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13. ABSTRACT *(Maximum 200 words)*
We have contributed in three general areas that are relevant to high performance control of aerospace vehicles. We have developed systematic techniques for real time optimization using the method of extremum seeking. This method is now highly general applicable to multivariable problems, to problems with time-varying parameters, to problems involving slow dynamics where fast convergence is demanded, to problems where the objective is convergence to any value of the gradient (not just zero). to problems that involve limit cycles (unsteady extrema). and to problems that evolve in discrete time. Examples of applications that we have pursued include formation flight, compressor stall and surge, arid thermoacoustic combustion instabilities. In 2003 we published a research monograph on extremum seeking. In the area of flow control we have pioneered the use of stabilization for drag reduction and mixing. Employing Lyapunov techniques, we have solved flow control problems in channels, pipe geometries, flows around bluff bodies, and jet flows. In 2002 we published the first book dedicated to control algorithm design for fluid flows. For a broad class of linear parabolic distributed parameter systems we pioneered a back-stepping method for solving the boundary control stabilization problems. This is the first method that yields explicit solutions for both the control laws and for the closed-loop solutions. The implications of this are impossible to overstate-the design is direct and free of numerical issues, and the well posedness analysis is trivial (it is a bonus to the explicit design process). For several nonlinear PDEs we solved global stabilization problems in the presence of parametric uncertainties and input dynamics, paving the way for a future general theoretical development for nonlinear PDEs.

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Stabilization of Nonlinear PDEs and Applications to Control of Flows

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Contents

Abstract

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We have developed systematic techniques for real time optimization using the method of extremum seeking. This method is now highly general—applicable to multivariable problems, to problems with time-varying parameters, to problems involving slow dynamics where fast convergence is demanded, to problems where the objective is convergence to any value of the gradient (not just zero), to problems that involve limit cycles (unsteady extrema), and to problems that evolve in discrete time. Examples of applications that we have pursued include formation flight, compressor stall and surge, and thermoacoustic combustion instabilities. In 2003 we published a research monograph on extremum seeking.

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For a broad class of linear parabolic distributed parameter systems we pioneered a backstepping method for solving the boundary control stabilization problems. This is the first method that yields explicit solutions for both the control laws and for the closed-loop solutions. The implications of this are impossible to overstate—the design is direct and free of numerical issues, and the well posedness analysis is trivial (it is a bonus to the explicit design process). For several *nonlinear* PDEs we solved *global* stabilization problems in the presence of parametric uncertainties and input dynamics, paving the way for a future general theoretical development for nonlinear PDEs.

1 Summary of Results

Extremum Seeking Control [K2]. Motivated by on-line optimization problems in aero-engines, formation flight, bioreactors, and automotive traction control, the PI developed the technique of extremum seeking control. Extremum seeking emerged in the 1940's with a great deal of promise but was abandoned in the 1960's after efforts to make it systematic failed. The PI revived it by creating the necessary analytical methodology for performance improvement and an extension to multivariable optimization. The book [K2] summarizes the efforts of four of the PI's graduate students. Extremum seeking is currently used in applications in United Technologies, Pratt & Whittney, and Ford.

Flow Control [K1]. The PI's efforts in flow control concentrated on developing methods that are unique in their implementational simplicity, while avoiding any analytical simplifications of the problem (linearization, discretization) common in this area. The monograph [K1], the first book on the topic, overviews the results on stabilization by the PI's four students/postdocs and other contributors to the field of flow control, while also pioneering *feedback* methods for control of mixing.

Other Applications. The PI has made contributions in several other areas of control applications: rotating stall in jet engine *compressors* [K3, K10, K12], thermoacoustic instabilities in *combustors* [K5, K16, K30], *satellite* control [K4], *underwater vehicles* [K8], *helicopter blade-vortex interaction* noise [K9], control of *aircraft formations* [K25], *bioreactors* [K7], *chemical tubular reactors* [K22], *solid propellant rocket* instabilities [K23], *pulsed detonation engines* [K31], and *mine ventilation* networks [K26]. His techniques for compressors and combustors have been patented, after experimental verifications at Caltech, Louisiana State University, and United Technologies.

