

# Adaptive Beampattern Control Via Linear and Quadratic Constraints for Circular Array STAP

Kristine L. Bell

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## **Abstract**

A general framework for adaptive and non-adaptive space-time beampattern synthesis using quadratic beampattern constraints with linearly constrained minimum variance (LCMV) beamforming has been developed. Main beam and sidelobe pattern control is achieved by imposing a set of inequality constraints on the weighted mean-square error between the adaptive pattern and a desired beampattern over a set of angle-Doppler regions. An iterative procedure for satisfying the constraints is developed which can be applied as post-processing to standard LCMV beamformers. The algorithm is used to synthesize a nearly uniform sidelobe level quiescent pattern for the circular UHF Electronically Scanned Array (UESA), and to control sidelobe levels for the same array in an adaptive manner. The technique has been generalized for general rank reducing transformations to reduce computational complexity. Performance results using data provided by Lincoln Lab show that under low sample support conditions, sidelobes can be effectively suppressed while maintaining high signal-to-interference plus noise ratio, and deep nulls on clutter and interferers.

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# 1 Introduction

Space-Time Adaptive Processing (STAP) used in airborne radar systems combines signals from  $N$  antenna array elements and  $M$  pulses to adaptively suppress clutter and jamming in both the space (angle) and time (Doppler frequency) dimension [1]. Traditionally, STAP systems have used a rotating linear array configuration, however a fixed circular ring array is currently under development under the UHF Electronically Scanned Array (UESA) program sponsored by the Office of Naval Research (ONR). The array consists of 54 directional antenna elements with suppressed backlobes. Only 20 of the elements will be used at a time to transmit and receive [2]. With this configuration, the antenna can be scanned mechanically in  $6.67^\circ$  increments by choosing the appropriate 20-element sector, and scanned electronically  $\pm 3.33^\circ$  with the chosen sector of elements. The circular array configuration has the potential to provide continuous  $360^\circ$  availability, however it has some potentially negative impacts for STAP algorithms [2]. First, the clutter rank is increased in a manner similar to the increase from misalignment with the velocity vector in linear arrays. Second, the clutter locus varies with range. This decreases the number of range gates that can be averaged to reliably estimate the clutter covariance matrix.

The foundation of most STAP techniques is the Minimum Variance Distortionless Response (MVDR) processor [1]. The standard MVDR processor weights are designed to minimize the processor output power subject to a linear distortionless constraint in the angle-Doppler steering direction. The MVDR beamformer can have unacceptably large sidelobes and mainlobe squinting due to sensor perturbations, pointing error, and low sample support. In radar systems, this behavior can lead to increased false alarms from clutter and unexpected interferers. Techniques for improved pattern control include steering vector tapering using a quiescent pattern with desirable main beam and sidelobe characteristics [3], linear main beam constraints [4], white noise gain constraints [5], and quadratic quiescent pattern constraints [6]-[10], and reduced rank subspace techniques [11]-[13].

To mitigate this problem, we have developed a general framework for adaptive and non-adaptive beampattern synthesis for non-linear arrays based on LCMV beamforming with quadratic beampattern constraints (QPC). Some of the results have been published in [14]-[17]. Additional publications are in preparation (including [18]-[19]). In this technique, main beam and sidelobe pattern control is achieved by imposing a set of inequality constraints on the weighted mean-square error between the adaptive pattern and a desired beampattern over a set of angle-Doppler regions. An important feature of the LCMV-QPC formulation is the specification of multiple quadratic pattern constraints. By proper choice of the number of constraints, the angle-Doppler regions to which they apply, and the desired beampatterns in those regions, the level of pattern control can be traded off against algorithmic complexity. At one extreme, low-complexity techniques can be obtained based on one or two constraints similar to the adaptive pattern control methods in [6]-[9]. At the other extreme, we can achieve tight pattern control using

