the allowed contiguous volume of the particle field. The total property-specific illumination energy exiting the allowed contiguous volume of the particle field is an integrated quantity over the intensity distribution, which does not require the intensity distribution to be known as long as the integral is known. Here the optional item is the distribution of intensity itself, as a function of wavelength, state of polarization, time, direction, and location in the beam.

4.2 Variations of the General Method of Performing the Required Measurements

The general method of performing the required measurements now having been described, variations of the general method of performing the required measurements may now be introduced. A large number of variations are possible. A partial list of variations of the general method of performing the required measurements includes a set of variations for performing the required measurements along a single property-specific probe path, and a set of variations for performing the required measurements involving more than one property-specific probe path. These sets of variations are described in more detail next.

4.2.1 Variations for Performing the Required Measurements Along a Single Property-Specific Probe Path. One set of variations of the general method of performing the required measurements is a set of variations for performing the required measurements along a single property-specific probe path. Variations for performing the required measurements along a single property-specific probe path include simultaneously performing more than one property-specific single measurement using more than one property-specific single detector, simultaneously performing more than one property-specific single measurement using more than one sensitive element behind a common set of collection optics, and combining more than one simultaneous property-specific detection into a single virtual property-specific detection.

4.2.1.1 The Variation of Simultaneously Performing More than One Property-Specific Single Measurement Using More than One Property-Specific Single Detector. One variation of performing the required measurements along a single property-specific probe path is the variation of simultaneously performing more than one property-specific single measurement using more than one property-specific single detector. This class of variations is illustrated in Figure 6, where several property-specific detectors are shown performing detections simultaneously along a single property-specific probe path. Provided that the property-specific detectors are available, this class of variations could, for example, speed up the rate at which measurements can be made.

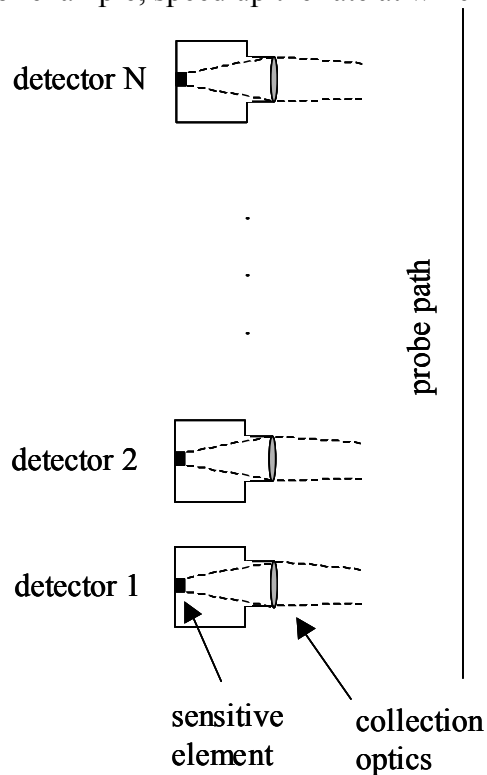


Figure 6. Simultaneously performing more than one property-specific single measurement using more than one property-specific single detector.

4.2.1.2 The Variation of Simultaneously Performing More than One Property-Specific Single Measurement Using More than One Sensitive Element Behind a Common Set of Collection Optics. Another variation of performing the required measurements along a single property-specific probe path is the variation of simultaneously performing more than one property-specific single measurement using more than one sensitive element behind a common set of collection optics. This class of variations is shown in Figure 7, where more than one sensitive element is shown to be arrayed behind a single set of collection optics. Each sensitive element will in general have associated with it its own effective probe volume. Arraying more than one sensitive element behind a single set of collection optics will in most cases restrict the sensitive elements to all detect the same property-specific signal. Each sensitive element will still be considered herein to constitute a separate “detector,” despite the fact that are all arrayed behind a common set of collection

optics. A large number of variations are possible. A partial list of variations of simultaneously performing more than one property-specific single measurement using more than one sensitive element behind a common set of collection optics includes the variation of arranging more than one sensitive element behind the common set of collection optics in a one dimensional array, and a variation of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array.

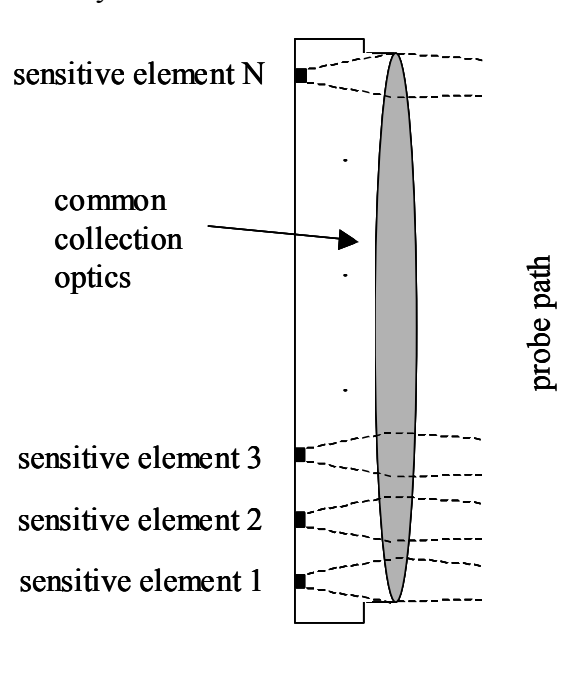


Figure 7. More than one sensitive element behind a common set of collection optics.

4.2.1.2.1 *The variation of arranging more than one sensitive element behind a common set of collection optics in a one dimensional array.* One variation of arranging more than one sensitive element behind a common set of collection optics is the variation of arranging more than one sensitive element behind a common set of collection optics in a one dimensional array. Arranging more than one sensitive element behind a common set of collection optics in a one dimensional array is what is illustrated in Figure 7. An example of arranging more than one set of collection optics behind a common set of collection optics in a one dimensional array is a linear photodiode array. In digital technology, each sensitive element in an array is commonly referred to as a “picture element,” or “pixel.” Each pixel will in general have its own effective probe volume. A large number of variations are possible. One variation of arranging more than one sensitive element behind a common set of collection optics in a one dimensional array includes having a sufficient number of pixels to detect the entire length of the property-specific probe path, to within the required degree of spatial resolution.

4.2.1.2.2 *The variation of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array.* Another variation of arranging more than one sensitive element behind a common set of collection optics is the variation of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array. Arranging more than one sensitive element behind a common set of collection optics in a two dimen-

sional array is illustrated in Figure 8 for the case where a single beam of illuminating light is used to probe the particle field. In Figure 8, the shaded gray area represents the single beam of illuminating light viewed from the direction of the detection optics, where the vertical direction represents the breadth of the beam, the horizontal direction represents the direction of propagation of the beam, and the circles represent the effective probe volumes associated with each sensitive element behind the common set of collection optics. Examples of arranging more than one set of collection optics behind a common set of collection optics in a two dimensional array include a photographic camera, a digital camera, a movie camera, and a video camera. A large number of variations are possible. A partial list of variations of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array includes having a sufficient number of pixels in one direction to detect the entire length of the property-specific probe path, having a sufficient number of pixels in one direction for more than one effective probe volume to span the width of a single beam of illuminating light, and having a sufficient number of pixels in both directions to do both.

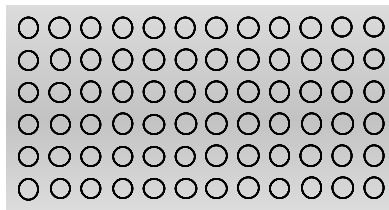


Figure 8. More than one sensitive element arranged behind a common set of collection optics in a two dimensional array.

The variation of having a sufficient number of pixels in one direction to detect the entire length of the property-specific probe path. One variation of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array is the variation of having a sufficient number of pixels in one direction of the two dimensional array to detect the entire length of the property-specific probe path, to within the required degree of spatial resolution.

The variation of having a sufficient number of pixels in one direction for more than one effective probe volume to span the width of a single beam of illuminating light. Another variation of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array is the variation of having a sufficient number of pixels in one direction of the two dimensional array for more than one effective probe volume to span the width of a single beam of illuminating light, in a direction perpendicular to property-specific probe path, to within the required degree of spatial resolution. This is the variation illustrated in Figure 8.

The variation of having a sufficient number of pixels in both directions to do both. Yet another variation of arranging more than one sensitive element behind a common set of collection optics in a two dimensional array is the variation of having a sufficient number of pixels in both directions of the two dimensional array to detect the entire length of the property-specific probe path, and for more than one effective probe volume to span the width of a single beam of illuminating light in a direction perpendicular to property-specific probe path, both to within the re-

quired degree of spatial resolution. This variation is evidently a combination of the variations in previous two paragraphs.

4.2.1.3 The Variation of Combining More than One Simultaneous Property-Specific Detection into a Single Virtual Property-Specific Detection. Yet another variation of performing the required measurements along a single property-specific probe path is the variation of combining more than one simultaneous property-specific detection into a single virtual property-specific detection. When more than one property-specific detection is performed simultaneously at the same or nearly the same distance along a single property-specific probe path, the only difference between the associated property-specific measurements (*see* section 4.1 of this chapter) can be the difference between the property-specific detections themselves. An example of performing more than one property-specific detection simultaneously at the same or nearly the same distance along a single property-specific probe path is when more than one effective probe volumes are located across the single beam of illuminating light in a direction perpendicular to the property-specific probe path. By adding the property-specific responses of more than one property-specific single detectors into a single property-specific virtual response, and defining a single property-specific virtual effective probe volume consisting of the sum of the individual property-specific effective probe volumes, it is possible to define a single property-specific virtual detection originating from a single virtual property-specific effective probe volume. As long as the virtual property-specific effective probe volume satisfies all the considerations in Chapters 3 and 4, it will be possible to consider the single property-specific virtual detection to be equivalent to a single property-specific detection.

4.2.2 Variations of Performing the Required Measurements Involving More than One Property-Specific Probe Path. Another set of variations of the general method of performing the required measurements is the set of variations of performing the required measurements involving more than one property-specific probe path. Variations of performing the required measurements involving more than one property-specific probe path include variations where detections along different property-specific probe paths are performed using the same detectors, and variations where detections along different property-specific probe paths are performed using different detectors.

4.2.2.1 Variations where Detections Along Different Property-Specific Probe Paths are Performed Using the Same Detectors. One set of variations of performing the required measurements involving more than one property-specific probe path is variations where detections along different probe paths are performed using the same detectors. Variations where detections along different probe paths are performed using the same detectors include variations where the locations of the detectors change for different probe paths, and variations where the locations of the detectors remain fixed for different probe paths.

4.2.2.1.1 *Variations where the locations of the detectors change for different probe paths.* One set of variations where detections along different probe paths are performed using the same detectors is the set of variations where the locations of the detectors change for different probe paths. A large number of variations are possible. One variation where the locations of the detectors change for different probe paths is the variation of maintaining fixed orientations between the detectors and the single beam of illuminating light for corresponding locations along prop-

erty-specific probe paths. This means that each detector responds to the same location relative to each probe path, and in the same orientation (the same distance and angles) relative to each probe path. An example of this variation would be maintaining the detector and the single beam of illuminating light in a fixed relationship a table, while the entire table is traversed through a stationary particle field. The variation of maintaining fixed orientations between the detectors and the single beam of illuminating light for corresponding locations along property-specific probe paths is nothing more than “doing it the same way” for each probe path. A large number of variations are possible, including different ways of combining the different variations of probing the particle field discussed in section 3.2 of Chapter 3 with the different variations for performing the required measurements along a single property-specific probe path discussed in section 4.2.1 of this chapter.

4.2.2.1.2 *Variations where the locations of the detectors remain fixed for different probe paths.* Another set of variations where detections along different probe paths are performed using the same detectors is the set of variations where the locations of the detectors remain fixed for different probe paths. Under this set of variations, the distances and relative orientations between the detectors and the probe paths will change for each probe path. An example of these variations would be orienting detector perpendicular to a probe path, and then moving the probe path through the particle field in a directions which are to and from the detector, without changing the location of the detector. As this occurs, the effective probe volume correction factor will in general be different for each probe path, but this is acceptable as long as the effective probe volume correction factor is repeatable along each probe path. Also, as each probe path corresponds to a different distance from the focal plane of the detector, the response of the detector may change, but this can be calibrated. In general the variations where the locations of the detectors remain fixed for different probe paths will be complicated to implement, and so will not generally be chosen. However, there may be some cases where implementation may be worthwhile. One such case is illustrated next.

A variation associated with probing the particle field only within a single plane of interest using single beams of illuminating light which successively fan out from a single apex. One variation where the locations of the detectors remain fixed for different probe path is a variation associated with probing the particle field only within a single plane of interest using single beams of illuminating light which successively fan out from a single apex. The corresponding variation of probing the particle field only within a single plane of interest was discussed in section 3.2.2.1 of Chapter 3. As discussed there, this method of probing a particle field would require only a single set of illumination optics, where the beam is swept by a rotating mirror, and therefore would have an extremely simple physical configuration. Performing detections only within a single plane of interest would require the detector to be located in the plane of the fan. However, rotating the detector to maintain a constant orientation with the beam would probably introduce as much complexity as the simplicity of the rotating mirror removed. Therefore the simplicity of using a fixed detector could make implementation of this variation worthwhile.

4.2.2.2 Variations Where Detections Along Different Property-Specific Probe Paths are Performed using Different Detectors. Another set of variations of performing the required measurements involving more than one property-specific probe path is the set of variations where detections along different property-specific probe paths are performed using different detectors. Variations where detections along different property-specific probe paths are performed using differ-

ent detectors may optionally be used when the particle field is probed sequentially with single beams of illuminating light, and are required to be used whenever the particle field is probed simultaneously with more than one single beam of illuminating light. A large number of variations are possible. One variation where detections along different probe paths are performed using the different detectors includes using different sensitive elements behind a common set of collection optics to detect different property-specific probe paths. A large number of variations of this variation is also possible. A partial list of variations where different sensitive elements behind a common set of collection optics are used to detect different property-specific probe paths includes using different sensitive elements behind a common set of collection optics in a two dimensional array, and using different sensitive elements behind a common set of collection optics in a one dimensional array.

4.2.2.2.1 The variation of using different sensitive elements behind a common set of collection optics in a two dimensional array. One variation where different sensitive elements behind a common set of collection optics are used to detect different property-specific probe paths is the variation of using different sensitive elements behind a common set of collection optics in a two dimensional array. An example of this variation would be using a two dimensional digital camera to visualize a planar sheet of light from a direction perpendicular to the sheet, where the planar sheet of light can be considered to be made up of multiple single beams of illuminating light propagating in parallel, as illustrated in Figure 4 of Chapter 3. Different elements will detect different probe paths. A large number of variations of this variation are possible as well. One variation of using different sensitive elements behind a common set of collection optics in a two dimensional array includes aligning the rows of the array to correspond with the direction of the probe paths. If the probe paths have some predominant direction, as in multiple parallel-propagating beams of illuminating light making up a sheet of light, then measurement and analysis can be made simpler if the rows of the array are aligned to correspond with the direction of the probe path.

4.2.2.2.2 The variation of using different sensitive elements behind a common set of collection optics in a one dimensional array. Another variation where different sensitive elements behind a common set of collection optics are used to detect different property-specific probe paths is the variation of using different sensitive elements behind a common set of collection optics in a one dimensional array. Again using the example of multiple parallel-propagating beams of illuminating light making up a sheet of light, different parts of a one dimensional array will detect different probe paths if the array is not aligned with the probe paths. Detections along each probe path could then be performed by sweeping the array along the probe path. A large number of variations of this variation are also possible. One variation of using different sensitive elements behind a common set of collection optics in a one dimensional array includes aligning the one dimensional array to be perpendicular to the directions of the probe paths. If the probe paths have some predominant direction, then measurement and analysis can be made simpler if the probe paths are swept using a linear array which is perpendicular to the direction of the probe paths.

4.3 The Most Recommended Method of Performing the Required Measurements

With the general method of performing the required measurements and variations of the general method of performing the required measurements now having been described, the most rec-

ommended method of performing the required measurements may now be introduced. The most recommended method of performing the required measurements contains the following elements:

- 4.3.1 Performing property-specific single measurements according to the general considerations of section 4.1 of this chapter.
 - 4.3.1.1 Using wavelength filters to allow the signals of interest to pass, while filtering out all other wavelengths.
 - 4.3.1.1.1 Detecting elastic scattering band wavelengths.
 - 4.3.1.1.2 Detecting fluorescence band wavelengths.
 - 4.3.1.2 Using detectors such that the small detector approximation of the first kind can be applied.
 - 4.3.1.3 Minimizing the collection of signals having traversed paths where attenuation of the signal light could have occurred by particles that are not contained within the allowed contiguous sub-volume of the particle field by masking off the light coming from those paths so that this light does not reach the sensitive element.
 - 4.3.1.4 Using spatial filters to minimize errors caused by secondary emission at the same wavelengths as the property-specific illumination in measuring the total property-specific illumination energy exiting the allowed contiguous sub-volume of the particle field.
 - 4.3.1.5 Using flow cells to detect the drift in the relationship between the fluorescence band coefficient and the particle volume concentration.
 - 4.3.1.6 Creating effective probe volumes which are as repeatable as possible along property-specific probe paths.
 - 4.3.1.7 Locating the detector a suitably large distance from the property-specific probe path that the small detector approximation of the second kind can be applied.
 - 4.3.1.8 Measuring the total property-specific illumination energy entering and exiting the allowed contiguous sub-volume of the particle field by using a partially reflecting mirror to direct a small fraction of the illumination energy towards the detector.
- 4.3.2 Performing property-specific single measurements along property-specific probe paths using a CCD detector.
 - 4.3.2.1 Using intensified CCD detectors, as required.
 - 4.3.2.2 Imaging the entire length along the property-specific probe path.
 - 4.3.2.3 Imaging several pixels across the breadth of the beam at each location along the probe path.
 - 4.3.2.4 Maintaining the CCD detector fixed relative to the laser beam while the particle field is traversed within the plane of interest.

5.0 THE METHOD OF ANALYZING THE DATA

The method of analyzing the data is described below. The assumed properties of the particle field will first be described, followed by a description of the approximations related to the development of the theory. Then the theory applicable to the method of analyzing the data will be developed. Finally, the general method of analyzing the data will be described, followed by variations of the general method, followed by the most recommended method of analyzing the data. Acronyms and mathematical definitions not explicitly defined in this chapter are defined in Appendix A.

5.1 Assumed Properties of the Particle Field

The assumed properties of the particle field used to develop the method of analyzing the data are described next. The assumed properties of the particle field may also be distinguished from the approximations related to the development of the theory to be described below in section 5.2, in that approximations depend in part on the characteristics chosen for the single beam of illuminating light and on the design of the detector, and are therefore at least partially under the control of the measurer. Assumed properties of the particle field depend entirely on the properties existing in the particle field. The assumed properties of the particle field used to develop the method of analyzing the data include the assumed property of possessing a suitable property-specific band coefficient, the assumed property of statistical stationarity, the assumed property of local statistical isotropy, the assumed property of statistically universal TAMPSEC component distribution functions, the assumed property of separable TAMPSEC components, the assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path, the assumed property that the IMPSEC component distribution function is uncorrelated with the IMPSEC component, and the assumed property that the TAMPSEC at the property-specific illumination wavelengths is equal to the TAMPSEC at the property-specific signal wavelengths. These assumed properties are described in detail below.

5.1.1 The Assumed Property of Possessing a Suitable Property-Specific Band Coefficient. One assumed property of the particle field is the assumed property of possessing a suitable property-specific band coefficient. A *property-specific band coefficient* was defined in section 3.1.1.1.1 of Chapter 3 to be a band coefficient which is proportional to the quantity of the particle property of interest present in the particle field. A *suitable* property-specific band coefficient is defined to be a property-specific band coefficient for which it is possible to condition the single beam of illuminating light so that at least one component of the illuminating light is property-specific as required by section 3.1.1.1.1 of Chapter 3, and one for which it is possible to further condition the single beam of illuminating light so that a property-specific signal is produced which can be detected by a property-specific single detector, as required by section 3.1.1.1.2 of Chapter 3. Variations of the property of possessing a suitable property-specific band coefficient include a variation where the particle property of interest is the particle surface area concentration, and a variation where the particle property of interest is the particle volume concentration.

5.1.1.1 A Variation Where the Particle Property of Interest is the Particle Surface Area Concentration. One variation of the assumed property of possessing a suitable property-specific band coefficient is the variation where the particle property of interest is the particle surface area con-

centration. It will be recalled that the elastic scattering band coefficient was selected as the surface-area specific band coefficient in the variation of the considerations governing the range of wavelengths present in the illumination where the particle property of interest is the particle surface area, as discussed in section 3.1.1.1.5 of Chapter 3. The corresponding assumed property of the particle field is that the particles in the particle field are such that the particle surface area per unit volume will be proportional to the elastic scattering band coefficient, *i.e.*, $a = g_s \xi_s$, where a is the particle surface area per unit volume and g_s is the constant of proportionality. This requirement is often satisfied increasingly well as the shape of the particles in the particle field approach a spherical shape and the wavelengths present in the surface-area-specific illumination are much smaller than the particle size. Therefore one variation of the assumed property of possessing a suitable property-specific band coefficient when the particle property of interest is the particle surface area is to assume the particles to have spherical shapes which are much larger than the wavelengths in the surface-area-specific illumination.

5.1.1.2 A Variation where the Particle Property of Interest is the Particle Volume Concentration. Another variation of the assumed property of possessing a suitable property-specific band coefficient is the variation where the particle property of interest is the particle volume concentration. It will be recalled that the fluorescence band coefficient was selected as the volume-concentration-specific band coefficient in the variation of the considerations governing the range of wavelengths present in the illumination where the particle property of interest is the particle volume concentration, as discussed in section 3.1.1.1.5. The corresponding assumed property of the particle field is that the particles are such that the fluorescence band coefficient will be proportional to the particle volume concentration, *i.e.*, $\xi_f = k_f \rho$, where ρ is the particle volume concentration and k_f is a constant of proportionality. This requirement may be satisfied if the particles contain fluorescent molecules that are either naturally present or are artificially doped into the particles at the right concentration. Therefore another variation of the assumed property of possessing a suitable property-specific band coefficient when the particle property of interest is the particle volume concentration is to assume that fluorescent molecules are present in the particles at the right concentration.

