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Aircraft Turbofan Engine Health Estimation Using Constrained Kalman Filtering

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USING CONSTRAINED KALMAN FILTERING

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ABSTRACT

Kalman filters are often used to estimate the state variables of a dynamic system. However, in the application of Kalman filters some known signal information is often either ignored or dealt with heuristically. For instance, state variable constraints (which may be based on physical considerations) are often neglected because they do not fit easily into the structure of the Kalman filter. This paper develops an analytic method of incorporating state variable inequality constraints in the Kalman filter. The resultant filter is a combination of a standard Kalman filter and a quadratic programming problem. The incorporation of state variable constraints increases the computational effort of the filter but significantly improves its estimation accuracy. The improvement is proven theoretically and shown via simulation results obtained from application to a turbofan engine model. This model contains 16 state variables, 12 measurements, and 8 component health parameters. It is shown that the new algorithms provide improved performance in this example over unconstrained Kalman filtering.

INTRODUCTION

For linear dynamic systems with white process and measurement noise, the Kalman filter is known to be an optimal estimator. However, in the application of Kalman filters there is often known model or signal information that is either ignored or dealt with heuristically [1]. This paper presents a way to generalize the Kalman filter in such a way that known inequality constraints among the state variables are satisfied by the state variable estimates.

The method presented here for enforcing inequality constraints on the state variable estimates uses hard constraints. It is based on a generalization of the approach presented in [2], which dealt with the incorporation of state variable equality constraints in the Kalman filter. Inequality constraints are inherently more complicated than equality constraints, but standard quadratic programming results can be used to solve the Kalman filter problem with inequality constraints. At each time step of the constrained Kalman filter, we solve a quadratic programming problem to obtain the constrained state estimate. A family of constrained state estimates is obtained, where the weighting matrix of the quadratic programming problem determines which family member forms the desired solution. It is stated in this paper, on the basis of [2], that the constrained estimate has several important properties. The constrained state estimate is unbiased (Theorem 1 below) and has a smaller error covariance than the unconstrained estimate (Theorem 2 below). We show which member of all possible constrained solutions has the smallest error covariance (Theorem 3 below). We also show the one particular member that is always (i.e., at each time step) closer to the true state than the unconstrained estimate (Theorem 4 below). Finally, we show that the variation of the constrained estimate is smaller than the variation of the unconstrained estimate (Theorem 5 below).

The application considered in this paper is turbofan engine health parameter estimation [3]. The performance of gas turbine engines deteriorates over time. This deterioration can affect the fuel economy, and impact emissions, component life consumption, and thrust response of the engine. Airlines periodically collect engine data in order to evaluate the health of the engine and its components. The

health evaluation is then used to determine maintenance schedules. Reliable health evaluations are used to anticipate future maintenance needs. This offers the benefits of improved safety and reduced operating costs. The money-saving potential of such health evaluations is substantial, but only if the evaluations are reliable. The data used to perform health evaluations are typically collected during flight and later transferred to ground-based computers for post-flight analysis. Data are collected each flight at approximately the same engine operating conditions and corrected to account for variability in ambient conditions and power setting levels. Typically, data are collected for a period of about 3 seconds at a rate of about 10 or 20 Hz. Various algorithms have been proposed to estimate engine health parameters, such as weighted least squares [4], expert systems [5], Kalman filters [6], neural networks [6], and genetic algorithms [7].

This paper applies constrained Kalman filtering to estimate engine component efficiencies and flow capacities, which are referred to as health parameters. We can use our knowledge of the physics of the turbofan engine in order to obtain a dynamic model [8, 9]. The health parameters that we try to estimate can be modelled as slowly varying biases. The state vector of the dynamic model is augmented to include the health parameters, which are then estimated with a Kalman filter [10]. The model formulation in this paper is similar to previous NASA work [11]. However, [11] was limited to a 3-state dynamic model and 2 health parameters, whereas this present work includes a more complete 16-state model and 8 health parameters. In addition, we have some *a priori* knowledge of the engine's health parameters: we know that they never improve. Engine health always degrades over time, and we can incorporate this information into state constraints to improve our health parameter estimation. (This is assuming that no maintenance or engine overhaul is performed.) This is similar to the probabilistic approach to turbofan prognostics proposed in [12]. The simulation results that we present here show that the Kalman filter can estimate health parameter deviations with an average error of less than 5%, and the constrained Kalman filter performs even better than the unconstrained filter.