Control of Systems Modeled by Partial Differential Equations. The PI and his two postdocs (with degrees in mathematics) have open a new area of control of distributed parameter systems—boundary control by the method of backstepping. In [K19, K21, K24] they developed feedback transformations in the form of integral operators, combined with boundary control, that transform a wide variety of parabolic equations into simple heat equations. They also developed nonlinear controllers for several benchmark PDE systems: Burgers' equation [K6, K13], Korteweg-de Vries equation [K14, K20], Kuramoto-Sivashinsky equation [K11], and the thermal convection loop [K18]. They addressed PDE problems with not only nonlinearities but also other practical constraints like actuator dynamics [K15] and unknown viscosity parameters [K17], obtaining results valid for arbitrarily large initial data.

2 Example of Results: Boundary Control of Distributed Parameter Systems

State Feedback. We have developed boundary control laws for a wide class of linear parabolic differential equations. An example that is light on notation but still quite illustrative is the plant

$$u_t(x,t) = u_{xx}(x,t) + \lambda_0 u(x,t)\,, \tag{1}$$

where $x \in (0,1)$ and λ_0 is constant and the uncontrolled boundary condition is $u(0,t) = 0$.[1] The control law

$$u(1,t) = -\int_0^1 \lambda y \frac{I_1(\sqrt{\lambda(1-y^2)})}{\sqrt{\lambda(1-y^2)}} u(y)\,dy\,, \qquad \lambda = \lambda_0 + c \tag{2}$$

achieves asymptotic stabilization. The symbol I_1 denotes the *modified* Bessel function of the first order. What is particularly striking about backstepping approach is that the closed loop solutions can be found explicitly:

$$\begin{aligned} u(x,t) &= \sum_{n=1}^{\infty} \mathrm{e}^{-(c+\pi^2 n^2)t} \frac{2\pi n}{\sqrt{\lambda + \pi^2 n^2}} \sin(\sqrt{\lambda + \pi^2 n^2}\,x) \\ &\quad \times \int_0^1 \left(\sin(\pi n \xi) - \int_\xi^1 \xi \frac{I_1(\sqrt{\lambda(\eta^2 - \xi^2)})}{\sqrt{\lambda(\eta^2 - \xi^2)}} \sin(\pi n \eta)\,d\eta \right) u_0(\xi)\,d\xi \end{aligned} \tag{3}$$

and the closed-loop control signal is obtained by evaluating (3) at $x = 1$. The function $u_0(\xi)$ denotes the initial condition/profile.

Another quick example is the system

$$u_t(x,t) = u_{xx}(x,t) + g u(0,t)\,, \tag{4}$$

where g is constant and $u_x(0,t) = 0$ at the boundary. This type of dynamics arise in instabilities in *solid propellant rockets.* The term $u(x = 0, t)$ appearing on the right hand side of the PDE is the result of end-domain burning affecting the thermal dynamics within the domain. We design the control law

$$u(1,t) = -\int_0^1 \sqrt{g} \sinh(\sqrt{g}(1-y)) u(y,t)\,dy\,. \tag{5}$$

In addition to the explicit control law, the closed-loop solution is compact enough to write:

$$\begin{aligned} u(x,t) &= 2\sum_{n=0}^{\infty} e^{-\left(\pi n + \frac{\pi}{2}\right)^2 t} \frac{\frac{g}{\left(\pi n + \frac{\pi}{2}\right)^2} \left(\cos\left(\left(\pi n + \frac{\pi}{2}\right)x\right) - 1\right) + \cos\left(\left(\pi n + \frac{\pi}{2}\right)x\right)}{1 + \frac{g}{\left(\pi n + \frac{\pi}{2}\right)^2}} \\ &\quad \times \int_0^1 u_0(\xi) \left\{ \cos\left(\left(\pi n + \frac{\pi}{2}\right)x\right) + (-1)^n \frac{\sqrt{g}}{\pi n + \frac{\pi}{2}} \sinh(\sqrt{g}(1-\xi)) \right\} d\xi\,. \end{aligned} \tag{6}$$

A third example is the system

$$u_t(x,t) = u_{xx}(x,t) + \frac{2\alpha^2}{\cosh^2(\alpha x - \beta)} u(x,t) \tag{7}$$

[1] The open-loop system (with homogeneous Dirichlet boundary conditions) is unstable with arbitrarily many unstable eigenvalues (as λ_0 grows).

with a boundary condition $u(0,t)=0$, for which we design the stabilizing control law

$$u(1,t) = -\int_0^1 \alpha e^{\alpha \tanh \beta(1-y)}[\tanh\beta - \tanh(\beta-\alpha y)]u(y,t)dy\,. \tag{8}$$

The right hand side of the system (7) may appear too exotic to be general but it is actually quite general. The x-dependent "coefficient" multiplying $u(x,t)$ is parametrized by α and β and this parametrization spans a family of "peak"-shaped functions with an arbitrary location and height of the peak parametrized by α and β. For instance, the widely studied *chemical tubular reactor* PDE control models can be described using this parametrization (the spatially-varying "coefficient" arises due to a nonlinearity in the problem).