5.1.2 The Assumed Property of Statistical Stationarity. Another assumed property of the particle field is the assumed property of statistical stationarity. If any property of a particle field is observed as a function of time at a fixed position, the value of that property will in general vary in time as if fluctuating about some average value. If at each instant of time a running average of that property is performed over a time interval which is at least as long as several periods of the highest frequency fluctuations, the highest frequency fluctuations will be smoothed out of the running average, but the running average may still vary over longer periods of time. A particle field will be defined here to be *statistically stationary* if the running averages of its properties do not vary over the period of time it takes to perform a complete series of measurements at all required and desired locations in the particle field. As a consequence of the assumed property of statistical stationarity, the TAMPSEC $\bar{\xi}(\lambda, \mathcal{P}, \bar{x}, \hat{n})$ will not vary in time, the TAMPSEC components $\bar{\xi}_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n})$ will not vary in time, the time-averaged band coefficients $\bar{\xi}_n(\lambda, \mathcal{P}, \bar{x}, \hat{n})$ will not vary in time, and the TAMPSEC component distribution function

$\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x})$ will not vary in time, where the overbar in each expression indicates a time average over at least several periods of the highest frequency fluctuations.

5.1.3 The Assumed Property of Local Statistical Isotropy. Yet another assumed property of the particle field is the assumed property of local statistical isotropy. At any instant in time, either the particle shapes or the local arrangement of the particles could give rise to certain naturally defined directions which could cause the TAMSEC or its components to depend on the direction of propagation of the illumination. For example, the particles might instantaneously be arranged locally in the shape of a cone, for which elastic scattering from the direction of the base of the cone might be different than elastic scattering from the direction of the apex of the cone. However, due to random fluctuations over time, the particles may locally experience a large number of orientations and different spatial configurations, such that no naturally defined local direction arises in the average. If a time scale and a length scale exists over which no naturally defined direction arises in the average over time, then *local statistical isotropy* will be considered to exist over that time scale and that length scale. Note that the particle field itself is not required to be isotropic. The particle field itself might indeed be shaped like a cone, as many particle fields in the form of a spray often approximately are, nor are spatial variations within the particle field prohibited. Local statistical isotropy is required to exist only over length scales equivalent to the size of the effective probe volume. In many particle fields exhibiting statistical properties, random fluctuations over the relatively longer period of time required to satisfy the assumed property of statistical stationarity defined in section 5.1.2 above will often easily satisfy the requirement for local statistical isotropy over the length scale of the size of the effective probe volume assumed in this section. As a result of the property of local statistical isotropy, the TAMPSEC, the TAMPSEC components, and the time-averaged band coefficients will no longer depend on the incident direction \hat{n}' nor on the state of incident polarization \mathcal{P} (the particle field will look the same in the average to all states of polarization). Thus $\bar{\xi}(\lambda, \mathcal{P}, \bar{x}, \hat{n}) = \bar{\xi}(\lambda, \bar{x})$, $\bar{\xi}_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}) = \bar{\xi}_c(\lambda_c, \mathcal{P}_c, \lambda, \bar{x})$, and $\bar{\xi}_n(\lambda, \mathcal{P}, \bar{x}, \hat{n}) = \bar{\xi}_n(\lambda, \bar{x})$. The TAMPSEC component distribution function $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x})$, on the other hand, may still depend on the incident state of polarization and the direction of the incident optical radiation, but the directional dependence \hat{n}_c of the distributed radiation relative to the incident direction \hat{n} will be changed. If the directional dependence of $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x})$ is envisioned to be analogous to the directions of the brushes of a broom, then the current general form $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x})$ allows the direction of the brushes of the broom \hat{n}_c to be different for each direction \hat{n} of the handlestick of the broom. In a local statistically isotropic medium, the directions of the brushes of the broom relative to the broom should be the same for any direction \hat{n} of the handle stick of the broom. The functional form which accomplishes this requirement is $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}) = \bar{P}(\hat{n}_c - \hat{n}, \lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x})$. This functional form allows the directions of the brushes of the broom to remain fixed relative to the broom regardless of the direction \hat{n} of the handlestick of the broom.

5.1.4 The Assumed Property of Statistically Universal TAMPSEC Component Distribution Functions. A further assumed property of the particle field is the assumed property of statistically universal TAMPSEC component distribution functions. The TAMPSEC component distribution function is defined to be *statistically universal* if it does not depend on the position \bar{x}

or time t for the property-specific directions and the property-specific wavelengths and states of polarization involved. Thus $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}) = \bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n})$. Under the assumed property of local statistical isotropy of section 5.1.3 above, this expression further simplifies to $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}) = \bar{P}(\hat{n}_c - \hat{n}, \lambda_c, \mathcal{P}_c, \lambda, \mathcal{P})$. The statistically universal TAMPSEC component distribution function under the assumed property of local statistical isotropy will be fixed by the nature of the particle field and will apply globally over it; it may in some instances be a known function, at least over the range of directions from the effective probe volume to the effective collection optics of the property-specific single detector.

5.1.5 The Assumed Property of Separable TAMPSEC Components. An even further assumed property of the particle field is the assumed property of separable TAMPSEC components $\bar{\xi}_c$. Under the assumed property of separable TAMPSEC components, the TAMPSEC components are assumed to be separable into a time-averaged shape function $\bar{\xi}_c^s$ which depends only on wavelengths, direction, and states of polarization, and a time-averaged magnitude function $\bar{\xi}_c^0$ which depends only on position. Thus $\bar{\xi}_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}) = \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \hat{n}) \bar{\xi}_c^0(\bar{x})$. Under this assumption, the time-averaged shape function $\bar{\xi}_c^s$ is considered to be uncorrelated with the time-averaged magnitude function $\bar{\xi}_c^0$. Under the assumed property of local statistical isotropy of section 5.1.3 above, this expression further simplifies to $\bar{\xi}_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}) = \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c, \lambda) \bar{\xi}_c^0(\bar{x})$. The TAMPSEC component shape function under the assumed property of local statistical isotropy will also be fixed by the nature of the particle field and will apply globally over it; it may in some instances also be a known function.

5.1.6 The Assumed Property of Negligible Integrated Statistical Fluctuations of the IMPSEC's Along a Path. An additional assumed property of the particle field is the assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path. The IMPSEC's at any given position will in general be a function of time, *i.e.*, $\xi(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) = \bar{\xi}(\lambda, \mathcal{P}, \bar{x}, \hat{n}) + \xi'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$, where $\bar{\xi}(\lambda, \mathcal{P}, \bar{x}, \hat{n})$ is the time average of the IMPSEC and $\xi'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ is the fluctuating component about the average. The assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path means that at any given time t , the value of $\xi'(\lambda_b, \mathcal{P}_b, \bar{x}, \hat{n}, t) ds$ along any path will be positive over approximately the same distances and with the same average magnitudes as they are negative. The resulting integral will approximately zero, *i.e.*, $\left| \int_0^S \xi'(\lambda_b, \mathcal{P}_b, \bar{x}, \hat{n}, t) ds \right| \ll 1$ for any path length S . The assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path implies that the characteristic distances over which the statistical fluctuations are correlated are small compared to the length of the path.

5.1.7 The Assumed Property that the IMPSEC Component Distribution Function is Uncorrelated with the IMPSEC Component. Another additional assumed property of the particle field is the assumed property that the IMPSEC component distribution function is uncorrelated with the IMPSEC component. Although the assumption of statistical stationarity allows the TAMPSEC component distribution function and the TAMPSEC component to be separately assumed to be constant in time, the theory applicable to the method of analyzing the data to be de-

veloped in section 5.3 of this chapter involves products of these two quantities. If the time fluctuations of these two quantities are correlated in any way, the average of the product of these two quantities will in general not be equal to the product of the averages. If the time fluctuations of these two quantities are uncorrelated, then

$$\overline{P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, t)} = \overline{P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x})} \overline{\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x})}.$$

Under the assumed property of local statistical isotropy of section 5.1.3 of this chapter, this expression reduces further to

$$\overline{P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \bar{x}, t)} = \overline{P(\hat{n}_c - \hat{n}, \lambda_c, \mathcal{P}_c, \lambda, \mathcal{P})} \overline{\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \bar{x})}.$$

5.1.8 The Assumed Property that the TAMPSEC at the Property-Specific Illumination Wavelengths is Equal to the TAMPSEC at the Property-Specific Signal Wavelengths. Yet another additional assumed property of the particle field is the assumed property that the TAMPSEC at the property-specific illumination wavelengths is equal to the TAMPSEC at the property-specific signal wavelengths. In the wavelength-independent extinction coefficient approximation to be developed in section 5.2.8 of this chapter, the TAMPSEC is approximated as being independent of wavelength for property-specific wavelengths in the illumination, and also to be independent of wavelength for property-specific wavelengths in the signal. The property-specific wavelengths in the illuminating light will in general be different than the property-specific wavelengths in the signal. Here, the further assumption is made that the TAMPSEC is the same for each of the two bands of wavelengths.

5.2 Approximations Related to the Development of the Theory

Approximations related to the development of the theory are to be distinguished from the assumed properties of the particle field in that approximations depend in part on the characteristics chosen for the single beam of illuminating light and on the design of the detector, and are therefore at least partially under the control of the measurer. Approximations related to the development of the theory include the approximation that particle properties are approximately uniform across the single beam of illuminating light, the small effective probe volume approximation of the first kind, the small effective probe volume approximation of the second kind, the slender beam approximation of the first kind, the slender beam approximation of the second kind, the small detector approximation, and the wavelength-independent TAMPSEC approximation. These approximations are described in more detail below.

5.2.1 The Approximation that Particle Properties are Approximately Uniform Across the Single Beam of Illuminating Light. One approximation related to the development of the theory is the approximation that particle properties are approximately uniform across the single beam of illuminating light. This approximation is discussed further in the considerations governing the area over which the single beam of illuminating light is distributed, as described in section 3.1.1.2.1 of Chapter 3.

5.2.2 The Small Effective Probe Volume Approximation of the First Kind. Another approximation related to the development of the theory is the small effective probe volume approximation of the first kind. The small effective probe volume approximation of the first kind makes it possible to neglect attenuation of the illuminating light and attenuation of the signal light across the effective probe volume. This approximation arises out of the considerations gov-

erning the area over which the single beam of illuminating light is distributed as described in section 3.1.1.2.4 of Chapter 3.

5.2.3 The Small Effective Probe Volume Approximation of the Second Kind. Yet another approximation related to the development of the theory is the small effective probe volume approximation of the second kind. The small effective probe volume approximation of the second kind allows the range of directions defined from each point in the effective probe volume to the effective collection area of the detector optics to be considered to all be approximately the same. This approximation arises out of the considerations governing the area over which the single beam of illuminating light is distributed as described in section 3.1.1.2.5 of Chapter 3.

5.2.4 The Slender Beam Approximation of the First Kind. A further approximation related to the development of the theory is the slender beam approximation of the first kind. Probing the particle field with slender beams of illuminating light is the primary method of reducing secondary emission. The slender beam approximation of the first kind arises out of the considerations governing the area over which the single beam of illuminating light is distributed as described in section 3.1.1.2.6 of Chapter 3.

The slender beam approximation of the first kind leads to the simplification that the attenuation of the single beam of illuminating light is not significantly affected by multiple scattering events. Light which is removed from the beam may be considered to leave the beam forever, without further interacting with the part of the beam that interacts with the effective probe volume. To demonstrate this, let x be the fraction of the original illumination energy that is removed from the beam by the time it exits the particle field. Conservatively assume that all of the light is scattered, and none of it is absorbed, in order to maximize the probability that some of it might through multiple scattering interact again with the original beam at the effective probe volume. Consider first the light which has been scattered only once. Of all the light which has been scattered only once, some of it will be scattered in the direction of the original beam, and will potentially interact with the beam at the effective probe volume. Since the light is scattered in many directions, an estimate of the amount of light scattered in the direction of the beam is given by the ratio of the volume of the beam to the volume of the particle field. If b is a characteristic transverse dimension of the beam and L is a characteristic dimension of the particle field, then the amount of light which is scattered only once in the direction of the beam is $x(b/L)^2$. The slender beam approximation of the first kind provides that the ratio b/L will be a very small number. Of the rest of the light which has so far been scattered only once, some of it will exit the particle field without any further interactions. The amount of light removed from the original beam provides a measure for estimating that x is also the fraction of the light which has so far been scattered only once which will remain in the particle field to be scattered more than once. Thus x^2 is the approximate amount of the original light available to be scattered more than once. Since by the second scattering event the light can be traveling in virtually any direction, an estimate of the amount of additional light that might interfere with a detection in any probe volume would be the ratio of the volume of the probe volume to the volume of the particle field. Assuming the characteristic probe volume dimension is on the order of the characteristic transverse beam dimension, the amount of light scattered more than once which might interfere with a detection in a probe volume is given by $x^2(b/L)^3$. Thus the total amount of scattered light that might through multiple scattering contribute to interactions in the effective probe volume is on

the order of $x(b/L)^2 + x^2(b/L)^3$. For typical laser beams, the scale of many applications leads to beam width to particle field dimension ratios easily less than 100, *i.e.*, $b/L < 100$. Thus even for x as large as 1, errors in the effective probe volume associated with neglecting the scattered light from the original beam will be exceedingly small. This is not to say that the contribution of the scattered light to the measured signal is also small, however. The measured signal contains much less light than the beam, and must travel a distance comparable to the characteristic dimension of the particle field. Thus any scattered light which reaches the response volume of the detector can still contribute errors to the measured signal. The simplification here is that the *generation* of signals within the effective probe volume is not significantly affected by multiple scattering events.

5.2.5 The Slender Beam Approximation of the Second Kind. An even further approximation related to the development of the theory is the slender beam approximation of the second kind. The slender beam approximation of the second kind allows the number of directions of propagation possible within the single beam of illuminating light to be minimized. This approximation arises out of the considerations governing the area over which the single beam of illuminating light is distributed as described in section 3.1.1.2.7 of Chapter 3.

5.2.6 The Small Detector Approximation of the First Kind. An additional approximation related to the development of the theory is the small detector approximation of the first kind. The small detector approximation of the first kind allows the number of directions from the property-specific effective probe volume to the effective collection area of the property-specific collection optics to be minimized. This approximation arises out of the considerations governing the location of the detector and the size of the effective collection area of the collection optics discussed in section 4.1.1.3.1 of Chapter 4.

5.2.7 The Small Detector Approximation of the Second Kind. Yet another additional approximation related to the development of the theory is the small detector approximation of the second kind. The small detector approximation of the second kind allows the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor to be minimized (The effective probe volume correction factor will be defined later in section 5.3.7.6 of this chapter). This approximation arises out of the considerations governing the location of the detector and the size of the effective collection area of the collection optics discussed in section 4.1.1.3.4 of Chapter 4.

5.2.8 The Wavelength-Independent TAMPSEC Approximation. A further additional approximation related to the development of the theory is the wavelength-independent TAMPSEC approximation. This approximation arises out of the considerations governing the wavelengths present in the single beam of illuminating light as discussed in section 3.1.1.1.3 of Chapter 3, in which the TAMPSEC components are independent of beam wavelength for property-specific wavelengths in the illumination, and out of the considerations governing the response of a property-specific single detector discussed in section 4.1.1.1.2 of Chapter 4, in which the TAMPSEC is considered to be approximately independent of signal wavelength for the property-specific wavelengths in the signal. Recall that one of the items that must be known, measured, or calculable as part of the method of performing a single measurement was given in section 4.1.9 of Chapter 4 to be the TAMPSEC at the point along the property-specific probe path to which the

response of the property-specific single detector is assigned to correspond. All band coefficients which contribute significantly to the TAMPSEC are to be determined. The fact that the TAMPSEC components are each independent of beam wavelength for property-specific wavelengths in the beam therefore implies that the TAMPSEC itself will also be independent of the property-specific wavelengths in the beam. Finally, if the TAMPSEC and its components are independent of property-specific wavelengths in the beam, then it is reasonable to project that the TAMPSEC component distribution function will also not depend on beam wavelength, *i.e.*, $\bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \vec{x}) = \bar{P}(\lambda_c, \mathcal{P}_c, \hat{n}_c, \mathcal{P}, \hat{n}, \vec{x})$.

5.3 Development of the Theory Applicable to the Method of Analyzing the Data

The theory applicable to the method of analyzing the data is developed in this section. The theory applicable to the method of analyzing the data can be divided into the following categories: the geometry applicable to the method of analyzing the data, the intensities applicable to the method of analyzing the data, the general expression for the total property-specific energy delivered by the single beam of illuminating light at a point p along the probe path, including reduction to a simplified form, the general expression for the total band-specific signal which reaches the detector from an effective probe volume, including reduction to a simplified form, general and reduced expressions for the TAMPSEC, constraints on the band constants, determination of the particle property of interest, and summary. These aspects of the theory are described in detail next.

5.3.1 The Geometry Applicable to the Method of Analyzing the Data. The geometry applicable to the method of analyzing the data is illustrated in Figure 9. Let A_b be a surface that extends over the entire area of the single beam of illuminating light at some location along the single beam of illuminating light before the beam enters the particle field. Let A_f be a surface that extends over the entire area of the same single beam of illuminating light at some point in the particle field. Let the direction of the probe path which intersects these two surfaces be represented by the unit normal vector \hat{n}_{pp} . Let p represent the axial distance along the probe path, where $p = p_b$ is the coordinate where the area A_b intersects the probe path, and $p = p_f$ is the coordinate where the area A_f intersects the probe path. Let dA_b be a differential element of area located at \vec{x}_b on A_b having unit outward normal \hat{n}_b . Let dA_f be a differential element of area located at \vec{x}_f on A_f having unit outward normal \hat{n}_f . Then the direction from dA_b to dA_f is represented by the unit normal vector $\hat{n}_{bf} = (\vec{x}_f - \vec{x}_b) / |\vec{x}_f - \vec{x}_b|$, where the subscript notation has been adopted from section A.3 of Appendix A that the order of the subscripts denotes a direction from the first subscript to the second subscript. Also, the differential solid angle subtended by the area dA_f relative to \vec{x}_b is given by $d\omega_{bf} = dA_f |\hat{n}_f \cdot \hat{n}_{bf}| / |\vec{x}_f - \vec{x}_b|^2$. As a result of interactions with the particle field, some of the light which is removed from the single beam of illuminating light at A_f is converted into other optical radiation which emanates in the direction of the detector. Let A_d be the effective collection area of the collection optics of the detector. Let dA_d be a differential element of area located at \vec{x}_d on A_d having unit outward normal \hat{n}_d . Then the direc-

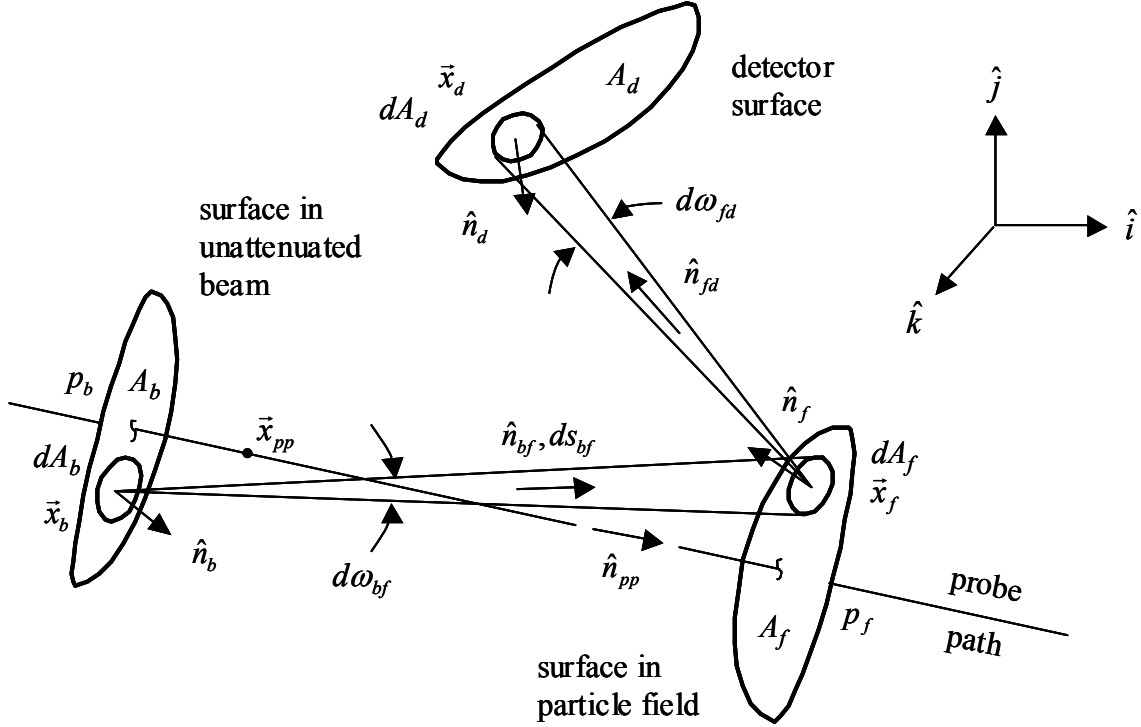


Figure 9. Geometry applicable to optical patterning.

tion from dA_f to dA_d is represented by the unit normal vector $\hat{n}_{fd} = (\vec{x}_d - \vec{x}_f) / |\vec{x}_d - \vec{x}_f|$. Also, the differential solid angle subtended by the area dA_d relative to \vec{x}_f is given by $d\omega_{fd} = dA_d |\hat{n}_d \cdot \hat{n}_{fd}| / |\vec{x}_d - \vec{x}_f|^2$. The application of optical radiation theory introduced in Appendix A to the method of analyzing the data will be developed relative to this geometry.

5.3.2 Intensities Applicable to Method of Analyzing the Data. Intensities applicable to the method of analyzing the data include the instantaneous monochromatic polarization-specific intensity of radiation emitted from dA_b in the direction of dA_f , the instantaneous monochromatic polarization-specific intensity of irradiation received at dA_f from the direction of dA_b , the instantaneous monochromatic polarization-specific intensity of radiation emitted from dA_f in the direction of dA_d , and the instantaneous monochromatic polarization-specific intensity of irradiation received at dA_d from the direction of dA_f . These intensities are described in further detail below. Note that none of the assumed properties of the particle field and none of the approximations related to the development of the theory will be applied in this section.

5.3.2.1 The Instantaneous Monochromatic Polarization-Specific Intensity of Radiation Emitted from dA_b in the Direction of dA_f . The instantaneous monochromatic polarization-specific

intensity of radiation emitted from dA_b in the direction of dA_f will be denoted $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$.

5.3.2.2 The Instantaneous Monochromatic Polarization-Specific Intensity of Irradiation Received at dA_f From the Direction of dA_b . The instantaneous monochromatic polarization-specific intensity of irradiation $I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t)$ received at dA_f from the direction of dA_b can be related to the instantaneous monochromatic polarization-specific intensity $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$ of radiation emitted from dA_b in the direction of dA_f through the relationship $dI' = -\xi ds$ developed in section A.9 of Appendix A. The result is

$$I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) = I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) \exp \left[- \int_0^{|\bar{x}_f - \bar{x}_b|} \xi(\lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_b + s\hat{n}_{bf}, t) ds \right], \quad (2)$$

where s is the distance along the direction defined by \hat{n}_{bf} , $s \equiv 0$ when $\bar{x} = \bar{x}_b$.