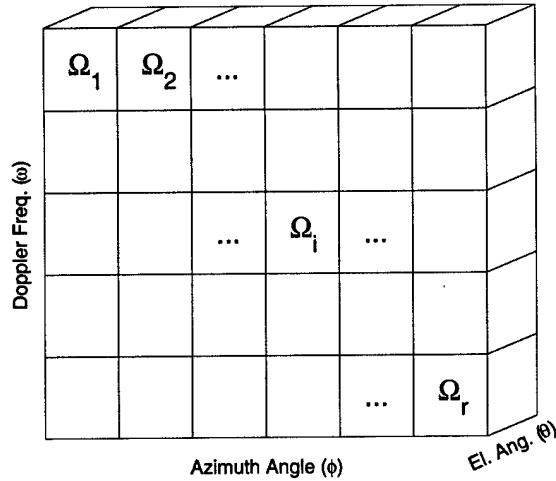


Figure 1: Partition of Angle-Doppler space

many constraints, in a manner similar to the technique in [10]. Between the two extremes, our approach using several constraints has been shown to achieve good pattern control and maintain a high SINR with reasonable complexity. The algorithm uses an iterative procedure for satisfying the constraints which can be applied as post-processing to standard STAP processors. The technique can also be used for non-adaptive pattern synthesis. It generalizes the techniques in [20] and [10] for developing low sidelobe quiescent patterns for arbitrary arrays, and can be used for developing the desired patterns used in the adaptive method. In this report, we describe the algorithm and present circular array STAP results with data provided by MIT Lincoln Lab [21].

## 2 LCMV Beamforming with Quadratic Pattern Constraints

### 2.1 Direct LCMV Problem Formulation

We assume a STAP model with  $N$  antenna elements and  $M$  pulses. Let  $\mathbf{v}(\theta, \phi, \omega)$  denote  $NM \times 1$  space-time array response vector to a signal arriving with elevation angle  $\theta$ , azimuth angle  $\phi$ , and Doppler frequency  $\omega$ . We partition azimuth angle-Doppler space into  $r$  sectors,  $\Omega_1, \dots, \Omega_r$ , as shown in Figure 1. In this illustration, the elevation angle space has only one partition and the sectors are cubes, however more general partitions of azimuth angle, elevation angle, and Doppler space can be used. Let  $B_{d,i}(\theta, \phi, \omega) = \mathbf{w}_{d,i}^H \mathbf{v}(\theta, \phi, \omega)$  be a desired beampattern in the region  $\Omega_i$ , and  $\mathbf{w}_{d,i}$  be the corresponding weight vector. The MSE between the beampattern generated by the adaptive weight vector  $\mathbf{w}$  and the desired beampattern over the region  $\Omega_i$  is given by

$$\epsilon_i^2 = \int_{\Omega_i} |\mathbf{w}^H \mathbf{v}(\theta, \phi, \omega) - \mathbf{w}_{d,i}^H \mathbf{v}(\theta, \phi, \omega)|^2 d\Omega. \quad (1)$$

The error can be written compactly as

$$\epsilon_i^2 = (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}) \quad (2)$$

where

$$\mathbf{Q}_i = \int_{\Omega_i} \mathbf{v}(\theta, \phi, \omega) \mathbf{v}(\theta, \phi, \omega)^H d\Omega. \quad (3)$$

Thus the pattern error is a quadratic function of the adaptive weight vector.

Adaptive weights are designed according to the standard LCMV criterion, while limiting the deviations from the desired pattern using quadratic pattern constraints. Let  $\mathbf{R}$  be the data covariance matrix,  $\mathbf{C}$  be the  $NM \times d$  constraint matrix, and  $\mathbf{f}$  be the  $d \times 1$  vector of constraint values. The LCMV-QPC optimization problem is

$$\begin{aligned} \min \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{st.} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \\ \text{st.} \quad & (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}) \leq L_i \quad i = 1, \dots, r \end{aligned} \quad (4)$$

The covariance matrix  $\mathbf{R}$  may be the noise and interference covariance matrix  $\mathbf{R}_n$  or the signal plus noise and interference covariance matrix  $\mathbf{R}_x$ , depending on the application. In practice, neither one is known exactly and must be estimated from data. In many radar problems, signal-free training data is used to estimate  $\hat{\mathbf{R}}_n$ . In most other applications, including active sonar and communications, the signal of interest is always present and  $\hat{\mathbf{R}}_x$  must be estimated. One commonly used estimate is the  $K$ -sample covariance matrix

$$\hat{\mathbf{R}}_x = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k) \mathbf{x}(k)^H \quad (5)$$

where  $\mathbf{x}(k)$  is the  $N \times 1$  vector of array data at time  $k$ .