It should be emphasized that in this paper we are confining the problem to the estimation of engine health parameters in the presence of degradation only. There are specific engine fault scenarios that can result in abrupt shifts in filter estimates, possibly even indicating an apparent improvement in some engine components. An actual engine performance monitoring system would need to include additional logic to detect and isolate such faults.

KALMAN FILTERING

Consider the discrete linear time-invariant system given by

$$x_{k+1} = Ax_k + Bu_k + w_k \qquad (1)$$
$$y_k = Cx_k + e_k$$

where k is the time index, x is the state vector, u is the known control input, y is the measurement, and $\{w_k\}$ and $\{e_k\}$ are uncorrelated zero-mean white noise input sequences. We use Q to denote the covariance of $\{w_k\}$ and R to denote the covariance of $\{e_k\}$, and $\bar{x}$ to denote the expected value of x. The problem is to find an estimate $\hat{x}_{k+1}$ of x_{k+1} given the measurements $\{y_0, y_1, \cdots, y_k\}$. We will use the symbol Y_k to denote the column vector that contains the measurements $\{y_0, y_1, \cdots, y_k\}$. The Kalman filter can be used to solve this problem as follows.

$$K_k = A\Sigma_k C^T (C\Sigma_k C^T + R)^{-1} \qquad (2)$$
$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K_k(y_k - C\hat{x}_k) \qquad (3)$$
$$\Sigma_{k+1} = (A\Sigma_k - K_k C\Sigma_k)A^T + Q \qquad (4)$$

where the filter is initialized with $\hat{x}_0 = \bar{x}_0$, and $\Sigma_0 = E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T]$. It can be shown [13] that the Kalman filter estimate has several attractive properties. It is unbiased, and of all affine estimators, it is the one that minimizes the variance of the estimation error. In addition, if x_0, $\{w_k\}$, and $\{e_k\}$ are jointly gaussian, then the Kalman filter estimate is the one that maximizes the conditional probability density function of the state given the measurement history.

KALMAN FILTERING WITH INEQUALITY CONSTRAINTS

This section extends the well known results of the previous section to cases where there are known linear inequality constraints among the state components. Also, several important properties of the constrained filter are discussed. Consider the dynamic system of (1) where we are given the additional constraint

$$Dx_k \leq d_k \qquad (5)$$

where D is a known $s \times n$ constant matrix, s is the number of constraints, n is the number of state variables, and $s \leq n$. It is assumed in this paper that D is full rank, i.e., that D has rank s. This is an easily satisfied assumption. If D is not full rank that means we have redundant state constraints.

In that case we can simply remove linearly dependent rows from D (i.e., remove redundant state constraints) until D is full rank. Three different approaches to the constrained state estimation problem are given in this section. The time index k is omitted in the remainder of this section for ease of notation.

The Maximum Probability Method

In this section we derive the constrained Kalman filtering problem by using a maximum probability method. From [13, pp. 93 ff.] we know that the Kalman filter estimate is that value of x that maximizes the conditional probability density function $P(x|Y)$. The constrained Kalman filter can be derived by finding an estimate $\tilde{x}$ such that the conditional probability $P(\tilde{x}|Y)$ is maximized and $\tilde{x}$ satisfies the constraint (5). Maximizing $P(\tilde{x}|Y)$ is the same as maximizing its natural logarithm. So the problem we want to solve can be given by

$$\max \ln P(\tilde{x}|Y) \Longrightarrow \min_{\tilde{x}} (\tilde{x} - \bar{\bar{x}})^T \Sigma^{-1} (\tilde{x} - \bar{\bar{x}}) \qquad (6)$$
$$\text{such that } D\tilde{x} \leq d$$