5.3.2.3 The Instantaneous Monochromatic Polarization-Specific Intensity of Radiation Emitted from dA_f in the Direction of dA_d . The instantaneous monochromatic polarization-specific intensity $dI_c(\lambda_c, \mathcal{P}_c, \bar{x}_f, \hat{n}_{fd}, t)$ of radiation emanating from dA_f in the direction of dA_d may be computed by first considering the time rate at which monochromatic polarization-specific energy is transferred from dA_b to dA_f . Adopting equation (A.1) of Appendix A to the notation of Figure 9, the time rate at which energy is transferred from dA_b to dA_f is $I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_b| |\hat{n}_{bf} \cdot \hat{n}_f| dA_b dA_f d\lambda_b d\mathcal{P}_b / |\bar{x}_f - \bar{x}_b|^2$. The energy converted into a differential range of wavelengths $d\lambda_c$ around the wavelength λ_c and into a differential range of polarization states $d\mathcal{P}_c$ around the polarization state \mathcal{P}_c will be this amount multiplied by $\xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) d\lambda_c d\mathcal{P}_c ds_{bf}$, where ds_{bf} is the differential distance in the direction of \hat{n}_{bf} at \bar{x}_f . However, the distance ds_f transited through dA_f in the direction \hat{n}_f will be somewhat shorter than ds_{bf} , namely $ds_f = ds_{bf} |\hat{n}_{bf} \cdot \hat{n}_f|$. Also, it will be convenient below to relate the distance ds_f to the distance dp along the probe path. Thus $ds_f = dp |\hat{n}_{pp} \cdot \hat{n}_f|$, so $ds_{bf} = dp |\hat{n}_{pp} \cdot \hat{n}_f| / |\hat{n}_{bf} \cdot \hat{n}_f|$. The fraction of the total energy converted into a differential range of wavelengths $d\lambda_c$ around the wavelength λ_c and in a differential range of polarization states $d\mathcal{P}_c$ around the polarization state \mathcal{P}_c directed specifically in the direction of dA_d is $P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) d\omega_{fd}$, where $d\omega_{fd}$ is the differential solid angle subtended by dA_d around \hat{n}_{fd} . After combining all these terms, the intensity of the light originating at dA_b which is scattered in the direction of dA_d is then obtained by dividing by the solid angle $d\omega_{fd}$, dividing by the component of the area dA_f which is perpendicular to \hat{n}_{fd} , namely $|\hat{n}_{fd} \cdot \hat{n}_f| dA_f$, dividing by the wavelength interval $d\lambda_c$, and dividing by the interval in polarization states $d\mathcal{P}_c$. The result is

$$dI_{c,dA_b}(\lambda_c, \mathcal{P}_c, \bar{x}_f, \hat{n}_{fd}, t) = dA_b d\lambda_b d\mathcal{P}_b dp \times \frac{P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_b| |\hat{n}_{pp} \cdot \hat{n}_f|}{|\hat{n}_f \cdot \hat{n}_{fd}| |\bar{x}_f - \bar{x}_b|^2}$$

However, this only represents the contribution from dA_b . The total intensity of the light scattered from dA_f in the direction of dA_d is the sum of the contributions from all A_b . Thus the total intensity of the light scattered from dA_f in the direction of dA_d is

$$dI_c(\lambda_c, \mathcal{P}_c, \bar{x}_f, \hat{n}_{fd}, t) = d\lambda_b d\mathcal{P}_b dp \frac{|\hat{n}_{pp} \cdot \hat{n}_f|}{|\hat{n}_f \cdot \hat{n}_{fd}|} \times \int_{A_b} \frac{P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} dA_b \quad (2)$$

5.3.2.4 The Instantaneous Monochromatic Polarization-Specific Intensity of Irradiation Received at dA_d from the Direction of dA_f . The instantaneous monochromatic polarization-specific intensity $dI'_c(\lambda_c, \mathcal{P}_c, \bar{x}_d, \hat{n}_{fd}, t)$ of irradiation received at dA_d from the direction of dA_f can be related to the instantaneous monochromatic polarization-specific intensity $dI_c(\lambda_c, \mathcal{P}_c, \bar{x}_f, \hat{n}_{fd}, t)$ of radiation emitted from dA_f in the direction of dA_d again through the relationship $dI' = -\xi I' ds$. The result is

$$dI'_c(\lambda_c, \mathcal{P}_c, \bar{x}_d, \hat{n}_{fd}, t) = dI_c(\lambda_c, \mathcal{P}_c, \bar{x}_f, \hat{n}_{fd}, t) \exp\left[-\int_0^{|\bar{x}_d - \bar{x}_f|} \xi(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \bar{x}_f + s\hat{n}_{fd}, t) ds\right]. \quad (3)$$

An *effective IMPSEC component distribution function* $P'(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t)$ can then be defined as

$$P'(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_d, t) = P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) \exp\left[-\int_0^{|\bar{x}_d - \bar{x}_f|} \xi(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \bar{x}_f + s\hat{n}_{fd}, t) ds\right]. \quad (4)$$

The instantaneous monochromatic polarization-specific intensity $dI'_c(\lambda_c, \mathcal{P}_c, \bar{x}_d, \hat{n}_{fd}, t)$ of irradiation received at dA_d from the direction of dA_f can then be written in the alternate form

$$dI'_c(\lambda_c, \mathcal{P}_c, \bar{x}_f, \hat{n}_{fd}, t) = d\lambda_b d\mathcal{P}_b dp \frac{|\hat{n}_{pp} \cdot \hat{n}_f|}{|\hat{n}_f \cdot \hat{n}_{fd}|} \times \int_{A_b} \frac{P'(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} dA_b \quad (5)$$

5.3.3 General Expression for the Total Property-Specific Energy Delivered by the Single Beam of Illuminating Light at a Point p along the Probe Path. The total property-specific energy of the single beam of illuminating light at any point p along the probe path can be calcu-

lated. First let A_f be perpendicular everywhere to the probe path at p , *i.e.*, let \hat{n}_f to be parallel to \hat{n}_p everywhere on A_f , and let A_f extend across the entire breadth of the beam there. That these two conditions have been satisfied shall be denoted herein by referring to the integral over the area A_f under these particular circumstances as the integral over the area $A_p(p)$, where the area of the beam may in general depend on p . A general expression for the total property-specific energy $E_b(p)$ of the single beam of illuminating light which crosses A_p at point p is then given by

$$E_b(p) = \int_{t_b} \int_{\lambda_{b,ps}} \int_{\mathcal{P}_{b,ps}} \int_{A_b} \int_{A_p(p)} I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_p| |\hat{n}_{bf} \cdot \hat{n}_b| \frac{1}{|\bar{x}_f - \bar{x}_b|^2} dA_f dA_b d\mathcal{P}_b d\lambda_b dt, \quad (6)$$

where t_b is the period of illumination, $\lambda_{b,ps}$ is the property-specific range of wavelengths in the beam, and $\mathcal{P}_{b,ps}$ is the property-specific range of states of polarization at each property-specific wavelength.

5.3.4 Reduced Form of the General Expression for the Total Property-Specific Energy Delivered by the Single Beam of Illuminating Light at a Point p along the Probe Path. The general expression given in equation (6) for the total property-specific energy of the single beam of illuminating light at a point p along the probe path may be reduced using the assumed properties of the particle field and the approximations related to the development of the theory. Recall first that the instantaneous monochromatic polarization-specific intensity of irradiation I'_b at A_f is the product of the instantaneous monochromatic polarization-specific intensity of radiation I_b before the beam enters the particle field times an exponential term which contains an integration of the IMPSEC ξ along the path of the beam, as given in equation (2). If ξ is decomposed into the sum of its time-averaged and fluctuating components, then under the assumption of statistical local isotropy, the time averaged component will depend only on the position and on the wavelength. However, under the wavelength-independent TAMPSEC approximation of section 5.2.8 of this chapter, ξ will not depend on wavelength for property-specific wavelengths in the illumination. Thus $\xi(\lambda_b, \mathcal{P}, \bar{x}, \hat{n}, t) = \bar{\xi}^b(\bar{x}) + \xi'(\lambda_b, \mathcal{P}, \bar{x}, \hat{n}, t)$, where the superscript “ b ” in $\bar{\xi}^b$ denotes the value of the TAMPSEC at the property-specific wavelengths in the illumination. When substituted into equation (6), the result is

$$I'_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) = I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) \exp\left[-\int_0^{|\bar{x}_f - \bar{x}_b|} \bar{\xi}^b(\bar{x}_b + s\hat{n}_{bf}) ds\right] \exp\left[-\int_0^{|\bar{x}_f - \bar{x}_b|} \xi'(\lambda_b, \mathcal{P}_b, \bar{x}, \hat{n}, t) ds\right]. \quad (7)$$

The second exponential term involving ξ' in equation (7) involves an integral over a path length of a quantity which is sometimes positive, sometimes negative, and zero in the average. Under the assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path discussed in section 5.1.6 above in this chapter, the integral of ξ' in this term should be much less than unity. Therefore the result of this assumption is that

$$\exp\left[-\int_0^{|\bar{x}_f - \bar{x}_b|} \xi'(\lambda_b \dots) ds\right] \approx 1. \quad (8)$$

Substituting equations (7) and (8) into equation (6), the result is

$$E_b(p) = \int_{A_b} \int_{A_p(p)} \exp\left[-\int_0^{|\bar{x}_f - \bar{x}_b|} \bar{\xi}^b(\bar{x}_b + s\hat{n}_{bf}) ds\right] \times \int_{I_b} \int_{\lambda_{b,ps}} \int_{\mathcal{P}_{b,ps}} I_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_p| |\hat{n}_{bf} \cdot \hat{n}_b| \frac{1}{|\bar{x}_f - \bar{x}_b|^2} d\mathcal{P}_b d\lambda_b dt dA_f dA_b \quad (9)$$

Now further consider that the TAMPSEC will be approximately uniform across the beam at any position p along the beam, as required by the approximation that particle properties are approximately uniform across the single beam of illuminating light, as discussed in section 5.2.1 of this chapter. Even further consider that if the single beam of illuminating light is collimated, or alternatively if the slender beam approximation of the second kind applies so that the number of directions of propagation possible within the single beam of illuminating light is minimized as discussed in section 5.2.5 of this chapter, then the unit vector \hat{n}_{bf} will be approximately constant.

The result of all these considerations is that the exponential term in equation (9) is approximately constant for all paths through the beam and might be taken. Therefore, define the *beam extinction function* $e_b(p)$ such that

$$e_b(p) \equiv \exp\left[-\int_0^{p-p_b} \bar{\xi}^b(\bar{x}_{bp} + s\hat{n}_{pp}) ds\right] \quad (10)$$

represents the total extinction along the probe path at point p . Here \bar{x}_{bp} is the point where the probe path intersects A_b . Then equation (9) can be further reduced to the alternate form

$$E_b(p) = e_b(p) E_b^0, \quad (11)$$

where

$$E_b^0 = \int_{I_b} \int_{\lambda_{b,ps}} \int_{\mathcal{P}_{b,ps}} \int_{A_b} \int_{A_p(p)} \frac{I_b(\lambda_b, \mathcal{P}_b, \bar{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_p| |\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} dA_f dA_b d\mathcal{P}_b d\lambda_b dt \quad (12)$$

is the property-specific energy of the unattenuated beam. Equation (11) can be written in the alternate form

$$E_b(p) = e_b(p) \int_{\mathcal{P}_{b,ps}} E_b^0(\mathcal{P}_b) d\mathcal{P}_b, \quad (13)$$

where $E_b^0(\mathcal{P}_b)$ is the property-specific energy of a particular state of polarization of the unattenuated beam,

$$E_b^0(\mathcal{P}_b) = \int_{t_b} \int_{\lambda_{b,ps}} \int_{A_b} \int_{A_p(p)} \frac{I_b(\lambda_b, \mathcal{P}_b, \vec{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_p| |\hat{n}_{bf} \cdot \hat{n}_b|}{|\vec{x}_f - \vec{x}_b|^2} dA_f dA_b d\lambda_b dt \quad (14)$$

If the property-specific illumination contains only one state of polarization, then $E_b^0(\mathcal{P}_b) = E_b^0$.

5.3.5 Differential Form for the Reduced Form of the Total Property-Specific Energy Delivered by the Single Beam of Illuminating Light at a Point p along the Probe Path. A differential expression for the reduced expression for the total property-specific energy of the single beam of illuminating light at a point p along the probe path can be derived. Taking the derivative of equation (11) with respect to p and using the definition given in equation (10), the result is

$$\frac{dE_b}{dp} = E_b^0 \frac{de_b(p)}{dp} = -E_b^0 e_b(p) \bar{\xi}^b(\vec{x}_{bp} + (p - p_b) \hat{n}_{pp}) = -E_b \bar{\xi}^b(p) \quad (15)$$

or

$$-\frac{1}{E_b} \frac{dE_b}{dp} = -\frac{1}{e_b(p)} \frac{de_b(p)}{dp} = \bar{\xi}^b(p). \quad (16)$$

5.3.6 General Expression for the Total Band-Specific Signal Which Reaches the Detector from an Effective Probe Volume. To calculate the total band-specific signal which reaches the detector from an effective probe volume, the area A_f in the particle field must again be taken to be perpendicular everywhere to the probe path direction \hat{n}_{pp} so that $|\hat{n}_{pp} \cdot \hat{n}_f| = 1$, and to extend over the area $A_{pv}(p)$ of the single beam of illuminating light which is locally occupied by the effective probe volume at p . In general, the shape of the effective probe volume will not conform exactly to the shape of the beam, so $A_{pv}(p) \leq A_p(p)$. Equation (5) must then be multiplied by the differential solid angle $d\omega_{fd} = dA_d |\hat{n}_{fd} \cdot \hat{n}_d| / |\vec{x}_d - \vec{x}_f|^2$, multiplied by the differential component of dA_f perpendicular to \hat{n}_{fd} , i.e., $dA_f |\hat{n}_{fd} \cdot \hat{n}_f|$, multiplied by the differential range of wavelengths $d\lambda_c$ and the differential range of states of polarization $d\mathcal{P}_c$, and integrated over all appropriate variables including the distance Δp occupied by the effective probe volume. The resulting expression is

$$E'_c = \int_{\Delta p} \int_{t_d} \int_{\lambda_{c,ps}} \int_{\lambda_{b,ps}} \int_{A_b} \int_{A_{pv}(p)} \int_{A_d} \int_{\mathcal{P}_{c,bs}} \int_{\lambda_{c,bs}} \frac{P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \vec{x}_f, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \vec{x}_f, t) I'_b(\lambda_b, \mathcal{P}_b, \vec{x}_f, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_b| |\hat{n}_{fd} \cdot \hat{n}_d|}{|\vec{x}_f - \vec{x}_b|^2 |\vec{x}_d - \vec{x}_f|^2} \times d\lambda_c d\mathcal{P}_c dA_d dA_f dA_b d\lambda_b d\mathcal{P}_b dt dp, \quad (17)$$

where the subscript “ps” stands for “property-specific” and the subscript “bs” stands for “band-specific.”

5.3.7 Reduced Form of the General Expression for the Total Band-Specific Signal Which Reaches the Detector from an Effective Probe Volume. The expression for the total

band-specific signal which reaches the detector from an effective probe volume may be reduced using the assumed properties of the particle field and the approximations related to the development of the theory. A reduction to a form of equivalent simplicity to equation (1) of Chapter 1 is sought. To begin this task, the time integral in equation (17) will first be considered.

5.3.7.1 The Time Integral Term. The time integral in equation (17) is actually an integral of the product of five terms, namely

$$P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t), \quad \exp\left[-\int_0^{|\bar{x}_d - \bar{x}_f|} \xi(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \bar{x}_f + s\hat{n}_{fd}, t) ds\right],$$

$$\xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t), \quad I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t), \quad \text{and}$$

$$\exp\left[-\int_0^{|\bar{x}_f - \bar{x}_b|} \xi(\lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_b + s\hat{n}_{bf}, t) ds\right].$$

The time variation of these terms will essentially all be random except for the beam intensity term $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$. Unlike the other terms, the time variation in $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$ will either be more or less absent in steady illumination, or will exhibit a rise time and a fall time for pulsed illumination, but in either case no part of the time variation in $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$ will in general be random, nor in general will the time variation in $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$ be correlated in any way with the random time variations in the other terms. On top of these considerations, recall also that performing a representative number of property-specific single measurements at each distance measured along the property-specific probe path requires that the number of measurements performed be sufficient to determine the time-averaged property-specific band coefficient to within the required degree of accuracy, as discussed in section 3.1.2.2 of Chapter 3. If long duration illuminating light and long periods of detection are used so that the time-averaged property-specific band coefficient can be determined in a single detection, then the statistical fluctuations of the other four terms will be integrated over a long period of time compared with the characteristic time of the fluctuations, and it will be appropriate to decompose the statistical quantities into their mean and fluctuating components in order to determine the time integral. On the other hand, if either the duration of the illuminating light or the period of detection or both are short, then decomposing the statistical quantities into mean and fluctuating components may not be appropriate for any single detection. However, determining the time-averaged property-specific band coefficient will then require adding many single detections. In determining the sum it will again be appropriate to decompose the statistical quantities into their mean and fluctuating components. Therefore, under the assumed property of statistical stationarity discussed in section 5.1.2 of this chapter, the assumed property of local statistical isotropy discussed in section 5.1.3, the assumed property of statistically universal TAMPSEC component distribution functions discussed in section 5.1.4, the assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path as discussed in section 5.1.6, and the wavelength-independent TAMPSEC approximation discussed in section 5.2.8, expressions for these quantities can be written as

$$P(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) = \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b) + P' \quad (18)$$

$$\exp\left[-\int_0^{|\bar{x}_d-\bar{x}_f|}\xi(\lambda_c, \mathcal{P}_c, \hat{n}_{fd}, \bar{x}_f + s\hat{n}_{fd}, t)ds\right] \approx \exp\left[-\int_0^{|\bar{x}_d-\bar{x}_f|}\bar{\xi}^c(\bar{x}_f + s\hat{n}_{fd})ds\right] \quad (19)$$

$$\xi_c(\lambda_c, \mathcal{P}_c, \lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_f, t) = \bar{\xi}^c(\lambda_c, \mathcal{P}_c, \bar{x}_f) + \xi'_c \quad (20)$$

$$\exp\left[-\int_0^{|\bar{x}_f-\bar{x}_b|}\xi(\lambda_b, \mathcal{P}_b, \hat{n}_{bf}, \bar{x}_b + s\hat{n}_{bf}, t)ds\right] \approx \exp\left[-\int_0^{|\bar{x}_f-\bar{x}_b|}\bar{\xi}^b(\bar{x}_b + s\hat{n}_{bf})ds\right], \quad (21)$$

where $\bar{\xi}^c(\bar{x}_f + s\hat{n}_{fd}) = \bar{\xi}^c(\lambda_c, \bar{x}_f + s\hat{n}_{fd})$ is the wavelength-independent extinction coefficient at the property-specific signal wavelengths, and where it has been assumed that

$$\exp\left[-\int_0^{|\bar{x}_d-\bar{x}_f|}\xi'(\lambda_c, \dots)ds\right] \approx 1, \quad (22)$$

in accordance with the assumed property of negligible integrated statistical fluctuations of the IMPSEC's along a path, as discussed in section 5.1.6 above in this chapter. In substituting equations (18)-(21) into equation (17), additional use may be made of the assumed property that the IMPSEC component distribution function is uncorrelated with the IMPSEC component, as discussed in section 5.1.7 of this chapter, so that the fluctuating terms in equations (18) and (20) may be neglected. Use may also be made of the assumed property that the TAMPSEC at the property-specific illumination wavelengths is equal to the TAMPSEC at the property-specific signal wavelengths, as discussed in section 5.1.8 of this chapter, so that one may simply write $\bar{\xi} = \bar{\xi}^b = \bar{\xi}^c$. Rearranging terms to prepare for the next series of reductions, the result is

$$E'_c = \quad (23)$$

$$\int_{\mathcal{P}_b, ps} \int_{\Delta p} \int_{A_b} \int_{A_{ps}(p)} \exp\left[-\int_0^{|\bar{x}_f-\bar{x}_b|}\bar{\xi}(\bar{x}_b + s\hat{n}_{bf})ds\right] \frac{|\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} \left[\int_{t_d} \int_{\lambda_b, ps} I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) d\lambda_b dt \right]$$

$$\left[\int_{A_d} \int_{\lambda_c, bs} \int_{\mathcal{P}_c, bs} \exp\left[-\int_0^{|\bar{x}_d-\bar{x}_f|}\bar{\xi}(\bar{x}_f + s\hat{n}_{fd})ds\right] \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b) \bar{\xi}^c(\lambda_c, \mathcal{P}_c, \bar{x}_f) \frac{|\hat{n}_{fd} \cdot \hat{n}_d|}{|\bar{x}_d - \bar{x}_f|^2} d\mathcal{P}_c d\lambda_c dA_d \right]$$

$$dA_f dA_b dp d\mathcal{P}_b$$

5.3.7.2 Introduction of the Beam Extinction Function. The use of $e_b(p)$ as defined in equation (10) to approximate the first exponential term in equation (7) is examined next. The question arises about which value of p in the interval Δp is the correct one to use. Note first that

$$e_b(p + \Delta p) = \exp\left[-\int_0^{p+\Delta p-p_b}\bar{\xi}^b(\bar{x}_{bp} + s\hat{n}_{pp})ds\right]$$

$$= \exp\left[-\int_0^{p-p_b}\bar{\xi}^b(\bar{x}_{bp} + s\hat{n}_{pp})ds\right] \exp\left[-\int_p^{p+\Delta p}\bar{\xi}^b(\bar{x}_{bp} + s\hat{n}_{pp})ds\right] \quad (24)$$

$$= e_b(p) \exp\left[-\int_p^{p+\Delta p}\bar{\xi}^b(\bar{x}_{bp} + s\hat{n}_{pp})ds\right].$$

Now according to the small effective probe volume approximation of the first kind described in section 5.2.2 of this chapter, attenuation of the illuminating light and attenuation of the signal light across the effective probe volume may be neglected. Thus the exponential term in the second equality above is approximately unity, and $e_b(p + \Delta p) \approx e_b(p)$. Therefore $e_b(p)$ will be approximately constant within the interval Δp , and this term may therefore be taken outside all the integrals. If p_{pv} is an arbitrary point within the interval Δp selected to represent the ‘‘location’’ of the probe volume along the property-specific probe path, then equation (23) can be shown to reduce further to

$$E'_c = e_b(p_{pv}) \int_{\mathcal{P}_{b,ps}} \int_{\Delta p} \int_{A_b} \int_{A_{pv}(p)} \left[\int_{t_d} \int_{\lambda_{b,ps}} I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) \frac{|\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} d\lambda_b dt \right] \int_{A_d} \int_{\lambda_{c,bs}} \int_{\mathcal{P}_{c,bs}} \exp \left[- \int_0^{|\bar{x}_d - \bar{x}_f|} \bar{\xi}(\bar{x}_f + s\hat{n}_{fd}) ds \right] \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b) \bar{\xi}_c(\lambda_c, \mathcal{P}_c, \bar{x}_f) \frac{|\hat{n}_{fd} \cdot \hat{n}_d|}{|\bar{x}_d - \bar{x}_f|^2} d\mathcal{P}_c d\lambda_c dA_d \Bigg] dA_f dA_b dp d\mathcal{P}_b, \quad (25)$$

5.3.7.3 The Signal Attenuation Integral. Under the assumed property of separable TAMPSEC components as discussed in section 5.1.5 of this chapter, the TAMPSEC component $\bar{\xi}_c(\lambda_c, \mathcal{P}_c, \bar{x}_f)$ can be separated into the product of a time-averaged shape function $\bar{\xi}_c^s(\lambda_c, \mathcal{P}_c)$ and a time-averaged magnitude function $\bar{\xi}_c^0(\bar{x}_f)$. Furthermore, under the approximation that particle properties are approximately uniform across the beam as discussed in section 5.2.1 of this chapter, the magnitude function $\bar{\xi}_c^0(\bar{x}_f)$ will not vary within the effective probe volume, and can be taken outside all of the integrals in equation (25). The result can be written

$$E'_c = e_b(p_{pv}) \bar{\xi}_c^0(p_{pv}) \Gamma(p_{pv}) E_b^0, \quad (26)$$

where $\Gamma(p_{pv})$ is the *signal attenuation integral*

$$\Gamma(p_{pv}) \equiv \frac{1}{E_b^0} \int_{\mathcal{P}_{b,ps}} \int_{\Delta p} \int_{A_b} \int_{A_{pv}(p)} \left[\int_{t_d} \int_{\lambda_{b,ps}} I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) \frac{|\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} d\lambda_b dt \right] \int_{A_d} \int_{\lambda_{c,bs}} \int_{\mathcal{P}_{c,bs}} \exp \left[- \int_0^{|\bar{x}_d - \bar{x}_f|} \bar{\xi}(\bar{x}_f + s\hat{n}_{fd}) ds \right] \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b) \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c) \frac{|\hat{n}_{fd} \cdot \hat{n}_d|}{|\bar{x}_d - \bar{x}_f|^2} d\mathcal{P}_c d\lambda_c dA_d \Bigg] dA_f dA_b dp d\mathcal{P}_b. \quad (27)$$

Recall that it was allowed in sections 5.1.4 and 5.1.5 of this chapter that $\bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b)$ and $\bar{\xi}_c^s(\lambda_c, \mathcal{P}_c)$ could potentially be known functions. Somewhat less restrictively, if only the integral $\int_{\mathcal{P}_{c,bs}} \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b) \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c) d\mathcal{P}_c$ is known, if it is assumed that the probing of the particle field can be done in such an order that the integral inside the exponential term in equation (27) can be calculated, which in fact will be demonstrated to be true later, then the entire

signal attenuation integral can be calculated if the initial intensity distribution in the single beam of illuminating light is known. Recall from section 4.1.10.4 that the initial intensity distribution of the single beam of illuminating light is one of the items in the method of performing the required measurements that may optionally be known, measurable, or calculable. The further reductions below are performed in order to transform the signal attenuation integral into a more useful form when the initial intensity distribution of the single beam of illuminating light is not known.