The LCMV-QPC weight vector has the form

$$\begin{aligned} \mathbf{w} = & \mathbf{R}_Q^{-1} \mathbf{C} \left( \mathbf{C}^H \mathbf{R}_Q^{-1} \mathbf{C} \right)^{-1} \mathbf{f} \\ & + \left[ \mathbf{R}_Q^{-1} - \mathbf{R}_Q^{-1} \mathbf{C} \left( \mathbf{C}^H \mathbf{R}_Q^{-1} \mathbf{C} \right)^{-1} \mathbf{C}^H \mathbf{R}_Q^{-1} \right] \mathbf{w}_Q \end{aligned} \quad (6)$$

where

$$\mathbf{R}_Q = \mathbf{R} + \sum_{i=1}^r \lambda_i \mathbf{Q}_i \quad (7)$$

$$\mathbf{w}_Q = \sum_{i=1}^r \lambda_i \mathbf{Q}_i \mathbf{w}_{d,i}. \quad (8)$$

This is the multiple constraint extension of the quadratically constrained MVDR processor developed in [8], [9].

It can be simplified further by defining

$$\mathbf{P}_Q = \mathbf{R}_Q^{-1} - \mathbf{R}_Q^{-1} \mathbf{C} \left( \mathbf{C}^H \mathbf{R}_Q^{-1} \mathbf{C} \right)^{-1} \mathbf{C}^H \mathbf{R}_Q^{-1}. \quad (9)$$

The solution then becomes

$$\mathbf{w} = \mathbf{R}_Q^{-1} \mathbf{C} \left( \mathbf{C}^H \mathbf{R}_Q^{-1} \mathbf{C} \right)^{-1} \mathbf{f} + \mathbf{P}_Q \mathbf{w}_Q. \quad (10)$$

In this form, we see that the first term satisfies the linear constraint, while the second term is orthogonal to the constraint vector  $\mathbf{C}$  and provides additional pattern control.

In this processor a weighted sum of 'loading' matrices  $\mathbf{Q}_i, i = 1 \dots r$  are added to  $\mathbf{R}$ , and a weighted sum of desired weight vector terms  $\mathbf{Q}_i \mathbf{w}_{d,i}, i = 1 \dots r$  appears in the second term. The loading factors balance the adaptive pattern with the desired pattern. The relative contribution of these terms can be adjusted to achieve pattern control while maintaining high signal-to-interference-plus-noise ratio (SINR). There are generally a set of optimum loading levels  $\lambda_i, i = 1 \dots r$  that satisfy the constraints, however there is no closed form solution for the loading levels, even when  $r = 1$ . It can be shown that the mean-square pattern error decreases with increasing  $\lambda_i$ , but at the expense of decreased interference suppression. The loading levels must be chosen judiciously to achieve pattern control while maintaining high signal-to-interference-plus-noise ratio (SINR).

When the array is a uniformly spaced linear array, and there is one sector which covers the entire angle-Doppler space,  $\mathbf{Q} = \mathbf{I}$ , and the technique is the same as diagonal loading. When one or two quadratic constraints are imposed over the main beam and/or sidelobe regions, beamformers similar to those developed in [6]-[8] are obtained. When there is a single main beam constraint and many constraints over a dense grid of discrete points in the sidelobe region, the technique in [10] is obtained. We take a moderate approach using several constraints to maintain a high SINR and good pattern behavior with reasonable complexity.

## 2.2 Iterative Implementation

Here we present an iterative update procedure for the partially adaptive LCMV-QPC processor. First define

$$\mathbf{q}_i = \mathbf{Q}_i \mathbf{w}_{d,i}. \quad (11)$$

Now the quadratic pattern constraints in (5) can be written as

$$\mathbf{w}^H \mathbf{Q}_i \mathbf{w} - 2\Re e(\mathbf{q}_i^H \mathbf{w}) \leq \eta_i \quad i = 1 \dots r \quad (12)$$

where

$$\eta_i = L_i - \mathbf{w}_{d,i}^H \mathbf{Q}_i \mathbf{w}_{d,i}. \quad (13)$$

At each iteration, the pattern errors are checked against the constraints. If a constraint is exceeded, the loading for that sector is increased by an incremental factor  $\Delta_i^{(p)}$ , i.e.  $\lambda_i^{(p)} = \lambda_i^{(p-1)} + \Delta_i^{(p)}$ . The loaded covariance matrix