Using the fact that the unconstrained state estimate $\hat{x} = \bar{\bar{x}}$ (the conditional mean of x), we rewrite the above equation as

$$\min_{\tilde{x}} (\tilde{x}^T \Sigma^{-1} \tilde{x} - 2\hat{x}^T \Sigma^{-1} \tilde{x}) \text{ such that } D\tilde{x} \leq d \qquad (7)$$

Note that this problem statement depends on the conditional gaussian nature of $\hat{x}$, which in turn depends on the gaussian nature of x_0, $\{w_k\}$, and $\{e_k\}$ in (1).

The Mean Square Method

In this section we derive the constrained Kalman filtering problem by using a mean square minimization method. We seek to minimize the conditional mean square error subject to the state constraints.

$$\min_{\tilde{x}} E(\|x - \tilde{x}\|^2 | Y) \text{ such that } D\tilde{x} \leq d \qquad (8)$$

where $\|\cdot\|$ denotes the vector two-norm. If we assume that x and Y are jointly gaussian, the mean square error can be written as

$$E(\|x - \tilde{x}\|^2 | Y) = \int (x - \tilde{x})^T (x - \tilde{x}) P(x|Y) dx \qquad (9)$$

$$= \int x^T x P(x|Y) dx - \qquad (10)$$
$$2\tilde{x}^T \int x P(x|Y) dx + \tilde{x}^T \tilde{x}$$

Noting that the Kalman filter estimate is the conditional mean of x, i.e.,

$$\hat{x} = \int x P(x|Y) dx \qquad (11)$$

we formulate the first order conditions necessary for a minimum as

$$\min_{\tilde{x}} (\tilde{x}^T \tilde{x} - 2\hat{x}^T \tilde{x}) \text{ such that } D\tilde{x} \leq d \qquad (12)$$

Again, this problem statement depends on the conditional gaussian nature of $\hat{x}$, which in turn depends on the gaussian nature of x_0, $\{w_k\}$, and $\{e_k\}$ in (1).

The Projection Method

In this section we derive the constrained Kalman filtering problem by directly projecting the unconstrained state estimate $\hat{x}$ onto the constraint surface. That is, we solve the problem

$$\min_{\tilde{x}} (\tilde{x} - \hat{x})^T W (\tilde{x} - \hat{x}) \text{ such that } D\tilde{x} \leq d \qquad (13)$$

where W is any symmetric positive definite weighting matrix. This problem can be rewritten as

$$\min_{\tilde{x}} (\tilde{x}^T W \tilde{x} - 2\hat{x}^T W \tilde{x}) \text{ such that } D\tilde{x} \leq d \qquad (14)$$

The constrained estimation problems derived by the maximum probability method (7) and the mean square method (12) can be obtained from this equation by setting $W = \Sigma^{-1}$ and $W = I$ respectively. Note that this derivation of the constrained estimation problem does not depend on the conditional gaussian nature of $\hat{x}$; i.e., x_0, $\{w_k\}$, and $\{e_k\}$ in (1) are not assumed to be gaussian.

The Solution of the Constrained State Estimation Problem

The problem defined by (14) is known as a quadratic programming problem [14, 15]. There are many algorithms for solving quadratic programming problems, almost all of which fall in the category known as active set methods. An

active set method uses the fact that it is only those constraints that are active at the solution of the problem that are significant in the optimality conditions. Assume that t of the s inequality constraints are active at the solution of (14), and denote by $\hat{D}$ and $\hat{d}$ the t rows of D and t elements of d corresponding to the active constraints. If the correct set of active constraints was known *a priori* then the solution of (14) would also be a solution of the equality-constrained problem

$$\min_{\tilde{x}} (\tilde{x}^T W \tilde{x} - 2\hat{x}^T W \tilde{x}) \text{ such that } \hat{D}\tilde{x} = \hat{d} \qquad (15)$$