5.3.7.4 The Integral over the Detector Area. In this section, the second square-bracketed term in equation (27) will be reduced. Attention may be drawn to the dependence of this term on the four quantities over which the leading integrals in equation (27) are to be integrated. There is a dependence on the beam polarization $\mathcal{P}_{b,ps}$, but there is no dependence on the original beam area A_b . The dependence on the other two quantities $A_{pv}(p)$, Δp is a dependence on the location inside the effective probe volume. However, under the small effective probe volume approximation of the second kind described in section 5.2.3 of this chapter, the range of directions defined from each point in the effective probe volume to the effective collection area of the detector optics will not vary by very much, nor will the range of distances. What remains is only the dependence on the state of polarization \mathcal{P}_b . Thus the square-bracketed term in the second line of equation (27) may be considered to be approximately constant with regard to the three other outer integrals. Therefore, for the same point inside the effective probe volume that was used define $e_b(p_{pv})$, define

$$h'(p_{pv}, \mathcal{P}_b) \equiv \int_{A_d} \int_{\lambda_{c,bs}} \int_{\mathcal{P}_{c,bs}} \exp \left[- \int_0^{|\vec{x}_d - \vec{x}_{f,pv}|} \bar{\xi}(\vec{x}_{f,pv} + s\hat{n}_{fd}) ds \right] \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b) \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c) \frac{|\hat{n}_{fd} \cdot \hat{n}_d|}{|\vec{x}_d - \vec{x}_f|^2} d\mathcal{P}_c d\lambda_c dA_d \quad (28)$$

and

$$h(p_{pv}, \mathcal{P}_b) \equiv \int_{A_d} \int_{\lambda_{c,bs}} \int_{\mathcal{P}_{c,bs}} \bar{P}(\hat{n}_{fd} - \hat{n}_{bf}, \lambda_c, \mathcal{P}_c, \mathcal{P}_b) \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c) \frac{|\hat{n}_{fd} \cdot \hat{n}_d|}{|\vec{x}_d - \vec{x}_f|^2} d\mathcal{P}_c d\lambda_c dA_d, \quad (29)$$

and define the *signal attenuation function* $e_c(p_{pv}, \mathcal{P}_b)$ as

$$e_c(p_{pv}, \mathcal{P}_b) \equiv h'(p_{pv}, \mathcal{P}_b) / h(p_{pv}, \mathcal{P}_b). \quad (30)$$

Then moving the terms discussed outside the outer integrals and regrouping for the next phase of reduction, the signal attenuation integral can be written approximately as

$$\Gamma(p_{pv}) = \frac{1}{E_b^0} \int_{\mathcal{P}_{b,ps}} e_c(p_{pv}, \mathcal{P}_b) h(p_{pv}, \mathcal{P}_b) \int_{\Delta p} \left[\int_{A_b} \int_{A_{pv}(p)} \int_{t_d} \int_{\lambda_{b,ps}} I_b(\lambda_b, \mathcal{P}_b, \vec{x}_b, \hat{n}_{bf}, t) \frac{|\hat{n}_{bf} \cdot \hat{n}_b|}{|\vec{x}_f - \vec{x}_b|^2} d\lambda_b dt dA_f dA_b \right] dp d\mathcal{P}_b \quad (31)$$

5.3.7.5 Form of the Signal Attenuation Function under the Small Detector Approximation of the First Kind. Under the small detector approximation of the first kind, the number of directions from the property-specific effective probe volume to the effective collection area of the collection optics is minimized, as discussed in section 5.2.6 of this chapter. The dependence of the variables in equations (28) and (29) on the collection area A_d will therefore also be minimized, so the contribution of the integral over A_d is simply A_d . The exponential term in equation (28) can be taken outside the wavelength and polarization state integrals, and the wavelength and polarization state integrals will cancel in determining the ratio $h'(p_{pv}, \mathcal{P}_b)/h(p_{pv}, \mathcal{P}_b)$. The simplified form of the signal attenuation function under the small detector approximation then becomes

$$e_c(p_{pv}, \mathcal{P}_b) = e_c(p_{pv}) = \exp\left[-\int_0^{|\bar{x}_d - \bar{x}_{f,pv}|} \bar{\xi}(\bar{x}_{f,pv} + s\hat{n}_{fd}) ds\right], \quad (32)$$

the form of which becomes very similar to the definition of the beam extinction function $e_b(p_{pv})$, given by equation (10).

5.3.7.6 The Effective Probe Volume Correction Factor. The group of terms inside the square-bracketed integrals of equation (31) is nearly the same as the group of terms inside the integrals in equation (14) for the property-specific energy $E_b^0(\mathcal{P}_b)$ of a particular state of polarization of the unattenuated beam, except for the missing term $|\hat{n}_{bf} \cdot \hat{n}_p|$. However, if the single beam of illuminating light is collimated, or alternatively if the slender beam approximation of the second kind applies so that the number of directions of propagation possible within the single beam of illumination is minimized, then this term will be nearly unity. Adding this term to the integrals inside the square brackets of equation (31) would still not cause the integrals to equal $E_b^0(\mathcal{P}_b)$, however, because the area $A_{pv}(p)$ over which the term inside the square brackets is integrated is the area of the effective probe volume of the beam, not the area of the beam itself. To relate the integrals inside the square brackets to $E_b^0(\mathcal{P}_b)$, it is necessary to introduce the correction factor $\sigma(p, \mathcal{P}_b)$,

$$\sigma(p, \mathcal{P}_b) \equiv \frac{\int_{A_b} \int_{A_{pv}(p)} \int_{t_d} \int_{\lambda_{b,ps}} \frac{I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_p| |\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} d\lambda_b dt dA_f dA_b}{\int_{A_b} \int_{A_p(p)} \int_{t_d} \int_{\lambda_{b,ps}} \frac{I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) |\hat{n}_{bf} \cdot \hat{n}_p| |\hat{n}_{bf} \cdot \hat{n}_b|}{|\bar{x}_f - \bar{x}_b|^2} d\lambda_b dt dA_f dA_b}. \quad (33)$$

Then equation (31) can be written as

$$\Gamma(p_{pv}) \equiv \frac{1}{E_b^0(\mathcal{P}_b)} \int_{\mathcal{P}_b, ps} e_c(p_{pv}, \mathcal{P}_b) h(p_{pv}, \mathcal{P}_b) E_b^0(\mathcal{P}_b) \left[\int_{\Delta p} \sigma(p, \mathcal{P}_b) dp \right] d\mathcal{P}_b. \quad (34)$$

Note that if the intensity $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t)$ can be separated into the product of a magnitude function $I_0(t)$ which depends only on time, and a shape function $I_s(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf})$ which is independent of time, such that $I_b(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf}, t) = I_0(t)I_s(\lambda_b, \mathcal{P}_b, \bar{x}_b, \hat{n}_{bf})$, as discussed in section 3.1.1.5 of Chapter 3, then the time integrals can be taken outside the other integrals in the numerator and denominator of equation (33), and will cancel. This implies that time variations in the source of the illuminating light will not cause $\sigma(p, \mathcal{P}_b)$ to vary from effective probe volume to effective probe volume. For instance if the magnitude function is different for one effective probe volume than another effective probe volume, the difference will not affect $\sigma(p, \mathcal{P}_b)$. Now introduce the *polarization-state-averaged signal attenuation function*

$$\bar{e}_c(p_{pv}) \equiv \frac{\int_{\mathcal{P}_{b,ps}} e_c(p_{pv}, \mathcal{P}_b) h(p_{pv}, \mathcal{P}_b) E_b^0(\mathcal{P}_b) \left[\int_{\Delta p} \sigma(p, \mathcal{P}_b) dp \right] d\mathcal{P}_b}{\int_{\mathcal{P}_{b,ps}} h(p_{pv}, \mathcal{P}_b) E_b^0(\mathcal{P}_b) \left[\int_{\Delta p} \sigma(p, \mathcal{P}_b) dp \right] d\mathcal{P}_b}, \quad (35)$$

and define the *effective probe volume correction factor*

$$\Sigma(p_{pv}) \equiv \int_{\mathcal{P}_{b,ps}} h(p_{pv}, \mathcal{P}_b) E_b^0(\mathcal{P}_b) \left[\int_{\Delta p} \sigma(p, \mathcal{P}_b) dp \right] d\mathcal{P}_b. \quad (36)$$

Then equation (34) can be rewritten as

$$\Gamma(p_{pv}) \equiv \bar{e}_c(p_{pv}) \Sigma(p_{pv}). \quad (37)$$

The reduced form of the total band-specific signal which reaches the detector from an effective probe volume is therefore obtained by using equation (37) in equation (26):

$$E'_c = e_b(p_{pv}) \bar{e}_c(p_{pv}) \bar{\xi}_c^0(p_{pv}) E_b^0 \Sigma(p_{pv}) \quad (38)$$

In general, neither the polarization-state-averaged signal attenuation function $\bar{e}_c(p_{pv})$ nor the effective probe volume correction factor $\Sigma(p_{pv})$ are calculable unless the initial intensity distribution of the beam is known. When the initial intensity distribution of the beam is not known, application of the theory developed herein will be possible only in certain limiting cases.

5.3.7.6.1 The limiting case where only a single state of polarization exists in the illumination. One limiting case where application of the theory developed herein will be possible if the initial intensity distribution of the beam is not known is the limiting case where only a single state of polarization exists in the illumination. When only a single state of polarization exists in the illumination, the polarization-state-averaged signal attenuation function will be equal to the signal attenuation function for the single state of polarization in the illumination and can therefore be known independent of the initial intensity distribution of the beam. The limiting case where only a single state of polarization exists in the illumination is a necessary but not sufficient condition

to make it possible to apply the theory developed herein, because this by itself does not guarantee that the effective probe volume correction factor will be known.

5.3.7.6.2 The limiting case where the small detector approximation of the first kind applies. Another limiting case where application of the theory developed herein will be possible if the initial intensity distribution of the beam is not known is the limiting case where the small detector approximation of the first kind applies. When the small detector approximation of the first kind applies, the signal attenuation function will have the same value for all states of polarization, and will therefore can be known independent of the initial intensity distribution of the beam. The limiting case where the small detector approximation applies is a necessary but not sufficient condition to make it possible to apply the theory developed herein, because this by itself does not guarantee that the effective probe volume correction factor will be known.

5.3.7.6.3 The limiting case where the effective probe volume correction factor is repeatable along the property-specific probe path. Yet another limiting case where application of the theory developed herein will be possible if the initial intensity distribution of the beam is not known is the limiting case where the effective probe volume correction factor is repeatable along the property-specific probe path. When the effective probe volume correction factor is repeatable along the property-specific probe path, it turns out that the value of the effective probe volume correction factor need not be known. It can be lumped along with other constants into a single constant. The combination of the limiting case where the effective probe volume correction factor is repeatable along the property-specific probe path with either of the limiting cases described in the previous two paragraphs constitutes a necessary and sufficient condition that it will be possible to apply the theory developed herein. When this limiting case applies, the effective probe volume correction factor will be written simply as $\Sigma(p_{pv}) = \Sigma$.

As is evident from its definition in equation (36), the repeatability of the effective probe volume correction factor Σ is determined by a complicated relationship between the area over which the light in a single beam of light is distributed, the intensity distribution within the beam, the location and direction of the effective collection area of the detector, and the geometry of the effective probe volume. Many combinations of these factors could create a repeatable effective probe volume correction factor. The particular method taken herein is probably also the simplest approach, wherein the repeatability of each factor is separately set so that it no longer affects the repeatability of the effective probe volume correction factor. When all of these factors are so set, the effective probe volume correction factor will be repeatable along the property-specific probe path. Thus the area over which the light in a single beam of illuminating light is distributed was set in section 3.1.1.2.10 of Chapter 3 to minimize variations in area along the length of the beam, the source of the single beam of illuminating light was set in section 3.1.1.5 of Chapter 3 to use a source where the illumination intensity could be separated into the product of a magnitude function which depends only on time and a shape function which is independent of time, the effective probe volume geometries were set in section 4.1.1.2.8 of Chapter 4 to be as repeatable as possible, and the location of the detector was set in section 4.1.1.3.3 to maintain a fixed relationship between the detector and the single beam of illuminating light. While these methods should be theoretically sufficient to ensure that the effective probe volume correction factor is repeatable along the property-specific probe path, practical considerations indicate that the below analytical method will also be desirable.

5.3.7.6.4 *An Analytical Method for Helping to Ensure that the Effective Probe Volume Correction Factor is Repeatable along the Property-Specific Probe Path.* Even when the intensity distribution of the single beam of illuminating light is repeatable and there is no drift in the properties of the property-specific single detector, small changes in alignment between the detector and beam along the property-specific probe path may cause the effective probe volume to be located at positions of different local intensities within the beam. If this occurs, the effective probe volume correction factor may not be repeatable along the probe path. One method for compensating for this effect is to locate more than one sensitive element behind a common set of collection optics as discussed in section 4.2.1.2 of Chapter 4, such that several effective probe volumes represent each axial distance along the property-specific probe path. A search among the several effective probe volumes based on the response of the effective probe volumes can then be performed to identify the effective probe volume for which the effective probe volume correction factor will be the most repeatable. This is illustrated in Figure 10, where the intensity variations in the beam are represented by variations in gray level, and multiple effective probe volumes at two axial locations are represented by the circles. A small change in the alignment between the detector and the beam is represented by a vertical shift of the circles. Choosing which effective probe volume to represent each axial location by some simple rule such as always choosing the third effective probe volume from the bottom would result in a different part of the beam having a different local intensity being selected. As a result, the effective probe volume correction factor might not be repeatable. However, selecting the effective probe volume by searching based on the response of the effective probe volume can result in a more repeatable effective probe volume correction factor. In Figure 10, the effective probe volume having the maximum response was selected, as represented by the black circles, resulting in a more repeatable effective probe volume correction factor.

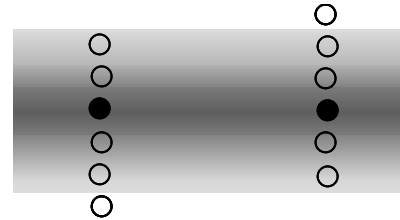


Figure 10. Displacement of effective probe volumes caused by a change in the alignment between the collection optics and the beam.

In the most recommended method of performing the required measurements described in section 4.3 of Chapter 4, a two dimensional array of sensitive elements is located behind a common set of collection optics, with enough pixel resolution to simultaneously perform measurements along the entire length of the property-specific probe path. When this is done, it is no longer possible to maintain a strictly fixed relationship between the detector and the single beam of illumination light, as required in section 4.1.1.3.3 to cause the repeatability of the effective probe volume correction factor to be independent of the repeatability of the location of the detector. The small detector approximation of the second kind given in section 5.2.7 allows the effect of locating more than one sensitive element behind a common set of collection optics on the repeatability of the effective probe volume correction factor to be minimized.

5.3.7.7 Final Reduced Form used by the Method of Analyzing the Data. Equation (38) provides a relationship between the total band-specific radiation E'_c which reaches the detector from an effective probe volume and the time-averaged magnitude function $\bar{\xi}_c^0$ of the band coefficient. What it is desired to be known, however, is the relationship between the response of the detector

and the property-specific band coefficient. This relationship is derived here. If G is the response of the detector, then E'_c will be related to G according to the relationship

$$E'_c = k_d f(G) \quad (39)$$

where k_d is a detector constant, not necessarily known, and $f(G)$ is some function of G which must be known (through calibration, for example). Equation (39) relates the response of the detector to the total band-specific energy which reaches the detector. The relationship between the time-averaged magnitude function and the time-averaged band coefficient is obtained by multiplying the time-averaged magnitude function by the time averaged shape function and integrating over all property-specific wavelengths and states of polarization. Thus

$$\bar{\xi}_n(p_{pv}) = \bar{\xi}_c^0(p_{pv}) \int_{\lambda_{c,ps}} \int_{\mathcal{P}_{c,ps}} \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c) d\mathcal{P}_c d\lambda_c \quad (40)$$

Now define the *reduced detector response* $\tilde{E}'_n \equiv f(G)/E_b^0$ and the *band constant* $k_n \equiv \Sigma/k_d \int_{\lambda_{c,ps}} \int_{\mathcal{P}_{c,ps}} \bar{\xi}_c^s(\lambda_c, \mathcal{P}_c) d\mathcal{P}_c d\lambda_c$. Then equation (38) may be written in its final reduced form as

$$\tilde{E}'_n(p) = k_n e_b(p) \bar{e}_c(p) \bar{\xi}_n(p), \quad (41)$$

where the subscript “ pv ” has been dropped from “ p_{pv} ” for brevity.

5.3.8 Reduced Expressions for the TAMSEC. The general expression for the IMPSEC has already been given in equation (A.9) of Appendix A, where the IMPSEC given by the integration of the IMPSEC components over every possible wavelength and state of polarization. The TAMSEC was introduced in section 3.1.1.1.3 of Chapter 3 to be the time-averaged IMPSEC, and the TAMSEC can similarly be given as the integration of the IMPSEC components over every possible wavelength and state of polarization. The reduced expression for the TAMSEC considers the TAMSEC to be a sum of only those band coefficients which contribute significantly to the TAMSEC,

$$\bar{\xi}(p) = \sum_N \bar{\xi}_n(p) \quad (42)$$

where N is the number of band coefficients which contribute significantly to the TAMSEC.

5.3.9 Constraints on the Band Constants. The band constants k_n have thus far been defined in terms that are theoretically knowable if the intensity distribution in the beam of illuminating light is known, but the calculations would be extremely tedious and prone to error. Furthermore, it is desired to apply the method of analyzing the data without having to know the initial intensity distribution of the illumination. It would therefore be preferable if the band constants can be determined in a simpler way. One constraint which can be used to determine one of the band constants can be found by combining equations (16), (41), and (42). The result is

$$de_b/dp = -e_b(p)\bar{\xi}(p) = -e_b(p)\sum_N \bar{\xi}_n(p) = -\sum_N [\tilde{E}'_n(p)/k_n \bar{e}_c(p)]. \quad (43)$$

Integrating over the entire length of the property-specific probe path in the allowed contiguous sub-volume of the particle field, the result is

$$1 - e_{bf} = \int_{p_0}^{p_f} \sum_N [\tilde{E}'_n(p)/k_n \bar{e}_c(p)] dp, \quad (44)$$

where p_0 is the value of p where the single beam of illuminating light enters the allowed contiguous sub-volume of the particle field, and p_f is the value of p where the single beam of illuminating light exits the allowed contiguous sub-volume of the particle field. Both p_0 and p_f will be known values because the size, shape, extent, and location of the allowed contiguous sub-volume of the particle field is one of the quantities that must be known, measured, or calculable in the method of performing the required measurements, as discussed in section 4.1.8 of Chapter 4. The quantity $e_{bf} = E_b^f / E_b^0$ is the final value of e_b , which is also known because the total property-specific illumination energy exiting the allowed contiguous sub-volume of the particle field E_b^f is one of the quantities that must be known, measured, or calculable according to the method of performing the required measurements, as discussed in section 4.1.6 of Chapter 4. Equation (44) can be used to determine the band coefficient if the TAMPSEC is dominated by only one band coefficient ($N = 1$); otherwise, additional relationships between the band coefficients or the band constants must be known. For example, by carefully adjusting effective probe volume correction factors or other properties, it may be possible to make other band constants have the same value, or at least to be known functions of each other. As a minimum, a sufficient number of relationships must exist to determine the time-averaged band constant of each band coefficient that makes a significant contribution to the TAMPSEC.