and desired weight term can be updated according to

$$\begin{aligned}\mathbf{R}_Q^{(p)} &= \mathbf{R} + \sum_{i=1}^r \lambda_i^{(p)} \mathbf{Q}_i \\ &= \mathbf{R}_Q^{(p-1)} + \sum_{i=1}^r \Delta_i^{(p)} \mathbf{Q}_i\end{aligned}\quad (14)$$

$$\begin{aligned}\mathbf{w}_Q^{(p)} &= \sum_{i=1}^r \lambda_i^{(p)} \mathbf{q}_i \\ &= \mathbf{w}_Q^{(p-1)} + \sum_{i=1}^r \Delta_i^{(p)} \mathbf{q}_i.\end{aligned}\quad (15)$$

The adaptive weights can then be computed from

$$\mathbf{S}^{(p)} = \left(\mathbf{R}_Q^{(p)}\right)^{-1}\quad (16)$$

$$\mathbf{P}^{(p)} = \mathbf{S}^{(p)} - \mathbf{S}^{(p)} \mathbf{C} \left(\mathbf{C}^H \mathbf{S}^{(p)} \mathbf{C}\right)^{-1} \mathbf{C}^H \mathbf{S}^{(p)}\quad (17)$$

$$\mathbf{w}^{(p)} = \mathbf{S}^{(p)} \mathbf{C} \left(\mathbf{C}^H \mathbf{S}^{(p)} \mathbf{C}\right)^{-1} \mathbf{f} + \mathbf{P}^{(p)} \mathbf{w}_Q^{(p)}.\quad (18)$$

This procedure isn't particularly attractive because of the computational complexity. However, if the incremental loading levels  $\Delta_i^{(p)}$  are small, the update can be accomplished with an approximation that reduces the complexity. The covariance matrix inverse at the  $p$ th iteration can be expressed as,

$$\mathbf{S}^{(p)} = \left( \left(\mathbf{S}^{(p-1)}\right)^{-1} + \sum_{i=1}^r \Delta_i^{(p)} \mathbf{Q}_i \right)^{-1}.\quad (19)$$

Substituting (19) and (15) into (17)-(18), we can expand the weight vector in a first order Taylor series approximation about the incremental loading levels  $\Delta_i^{(p)}$ ,  $i = 1 \dots r$ . After some fairly tedious manipulations, this gives a direct update for  $\mathbf{w}^{(p)}$  and  $\mathbf{P}^{(p)}$  as follows:

$$\mathbf{w}^{(p)} = \mathbf{w}^{(p-1)} - \mathbf{P}^{(p-1)} \sum_{i=1}^r \Delta_i^{(p)} \left( \mathbf{Q}_i \mathbf{w}^{(p-1)} - \mathbf{q}_i \right)\quad (20)$$

$$\mathbf{P}^{(p)} = \mathbf{P}^{(p-1)} - \mathbf{P}^{(p-1)} \left( \sum_{i=1}^r \Delta_i^{(p)} \mathbf{Q}_i \right) \mathbf{P}^{(p-1)}.\quad (21)$$

One way to achieve fast convergence while ensuring that the small update assumption is valid is to let  $\Delta_i^{(p)}$  be a fraction of the of the current loading value, i.e.  $\Delta_i^{(p)} = \alpha \lambda_i^{(p)}$ , where  $\alpha$  in the range 0.3 to 1 seems to work well. This requires that the initial loading level be non-zero. One possibility is to initialize all of the loading levels to some small value, i.e.  $\lambda_i^{(0)} = \lambda_0$ ,  $i = 1 \dots r$ . If the initial loading is small enough, the initial weight vector is essentially the standard adaptive LCMV weight vector given in (7).