Output Feedback. Consider, as an example, the plant $u_t(x,t) = u_{xx}(x,t) + \lambda_0 u(x,t)$, $u(0,t)=0$, $u_x(1,t) = U(t)$, where $U(t)$ is the control input (open-loop in the observer problem; feedback in the stabilization problem). Our observer design procedure results in

$$\hat{u}_t(x,t) = \hat{u}_{xx}(x,t) + \lambda_0\hat{u}(x,t) + \underbrace{\frac{\lambda x}{1-x^2} I_2\left(\sqrt{\lambda(1-x^2)}\right)}[u(1,t)-\hat{u}(1,t)] \tag{9}$$

$$\hat{u}_x(1,t) = U(t) \underbrace{-\frac{\lambda}{2}}[\hat{u}(1,t)-u(1,t)]\,, \tag{10}$$

with $\hat{u}(0,t)=0$, where $\lambda = \lambda_0 + c, c \geq 0$. We stress that the observer is convergent for any type of input $U(t)$ applied. In particular, if the system is unstable and $U(t)=0$ is applied, the observer state $\hat{u}(x,t)$ will converge to the (diverging) $u(x,t)$ (in maximum norm, for example). The backstepping controller employing the state estimates is designed as

$$u_x(1,t) = U(t) = -\frac{\lambda}{2}u(1,t) - \int_0^1 \frac{\lambda y}{1-y^2} I_2\left(\sqrt{\lambda(1-y^2)}\right)\hat{u}(y)\,dy\,. \tag{11}$$

The central point of our backstepping observer design procedure is the selection of the *output injection* gain functions, which are underbraced in (9) and (10). A quick comparison between the controller and the observer shows that they are dual in a particular sense.

We re-emphasize that, with every element of our design being explicit, the closed loop solution can be derived, it just takes a bit more effort due to the presence of a second PDE—the observer. The solution is given by

$$\begin{aligned} u(x,t) &= \sum_{n=0}^{\infty} e^{-\left(c+\left(\pi n+\frac{\pi}{2}\right)^2\right)t}\phi_n(x) \\ &\times \left(\int_0^1 \psi_n(\xi)u_0(\xi)d\xi - (-1)^n\left(tC_n + \sum_{\substack{m=0\\ m\neq n}} \frac{1-e^{\pi^2(n-m)(n+m+1)t}}{\pi^2(n-m)(n+m+1)}C_m\right)\right) \end{aligned} \tag{12}$$

where

$$\phi_n(x) = \frac{2\pi n + \pi}{\sqrt{\lambda + \left(\pi n + \frac{\pi}{2}\right)^2}} \sin\left(\sqrt{\lambda + \left(\pi n + \frac{\pi}{2}\right)^2}\, x\right) \tag{13}$$

$$\psi_n(x) = \sin\left(\left(\pi n + \frac{\pi}{2}\right) x\right) + \int_x^1 \lambda x \frac{I_1(\sqrt{\lambda(\xi^2 - x^2)})}{\sqrt{\lambda(\xi^2 - x^2)}} \sin\left(\left(\pi n + \frac{\pi}{2}\right) x\right) d\xi \tag{14}$$

$$C_n = \left(\int_0^1 \frac{\lambda\xi}{1-\xi^2} I_2\left(\sqrt{\lambda(1-\xi^2)}\right) \psi_n(\xi)\, d\xi\right) \left(\int_0^1 \phi_n(\xi)(u_0(\xi) - \hat{u}_0(\xi))\, d\xi\right). \tag{15}$$

Another example is the PDE $u_t = u_{xx}$ with a boundary condition $u_x(0,t) = qu(0,t)$, where q is a constant of any sign. Negative q gives rise to instability, despite the fact that the system is governed by a plain heat equation in the interior of the domain. This type of a boundary condition comes up in *solid propellant rocket* instabilities and it models the heat release on the burning end of the propellant. We choose to actuate through $u(1,t)$, and for sensing, we select the anti-collocated architecture (measuring $u(0,t)$). The stabilizing compensator can be derived in the transfer function form as

$$C(s) = \frac{g}{s}\left(-1 + \frac{(s-g)\cosh(\sqrt{s})\cosh(\sqrt{g})}{s\cosh(\sqrt{s}) - g\cosh(\sqrt{g})}\right). \tag{16}$$