Special consideration must be given when the time-averaged absorption band coefficient $\bar{\xi}_{abs}$ contributes significantly to the TAMPSEC. It will be recalled from section A.9 of Appendix A that no portion of the extinguished optical radiation accounted for by $\bar{\xi}_{abs}$ is emitted again as any other form of optical radiation. Therefore the time-averaged absorption band coefficient cannot be directly measured from a detector response, and must be determined by some other means. For example, the absorption band coefficient may be a known function of some other band coefficient. One specific case that is often true is that the absorption band coefficient will be proportional to the fluorescence band coefficient, $\bar{\xi}_{abs} = k_{abs} \bar{\xi}_f$, where k_{abs} is the constant of proportionality between $\bar{\xi}_{abs}$ and $\bar{\xi}_f$.

5.3.10 Determination of the Particle Property of Interest. Given that one of the band coefficients will be proportional to the particle property of interest due to the various conditionings which have been applied, the particle property of interest will be given by

$$\bar{\zeta} = g_\zeta \bar{\xi}_\zeta \quad (45)$$

where $\bar{\zeta}$ is the time-averaged particle property of interest, $\bar{\xi}_c$ is its time averaged band coefficient, and g_c is the constant of proportionality. It is assumed that g_c is either known, or it is acceptable to know the distribution of the particle property of interest only to within an unknown constant. Note that $\bar{\xi}_c$ may not be any of the band coefficients which contribute significantly to the TAMPSEC. Further note that it is not necessary to know which particle properties are responsible for any of the band coefficients which contribute significantly to the TAMPSEC. It is only necessary to know the relationship of equation (45) for the particle property of interest. For example, if scattering is the dominant extinction mechanism, but $\bar{\xi}_c$ is the fluorescence band coefficient, then only the scattering band coefficient needs to be determined. It is not necessary to have an equation like equation (45) to relate the scattering coefficient to any particle property. In particular, this means that $\bar{\xi}_c$ can be determined *even if the particles are non-spherical*, and there is no clear relationship between the scattering coefficient and the particle surface area.

5.3.11 Implications of the Theory Applicable to the Method of Analyzing the Data.

When attenuation of the illumination light and attenuation of the signal light are significant, measuring the particle property of interest at each measurement location in the allowed contiguous sub-volume of the particle field requires computing the TAMPSEC at each location. If N band coefficients contribute significantly to the TAMPSEC, then at each location there will be $2N + 3$, unknowns, namely $\bar{\zeta}$, $\bar{\xi}_n$, k_n , e_b , and \bar{e}_c . To compute the $2N + 3$ unknowns, a total of $4 + N$ equations are provided, namely the $3 + N$ equations

$$\bar{\zeta}(p) = \sum_N \bar{\xi}_n(p), \quad (46)$$

$$\tilde{E}'_n(p) = k_n e_b(p) \bar{e}_c(p) \bar{\xi}_n(p), \quad (47)$$

$$1 - e_{bf} = \int_{p_0}^{p_f} \sum_N [\tilde{E}'_n(p) / k_n \bar{e}_c(p)] dp, \quad (48)$$

$$1 - e_b(p) = \int_{p_0}^p \sum_N [\tilde{E}'_n(p) / k_n \bar{e}_c(p)] dp, \quad (49)$$

plus an additional equation for \bar{e}_c which takes different forms depending on the limiting case chosen. In its simplest form (using the small detector approximation of the first kind), the expression for \bar{e}_c is

$$\bar{e}_c(p) = \exp \left[- \int_0^{|\bar{x}_d - \bar{x}_{f,pv}|} \bar{\xi}(\bar{x}_{f,pv} + s \hat{n}_{fd}) ds \right]. \quad (50)$$

Application of equation (50) requires that the TAMPSEC's be known at locations other than at the point p , namely along the path between the point p and the detector. These TAMPSEC's may

for the present be considered to be known. The means by which the TAMPSEC's along the path between the point p and the detector may be considered to be known will be described in detail further below in section 5.4 of this chapter. It is evident from the above that a sufficient number of equations to calculate all the unknowns will in general exist only when $N = 1$. When $N > 1$, additional relationships will need to be known or assumed.

Given that the TAMPSEC is known, two equations are available to compute each particle property of interest, namely

$$\tilde{E}'_{\zeta}(p) = k_{\zeta} e_b(p) \bar{e}_{\zeta}(p) \bar{\xi}_{\zeta}(p), \quad (51)$$

$$\bar{\zeta} = g_{\zeta} \bar{\xi}_{\zeta}. \quad (52)$$

However, these two equations contain four unknowns, namely k_{ζ} , $\bar{\xi}_{\zeta}$, g_{ζ} , and $\bar{\zeta}$ (recall that e_b and \bar{e}_{ζ} will be known from the TAMPSEC calculations). Combining equations (51) and (52), the result is

$$\bar{\zeta} = g_{\zeta} \bar{\xi}_{\zeta} = \left(\frac{g_{\zeta}}{k_{\zeta}} \right) \frac{\tilde{E}'_{\zeta}(p)}{e_b(p) \bar{e}_{\zeta}(p)} \quad (53)$$

Since all of the quantities outside of the parentheses on the right hand side of equation (53) will have been measured or calculated, the consequence of having only two equations for four unknowns is that it will in general be possible to determine the particle property of interest only to within an unknown constant, unless additional relationships are known. It may be noted that if $\bar{\xi}_{\zeta}$ is not one of the band coefficients that contribute significantly to the TAMPSEC, then additional measurements will be required according to the method of performing the required measurements in Chapter 4 so that \tilde{E}'_{ζ} will be known.

It is clear from the above that the number of additional relationships which need to be determined can be significantly reduced if the number of band coefficients which contribute significantly to the TAMPSEC can be reduced to just one, *i.e.* $N = 1$. At least two cases where only one band coefficient contributes significantly to the TAMPSEC can be identified which commonly occur in practice. Cases where only one band coefficient contributes significantly to the TAMPSEC which commonly occur in practice include the case where only the scattering band coefficient contributes significantly to the TAMPSEC, and the case where only the absorption band coefficient contributes significantly to the TAMPSEC. These cases are described next.

5.3.11.1 The Case Where only the Scattering Band Coefficient Contributes Significantly to the TAMPSEC. One case where only one band coefficient contributes significantly to the TAMPSEC which commonly occurs in practice is the case where only the scattering band coefficient contributes significantly to the TAMPSEC. Scattering dominates the TAMPSEC in many particle fields of interest, and this case is in fact the target case for the method presented in this report. When only the scattering band coefficient contributes significantly to the TAMPSEC, both the scattering band constant k_s and the scattering band coefficient ξ_s will be determined in the process of calculating the TAMPSEC. Therefore all the unknowns in equation (51) will be

determined for the scattering band. The only unknown remaining to prevent equation (52) from being used to compute the particle surface area concentration a is the constant g_s . As is observed in section A.11 of Appendix A, for spherical particles in the geometric optics regime, where the particle sizes are much larger than the wavelength of the light, the elastic scattering cross section of a particle will be equal to twice its projected area, or half its total surface area, giving $g_s = 2$. This additional relationship for g_s completes the system of equations required to determine the particle surface area concentration a .

When only the scattering band coefficient contributes significantly to the TAMPSEC, and when the particle volume concentration is to be determined, other relationships will in general need to be known in addition to equations (51) and (52) to determine the particle volume concentration. If these relationships are not known, then it will be possible to determine the particle volume concentration only to within an unknown constant, as in equation (53) above. As was observed previously in section 5.3.10 of this chapter, it is not necessary to relate the scattering band coefficient to any particle property in connection with determining the particle volume concentration. In particular, it is not necessary that the particles be spherical, nor need there be any clear relationship between the scattering band coefficient and the particle surface area concentration. There need only be at least a clear proportional relationship between the particle volume concentration and the fluorescence band coefficient.

5.3.11.2 The Case Where only the Absorption Band Coefficient Contributes Significantly to the TAMPSEC. Another case where only one band coefficient contributes significantly to the TAMPSEC which commonly occurs in practice is the case where only the absorption band coefficient contributes significantly to the TAMPSEC. The case where only the absorption band coefficient contributes significantly to the TAMPSEC in fact does not occur very frequently when particles are present, but this case does occur frequently in purely absorbing fields such as purely gaseous or purely liquid fields. Since absorption is frequently accompanied by fluorescence, the method developed in this report can still be applied to determine the volume concentration of the fluorescence molecules, which in turn will be proportional to the field density.

As was discussed above in section 5.3.9 of this chapter and in section A.9 of Appendix A, the absorption band coefficient cannot be measured directly by the detection of an optical signal as can the other band coefficients. Here the assumption can be made that the absorption band coefficient is proportional to the fluorescence band coefficient, $\bar{\xi}_{abs} = k_{abs}\bar{\xi}_f$, where k_{abs} is the constant of proportionality between $\bar{\xi}_{abs}$ and $\bar{\xi}_f$. Then $\sum_N \bar{\xi}_n(p) \approx \bar{\xi}_{abs} = k_{abs}(\bar{E}'_f(p)/k_f\bar{e}_c(p))$, where equation (44), with $N=1$, can give the ratio k_{abs}/k_f . Thus $\bar{\xi}_{abs}$ can still be calculated, and the volume concentration $\bar{\rho}_{abs} = g_{abs}\bar{\xi}_{abs}$ of the absorbing molecules can be known at least to within the constant g_{abs} . Determination of g_{abs} may typically be done using calibration methods.

5.4 General Method of Analyzing the Data.

The general method of analyzing the data is described in this section. The general method of analyzing the data is described only for the case where the initial intensity distribution of the single beam of illuminating light is not known. The general method of analyzing the data includes

the method of selecting the order in which property-specific probe paths are analyzed, the method of determining the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond, the method of selecting which detector responses to analyze, the method of discretizing a property-specific probe path, and the method of analyzing the data along a property specific probe path.

5.4.1 The Method of Selecting the Order in which Property-Specific Probe Paths are Analyzed. The first part of the general method of analyzing the data is the method of selecting the order in which property-specific probe paths are analyzed. The method of selecting the order in which property-specific probe paths are analyzed consists of selecting the probe paths in such an order that the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can always be calculated. The method of selecting the probe paths in such an order that the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can always be calculated consists of a series of steps, as follows.

5.4.1.1 The First Step. The first step of the method of selecting the probe paths in such an order that the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can always be calculated is to select probe paths for which the polarization-state-averaged signal attenuation function is known to be unity, $\bar{\epsilon}_c(p) = 1$. Recall that in the general method probing the particle field with illuminating light, the sum total of all the probe paths within the allowed contiguous sub-volume of the particle field was described in section 3.1.2.3 of Chapter 3 to be like a random stack of hay or a random pile of toothpicks. Some of the hay stalks or toothpicks will be located on the very outside of the stack. Signals emanating from them will reach the detector without propagating through any portion of the allowed contiguous sub-volume of the particle field, and will not be attenuated. The polarization-state-averaged signal attenuation function is therefore known to be unity for these particular paths. With $\bar{\epsilon}_c(p)$ now known, the rest of the method of analyzing the data can be completed in entirety along these probe paths. As a result, the TAMPSEC $\bar{\xi}$ will be known along the probe paths which are selected in this first step. The sum total of the locations where the TAMPSEC will then be known after this first step will constitute an outermost layer of the allowed contiguous sub-volume of the particle field.

5.4.1.2 The Second Step. The second step of the method of selecting the probe paths in such an order that the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can always be calculated is to select property-specific probe paths for which the signals have to pass only through the outermost layer of the allowed contiguous sub-volume of the particle field which was identified in the first step before the signals reach the detector. Because the signals pass only through this outermost layer of the allowed contiguous sub-volume of the particle field, and because the TAMPSEC is known in this outermost layer, equation (50) or one of its other forms given in section 5.3.7.6 of this Chapter can be used to calculate $\bar{\epsilon}_c(p)$. With $\bar{\epsilon}_c(p)$ now known for these probe paths, the rest of the method of analyzing the data can be completed in entirety along the probe paths identified in this second step. As a result, the TAMPSEC $\bar{\xi}$ will now also be known along these probe paths. The sum total of the locations where the TAMPSEC will then be known after this second step will constitute a second layer of the allowed contiguous sub-volume of the particle field.

5.4.1.3 The Third Step. The third step of the method of selecting the probe paths in such an order that the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can always be calculated is to select property-specific probe paths for which the signals have to pass only through the first and second outermost layers of the allowed contiguous sub-volume of the particle field identified in the previous two steps before reaching the detector. Because the TAMPSEC is known in the first and second outermost layers, the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can be calculated in this third layer, and the rest of the method of analyzing the data can be completed in entirety within the third layer.

5.4.1.4 Subsequent Steps. Subsequent steps of the method of selecting the probe paths in such an order that the polarization-state-averaged signal attenuation function $\bar{\epsilon}_c(p)$ can always be calculated include the process of proceeding successively to the next inner layers, like peeling the layers of an onion, until the polarization-state-averaged signal attenuation function can be calculated for all the property-specific probe paths within the allowed contiguous sub-volumes of the particle field.

5.4.2 The Method of Determining the Distance Along the Property-Specific Probe Path to which The Response of the Property-Specific Single Detector is Assigned to Correspond. The second part of the general method of analyzing the data is the method of determining the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond. The method of determining the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond is to select the point on the probe path which is nearest to any point in the effective probe volume. This method is discussed in detail in section 4.1.5 of Chapter 4.

5.4.3 The Method of Selecting Which Detector Responses to Analyze. The third part of the most general embodiment of the method of analyzing the data is the method of selecting which detector responses to analyze. The method of selecting which detector responses to analyze consists of using the relative differences between the responses of detectors assigned to the same or nearly the same distance along the property-specific probe path to select the response for which the effective probe volume correction factor Σ is the most repeatable. Recall that in the most recommended method of performing the required measurements, two or more effective probe volumes will be created to span the single beam of illuminating light in a direction perpendicular to the probe path, as indicated in section 4.3.2.3 of Chapter 4. The reason for causing two or more effective probe volumes to be created to span the single beam of illuminating light in a direction perpendicular to the probe path is to help ensure that the effective probe volume correction factor Σ can be made to be as repeatable as possible along the probe path. Using the relative differences between the responses of detectors assigned to the same or nearly the same distance along the property-specific probe path to select the response for which the effective probe volume correction factor Σ is the most repeatable was discussed in detail in section 5.3.7.6.4 of this chapter.

Many variations of this method are possible. One variation is the variation of selecting the maximum response. The variation of selecting the maximum response is simply to select the response of the detector that has the largest magnitude. This variation was illustrated in Figure 10 and the discussion related to it. To the extent that the beam has a Gaussian intensity distribution,

this method will tend to select the effective probe volume which is located at the center of the beam.

5.4.4 The Method of Discretizing a Property-Specific Probe Path. The fourth part of the general method of analyzing the data is the method of discretizing a property-specific probe path. The method of discretizing the probe path is to discretize the probe path into equal intervals. The intervals should be roughly no further apart than an average distance between measurement locations along the probe path. In some cases, the measurements themselves might be made over equal intervals along the probe path. In those cases, no further action is required under this section. In other cases, the measurements might not be made over equal intervals, or it might be numerically convenient to discretize the probe path into smaller intervals than the spacing between the measurements (for more convenient calculation of relationships with other probe paths, for example). In these cases, the method of discretizing the probe path is to discretize the probe path into equal intervals and to use interpolation and extrapolation to project what the reduced detector response $\tilde{E}'_n(p)$ would have been at those intervals had measurements been made there. This process is illustrated in Figure 11, where the measurements are shown as the dark circles and are at unequal intervals in the example in the figure, and estimates through interpolation and extrapolation of what the measurements of $\tilde{E}'_n(p)$ would have been at each interval are illustrated as the open circles. One of the measurements is shown to fall exactly on an interval, so interpolation is not required at that point. The method of interpolation and extrapolation illustrated in Figure 11 is linear interpolation and extrapolation. However, any method of interpolation and extrapolation may be used.

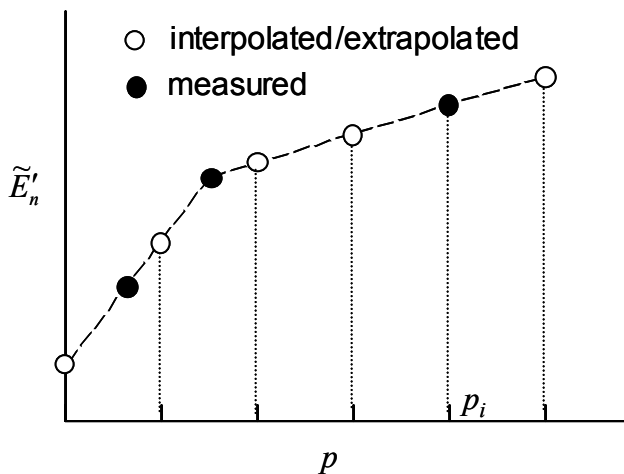


Figure 11 The method of discretizing a probe path.

5.4.5 The Method of Analyzing the Data along a Property-Specific Probe Path. The fifth and final part of the general method of analyzing the data is the method of analyzing the data along a property-specific probe path. The method of analyzing the data along a property-specific probe path consists of the following series of seven steps.

5.4.5.1 The First Step. The first step of the method of analyzing the data along a property-specific probe path is to use equation (50) or one of its other forms given in section 5.3.7.6 of this Chapter to calculate the polarization-state-averaged signal attenuation function $\bar{e}_c(p)$ at each point along the probe path. If equation (50) is used to calculate $\bar{e}_c(p)$, its discretized form would be

$$\bar{e}_c(p) = \exp \left[- \sum_{i=1}^S \bar{\xi} (\bar{x}_{f,pv} + s_i \hat{n}_{fd}) \Delta s \right], \quad (54)$$

where Δs is the discretization interval on the path connecting the point on the property-specific probe path to the detector, $s_i = (i-1)\Delta s$ is the distance at the beginning of the i th interval along this path, and S is equal to one plus the total number of intervals. Recall that the property-specific probe paths will have been selected for analysis in section 5.4.1 of this chapter in such an order that TAMPSEC $\bar{\xi}$ will be known at each point along the path connecting the property-specific probe path to the detector.

5.4.5.2 The Second Step. The second step of the method of analyzing the data along a property-specific probe path is to calculate the band constants k_n of those band coefficients which contribute significantly to the TAMPSEC $\bar{\xi}$. The band constants k_n of the band coefficients which contribute significantly to the TAMPSEC $\bar{\xi}$ are calculated using equation (48), plus $n-1$ other relationships. For $n=1$, solving the discretized form of this equation gives

$$k_1 = \frac{1}{1 - e_{bf}} \sum_{i=1}^N \tilde{E}'_1(p_i) / \bar{e}_c(p_i) \Delta p, \quad (55)$$

where Δp is the discretization interval on the property-specific probe path, $p_i = p_b + (i-1)\Delta p$ is the location at the beginning of the i th interval, and N is one plus the number of intervals. Recall that the final value e_{bf} of the beam extinction function and that the reduced detector responses $\tilde{E}'_n(p)$ are measured, and that the polarization-state-averaged signal attenuation function $\bar{e}_c(p)$ was calculated in the first step of the method of analyzing the data along a property-specific probe path. Therefore the band constants k_n of those band coefficients which contribute significantly to the TAMPSEC $\bar{\xi}$ can be calculated.

5.4.5.3 The Third Step. The third step of the method of analyzing the data along a property-specific probe path is to use equation (49) to calculate the beam extinction function $e_b(p)$ at each point along the property-specific probe path. At the beginning of the first interval of the property-specific probe path in the direction of propagation of the illumination, where the illumination is first entering the allowed contiguous sub-volume of the particle field, no extinction of the illumination has yet had a chance to occur, so the beam extinction function is initially unity there, *i.e.*, $e_b(p_1) = 1$. For $i \geq 2$, the value of $e_b(p_i)$ can be determined by solving the discretized form of equation (49). The result is

$$e_b(p_i) = 1 - \sum_1^i \left[\sum_n \left(\tilde{E}'_n(p) / k_n \bar{e}_c(p_i) \right) \right] \Delta p \quad (56)$$

For $n=1$, equation (56) reduces to

$$e_b(p_i) = 1 - (1/k_1) \sum_1^i \left(\tilde{E}'_1(p) / \bar{e}_c(p_i) \right) \Delta p \quad (57)$$

5.4.5.4 The Fourth Step. The fourth step of the method of analyzing the data along a property-specific probe path is to use equation (47) to calculate the time-averaged band coefficient $\bar{\xi}_n$ of every band coefficient that makes a significant contribution to the TAMPSEC $\bar{\xi}$. Recall that the polarization-state-averaged signal attenuation function $\bar{e}_c(p)$ was calculated in the first step of the method of analyzing the data along a property-specific probe path, the band constants k_n of those band coefficients which contribute significantly to the TAMPSEC $\bar{\xi}$ were calculated in the second step of the method of analyzing the data along a property-specific probe path, and the beam extinction function $e_b(p)$ was calculated in the third step of the method of analyzing the data along a property-specific probe path. Therefore the band coefficient $\bar{\xi}_n$ of every band coefficient that makes a significant contribution to the TAMPSEC $\bar{\xi}$ can be calculated using equation (47).

5.4.5.5 The Fifth Step. The fifth step of the method of analyzing the data along a property-specific probe path is to use equation (46) to calculate the TAMPSEC $\bar{\xi}$. Recall that the time-averaged band coefficient $\bar{\xi}_n$ of every band coefficient that makes a significant contribution to the TAMPSEC was calculated in the fourth step of the method of analyzing the data along a property-specific probe path.

5.4.5.6 The Sixth Step. The sixth step of the method of analyzing the data along a property-specific probe path is to use equation (47) to calculate the time-averaged band coefficient $\bar{\xi}_c$ of the time-averaged particle property $\bar{\zeta}$ of interest. This may already have been done if the time-averaged band coefficient of the particle property of interest is one of the time-averaged band coefficients which contributes significantly to the TAMPSEC. If the time-averaged band coefficient of the particle property of interest is one of the time-averaged band coefficients which contributes significantly to the TAMPSEC, then the band constant of the time-averaged band coefficient of the particle property of interest will be known, and it will be possible to calculate the time-averaged band coefficient of the particle property of interest. If the time-averaged band coefficient of the particle property of interest is not one of the time-averaged band coefficients which contributes significantly to the TAMPSEC, then the band constant of the time-averaged band coefficient of the particle property of interest will not be known, and it will be possible to calculate the time-averaged band coefficient of the particle property of interest only to within an unknown constant.

5.4.5.7 The Seventh and Final Step. The seventh and final step of the method of analyzing the data along a property-specific probe path is to use equation (52) to calculate the time-averaged particle property $\bar{\zeta}$ of interest. If the constant of proportionality k_ζ between the time-averaged particle property of interest and its time-averaged band coefficient is not known, then it will be possible to calculate the time-averaged particle property $\bar{\zeta}$ of interest only to within an unknown constant.