To summarize, the algorithm is initialized by

$$\mathbf{S}^{(0)} = \left( \mathbf{R} + \lambda_0 \sum_{i=1}^r \mathbf{Q}_i \right)^{-1} \quad (22)$$

$$\mathbf{P}^{(0)} = \mathbf{S}^{(0)} - \mathbf{S}^{(0)} \mathbf{C} \left( \mathbf{C}^H \mathbf{S}^{(0)} \mathbf{C} \right)^{-1} \mathbf{C}^H \mathbf{S}^{(0)} \quad (23)$$

$$\mathbf{w}^{(0)} = \mathbf{S}^{(0)} \mathbf{C} \left( \mathbf{C}^H \mathbf{S}^{(0)} \mathbf{C} \right)^{-1} \mathbf{f} + \mathbf{P}^{(0)} \left( \lambda_0 \sum_{i=1}^r \mathbf{q}_i \right). \quad (24)$$

At each iteration, the weights are updated by

1. for  $i = 1, \dots, r$

if  $\mathbf{w}^{(p-1)H} \mathbf{Q}_i \mathbf{w}^{(p-1)} - 2\Re e \left( \mathbf{q}_i^H \mathbf{w}^{(p-1)} \right) > \eta_i$

then  $\Delta_i^{(p)} = \alpha \lambda_i^{(p-1)}$ , else  $\Delta_i^{(p)} = 0$

$\lambda_i^{(p)} = \lambda_i^{(p-1)} + \Delta_i^{(p)}$

$$2. \mathbf{Q}^{(p)} = \sum_{i=1}^r \Delta_i^{(p)} \mathbf{Q}_i \quad (25)$$

$$3. \mathbf{q}^{(p)} = \mathbf{Q}^{(p)} \mathbf{w}^{(p-1)} - \sum_{i=1}^r \Delta_i^{(p)} \mathbf{q}_i \quad (26)$$

$$4. \mathbf{w}^{(p)} = \mathbf{w}^{(p-1)} - \mathbf{P}^{(p-1)} \mathbf{q}^{(p)} \quad (27)$$

$$5. \mathbf{P}^{(p)} = \mathbf{P}^{(p-1)} - \mathbf{P}^{(p-1)} \mathbf{Q}^{(p)} \mathbf{P}^{(p-1)}. \quad (28)$$

The LCMV-QPC technique can be used for non-adaptive pattern synthesis by letting

$$\mathbf{R} = \int \mathbf{v}(\theta, \phi, \omega) \mathbf{v}(\theta, \phi, \omega)^H d\Omega, \quad (29)$$

which is the same as (3) for a single sector covering the entire angle-Doppler space. It generalizes the techniques in [10] and [20] for developing low sidelobe quiescent patterns for arbitrary arrays, and can be used for developing a tapered steering vector for use in the adaptive methods.

### 3 Examples

In the MIT Lincoln Lab data set [21], there are  $N = 20$  elements and  $M = 18$  pulses with a 300 Hz pulse repetition frequency. The UESA spatial beampattern is shown in Figure 2 and the space-time beampattern is shown in Figure 3.

The beam-pattern is steered to  $\phi = 0^\circ$  and  $\omega = 60$  Hz for a range of 50 km, which corresponds to  $\theta = -10.5^\circ$ . Angle-Doppler space was partitioned into one elevation angle sector  $\theta \in (-11^\circ, -2^\circ)$ , 11 azimuth angle sectors  $\phi \in (-12^\circ, 12^\circ), \pm(12^\circ, 30^\circ), \pm(30^\circ, 60^\circ), \pm(60^\circ, 100^\circ), \pm(100^\circ, 140^\circ), \pm(140^\circ, 180^\circ)$ , and 5 Doppler sectors  $\omega \in (-30, 30), \pm(30, 90), \pm(90, 150)$  Hz for a total of  $1 \times 11 \times 5 = 55$  sectors. The desired pattern was set to zero outside of the mainlobe region, and the constraint levels were chosen for -35 dB sidelobe levels. No constraint was used in the mainlobe region. There were two 30 dB interference-to-noise ratio (INR) jammers at  $60^\circ$  and  $-20^\circ$ , in addition to clutter. An 8 km training window (200 snapshots) was used to estimate the covariance matrix. The standard LCMV processor weights were computed by adding -30 dB diagonal loading to allow the covariance matrix to be inverted. The resulting space-time beampattern, and beampattern cuts are shown in Figures 4 and 5. The beamformer has put nulls on the clutter ridge and the two jammers, however the sidelobes are quite high.

The LCMV-QPC adaptive beamformer was used to reduce the sidelobes. The initial loading levels were set to  $\lambda_0 = 0.0013$ , and then iteratively increased using  $\alpha = 0.8$ . In 10 iterations, the LCMV-QPC beamformer is able to reduce the sidelobes below the -35 dB level while maintaining a well behaved main-beam, and deep clutter and jammer nulls. The final beampattern is shown in Figures 6 and 7.