Advantages of the Backstepping Approach. The examples above show that the backstepping approach generates explicit boundary control laws for a large class of parabolic PDEs. Relative to the previous approaches this implies the following advantages:

- **Simplicity of Implementation.** The explicit form of the control law, where even the dependence on parameters appears explicitly (in addition to the dependence on the spatial coordinates), is achievable only with this method. This means that our controllers come as ready-to-implement families.

- **Less (Or None) a Priori Computation.** In the backstepping design there are neither open-loop eigenfunctions to compute (as in pole placement), nor operator Riccati equations to solve (as in LQR).

- **Assigning Infinitely Many Eigenvalues.** In contrast to the pole placement method, backstepping is capable of assigning infinitely many eigenvalues to the locations of a heat equation, with arbitrary extra damping $c > 0$.

- **Easier Analysis.** Possibly the most significant advantage. With backstepping, boundary control of PDEs becomes tractable using calculus. With control laws, and even the closed loop solutions, given explicitly, design and existence of solutions analysis become accessible to anyone who knows what an integral is and how to solve a heat equation.

3 PI's Selected Publications From the Grant Period

Books

[K1] O. M. Aamo and M. Krstic, *Flow Control by Feedback*, Springer, 2002.

[K2] K. Ariyur and M. Krstic, *Real Time Optimization by Extremum Seeking Control*, Wiley, 2003.

Selected Journal Papers
(on topics not covered by the above books)

[K3] M. Krstic, D. Fontaine, P. V. Kokotovic, and J. Paduano, "Useful nonlinearities and global bifurcation control of jet engine surge and stall," *IEEE Transactions on Automatic Control*, vol.43, p.1739-45, 1998.

[K4] M. Krstic and P. Tsiotras, "Inverse optimality results for the attitude motion of a rigid spacecraft," *IEEE Transactions on Automatic Control*, vol.44, p.1042-9, 1999.

[K5] M. Krstic, A. S. Krupadanam, and C. A. Jacobson, "Self-tuning control of a nonlinear model of combustion instabilities," *IEEE Transactions on Control Systems Technology*, vol.7, p.424-436, 1999.

[K6] M. Krstic, "On global stabilization of Burgers' equation by boundary control," *Systems and Control Letters*, vol.37, p.123-142, 1999.

[K7] H.-H. Wang, M. Krstic, and G. Bastin, "Optimizing bioreactors by extremum seeking," *International Journal of Adaptive Control and Signal Processing*, vol. 13, pp. 651-669, 1999.

[K8] D. Boskovic and M. Krstic, "Global attitude/position regulation for underwater vehicles," *International Journal of Systems Science*, vol. 30, pp. 939-946, 1999.

[K9] K.B.Ariyur and M. Krstic, "Feedback attenuation and adaptive cancellation of blade-vortex interaction noise on a helicopter blade element," *IEEE Transactions on Control Systems Technology*, vol. 7, pp. 424-426, 1999.

[K10] H.-H. Wang, S. Yeung, and M. Krstic, "Experimental application of extremum seeking on an axial-flow compressor," *IEEE Transactions on Control Systems Technology*, vol. 8, pp. 300-309, 1999.

[K11] W.-J. Liu and M. Krstic, "Stability enhancement by boundary control in the Kuramoto-Sivashinsky equation," *Nonlinear Analysis*, vol. 43, pp. 485-583, 2000.

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[K15] W.-J. Liu and M. Krstic, "Backstepping boundary control of Burgers' equation with actuator dynamics," *Systems and Control Letters*, vol. 41, pp. 291-303, 2000.

[K16] S. Murugappan, E. Gutmark, S. Acharya, and M. Krstic, "Extremum seeking adaptive controller of swirl-stabilized spray combustion," *Proceedings of the Combustion Institute*, vol. 28, 2000.

[K17] W.-J. Liu and M. Krstic, "Adaptive control of Burgers' equation with unknown viscosity," *International Journal of Adaptive Control and Signal Processing*, vol. 15, pp. 745-766, 2001.

[K18] D. Boskovic and M. Krstic, "Nonlinear stabilization of a thermal convection loop by state feedback," *Automatica*, vol. 37, pp. 2033-2040, 2001.