5.5 Variations of the Method of Analyzing the Data.

The general method of analyzing the data now having been described, variations of the method of analyzing the data may now be given. Variations of the method of analyzing the data include a variation where measurements are made over equal intervals along the probe path, a variation for property-specific probe paths oriented in the same direction in a regular Cartesian array, and a variation for property-specific probe paths oriented in the same direction in a linear array only within a plane of interest.

5.5.1 A Variation Where Measurements are Made over Equal Intervals Along the Probe Path. One variation of the method of analyzing the data is a variation where measurements are made over equal intervals along the probe path. Under the variation where measurements are made over equal intervals along the probe path, the method of using interpolation and extrapolation between the actual measurement points to project what the reduced detector response $\tilde{E}'_n(p)$ would have been measured to be over equal intervals along the probe volume is not required, unless it is numerically convenient to discretize the probe path into smaller intervals than the spacing between the measurements. Further variations of the most general embodiment of the method of analyzing the data depend mainly on variations in the method of probing the particle field with illuminating light.

5.5.2 A Variation for Property-Specific Probe Paths Oriented in the Same Direction in a Regular Cartesian Array. Another variation of the method of analyzing the data is a variation for property-specific probe paths oriented in the same direction in a regular Cartesian array. The variation for property-specific probe paths oriented in the same direction in a regular Cartesian array arises as a result of the variation of the general method of probing the particle field with illuminating light where more order or less randomness is introduced into the probing process, as discussed in section 3.2.2 of Chapter 3. Here, more order and less randomness is introduced by ordering all the property-specific probe paths in the same direction, and arranging all the probe paths into a regular Cartesian array. Ordering all the property-specific probe paths in the same direction, and arranging all the probe paths into a regular Cartesian array, is illustrated in Figure 12.

A perspective of the all the probe paths including an indication of the general location of the detector is shown in Figure 12(a), and the perspective of Figure 12(a) is rotated to show an end-on view of all the probe paths in the regular Cartesian array in Figure 12(b). When all the property-specific probe paths are oriented in the same direction, the coordinate “ p ” along the property-specific probe path of previous sections may be replaced with one of the Cartesian coordinates. In this case, the coordinate “ p ” is replaced by the coordinate “ x .” The discrete coordinates p_i may be replaced by the coordinates x_i , or more conveniently, simply with the index notation $i = 1, 2, 3, \dots$. The Cartesian coordinate “ y ” will therefore correspond to one of the offset directions between the property-specific probe paths, while the Cartesian coordinate “ z ” will correspond to the other offset direction, as illustrated in Figure 12. The discretized locations along the y and z axes are represented by the index notations $j = 1, 2, 3, \dots$ and $k = 1, 2, 3, \dots$, respectively. A detector located in such a way as to minimize the complexity of the analysis and to minimize the repeatability of the effective probe volume correction factor will predominantly view the property-specific probe paths through one of the planes defined by the Cartesian probing process. No at-

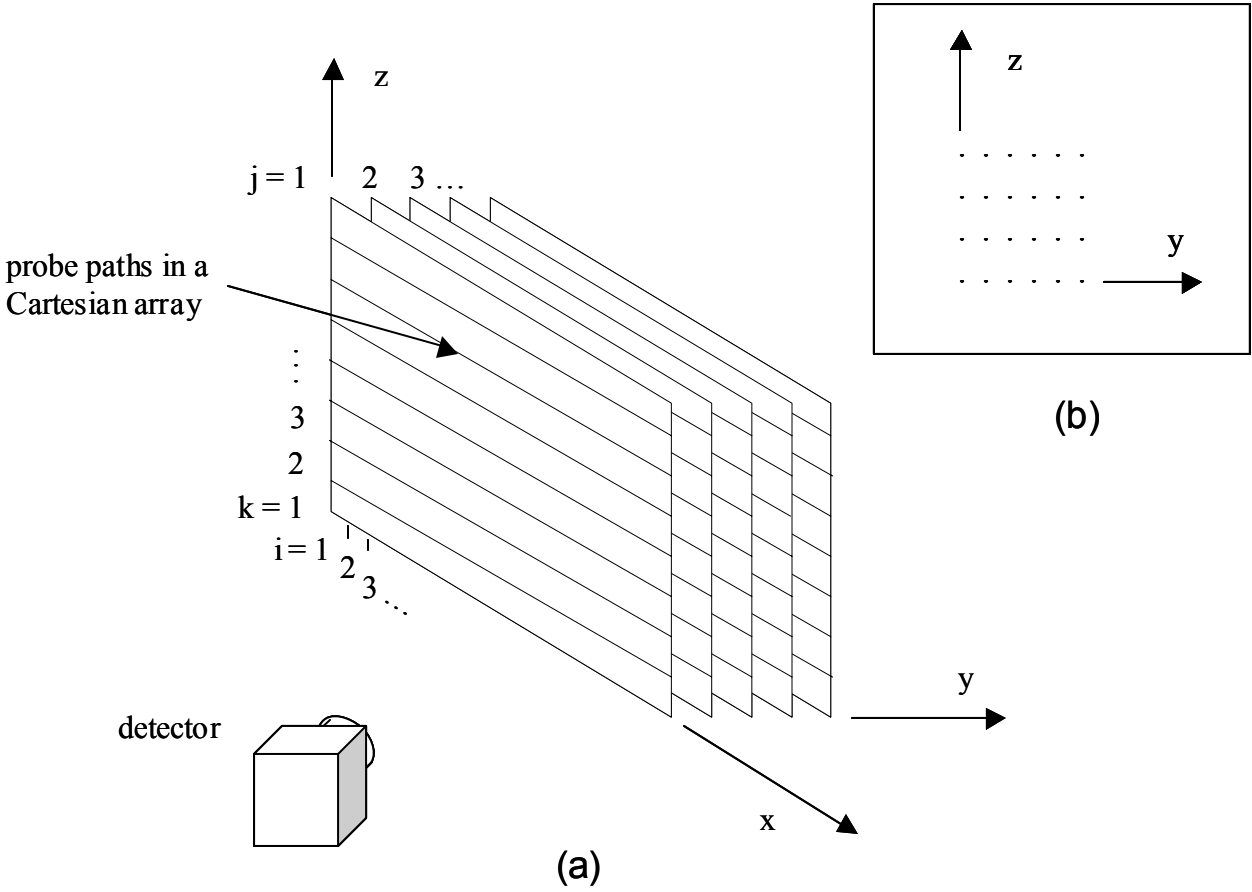


Figure 12. (a) Probe paths in a regular Cartesian array. (b) End-on view.

tempt has been made in Figure 12 to illustrate any repositionings of the detector, if any, which may be required to improve the repeatability of the effective probe volume correction factor. The plane defined by the Cartesian probing process through which the property-specific probe paths are predominantly viewed could be either the x - y plane or the x - z plane. The x - y plane is selected for illustration in Figure 12. The y - z plane would not normally be selected, because the small angle between the orientation of the property-specific probe paths and the viewing direction of the detector would probably be unacceptable by the criteria of sections 3.1.1.2.1 and 3.1.1.2.2 of Chapter 3, due to inadequate spatial resolution in the axial direction, and due to signal being produced from parts of the response volume where the detector response is non-uniform. The boundaries of the rectangular volume illustrated in Figure 12 will be assumed to represent the boundaries of the allowed contiguous sub-volume of the particle field. In this case, the method of selecting the order in which property-specific probe paths are analyzed will be exceedingly simple. The first layer of the allowed contiguous sub-volume of the particle field corresponding to the first step in section 5.4.1.1 of this chapter for which $\bar{e}_c = 1$ is simply the plane $j = 1$. Each probe path corresponding to a different k can be analyzed in turn, knowing that $\bar{e}_c(i,1,k) = 1$. The second layer corresponds to the plane $j = 2$. Although $\bar{e}_c(i,2,k)$ can be different from unity in this plane, all signals emanating from the plane $j = 2$ will travel only through the plane $j = 1$, where the TAMPSEC's are now known, before reaching the detector. Thus $\bar{e}_c(i,2,k)$ can be cal-

culated in the plane $j = 2$. The next layer is the plane $j = 3$, where signals will travel only through the planes $j = 1$ and $j = 2$, and so on, to successively higher values of j .

5.5.3 A Variation for Property-Specific Probe Paths Oriented in the Same Direction in a Linear Array only within a Single Plane of Interest. Yet another variation of the method of analyzing the data is a variation for property-specific probe paths oriented in the same direction in a linear array only within a single plane of interest. The variation for property-specific probe paths oriented in the same direction in a linear array only within a single plane of interest arises as a result of the most recommended method of probing the particle field with illumination, where the particle field is probed successively with a single beam of illuminating light only within the a single plane of interest, as indicated in section 3.3.2.1 of Chapter 3, and holding the illuminating optics fixed with the beam oriented in the same direction as the particle field is traversed relative to the beam, as indicated in section 3.3.2.2 of Chapter 3. Orienting the property-specific probe paths oriented in the same direction in a linear array only within a plane of interest is illustrated in Figure 13. In this case, the allowed contiguous sub-volume of the particle field is not the defined by the boundaries of the entire rectangular volume used in Figure 12. Rather, it is defined by the plane of interest plus a small distance above and below it defined by the thickness of the single beam of illuminating light. If signals are detected only from within this plane, then measurement and analysis of the entire three dimensional volume of Figure 12 will not be neces-

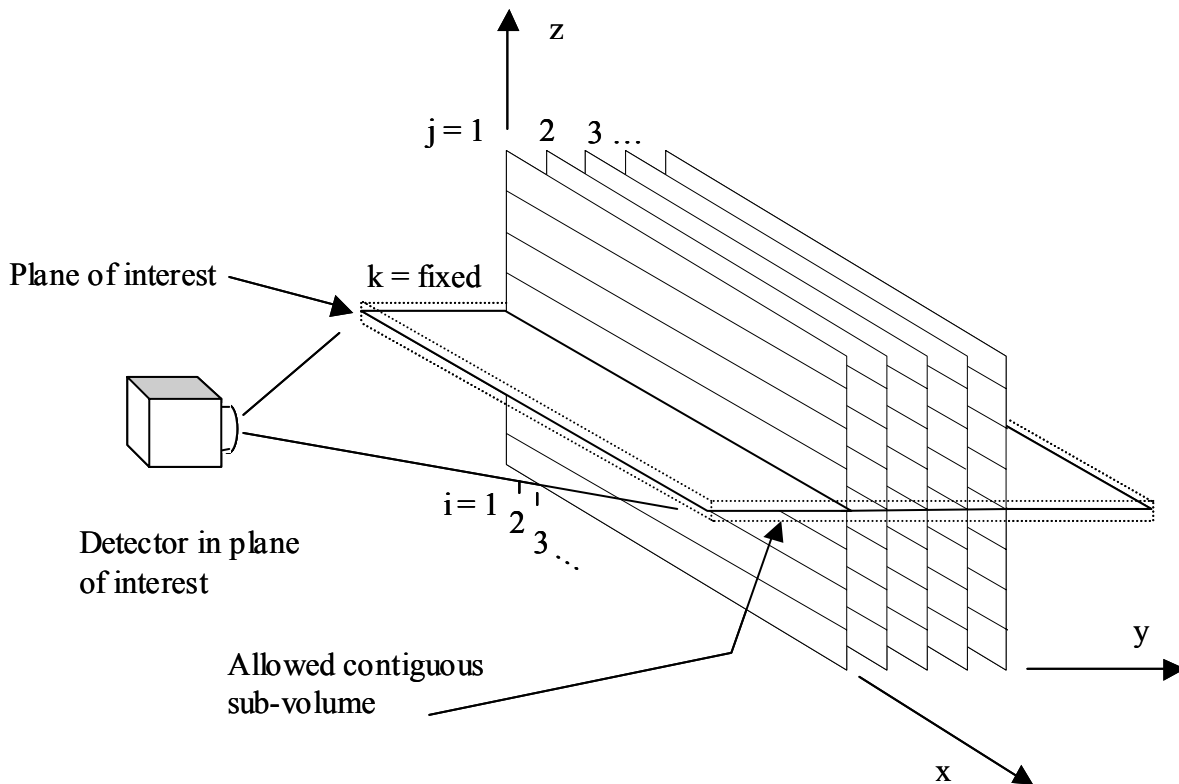


Figure 13. Probe paths and detector only within a plane of interest.

sary. The first layer where $\bar{e}_c = 1$ now corresponds to the line $j = 1$. Along this line, $\bar{e}_c(i,1) = 1$, where the k index has now been dropped because k is now fixed. The second layer is the line $j = 2$, where $\bar{e}_c(i,2)$ can now be calculated because the TAMPSEC's are now known along the line $j = 1$. The third and subsequent layers are the lines $j = 3$, $j = 4$, and so on, to successively higher values of j .

5.6 The Most Recommended Method of Analyzing the Data.

With the general method of analyzing the data and variations of the general method of analyzing the data now having been described, the most recommended method of analyzing the data may now be introduced. The most recommended method of analyzing the data contains the following elements:

- 5.6.1 Having either a single state of polarization in the illumination as discussed in section 5.3.7.6.1 or having the small detector approximation of the first kind apply as discussed in section 5.3.7.6.2, and having the effective probe volume correction factor be repeatable along the property-specific probe path as discussed in section 5.3.7.6.3, so that the initial intensity distribution of the single beam of illuminating light does not need to be known
- 5.6.2 Using the assumed properties of the particle field described in section 5.1 and the approximations related to the development of the theory described in section 5.2 to reduce the mathematical expression of the theory to the forms given in equations (46)-(49) and (51)-(52), reproduced below.

$$\bar{\xi}(p) = \sum_N \bar{\xi}_n(p), \quad (46)$$

$$\tilde{E}'_n(p) = k_n e_b(p) \bar{e}_c(p) \bar{\xi}_n(p), \quad (47)$$

$$1 - e_{bf} = \int_{p_0}^{p_f} \sum_N [\tilde{E}'_n(p) / k_n \bar{e}_c(p)] dp, \quad (48)$$

$$1 - e_b(p) = \int_{p_0}^p \sum_N [\tilde{E}'_n(p) / k_n \bar{e}_c(p)] dp, \quad (49)$$

$$\tilde{E}'_\zeta(p) = k_\zeta e_b(p) \bar{e}_c(p) \bar{\xi}_\zeta(p), \quad (51)$$

$$\bar{\xi} = g_\zeta \bar{\xi}_\zeta. \quad (52)$$

- 5.6.3 Using the small detector approximation of the first kind to reduce the expression for the polarization state averaged signal attenuation function to its simplest form given by equation (50), reproduced below.

$$\bar{e}_c(p) = \exp\left[-\int_0^{|\bar{x}_d - \bar{x}_{f,pv}|} \bar{\xi}(\bar{x}_{f,pv} + s\hat{n}_{fd}) ds\right] \quad (50)$$

- 5.6.4 Assuming that elastic scattering dominates the TAMPSEC such that $\bar{\xi} \approx \bar{\xi}_s$, giving $n = 1$ in equations (46)-(49) above.
- 5.6.4.1 When the particles are spherical, using the fact that the average particle surface area concentration will be proportional to the elastic scattering band coefficient, $\bar{a} = g_s \bar{\xi}_s$.
- 5.6.4.1.1 When geometric optics can be applied, taking $g_s = 2$.
- 5.6.5 Using fluorescence measurements to determine the distribution of the average particle volume concentration, to within an unknown constant.
- 5.6.6 Selecting the order in which property-specific probe paths are analyzed using the layering method described in section 5.4.1.
- 5.6.7 Determining the distance along the property-specific probe path to which the response of the property-specific single detector is assigned to correspond using the nearest distance approach described in section 5.4.2.
- 5.6.8 Selecting which detector responses to analyze by searching based on the responses of detectors assigned to the same or nearly the same distance along the property-specific probe path to determine the response for which the effective probe volume correction factor Σ is the most repeatable, as described in section 5.4.3.
- 5.6.8.1 Selecting the detector having the maximum response to be the detector response to analyze.
- 5.6.9 Discretizing the property-specific probe path into equal intervals according to the method described in section 5.4.4.
- 5.6.10 Analyzing the data along a property-specific probe path as described in section 5.4.5.
- 5.6.11 Analyzing the data when the property-specific probe paths are oriented in the same direction in a regular Cartesian array, as described in section 5.5.2.
- 5.6.11.1 Analyzing the data when the regular Cartesian array is a linear array in a single plane of interest, as described in section 5.5.3.

6.0 CONCLUSIONS

When optical radiation is incident on a particle field, some of the radiation may pass through the particle field without interacting with it, some of it may be absorbed, and the rest of it will be re-emitted, or “converted,” into other optical radiation over a range of wavelengths, states of polarization, and directions. The nature of the interaction will depend on the properties of the incident radiation, which in most cases will at least be partially under the control of the diagnostics technician, and on the properties of the particle field, which in most cases will not be. General conditions have been examined here under which detection of the converted radiation can be used to perform ensemble-based optical patterning. Special consideration was given to the conditions under which quantitative measurements might be possible in dense particle fields, where attenuation of the illumination light, attenuation of the signal light, and secondary emission can be significant. Emphasis was also placed on the case where the particle property of interest was the particle surface area concentration, for which the scattering band coefficient was selected to be the corresponding band coefficient, on the case where the particle property of interest was the particle volume concentration, for which the fluorescence band coefficient was selected to be the corresponding band coefficient, and the case where the particle properties of interest could be both.

It was anticipated that a successful measurement would require that the signal generated be proportional to the product of the local illumination energy times the concentration of the property at each point measured, as expressed in equation (1) from Chapter 1, reproduced below.

$$E_s \propto \zeta E \quad (1)$$

An examination of the general equations of optical radiation, however, found that such a simple relationship in general does not exist. A large number of assumptions, approximations, and conditions are required, as have been described in this report, before a relationship in the form of equation (1) can be derived. Fortunately, with modern instrumentation and sources of illumination, it appears that the required assumptions, approximations, and conditions can often be satisfied for many particle fields of practical interest.

It was found that a relationship in the form of equation (1) was not strictly required if the spatial distribution of the radiation intensity and other detailed measurements could be performed. For example, equations (26) and (27) from section 5.3.7.3 of Chapter 5 could be used. Given the practical limitations in performing such detailed measurements, however, it was desired to find a relationship where only the total illumination energy is involved. Further reduction of the general equations of optical radiation under conditions led, with additional required assumptions and approximations, to the expression given by equation (41) from section 5.3.7.7 of Chapter 5, reproduced below.

$$\tilde{E}'_n(p) = k_n e_b(p) \bar{e}_c(p) \bar{\xi}_n(p), \quad (41)$$

Here the band constant k_n is the constant or proportionality, and the beam extinction function $e_b(p)$ is effectively the local total illumination energy, since the definition of $\tilde{E}'_n(p)$ (section 5.3.7.7 of Chapter 5) includes normalization by the initial beam energy. One difference between equation (1) and equation (41) is the existence of the polarization-state-averaged signal attenua-

tion function $\bar{e}_c(p)$ in equation (41). However, equation (1) was cast in terms of the energy E_s produced at the measurement location, whereas equation (41) is cast in terms of the energy $\tilde{E}'_n(p)$ received by the detector. The quantity $\tilde{E}'_n(p)/\bar{e}_c(p)$ is the energy produced at the measurement location which is directed in the direction of the detector.

The other main difference is that $\bar{\xi}_n(p)$ is not the concentration of the particle property of interest, but is rather its associated band coefficient. Optical patterning by measuring converted optical radiation therefore directly measures only band coefficients. Relating the band coefficients to the particle property of interest requires that additional relationships be known, such as is given in equation (45) from section 5.3.10 of Chapter 5, reproduced below.

$$\bar{\zeta} = g_\zeta \bar{\xi}_\zeta \quad (45)$$

One implication of this is that in many cases it will not be possible to measure the concentration of the particle property of interest except to within an unknown constant. Determination of the constant g_ζ in general requires separate calibrations. An exception was pointed out for the case of spherical particles in the geometric optics regime, for which it is known that the constant relating the scattering band coefficient to the particle surface area concentration is equal to two, *i.e.*, $g_s = 2$.

When g_ζ is not known, it is tempting to combine the constants in equations (41) and (45) into a single constant. However, the corrections developed in this report require at least some of the band coefficients themselves to be known. The band constants that must be known are those that contribute significantly to the time-averaged monochromatic polarization-specific extinction coefficient, or the TAMPSEC. For these band constants, the band constants k need to be known independently from the property constants g .

The number of relationships required in addition to those given in this report was also found to depend on the number of band constants which contribute significantly to the TAMPSEC. Approaches that lead to the determination of the particle property of interest at least to within an unknown constant were given for the case where only the scattering band coefficient contributes significantly to the TAMPSEC, and the case where only the absorption band coefficient contributes significantly to the TAMPSEC. When more than one band coefficient contributes significantly to the TAMPSEC, relationships in addition to the ones given in this report will in general need to be determined.

It may be noted that the inability to quantitatively relate the band coefficients to particular particle properties is not always a disadvantage. One example of this is when ensemble-based optical patterning is used as a quality control measurement in the production of spray nozzles. In such applications, the pattern produced would be compared to some baseline pattern. Agreement with the baseline pattern to within certain parameters would constitute acceptance. In such cases, the distribution of the band coefficient itself could be compared, without the necessity to relate the band coefficient to any particle property. Another example was pointed out when it is desired to know the particle volume concentration to within a constant using fluorescence, where the scattering band coefficient is the only band coefficient that contributes significantly to the TAMPSEC. In this case only the scattering band coefficient itself needs to be determined in order to perform the required corrections without having to relate the scattering band coefficient to any particle property. In particular, for this case it is not necessary to relate the scattering band

coefficient to the particle surface area concentration. This in turn means that it is not necessary for the particles to be spherical. This is an important observation because non-spherical particles have tended to present serious difficulties for past optical patternation methods. In this case, the ability to relate the fluorescence band coefficient to the particle volume concentration when the particles are non-spherical would still need to be assessed, however.

Various methods of probing the particle field with illuminating light were considered. The most irreducible method was found to be by using a single beam of illuminating light. All other methods of probing the particle field with illuminating light, including planar light sheets and even volumetric probes, can be constructed from a combination of multiple simultaneous and sequential single beams of illuminating light.

Using single beams of illuminating light turns out to lead to two important advantages in addition to just being the most irreducible method of probing the particle field, however. First, by limiting the amount of light illuminating the particle field to just a single beam, the effects of secondary emission can be minimized. Thus using a single beam and sweeping the beam in a plane is expected to produce less secondary emission than illuminating the entire plane with a light sheet and collecting the data simultaneously over the entire plane. Secondly, using a single beam and sweeping the beam in a plane allows data to be collected within the same plane. The advantage of this is that tedious three dimensional mappings can be avoided, which otherwise would be required if data was collected outside the plane. This is an important consideration given that most applications of optical patternation only require information within a single plane, or within a limited number of planes. Three dimensional information is seldom required.