This shows that the inequality constrained problem defined by (14) is equivalent to the equality-constrained problem defined by (15). Note that $\tilde{x}$ depends entirely on $\hat{x}$ at each time step. Although the equation for $\hat{x}$ is recursive as seen in (3), the equation for $\tilde{x}$ is not recursive. Therefore, an inequality-constrained Kalman filter that uses active set methods to enforce the constraints completely reduces to an equality-constrained Kalman filter, even though the active set may change at each time step. The equality-constrained problem was discussed in [2], in which there is no assumption that the constraints remain constant from one time step to the next. Therefore, those results can be used to investigate the properties of the inequality-constrained filter.

Properties of the Constrained State Estimate

In this section we examine some of the statistical properties of the constrained Kalman filter. We use $\hat{x}$ to denote the state estimate of the unconstrained Kalman filter, and $\tilde{x}$ to denote the state estimate of the constrained Kalman filter as given by (14), recalling that (7) and (12) are special cases of (14).

Theorem 1. *The solution $\tilde{x}$ of the constrained state estimation problem given by (14) is an unbiased state estimator for the system (1) for any symmetric positive definite weighting matrix W. That is,*

$$E(\tilde{x}) = E(x) \qquad (16)$$

Theorem 2. *The solution $\tilde{x}$ of the constrained state estimation problem given by (14) with $W = \Sigma^{-1}$, where Σ is the error covariance of the unconstrained estimate given in (4), has an error covariance that is less than or equal to that of the unconstrained state estimate. That is,*

$$Cov(x - \tilde{x}) \le Cov(x - \hat{x}) \qquad (17)$$

At first this seems counterintuitive, since the standard Kalman filter is by definition the minimum variance filter. However, we have changed the problem by introducing state variable constraints. Therefore, the standard Kalman filter is no longer the minimum variance filter, and we can do better with the constrained Kalman filter.

Theorem 3. *Among all the constrained Kalman filters resulting from the solution of (14), the filter that uses $W = \Sigma^{-1}$ has the smallest estimation error covariance. That is,*

$$Cov(x - \tilde{x}_{\Sigma^{-1}}) \le Cov(x - \tilde{x}_W) \text{ for all } W \qquad (18)$$

Theorem 4. *The solution $\tilde{x}$ of the constrained state estimation problem given by (14) with $W = I$ satisfies the inequality*

$$\|x_k - \tilde{x}_k\| \le \|x_k - \hat{x}_k\| \text{ for all } k \qquad (19)$$

where $\|\cdot\|$ is the vector two-norm and $\hat{x}$ is the unconstrained Kalman filter estimate.

Theorem 5. *The error of the solution $\tilde{x}$ of the constrained state estimation problem given by (14) with $W = I$ is smaller than the unconstrained estimation error in the sense that*

$$Tr[Cov(\tilde{x})] \le Tr[Cov(\hat{x})] \qquad (20)$$

where $Tr[\cdot]$ indicates the trace of a matrix, and $Cov(\cdot)$ indicates the covariance matrix of a random vector.

The above theorems all follow from the equivalence of (14) and (15), and the proofs presented in [2]. We note that if any of the s constraints are active at the solution of (14), then strict inequalities hold in the statements of Theorems 2–5. The only time that equalities hold in the theorems is if there are no active constraints at the solution of (14); that is, if the unconstrained Kalman filter satisfies the inequality constraints.

TURBOFAN ENGINE HEALTH MONITORING

The high performance turbofan engine model used in this research is based on a gas turbine engine simulation software package called DIGTEM (Digital Turbofan Engine Model) [8, 16]. DIGTEM is written in Fortran and includes 16 state variables. It uses a backward difference integration

scheme because the turbofan model contains time constants that differ by up to four orders of magnitude.