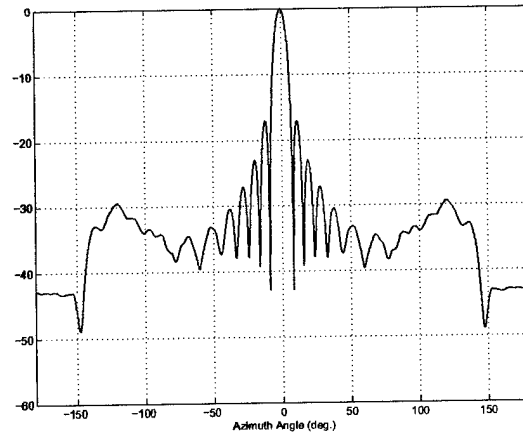


Figure 2: UESA Conventional Spatial Beampattern

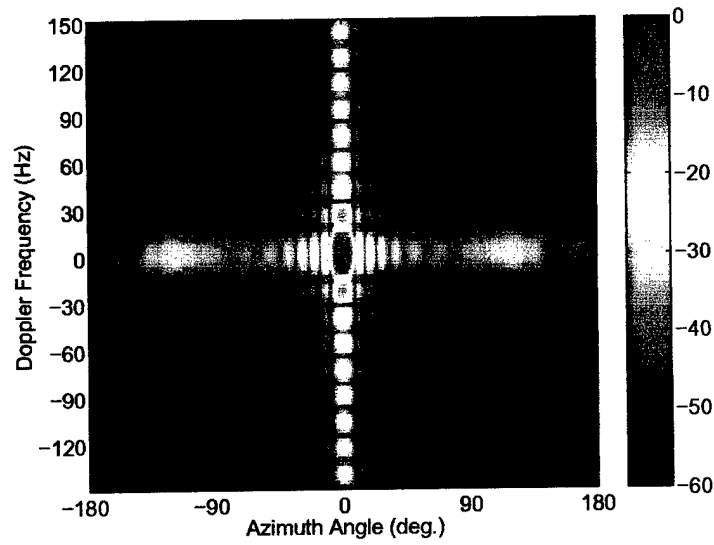


Figure 3: UESA Conventional Space-Time Beampattern

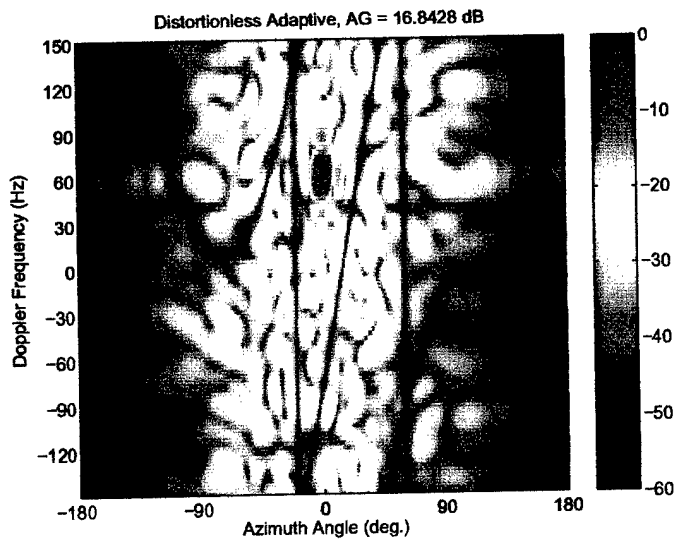


Figure 4: Initial Adaptive Beampattern

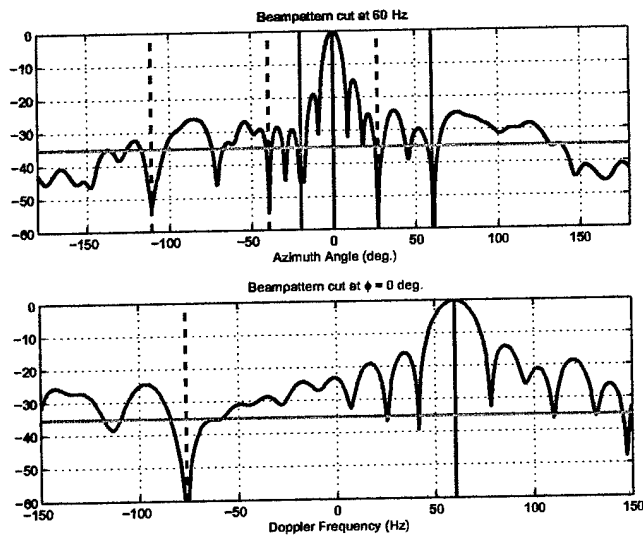


Figure 5: Initial Adaptive Beampattern

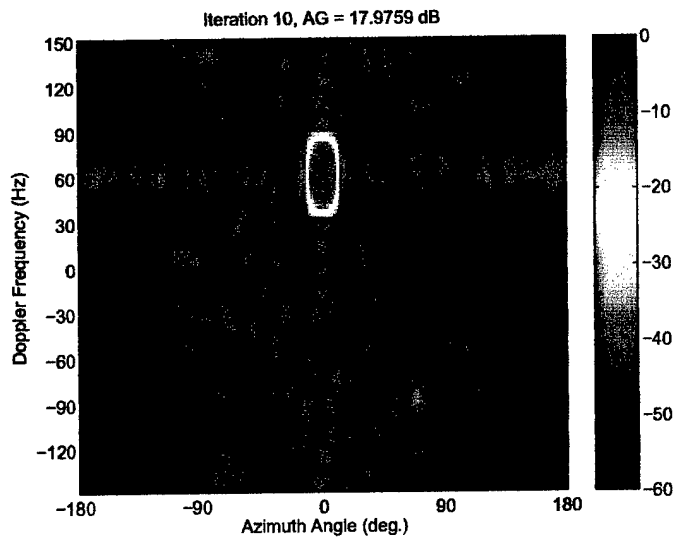


Figure 6: Final Adaptive Beampattern

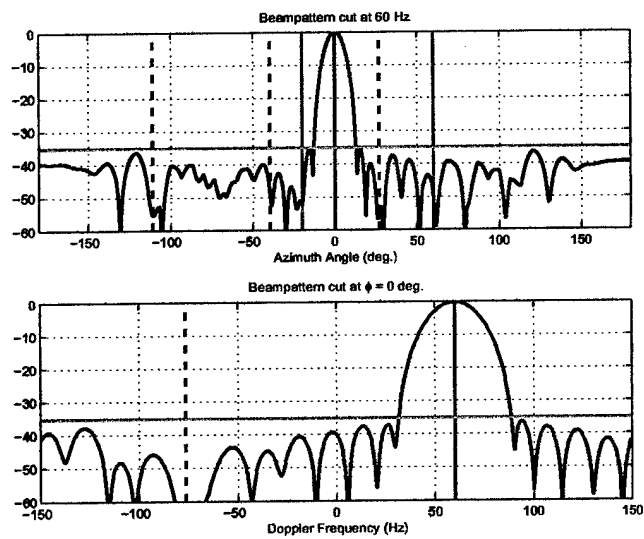


Figure 7: Final Adaptive Beampattern

## 4 Refinements and Enhancements

The core LCMV-QPC technique has also been formulated for the generalized sidelobe canceller (GSC) [22] form of the LCMV processor. These two formulations are mathematically equivalent and provide flexibility in implementation. They are based on an earlier technique developed under ONR Grant #N00014-99-1-0691 using a minimum mean square error (MMSE) approach for beamforming. Although the LCMV and MMSE approaches are similar, we prefer the LCMV approach because it allows specification of a distortionless constraint in the steering direction and scaling of the beampattern is not an issue.

The technique has been generalized for general rank reducing transformations. This is important because while full-dimension STAP algorithms provide excellent suppression of clutter and jammers, they require a large number of samples ( $\approx 2NM$ ) for optimum performance and are computationally expensive. Reduced-dimension, or partially-adaptive, techniques operate in a reduced-dimensional subspace by exploiting the low-rank properties of the clutter and interference. Techniques such as element-space pre-Doppler, element-space post-Doppler, beamspace pre-Doppler, beamspace post-Doppler, PRI-staggered post-Doppler, and eigenspace have been shown to reduce computational complexity and sample support requirements while providing performance close to full-dimension STAP. The reduced rank LCMV-QPC technique can be applied directly to these techniques to provide additional pattern control.

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