The method of probing the particle field with single beams of illuminating light was considered in the most general implementation possible. The result was the analogy that the total of all the probe paths could resemble a “random stack of hay or a random pile of toothpicks,” as discussed in section 3.1.2.3 of Chapter 3. The general method of analyzing the data was constructed to be consistent with this implementation. However, while technically feasible, in most cases there will be little motivation to implement the complexity of the most general method. Considerable simplification should almost always be possible. The most recommended method of probing the particle field with single beams of illuminating light was found to be systematically sweeping the beam only in a single plane.

A natural way to produce a single beam of illuminating light would be to use a laser beam. Considerable effort was made to allow for other possible sources of illumination. However, given all the considerations involved in conditioning the single beam of illuminating light for measuring the particle property of interest, as discussed in section 3.1.1 of Chapter 3, it is difficult to see how anything but a laser beam would be a practical method of satisfying all the considerations, especially the consideration of maintaining a repeatable effective probe volume correction factor, as discussed in section 3.1.1.2.10 of Chapter 3. Given the general availability of lasers, the most recommended method of probing the particle field with illuminating light provided in section 3.3 of Chapter 3 is to use a laser beam.

The most irreducible method of detecting the required signals was found in Chapter 4 to be to use a single detector, which was defined to produce only a single response to the entirety of signals it receives. All other methods of detecting the required signals can be constructed from a combination of multiple simultaneous and sequential single detectors. For example, a common CCD array can be considered to be an array of single detectors arranged behind a common set of collection optics.

As with probing the particle field with single beams of illuminating light, the most general implementation of performing the required measurements leads to a complexity which in almost all applications should not be required. Considerable simplification should almost always be possible. The most recommended method of performing the required measurements is to use a CCD detector, intensified or non-intensified as required.

Given that the most recommended method of probing the particle field with illuminating light is to use single beams of illuminating light, it is tempting to consider using linear arrays of single detectors instead of two dimensional arrays like CCD detectors. Linear arrays offer the natural advantage of reducing data collection times. Offsetting this natural advantage is the requirement to maintain a repeatable effective probe volume correction factor, as discussed in section 5.2.7.6.3 of Chapter 5. Using a two dimensional array with enough resolution to allow one to scan across a single axial location along the beam may allow one to perform comparisons to determine which detection to use in order to produce the most repeatable effective probe volume correction factor. The comparison selected in this report was to use the detection which gives the largest magnitude. Scanning a two dimensional array in this manner relaxes potentially significant alignment problems associated with the use of linear arrays. Conventional CCD cameras involve many hundreds of rows and columns of pixels. Two dimensional arrays having only a few rows (ten to twenty) and many hundreds of columns will offer the best compromise between data collection times and ease of alignment.

In summary, the method described in this report should improve the accuracy with which ensemble-based optical patterning can be performed in dense particle fields. A minimum requirement is that the all the particles in the path of illumination need to be illuminated, and the amount of illumination exiting the particle field needs to be measured. The particle field therefore cannot be so dense as to completely extinguish the illumination. Other limitations exist as well, as given by the assumptions, approximations, and conditions which were listed. The analysis which has been conducted provides a sound theoretical foundation from which to understand these limitations. Further insight into the method awaits experimental evaluation.

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APPENDIX A OPTICAL RADIATION THEORY

Radiation theory is the fundamental method of analysis applied to the optical patterning method. Radiation at optical wavelengths and exhibiting other optical properties is referred to here as *optical radiation*. The central concepts of optical radiation theory applicable to the analysis of optical patterning are developed in this Appendix. Further details not given here can be found in texts such as references [10-15].

A.1 Definition of a Single Beam of Illuminating Light

One of the central concepts of the optical patterning method developed in this report is the concept of a single beam of illuminating light. A *single beam of illuminating light* is defined to be light the propagation of which is limited to pass through two finite areas which are separated in space. Examples of single beams of illuminating light include a laser beam, a flashlight beam, a searchlight, and a sunbeam propagating through a window pane. A single beam of illuminating light and quantities applicable to it is illustrated in Figure A.1.

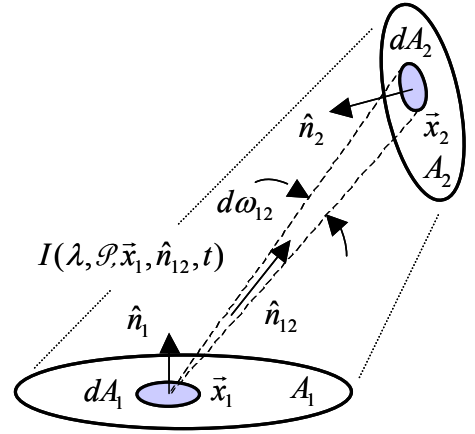


Figure A.1. A single beam of illuminating light.

A.2 General Properties of Optical Radiation

At each position \vec{x} in space and at each time t , optical radiation (illuminating light) will in general exist over a range of wavelengths λ . At each wavelength λ , optical radiation will in general also exist over a range of polarization states \mathcal{P} , where \mathcal{P} stands for the five polarization variables $\tilde{p}, a, b/a, \theta_a,$ and \tilde{c} . Here \tilde{p} is a polarization state variable which can take one of only two possible values, $\tilde{p} = \text{polarized}$ or $\tilde{p} = \text{unpolarized}$, a is the magnitude of the semi-major axis of the polarization ellipse, $0 \leq a \leq \infty$, b/a is the ratio of the magnitude of the semi-minor axis of the polarization ellipse to the semi-major axis, $0 \leq b/a \leq 1$, θ_a is an angle describing the orientation of the semi-major axis of the polarization ellipse relative to some reference angle, $0 \leq \theta_a \leq 2\pi$, and \tilde{c} is the handedness of the polarization ellipse which can take one of only two possible values, $\tilde{c} = \text{right-handed}$, or $\tilde{c} = \text{left-handed}$. Evidently, the latter four variables are not applicable when $\tilde{p} = \text{unpolarized}$. For each wavelength λ and each polarization state \mathcal{P} , optical radiation will in general be found to propagate over a range of directions \hat{n} , where \hat{n} is a unit vector representing the direction. Finally, for each wavelength λ , each polarization state \mathcal{P} , and each direction or propagation \hat{n} , optical radiation will have an instantaneous monochromatic polarization-specific intensity of radiation of $I(\lambda, \mathcal{P}, \vec{x}, \hat{n}, t)$.

A.3 The Instantaneous Monochromatic Polarization-Specific Intensity of Radiation

For the unpolarized component of optical radiation, the *instantaneous monochromatic polarization-specific intensity of radiation* $I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ is defined for each location \bar{x} in space and each instant t of time to be the instantaneous rate at which optical radiation energy leaves a surface, in a differential range of wavelengths $d\lambda$ around the wavelength λ , in the direction of the unit vector \hat{n} , per unit wavelength $d\lambda$, per unit solid angle $d\omega$ around \hat{n} , and per unit area perpendicular to \hat{n} . For the polarized component of optical radiation, the instantaneous monochromatic polarization-specific intensity of radiation $I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ is defined for each location \bar{x} in space and each instant t of time, to be the instantaneous rate at which optical radiation energy leaves a surface, at a specific handedness \tilde{c} of the polarization, in a differential range of wavelengths $d\lambda$ around the wavelength λ , in a differential range of polarization states da , $d(b/a)$, and $d\theta_a$ around the polarization states a , b/a , and θ_a respectively, in the direction of the unit vector \hat{n} , per unit polarization state a , b/a , and θ_a , per unit wavelength, per unit solid angle $d\omega$ around \hat{n} , and per unit area perpendicular to \hat{n} . Consider the instantaneous monochromatic polarization-specific energy in the single beam of illuminating light emitted from area A_1 in the direction of area A_2 , as illustrated in Figure A.1. Let dA_1 be a differential area on A_1 at position \bar{x}_1 having a unit outward normal \hat{n}_1 . Similarly, let dA_2 be a differential area on A_2 at position \bar{x}_2 having a unit outward normal vector \hat{n}_2 . Let \hat{n}_{12} be a unit vector describing the direction from dA_1 to dA_2 , where $\hat{n}_{12} = (\bar{x}_2 - \bar{x}_1) / |\bar{x}_2 - \bar{x}_1|$, and where a subscript convention has been adopted that the order of the subscripts implies a direction from the first subscript to the second subscript. For example, in this case, the subscript “12” implies a direction “1 \rightarrow 2.” If $I(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t)$ is the instantaneous monochromatic polarization-specific intensity of radiation at \bar{x}_1 emitted in the direction \hat{n}_{12} , then the instantaneous rate $d\dot{E}_{12}(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t)$ at which monochromatic polarization-specific energy is emitted from dA_1 towards dA_2 is

$$d\dot{E}_{12}(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t) = I(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t) |\hat{n}_{12} \cdot \hat{n}_1| dA_1 d\omega_{12} d\lambda d\mathcal{P}, \quad (\text{A.1})$$

where $|\hat{n}_{12} \cdot \hat{n}_1|$ is the magnitude of the dot product between \hat{n}_{12} and \hat{n}_1 , *i.e.*, the magnitude of the cosine of the angle between \hat{n}_{12} and \hat{n}_1 , and $d\mathcal{P}$ is a shorthand notation for the appropriate differential range of states of polarization, to be defined in Section A.4 next. Thus $|\hat{n}_{12} \cdot \hat{n}_1| dA_1$ is equal to the projection of dA_1 onto the plane perpendicular to \hat{n}_{12} . In the present configuration, the differential solid angle is given by $d\omega_{12} = |\hat{n}_{12} \cdot \hat{n}_2| dA_2 / |\bar{x}_2 - \bar{x}_1|^2$, where $|\bar{x}_2 - \bar{x}_1|$ is the distance between dA_1 and dA_2 , and where $|\hat{n}_{12} \cdot \hat{n}_2| dA_2$ is the projection of the area dA_2 normal to \hat{n}_{12} . Thus $d\dot{E}_{12}(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t) = I(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t) |\hat{n}_{12} \cdot \hat{n}_1| |\hat{n}_{12} \cdot \hat{n}_2| dA_1 dA_2 d\lambda d\mathcal{P} / |\bar{x}_2 - \bar{x}_1|^2$. Therefore, the total instantaneous rate at which monochromatic polarization-specific energy is radiated from A_1 in the direction of A_2 is given by

$$d\dot{E}_{12}(\lambda, \mathcal{P}, t) = d\lambda d\mathcal{P} \int_{A_2} \int_{A_1} \frac{I(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t) |\hat{n}_{12} \cdot \hat{n}_1| |\hat{n}_{12} \cdot \hat{n}_2|}{|\bar{x}_2 - \bar{x}_1|^2} dA_1 dA_2. \quad (\text{A.2})$$

The rate at which monochromatic polarization-specific energy crosses a surface is the *monochromatic polarization-specific power*. The total monochromatic polarization-specific power delivered over a period of time is the *monochromatic polarization-specific energy*.

A.4 A Shorthand Notation Convention for Denoting a Differential Range of States of Polarization

The mathematically correct way to write equation A.1 without resorting to shorthand notation is

$$\begin{aligned} d\dot{E}_{12}(\lambda, \mathcal{P}, \vec{x}_1, \hat{n}_{12}, t) = & [I(\lambda, \tilde{p} = \text{unpolarized}, a = n/a, b/a = n/a, \theta_a = n/a, \tilde{c} = n/a, \vec{x}, \hat{n}, t) \\ & + I(\lambda, \tilde{p} = \text{polarized}, a, b/a, \theta_a, \tilde{c} = \text{right - handed}, \vec{x}, \hat{n}, t) d\theta_a d(b/a) da \\ & + I(\lambda, \tilde{p} = \text{polarized}, a, b/a, \theta_a, \tilde{c} = \text{left - handed}, \vec{x}, \hat{n}, t) d\theta_a d(b/a) da] dA_1 d\omega_{12} d\lambda \end{aligned}$$

Given the tedious nature of this expression, a differential-like shorthand notation convention $d\mathcal{P}$ will be adopted that if $f(\mathcal{P}, \dots) = f(\tilde{p}, a, b/a, \theta_a, \tilde{c}, \dots)$ is some function that includes of the state of polarization, then

$$f(\mathcal{P}, \dots) d\mathcal{P}$$

will be by convention written as shorthand for

$$\begin{aligned} f(\tilde{p} = \text{unpolarized}, a = n/a, b/a = n/a, \theta_a = n/a, \tilde{c} = n/a, \dots) \\ + f(\tilde{p} = \text{polarized}, a, b/a, \theta_a, \tilde{c} = \text{right - handed}, \dots) d\theta_a d(b/a) da \\ + f(\tilde{p} = \text{polarized}, a, b/a, \theta_a, \tilde{c} = \text{left - handed}, \dots) d\theta_a d(b/a) da \end{aligned}$$

A.5 The Instantaneous Monochromatic Polarization-Specific Intensity of Irradiation

Optical *irradiation*, as distinguished from optical *radiation*, refers to optical radiation *impinging on* a surface, rather than leaving it. Irradiation properties are distinguished herein from radiation properties by the use of the prime symbol ($'$). For the unpolarized component of optical radiation, the *instantaneous monochromatic polarization-specific intensity of irradiation* $I'(\lambda, \mathcal{P}, \vec{x}, \hat{n}, t)$ is defined for each location \vec{x} in space and each instant t of time to be the instantaneous rate at which optical radiation energy impinges on a surface, in a differential range of wavelengths $d\lambda$ around the wavelength λ , from the direction of the unit vector \hat{n} , per unit wavelength $d\lambda$, per unit solid angle $d\omega$ around \hat{n} , and per unit area perpendicular to \hat{n} . For the polarized component of optical irradiation, the instantaneous monochromatic polarization-specific intensity of irradiation $I'(\lambda, \mathcal{P}, \vec{x}, \hat{n}, t)$ is defined for each location \vec{x} in space and each instant t of time to be the instantaneous rate at which optical radiation energy impinges upon a surface, at a specific handedness \tilde{c} of the polarization, in a differential range of wavelengths $d\lambda$ around the wavelength λ , in a differential range of polarization states da , $d(b/a)$, and $d\theta_a$ around the polarization states a , b/a , and θ_a , respectively, from the direction of the unit vector \hat{n} , per unit polarization state a , b/a , and θ_a , per unit wavelength, per unit solid angle $d\omega$ around \hat{n} , and per unit area perpendicular to \hat{n} . If $I'(\lambda, \mathcal{P}, \vec{x}, \hat{n}, t)$ is the monochromatic polariza-

tion-specific intensity of irradiation impinging on the differential area dA_2 from the direction of the differential area dA_1 in Figure A.1, then the instantaneous rate $d\dot{E}'_{12}(\lambda, \mathcal{P}, \bar{x}_2, \hat{n}_{12}, t)$ at which energy impinges on dA_2 from the direction of dA_1 is $I'(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t) |\hat{n}_{12} \cdot \hat{n}_2| dA_2 d\omega_{21} d\lambda d\mathcal{P}$, where $d\omega_{21} = |\hat{n}_{12} \cdot \hat{n}_1| dA_1 / |\bar{x}_2 - \bar{x}_1|^2$. Substituting in and integrating, the result is

$$d\dot{E}_{12}(\lambda, \mathcal{P}, t) = d\lambda d\mathcal{P} \int_{A_2} \int_{A_1} \frac{I'(\lambda, \mathcal{P}, \bar{x}_2, \hat{n}_{12}, t) |\hat{n}_{12} \cdot \hat{n}_1| |\hat{n}_{12} \cdot \hat{n}_2| dA_1 dA_2}{|\bar{x}_2 - \bar{x}_1|^2}. \quad (\text{A.3})$$

If the media between A_1 and A_2 is non-scattering, non-absorbing, and non-emitting, then conservation of energy requires that $\dot{E}'_{12}(\lambda, \mathcal{P}, t) = \dot{E}_{12}(\lambda, \mathcal{P}, t)$. A comparison of equation (A.2) and equation (A.3) then shows that $I'(\lambda, \mathcal{P}, \bar{x}_2, \hat{n}_{12}, t) = I(\lambda, \mathcal{P}, \bar{x}_1, \hat{n}_{12}, t)$ also.

A.6 Property-Specific Optical Radiation

It will often be desirable to consider only those components of the total optical radiation which are relevant in some way to a particular particle property. Such components of the total optical radiation will be referred to here as being *property-specific*. For example, property-specific optical radiation might include property-specific illumination or property-specific signals produced by the interaction of the property-specific illumination with the particle field. A *signal* is defined to be the components of optical radiation which have been identified for detection. In general, property-specific components of the total optical radiation may span over more than one wavelength or state of polarization. Determining property-specific quantities will therefore in general require integrations to be performed over a ranges of variables such as wavelengths or states of states of polarization. Thus a property-specific radiation intensity might be

$$I_{ps}(\bar{x}, \hat{n}, t) = \int_{\mathcal{P}_{ps}} \int_{\lambda_{ps}} I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) d\lambda d\mathcal{P}, \quad (\text{A.4})$$

where the subscript “*ps*” stands for “property-specific.”

A.7 Collimated and Nearly Collimated Optical Radiation

Collimated optical radiation is optical radiation which propagates only over a very narrow range of directions. Collimated optical radiation is to be distinguished from the optical radiation considered thus far which more generally has been considered to propagate over an arbitrarily wide range of directions. *Perfectly collimated optical radiation* is defined to be optical radiation which propagates in only a single direction direction. The instantaneous monochromatic polarization-specific intensity of perfectly collimated optical radiation can be expressed in the form $I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) = I(\lambda, \mathcal{P}, \bar{x}, t) \delta(\hat{n} - \hat{n}_0)$, where \hat{n}_0 is the single direction of propagation, and where $\delta(\bar{x}) \equiv \delta(x)\delta(y)\delta(z)$ is the product of three Dirac delta functions for each of the three coordinate directions. ($\delta(x) = \infty$ for any real scalar variable x when $x = 0$, $\delta(x) = 0$ when $x \neq 0$, and $\delta(x)$ has the property that $\int_{-\infty}^{\infty} \delta(x) dx = 1$). *Nearly collimated optical radiation* is defined to be optical radiation which propagates only over a very narrow range of directions at any location \bar{x} ,

and can be expressed in the form $I(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) = I(\lambda, \mathcal{P}, \bar{x}, t)\gamma(\hat{n} - \hat{n}_0(\bar{x}))$, where the direction of propagation is allowed to depend on \bar{x} in this case (picture a diverging or converging cone), and where $\gamma(\bar{x}) \equiv \gamma(x)\gamma(y)\gamma(z)$ is the product of three gamma functions for each of the three coordinate directions. ($\gamma(x)$ is very large but less than ∞ when $x = 0$, $\gamma(x)$ is allowed to be non-zero when $x \neq 0$, but only within a very narrow neighborhood of $x = 0$ (including a neighborhood of 0; $\delta(x)$ is a special case of $\gamma(x)$), and $\gamma(x)$ still has the property that $\int_{-\infty}^{\infty} \gamma(x)dx = 1$.) A consequence of collimated optical radiation may be understood with reference to Figure A.1. Previously, the differential area dA_1 could potentially emit optical radiation to any differential area dA_2 on A_2 . With perfectly collimated optical radiation, the differential area dA_1 can emit optical radiation to one and only one corresponding differential area dA_2 on A_2 . With nearly collimated optical radiation, the differential area dA_1 may be capable of emitting optical radiation to only a limited number of differential areas dA_2 on A_2 .

A.8 The Coherency of Optical Radiation

The *coherency of optical radiation* refers to the degree to which the phase of the electromagnetic radiation is correlated across various intervals of space and time. Whether or not the optical radiation is coherent will not need to be specifically considered in developing the current method of optical patterning. Therefore the topic of coherent optical radiation will not be discussed further.

A.9 Optical Radiation in Scattering, Absorbing, and Emitting Media

Optical radiation in scattering, absorbing, and emitting media needs to be considered because a particle field is a particular kind of scattering, absorbing, and emitting media. Consider incident irradiation having instantaneous monochromatic polarization-specific intensity of irradiation $I'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ at position \bar{x} and time t incident from some direction \hat{n} over some solid angle $d\omega$, in a scattering, absorbing, and emitting media, as illustrated in Figure A.2. If ds is the distance along the direction of propagation \hat{n} at \bar{x} , then the instantaneous decrease in beam intensity is given by $dI'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) = -\xi(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)I'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)ds$, where $\xi(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ is the *instantaneous monochromatic polarization-specific extinction coefficient*, written here for short as IMPSEC.

Some of the intensity removed from the beam will be converted into other kinds of optical radiation, for example, scattering or fluorescence. The rest of the intensity will simply be absorbed without being converted into any other kind of optical radiation. The IMSEC can be broken down into

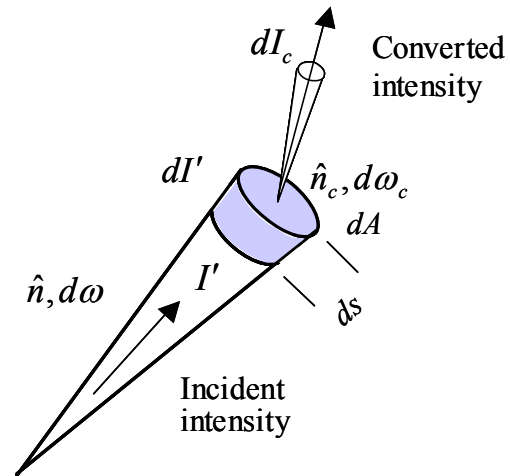


Figure A.2 Optical irradiation in a scattering, absorbing, and emitting media.

various components to reflect the various forms into which the removed intensity is converted. Define the *IMPSEC component* $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$, such that $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)I'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)d\lambda_c d\mathcal{P}_c ds$ gives the fraction of dI' which is converted into radiation propagating in all directions over a differential range of wavelengths $d\lambda_c$ around the wavelength λ_c and over a differential range of states of polarization $d\mathcal{P}_c$ around the state of polarization \mathcal{P}_c . An idealized plot of what $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ might look like as a function of λ_c for fixed values of all the other parameters is shown in Figure A.3.