The nonlinear equations used in DIGTEM can be found in [8, 9]. The time-invariant equations can be summarized as follows.

$$\dot{x} = f(x, u, p) + w_1(t) \qquad (21)$$
$$y = g(x, u, p) + e(t)$$

x is the 16-element state vector, u is the 6-element control vector, p is the 8-element vector of health parameters, and y is the 12-element vector of measurements. The noise term $w_1(t)$ represents inaccuracies in the model, and $e(t)$ represents measurement noise. The state variables and their nominal values at the selected operating point are as follows:

- Low Pressure Turbine (LPT) rotor speed (9200 RPM)
- High Pressure Turbine (HPT) rotor speed (11900 RPM)
- Compressor volume stored mass (0.91294 lbm)
- Combustor inlet temperature (1325 R)
- Combustor volume stored mass (0.460 lbm)
- HPT inlet temperature (2520 R)
- HPT volume stored mass (2.4575 lbm)
- LPT inlet temperature (1780 R)
- LPT volume stored mass (2.227 lbm)
- Augmentor inlet temperature (1160 R)
- Augmentor volume stored mass (1.7721 lbm)
- Nozzle inlet temperature (1160 R)
- Duct airflow (86.501 lbm/s)
- Augmentor airflow (194.94 lbm/s)
- Duct volume stored mass (6.7372 lbm)
- Duct temperature (696 R)

The turbofan controls and their nominal values are as follows:

- Combustor fuel flow (1.70 lbm/s)
- Augmentor fuel flow (0 lbm/s)
- Nozzle throat area (430 in^2)
- Nozzle exit area (492 in^2)
- Fan vane angle (–1.7 deg)
- Compressor vane angle (4.0 deg)

The health parameters and their nominal values are as follows:

- Fan airflow (193.5 lbm/s)
- Fan efficiency (0.8269)
- Compressor airflow (107.0 lbm/s)
- Compressor efficiency (0.8298)
- HPT airflow (89.8 lbm/s)
- HPT enthalpy change (167.0 Btu/lbm)

- LPT airflow (107.0 lbm/s)
- LPT enthalpy change (75.5 Btu/lbm)

The turbofan measurements and their nominal values and signal-to-noise ratios (SNRs) are as follows:

- LPT rotor speed (9200 RPM, SNR = 150)
- HPT rotor speed (11900 RPM, SNR = 150)
- Duct pressure (34.5 psia, SNR = 200)
- Duct temperature (696 R, SNR = 100)
- Compressor inlet pressure (36.0 psia, SNR = 200)
- Compressor inlet temperature (698 R, SNR = 100)
- Combustor pressure (267 psia, SNR = 200)
- Combustor inlet temperature (1325 R, SNR = 100)
- LPT inlet pressure (70.0 psia, SNR = 100)
- LPT inlet temperature (1780 R, SNR = 70)
- Augmentor inlet pressure (31.8 psia, SNR = 100)
- Augmentor inlet temperature (1160 R, SNR = 70)

The SNR values above are based on NASA experience and previously published data [17]. Sensor dynamics are assumed to be high enough bandwidth that they can be ignored in the dynamic equations [17]. Equation (21) can be linearized about the nominal operating point by using the first order approximation of the Taylor series expansion. This gives a linear small signal system model defined for small excursions from the nominal operating point.

$$\delta\dot{x} = A_1\delta x + B\delta u + A_2\delta p + w_1(t) \qquad (22)$$
$$\delta y = C_1\delta x + D\delta u + C_2\delta p + e(t)$$

We note that

$$A_1 = \frac{\partial f}{\partial x} \qquad (23)$$
$$A_1(i, j) \approx \frac{\Delta\dot{x}(i)}{\Delta x(j)}$$

Similar equations hold for the A_2, C_1, and C_2 matrices. We obtained numerical approximations to the A_1, A_2, C_1, and C_2 matrices by varying x and p from their nominal values (one element at a time) and recording the new $\dot{x}$ and y vectors in DIGTEM.