[K19] D. Boskovic, M. Krstic, and W.-J. Liu, "Boundary control of an unstable heat equation via measurement of domain-averaged temperature," *IEEE Transactions on Automatic Control*, vol. 46, pp. 2022-2028, 2001.

[K20] W.-J. Liu and M. Krstic, "Global boundary stabilization of the Korteweg-de Vries-Burgers equation," *Computational and Applied Mathematics*, vol. 21, pp. 315–354, 2002.

[K21] A. Balogh and M. Krstic, "Infinite-dimensional backstepping-style feedback transformations for a heat equation with an arbitrary level of instability," *European Journal of Control*, vol. 8, pp. 165-177, 2002.

[K22] D. Boskovic and M. Krstic, "Backstepping control of chemical tubular reactors," *Computers and Chemical Engineering*, vol. 26, pp. 1077–1085, 2002..

[K23] D. Boskovic and M. Krstic, "Stabilization of a solid propellant rocket instability by state feedback," *International Journal of Robust and Nonlinear Control*, vol. 13, pp. 483–495, 2003.

[K24] D. Boskovic, A. Balogh, and M. Krstic, "Backstepping in infinite dimension for a class of parabolic distributed parameter systems," *Mathematics of Control, Signals, and Systems*, vol. 16, pp. 44-75, 2003.

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[K26] H. Yu, O. Koroleva, and M. Krstic, "Nonlinear control of mine ventilation networks," *Systems and Control Letters*, vol. 49, pp. 239–254, 2003.

[K27] E. Schuster, M. Krstic, and G. Tynan, "Burn control in fusion reactors via nonlinear stabilization techniques," *Fusion Science and Technology*, vol. 43, pp. 18–37, 2003.

[K28] E. Schuster and M. Krstic, "Control of a nonlinear PDE system arising from non-burning tokamak plasma transport dynamics," *International Journal of Control*, vol. 76, pp. 1116–1124, 2003.

[K29] E. Schuster, M. L. Walker, D. A. Humphreys, and M. Krstic, "Plasma vertical stabilization in the presence of coil voltage saturation in the DIII-D tokamak," *2003 American Control Conference.*

[K30] A. Banaszuk, K. B. Ariyur, M. Krstic, and C. A. Jacobson, "An adaptive algorithm for control of combustion instability," *Automatica*, in press.

[K31] O. Sarrazin, K. B. Ariyur, A. Aliseda, J. C. Lasheras, and M. Krstic, "Tailored fuel injection for pulsed detonation engines via feedback control," *Journal of Propulsion and Power*, in press.

[K32] A. Balogh and M. Krstic, "Stability of partial difference equations governing control gains in infinite-dimensional backstepping," *Systems and Control Letters*, in press.

[K33] A. Smyshlyaev and M. Krstic, "Smooth gain kernels for optimal stabilization of partial integro-differential equations," submitted *IEEE Transactions on Automatic Control*, a conference version published at 2003 American Control Conference.

4 Additional Data

4.1 Personnel Supported

- Dr. Miroslav Krstic, Associate Professor, Principal Investigator
- Dr. Andras Balogh, postdoctoral researcher
- Kartik Ariyur, Graduate Student Researcher
- Dejan Boskovic, Graduate Student Researcher
- Eugenio Schuster, Graduate Student Researcher
- Lawrence Yuan, Graduate Student Researcher
- Hua Deng, Graduate Student Researcher
- Olga Koroleva, Graduate Student Researcher

4.2 Doctoral Dissertations Completed

1. Hua Deng, *Stochastic Nonlinear Stabilization*, 2000.
2. Dejan Boskovic, *Infinite Dimensional Backstepping for a Class of Parabolic Distributed Parameter Systems*, 2001.
3. Kartik Ariyur, *Multiparameter Extremum Seeking and Applications to Propulsion and Aerodynamics Problems*, 2002.
4. Lawrence Yuan, *Control of Jet Flow Mixing and Stabilization*, 2002.
5. Ole-Morten Aamo, *Modeling and Control of Fluid Flows and Marine Structures*, 2002.

4.3 Transitions

Our extremum seeking algorithms are in use for control of combustion instabilities and flow separation in diffusers at the *United Technologies Research Center.* Point of contact: Dr. Andrzej Banaszuk (phone: 860 610-7381).

4.4 Awards

Elected Fellow, IEEE, 2001.