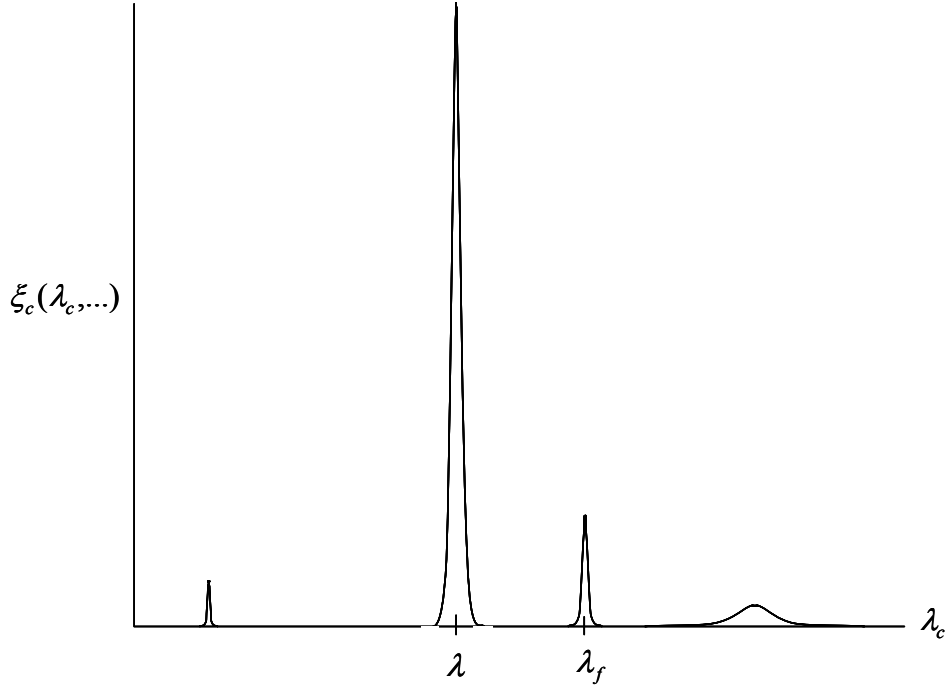


Figure A.3 Distribution of $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ with wavelength λ for fixed values of all other parameters.

Depending on the case, there can be large ranges, or bands, of wavelengths λ_c over which $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ will be zero. Over other wavelength bands, $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ can be non-zero. The different wavelength bands over which $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ is non-zero will in general correspond to different properties of the media. A wavelength band having a large peak is shown in Figure A.3 to exist over a narrow band of wavelengths where the wavelength of the converted radiation is nearly equal to the wavelength of the incident beam, *i.e.*, near where $\lambda_i \approx \lambda$. This would be due to a phenomenon known as *elastic scattering*, and the integral $\int_{\mathcal{P}_c} \int_{\lambda_{scat}} \xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t) d\lambda_c d\mathcal{P}_c$ over this range will be referred to here as the *elastic scattering band coefficient*, denoted $\xi_s(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$. The center of the elastic scattering band may not correspond exactly to λ if there has been a Doppler shift due to an average velocity of the media, and the width of the scattering peak will depend on the range of velocities in the media. Another

band of wavelengths is shown in Figure A.3 to occur around the wavelength denoted λ_f . This wavelength denotes radiation which is generated as fluorescence after absorption. The corresponding fluorescence band coefficient $\xi_f(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ is the integral of ξ_c over all the states of polarization and over all the wavelengths associated with the fluorescence. Any number of other wavelength bands may exist corresponding to inelastic scattering and to other linear and non-linear modes of interaction with the media. If a wavelength band has no other name, then the integral over all states of polarization and over all the wavelengths λ_c of the n^{th} band will be referred to as the n^{th} band coefficient ξ_n . The integral of $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ over all states of polarization \mathcal{P} and over all wavelengths λ will not in general add up to the total extinction coefficient $\xi(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$. Some of the optical radiation removed from the beam will simply be absorbed without being converted into any other form of radiation. Thus an *instantaneous monochromatic polarization-specific absorption coefficient* (IMPSEC) $\xi_{abs}(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t)$ may be defined as

$$\xi_{abs}(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) \equiv \xi(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) - \int_{\text{all } \mathcal{P}} \int_0^\infty \xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t) d\lambda_c d\mathcal{P}_c. \quad (\text{A.5})$$

In particle fields, it may often be that the elastic scattering band coefficient dominates all other band coefficients, including the absorption coefficient, such that the elastic scattering band coefficient will be approximately equal to the total extinction coefficient, *i.e.*, $\xi \approx \xi_s$. In other cases the IMPSEC may be dominated by only a limited number of band coefficients. Cases where the IMPSEC may be dominated by only a limited number of band coefficients greatly simplifies the method of performing the required measurements which was developed in section 3.

A.10 Distribution of the Converted Optical Radiation

Radiation removed from the beam which is converted into other kinds of radiation will in general emanate in different proportions in different directions. Define the *IMPSEC component distribution function* $P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}, t)$ such that $P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}, t) d\omega_c$ gives the fraction of the total amount of converted optical radiation emanating in all directions over a differential range of wavelengths $d\lambda_c$ around the wavelength λ_c and over a differential range of states of polarization $d\mathcal{P}_c$ around the state of polarization \mathcal{P}_c which emanates within a differential solid angle $d\omega_c$ around the \hat{n}_c direction. Since the sum of the fractions over all possible directions have to add up to the total, it must be that $\int_{4\pi} P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}, t) d\omega_c = 1$, where the total solid angle encompassed by all possible directions is 4π steradians. If the total energy converted in all directions over a differential range of wavelengths $d\lambda_c$ around the wavelength λ_c and over a differential range of states of polarization $d\mathcal{P}_c$ around the state of polarization \mathcal{P}_c is $\xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t) I'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) dA d\omega d\lambda_c d\mathcal{P}_c d\lambda d\mathcal{P} ds$, then the amount of energy emitted in the \hat{n}_c direction is

$$P(\lambda_c, \mathcal{P}_c, \hat{n}_c, \lambda, \mathcal{P}, \hat{n}, \bar{x}, t) \xi_c(\lambda_c, \mathcal{P}_c, \lambda, \mathcal{P}, \bar{x}, \hat{n}, t) I'(\lambda, \mathcal{P}, \bar{x}, \hat{n}, t) dA d\omega_c d\omega d\lambda_c d\mathcal{P}_c d\lambda d\mathcal{P} ds.$$

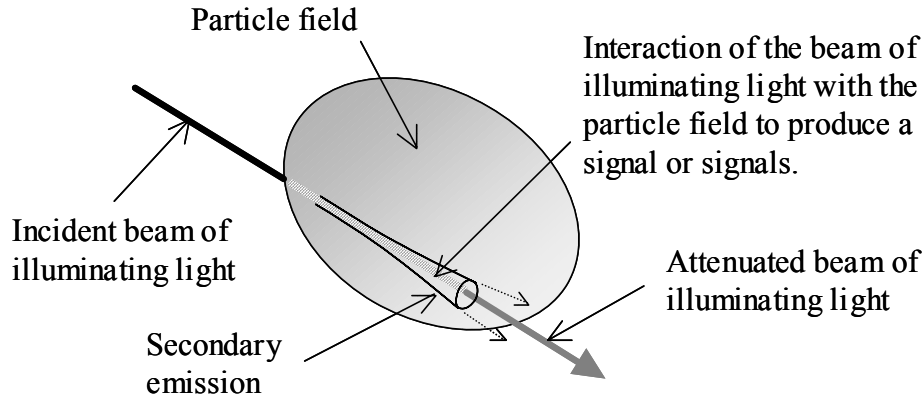


Figure A.4. A single beam of illuminating light interacting with a particle field.

A.11 The Particle Field as a Scattering, Absorbing, and Emitting Media

The scattering, absorbing, and emitting media in the present application is the particle field itself. The interaction of a single beam of illuminating light with a particle field is illustrated in Figure A.4, at a frozen instant in time. The particle field is shown for illustration purposes to have an ovoid shape in Figure A.4, although in general the particle field may have any shape. Spatial variations will in general be exhibited along the beam, due to two possible causes. One cause is attenuation of the beam by the particle field. The other cause of spatial variations along the beam may be time variations in the source of the beam itself, producing residual waves at the frozen instant in time, much like a bobbing boat can produce residual waves in water at a given instant in time. However, the latter cause of spatial variations would require time variations in the beam source to be on the order of the transit time of light across the particle field, or faster. The transit time of light across the particle field will in most applications be so much faster than any other characteristic time that any residual spatial variations will be negligible. Converted radiation emanating from the single beam of illuminating light will in general emanate in all directions. However, some directions may be preferred over others, leading to the IMPSEC component distribution function introduced in section A.10 above. The IMPSEC components, as well as the IMPSEC component distribution functions, will be related to the properties of the particles in the particle field. For example, for spherical particles, the particle surface area concentration will tend to be proportional to the elastic scattering band coefficient, *i.e.*, $a = g_s \xi_s$, where a is the particle surface area concentration (total particle surface area per unit volume in the particle field), and g_s is a constant of proportionality. For spherical particles in the geometric optics regime, where the particle sizes are much larger than the wavelength of the light, the elastic scattering cross section of a particle will be equal to twice its projected area, or half its total surface area [15], giving $g_s = 2$. The fluorescence band coefficient, on the other hand, will be more closely related to particle volume concentration (total particle volume per unit volume in the particle field). Although the fluorescence coefficient is not always linearly proportional to the total particle mass, Domann and Hardalupas [5] have found the relationship to be proportional for certain ranges of concentrations of fluorescent molecules. For these ranges, $\rho = g_f \xi_f$, where ρ is the particle volume concentration, and g_f is a constant of proportionality. In many particle ap-

plications including most sprays, the elastic scattering band coefficient will be so much larger than all the other coefficients that the other coefficients may be neglected in computing the total extinction coefficient, *i.e.*, $\xi \approx \xi_s$. Much of the converted radiation emanating from the single beam of illuminating light will leave the particle field entirely. However, some of this radiation will excite secondary emission. The regions where secondary emission occurs are represented by a growing conical shape in Figure A.4, and the emission is shown to emanate in the approximate direction of the cone. In some cases, secondary emission may in fact appear to be brightest in regions having approximately this same shape, and much of the light may be directed in the direction shown. However, secondary emission can in general come from any region in the particle field, and can emanate in any direction. All of the features mentioned above may vary at different instants of time.

APPENDIX B GENERAL CONCEPT OF A SINGLE DETECTOR

Another central concept of the optical patterning method developed in this report is the concept of a single detector. A *single detector* is defined to be a device which converts the selected signal or signals received into a single response, the magnitude of which can be uniquely related to the total signal energy received over the effective collection area of the collection optics and the period of detection. The *period of detection* is defined to be the total time over which the detector is sensitized to collect signals. A *signal* is defined to be the components of optical radiation which have been identified for detection. A *detection* is defined to be the single response of a single detector. A single detector will in general be composed of collection optics, a sensitive element, and a control system. The collection optics, sensitive element, and control system may often all be contained within the same enclosure, such as is the case with a conventional camera, but they may also be contained within separate enclosures. For example, the collection optics might include some components which are contained within the same enclosure as the sensitive element, and other components such as lenses contained in different enclosures which are close to the particle field. Many commercial devices allow the same hardware to easily vary certain features, such as apertures or exposure times. However, any change at all in any feature of a detector will, for the purposes of the discussion herein, be considered to create a different "detector."

B.1 Collection Optics

The function of the collection optics is to collect the signal or signals received over an effective collection area during the period of detection, condition the signal or signals according to the method of performing the required measurements developed in Section 3, and direct the result onto a sensitive element. The *effective collection area* is the area over which signals enter the collection optics that are actually detected, *i.e.*, that are actually directed onto the sensitive element. The effective collection area is to be distinguished from other areas of the collection optics over which signal might enter, but which might be prevented from reaching the sensitive element, for instance by a spatial mask. Components that might potentially be among the elements of the collection optics would include lenses (convex, concave, compound, cylindrical), lens coatings, mirrors (flat, concave, convex), prisms, gratings, spatial filters, masks, apertures, wavelength filters (high-pass, low-pass, band pass, notch), polarization filters, attenuators, beam splitters, and fiber optics.

B.2 The Sensitive Element

The function of the sensitive element is to convert the signal directed onto it by the collection optics into a single recordable response, the magnitude of which can be uniquely related to the total signal energy directed onto it over the effective collection area of the collection optics, and over the period of detection. Potential sensitive elements would include photographic film, magnetic tape, photomultiplier tubes, photo diodes, charge coupled devices, charge injection devices, and micro channel plates. Potential recordable responses of sensitive elements would include a voltage, a current, a frequency, a deflection of an electron beam, light of a particular wavelength or wavelengths, and a gray level or color on photographic film.

B.3 The Control System

The function of the control system is to control various detector settings, including the period of detection. Potential ways a control system might control the period of detection would include a shutter, an electronic gate of an image intensifier, or controlling the power provided to the sensitive element.

APPENDIX C THE EFFECTIVE PROBE VOLUME

Yet another central concept of the optical patterning method developed in this report is the concept of the effective probe volume. The effective probe volume is key to the method of relating the measurements to the properties of the particle field. It arises as a result of the most general method of probing the particle field with illuminating light, namely, by using a single beam of illuminating light, in combination with the most general method of performing the required measurements, namely, by using a single detector. The *effective probe volume* is defined to be a single volume selected to represent all the probe volumes created by the intersection of a single beam of illuminating light with the different response volumes of a single detector. In general a different effective probe volume will need to be defined for each particle property of interest. Definitions of the terms used in the above definition of the effective probe volume are developed below.

C.1 Definition of the Response Volume of a Detector to a Single Specific Particle Under Specific Conditions of Illumination

If the volume around a single detector is swept by a single specific particle under specific conditions of illumination (specifically defined intensity distribution, wavelength distribution, distribution of states of polarization, and time variation), the single detector will in general produce a response to the presence of the particle only when the particle is within a certain volume. When the particle is outside of this volume, the detector will produce no response to the presence of the particle. The volume over which a single detector will produce a response to a single specific particle under specific conditions of illumination will be referred to here as the *response volume* of the detector, to that specific particle, under those specific conditions of illumination.

C.2 Definition of the Volume of Nearly Uniform Response of a Detector to a Single Specific Particle Under Specific Conditions of Illumination

Even when a single specific particle is within the response volume of a detector, the magnitude of the response of the detector will in general not be the same at every position in the response volume. However, often a detector can be designed so that its response will be nearly uniform over some sub-volume of the response volume, to within some maximum variation. The volume over which the response of a detector is nearly uniform to within some maximum variation is defined here to be the *volume of nearly uniform response* of a detector, to a single specific particle, under specific conditions of illumination. In some cases, the boundaries of the volume of nearly uniform response will closely approach the boundaries of the response volume itself, and there will only be a small distance over which the response of the detector is non-uniform. In other cases, the boundary of the volume of nearly uniform response will not closely approach the boundaries of the response volume, producing large regions where the response of the detector is not uniform.

C.3 Example of the Response Volume and the Volume of Nearly Uniform Response of a Detector

To illustrate the response volume and the volume of nearly uniform response of a detector to a single specific particle under specific conditions of illumination, consider the examination of a speck of dust under a common microscope, as illustrated in Figure C.1. The illumination of a common microscope is continuous back lighting with diffuse light. When the speck of dust is in focus at the focal plane of the microscope, as in position 1 of Figure C.1, the lateral boundaries of the response volume are given by the familiar round field of view of a microscope. If the speck of dust is outside this round field of view, as in position 2 of Figure C.1, the speck of dust cannot be seen. The speck of dust at position 2 of Figure C.1 is outside of the response volume of the microscope. Within the field of view, the response of the microscope is uniform over most of the field of view. In other words, the speck of dust looks equally black and has the same apparent area and shape no matter where it is. Thus the lateral boundaries of the volume of nearly uniform detector response closely approach the lateral boundaries of the response volume itself when the speck of dust is in the focal plane of the microscope. Now let the axial distance of the speck of dust from the microscope lens be varied away from the focal plane by small amounts. Over a short axial distance around the focal plane, the particle will still appear to be in focus and to remain equally black and to have the same area and shape no matter where it is within the field of view. The cylindrically shaped volume formed by the round edges of the field of view and the small axial distance over which the response of the microscope does not change will define the volume of nearly uniform detector response for the microscope. As the axial distance of the speck of dust is further varied outside of this narrow range around the focal plane, the particle will no longer appear to be in focus, it will no longer appear to be as black, and its apparent size and shape may change, as illustrated at position 3 of Figure C.1. This is the region of non-uniform response of the response volume of the microscope. Eventually, if the speck of dust is placed too far away from or too close to the microscope, it will not be possible to see the speck of dust at all. The fore and aft distances beyond which the speck of dust can no longer be seen define the fore and aft boundaries of the response volume.

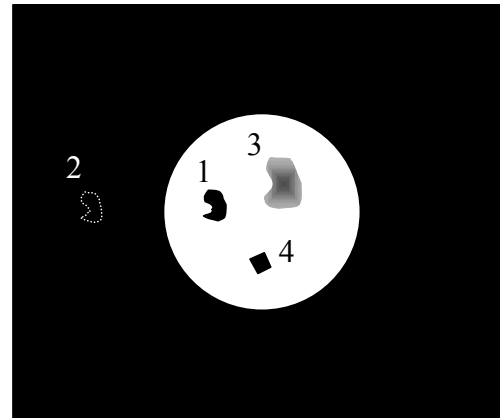


Figure C.1. Examination of a speck of dust under a microscope. 1. In the focal plane. 2. Outside the field of view. 3. Outside the focal plane. 4. Different speck of dust.

C.4 Dependency of the Response Volume of a Detector on the Specific Particle Properties and on the Specific Conditions of Illumination

The size, shape, and extent of the response volume of the detector and the uniformity of the detector response within it will depend in part on the design of the detector. However, even for a fixed detector, the size, shape, and extent of the response volume and the uniformity of the detector response within it will in general also depend on the specific particle properties and the specific conditions of illumination. For example, a useful kind of particle to consider is a particle

small enough that the light originating from it in response to illumination appears to come from a single point. If all the light originating from such a particle is spread equally over all directions, then the response volume might be contained between a limited fore and aft distance in the direction to the detector. On the other hand, if all the light originating from such a particle is directed only in the direction of the detector, as a mirror might reflect a laser beam, then the fore and aft extents of the response volume might become nearly unlimited. Now suppose the small particle is indeed a tiny mirror. If the illuminating light is a laser beam, then the fore and aft extents of the response volume might be nearly unlimited, as before. However, if the illuminating light is the slightly diverging incoherent light of a search light or a flash light, the fore and aft extents of the response volume might again become more limited. In this latter case where the illuminating light is a search light or a flashlight, the fore and aft extents of the response volume could vary depending on the size of the particle. In this case where the particle has the properties of a tiny mirror, the response volume would also depend sensitively on the orientation of the particle, becoming nearly zero for all except very specific orientations. If the illumination light varies during the detection period, all of the above response volumes could also become modified. Thus the size, shape and extent of the response volume and the uniformity of the detector response within it will in general depend on a large number of factors, including the intensity, direction, collimation, state of polarization, and the degree of time variation of each wavelength in the illumination, as well as particle properties such as the particle size, shape, optical properties, and orientation with respect to the detector and the beam of illuminating light. An important implication of this observation is that there can potentially be as many different response volumes as there are particle detections.

C.5 Dependency of the Volume of Nearly Uniform Response of a Detector on the Specific Particle properties and on the Specific Conditions of Illumination

The dependency of the response volume of a detector on the specific particle properties and the specific illumination conditions implies that the volume of nearly uniform response of the detector will also depend on the specific particle properties and the specific conditions of illumination. In application, there can also potentially be as many different volumes of nearly uniform response as there are particle detections.

C.6 Regions of Overlap Between the Different Volumes

Although the extents of each response volume may vary, the response volumes will in general tend to overlap in space. The regions of overlap will define a common volume. For example, particles of many sizes and shapes will produce a response proportional to their projected area in a microscope if they are all in the focal plane of the microscope. This is illustrated at position 4 of figure 4. The focal plane of the microscope is part of the volume which all the response volumes share in common. Likewise, the volumes of nearly uniform detector response will also tend to overlap in space. The regions of overlap will also define a common volume. The volume defined by the regions of overlap between the volumes of nearly uniform detector response will in general be a sub-volume of the volume defined by the regions of overlap between the response volumes. However, the volumes of nearly uniform detector response will often tend to vary in spatial extent around their common volume to a much smaller extent than the response volumes vary around their larger common volume. For example, the volumes of nearly uniform detector

response of a microscope will tend to depend on the size of the specks of dust to a much smaller extent than the response volumes themselves.

C.7 Consequences of the Variability of the Response Volume of a Detector on Measuring Particle Properties

The consequences of the variability of the response volume of a detector on measuring particle properties may now be assessed. There are three main consequences. First, when the detector response is variable over the response volume, errors can be introduced in relating the detector response to a particle property, because the same particle under the same conditions of illumination will produce a different response depending on where it is located in the response volume. Secondly, if the response volume is too large, the spatial resolution with which a given detection can be related to a specific location in space may be adversely impacted. Exacerbating these two errors is the dependency of the response volume on specific particle properties and on the specific conditions of illumination. The variability in the spatial extent of the detector response volumes can bias the measurement by favoring particles having larger response volumes over others.

C.8 Strategy for Minimizing Errors Caused by the Variability of Detector Response Volumes

One strategy for minimizing errors caused by the variability of detector response volumes would be to limit the illumination only to regions where the response of the detector is locally uniform and repeatable. This reduces error because regions where the detector response is different cannot produce a signal if they are not illuminated. The strategy of limiting illumination only to regions where the response of the detector is locally uniform and repeatable is one reason why single beams of illuminating light are used in the most general method of probing the particle field with illuminating light, as described in Section 3. The volume created by the intersection of the single beam of illuminating light with a response volume of a detector is defined to be a *probe volume*. Note that as long as the response of the detector is locally uniform and repeatable, and as long as it remains possible to interpret the response of the detector, it is not strictly necessary that the illumination correspond to regions where the volumes of nearly uniform detector response overlap. For example, consider a beam of illuminating light which is slightly beyond the focal plane of the detector optics, such that the beam is slightly out of focus. The probe volume thus formed may still be usable, depending on the case. However, in most instances it will be the most advantageous to locate the probe volume where the volumes of nearly uniform detector response overlap. In general, there can still be as many probe volumes as there are response volumes. However, the variability of the probe volumes will be much smaller than the variability of the response volumes. By further reducing the illumination only to small regions where the detector response is uniform and repeatable, it will often be possible to even further reduce the variability of the probe volumes to a point where most if not all of the probe volumes may be nearly identical. In many cases the lateral extent of the probe volume will be limited by the design of the detecting optics, while the fore and aft extent of the probe volume in the direction towards the detector will be limited by the thickness of the single beam of illuminating light.

C.9 Definition of the Effective Probe Volume

With particle based methods, which are capable of distinguishing single particles, it is sometimes possible to develop corrections for how the probe volume size changes in response to particle properties such as the particle size. However, with ensemble based methods such as the method described in this report, the response only to total particle properties without distinguishing individual particles makes it generally impossible to develop such corrections. With suitable design of the detector and the single beam of illuminating light, the variability in probe volumes can be significantly reduced, but it may not always be possible to entirely eliminate the variability. What variability remains will be a source of measurement error. As it is necessary to relate the ensemble measurement to *some* probe volume, a single volume must be selected to represent all of the probe volumes. This single volume will be referred to here as the *effective probe volume*. Some latitude will generally exist in determining which volume to select to be the effective probe volume. In general, the effective probe volume will not be larger than the largest expected probe volume, nor smaller than the smallest expected probe volume. It may be selected to represent some average of the expected probe volumes. Since different detectors will in general be used for different particle properties of interest, a different effective probe volume will in general need to be defined for each particle property of interest.

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