The goal of our turbofan engine health monitoring problem is to obtain an accurate estimate of δp, which varies slowly with time. We therefore assume that δp remains essentially constant during a single flight. We also assume that the control input is constant, so $\delta u = 0$. (In reality, $\delta u \neq 0$, which complicates the problem and will give different results than we present in this paper. This will be explored in further work.) This gives us the following equivalent discrete time system [18, pp. 90 ff.].

$$\delta x_{k+1} = A_{1d}\delta x_k + A_{2d}\delta p_k + w_{1k} \qquad (24)$$
$$\delta y_k = C_1\delta x_k + C_2\delta p_k + e_k$$

where $A_{1d} = \exp(A_1 T)$ and $A_{2d} = A_1^{-1}(A_{1d} - I)A_2$ (assuming that A_1 is invertible, which it is in our problem). We next augment the state vector with the health parameter vector [11] to obtain the system equation

$$\begin{bmatrix} \delta x_{k+1} \\ \delta p_{k+1} \end{bmatrix} = \begin{bmatrix} A_{1d} & A_{2d} \\ 0 & I \end{bmatrix}\begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix} \qquad (25)$$
$$\delta y_k = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + e_k$$

where w_{2k} is a small noise term (uncorrelated with w_{1k}) that represents model uncertainty and allows the Kalman filter to estimate time-varying health parameter variations. The discrete time small signal model can be written as

$$\begin{bmatrix} \delta x_{k+1} \\ \delta p_{k+1} \end{bmatrix} = A\begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + w_k \qquad (26)$$
$$\delta y_k = C\begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + e_k$$

where the definitions of A and C are apparent from a comparison of the two preceding equations. Now we can use a Kalman filter to estimate δx_k and δp_k. Actually, we are only interested in estimating δp_k (the health parameter deviations), but the Kalman filter gives us the bonus of also estimating δx_k (the excursions of the original turbofan state variables). Note that four of the state variables are directly measured with good SNRs. The Kalman filter equations automatically use this information to improve its estimate of all of the state variables and generate the optimal state estimate.

It is known that health parameters do not improve over time. That is, $\delta p(1)$, $\delta p(2)$, $\delta p(3)$, $\delta p(4)$, $\delta p(6)$, and $\delta p(8)$ are always less than or equal to zero and always decrease with time. Similarly, $\delta p(5)$ and $\delta p(7)$ (the two turbine airflow parameters) are always greater than or equal to zero and always increase with time. In addition, it is known that the health parameters vary slowly with time. As an example, since $\tilde{\delta p}(1)$ is the constrained estimate of $\delta p(1)$, we can enforce the following constraints on $\tilde{\delta p}(1)$.

$$\tilde{\delta p}(1) \leq 0 \qquad (27)$$
$$\tilde{\delta p}_{k+1}(1) \leq \tilde{\delta p}_k(1) + \gamma_1^+$$
$$\tilde{\delta p}_{k+1}(1) \geq \tilde{\delta p}_k(1) - \gamma_1^-$$

where γ_1^+ and γ_1^- are nonnegative factors chosen by the user that allows the state estimate to vary only within prescribed limits. Typically we choose $\gamma_1^- > \gamma_1^+$ so that the state estimate can change more in the negative direction than in the positive direction. This is in keeping with our *a priori* knowledge that this particular state variable never increases with time. Ideally we would have $\gamma_1^+ = 0$ since $\delta p(1)$ never increases. However, since the state variable estimate varies around the true value of the state variable, we choose $\gamma_1^+ > 0$. This allows some time-varying increase in the state variable estimate to compensate for a state variable estimate that is smaller than the true state variable value.

These constraints are linear and can therefore easily be incorporated into the form required in the constrained filtering problem statement (5). If the state constraints are nonlinear they can be linearized as discussed in [2]. Note that this does not take into account the possibility of abrupt changes in health parameters due to discrete damage events. That possibility must be addressed by some other means (e.g., residual checking [3]) in conjuction with the methods presented in this paper.

SIMULATION RESULTS

We simulated the methods discussed in this paper using MATLAB. We simulated a steady state 3 second burst of engine data measured at 10 Hz during each flight. The nonlinear DIGTEM software described in the previous section was used to generated the measurement data. Each data collection was performed at the single operating point shown earlier in this paper. The signal-to-noise ratios were determined on the basis of NASA experience and previously published data [17] and are shown earlier in this paper. The Kalman filter was re-linearized around the state estimates every 50 flights. We used a one-sigma process noise in the Kalman filter equal to approximately 1% of the nominal state values to allow the filter to be responsive to changes in the state variables. We set the one sigma process noise for each component of the health parameter portion of the state derivative equation to 0.01% of the nominal parameter value. This was obtained by tuning. It was small enough to give reasonably smooth estimates, and large enough to allow the filter to track slowly time-varying parameters. For the constrained filter, we chose the γ variables in (27) such that the maximum allowable rate of change in δp was the sum of a linear and exponential function that reached 9% after 500 flights in the direction of expected change, and 3% after 500 flights in the opposite direction. The true health parameter values never change in a direction opposite to the expected change. However, we allow the state estimate to change in the opposite direction to allow the Kalman filter

to compensate for the fact that the state estimate might be either too large or too small. We set the weighting matrix W in (14) equal to Σ^{-1} in accordance with Theorem 3.

We simulated an exponential degradation of the eight health parameters over 500 flights. The initial health parameter estimation errors were taken from one-sided normal distributions with standard deviations of 0.5% of their nominal values. The simulated health parameter degradations were representative of turbofan performance data reported in the literature [19]. We ran 30 Monte Carlo simulations like this, each with a different noise history and different initial estimation errors. Figures 1 and 2 show the Kalman filter's performance in a typical case when the initial estimation errors are zero. Both simulations represented in the figures have the same "random" measurement noise. Table 1 shows the performance of the filters averaged over all 30 simulations. The standard Kalman filter estimates the health parameters to within 4.66% of their final degradations. The constrained filter estimates the health parameters to within 3.90% of their final degradations. These numbers show the improvement that is possible with the constrained Kalman filter.

The improved performance of the constrained filter comes with a price, and that price is computational effort. The constrained filter requires about four times the computational effort of the unconstrained filter. This is because of the additional quadratic programming problem that is required. However, computational effort is not a critical issue for turbofan health estimation since the filtering is performed on ground-based computers after each flight.

Note that the Kalman filter works well only if the assumed system model matches reality fairly closely. The method presented in this paper, by itself, will not work well if there are large sensor biases or hard faults due to severe component failures. A mission-critical implementation of a Kalman filter should always include some sort of residual check to verify the validity of the Kalman filter results [20], particularly for the application of turbofan engine health estimation considered in this paper [3].

It can be seen from the figures that although the constrained filter improves the estimation accuracy, the general trend of the state variable estimates does not change with the introduction of state constraints. This is because the constrained filter is based on the unconstrained Kalman filter. The constrained filter estimates therefore have the same shape as the unconstrained estimates *until* the constraints are violated, at which point the state variable estimates are projected onto the edge of the constraint boundary. The constrained filter presented in this paper is not qualitatively different than the standard Kalman filter; it is rather a quantitative improvement in the standard Kalman filter.

Table 1. KALMAN FILTER ESTIMATION ERRORS. THE NUMBERS SHOWN ARE RMS ESTIMATION ERRORS AVERAGED OVER 30 SIMULATIONS WHERE EACH SIMULATION HAD A LINEAR-PLUS-EXPONENTIAL DEGRADATION OF ALL EIGHT HEALTH PARAMETERS. THE NUMBERS SHOW THE ERROR BETWEEN THE ESTIMATED AND ACTUAL DEGRADATION AS PERCENTAGES OF THE DEGRADATION AT THE FINAL TIME.

	Estimation Error (%)	
Health Parameter	Unconstrained	Constrained
Fan Airflow	4.81	4.41
Fan Efficiency	5.85	4.60
Compressor Airflow	3.43	2.73
Compressor Efficiency	4.82	3.80
HPT Airflow	3.09	2.39
HPT Enthalpy Change	4.48	3.76
LPT Airflow	4.54	4.26
LPT Enthalpy Change	6.28	5.22
Average	4.66	3.90

CONCLUSION

We have presented an analytic method for incorporating linear state inequality constraints in a Kalman filter. This maintains the state variable estimates within a user-defined envelope. The simulation results demonstrate the effectiveness of this method, particularly for turbofan engine health estimation.

If the system whose state variables are being estimated has known state variable constraints, then those constraints can be incorporated into the Kalman filter as shown in this paper. However, in implementation, the constraints enforced in the filter might be more relaxed than the true constraints. This allows the filter to correct state variable estimates in a direction that the true state variables might never change. This is a departure from strict adherence to theory, but in practice this improves the performance of the filter. This is an implementation issue that is conceptually similar to tuning a standard Kalman filter.

We saw that the constrained filter requires a much larger computational effort than the standard Kalman filter. This is due to the addition of the quadratic programming problem that must be solved in the constrained Kalman filter. The engineer must therefore perform a tradeoff between computational effort and estimation accuracy.
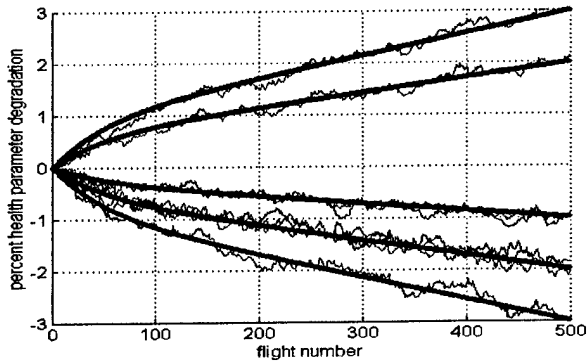
Figure 1. UNCONSTRAINED KALMAN FILTER ESTIMATES OF HEALTH PARAMETERS. TRUE HEALTH PARAMETER CHANGES ARE SHOWN AS HEAVY LINES. FILTER ESTIMATES ARE SHOWN AS LIGHTER LINES.
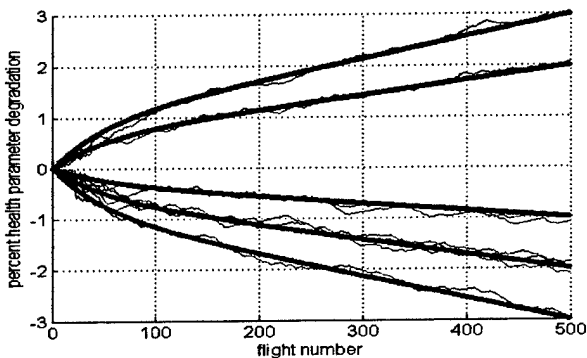

Figure 2. CONSTRAINED KALMAN FILTER ESTIMATES OF HEALTH PARAMETERS. TRUE HEALTH PARAMETER CHANGES ARE SHOWN AS HEAVY LINES. FILTER ESTIMATES ARE SHOWN AS LIGHTER LINES.

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13. ABSTRACT (Maximum 200 words)

Kalman filters are often used to estimate the state variables of a dynamic system. However, in the application of Kalman filters some known signal information is often either ignored or dealt with heuristically. For instance, state variable constraints (which may be based on physical considerations) are often neglected because they do not fit easily into the structure of the Kalman filter. This paper develops an analytic method of incorporating state variable inequality constraints in the Kalman filter. The resultant filter is a combination of a standard Kalman filter and a quadratic programming problem. The incorporation of state variable constraints increases the computational effort of the filter but significantly improves its estimation accuracy. The improvement is proven theoretically and shown via simulation results obtained from application to a turbofan engine model. This model contains 16 state variables, 12 measurements, and 8 component health parameters. It is shown that the new algorithms provide improved performance in this example over unconstrained Kalman filtering.

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