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13. ABSTRACT (Maximum 200 Words) This report describes progress made during the second year of study. Our goal is to detect masses in dense mammograms having a diameter less than 1 cm. The idea of this project is to detect subtle masses by tuning the central frequency and width of a basis function used in an overcomplete expansion. By modeling the shape of a mass through this flexibility we hope to detect small and subtle masses in dense breasts and improve the chances of early detection in screening mammography. During this year of the project we focused on an enhancement method to be carried out prior the detection algorithm and have begun to incorporate an expansion based method of detecting masses in digital radiographs. We showed that the method of adaptive multi-scale histogram equalization can be integrated into our existing expansion framework as a preprocessing step to assist detection. This method will be tested using a local database of digital mammograms during the final year of the project.				
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1. INTRODCUTION

From a signal processing prospective, masses within a digital mammogram represent low frequency signals. Here we take this into account in order to choose among a family of existing wavelet basis functions. In general, the functions we want to use should be smooth in terms of shape and can, for instance be first or second derivative functions. The fractional spline family appeared most promising in this regard since it has one continuously varying order parameter: one can therefore "tune" the central frequency of the wavelet. In our previous report, this flexibility let us think of designing a wavelet that would be the most efficient for our purpose of mass detection. In this report, we recognize and address the advantages of contrast enhancement as a preprocessing step towards automated detection. Also, we have begun to develop and implement a detection algorithm based on level set theory that will be integrated into the existing multi-resolution framework provided by the continuous wavelet expansions described in our previous report.

Previously in this study we showed that the dyadic transform was not sufficient for our case mass of detection [1]. Last year, we developed a 2D-continuous discrete transform (implementing the "*a trou* algorithm"). This provided a framework to design the fractional spline wavelet. In this phase, we introduce a multi-scale contrast enhancement algorithm by embedding an adaptive histogram equalization operator into an over-completed dyadic wavelet framework. The over-completed wavelet expansion preserves essential translation invariance and provides a degree of selectivity between signal and noise in a spatial-frequency domain representation. By comparison, adaptive histogram equalization (AHE) achieves a global contrast enhancement by adjusting contrast locally through reassigning a central pixel the value through a local histogram equalization mapping function. Traditional AHE methods usually generate undesirable noise amplification and artifacts. However, by taking advantage of the signal and noise selectivity of a spatial-frequency expansion, a multi-scale adaptive histogram equalization process can be realized. This report describes some remarkable contrast enhancement results that do not amplify noise or introduce artifacts. As a local enhancement, adaptive histogram equalization provides the power to enhance features within distinct dynamic ranges. This property is desirable for clinical screening of mammograms as well as a preprocessing step for automated detection of subtle masses that contain multiple tissue types across a narrow band of attenuation levels. In addition, we also describe an improvement to the original implementation of AHE first published in 1987 that removes artifacts due to the use of bi-linear interpolation. This new implementation of the core algorithm also avoids quantization errors associated with the computation of discrete histograms.

2. BODY

The visual interpretation of images remains the basis of diagnostic decisions and surgical screening for many diseases including lung and breast cancer, and stroke. In modalities such as computed tomography (CT), chest radiography and digital mammography, the images from the acquisition system usually detect 12 to 16 bits of gray level intensity information (2 to the 12th thru 16th power distinct gray levels) while at best the human visual system typically can distinguish less than 100 distinct shades of gray [2]. Moreover, the contrast within a tissue or organ of interest is visually low due to the small differences in X-ray attenuation.

The contrast between different tissue types dominate the available dynamic range, which can suppress contrast needed for detecting local details (e.g., boundaries of a mass) within a tissue or organ of interest. Contrast enhancement using linear intensity windowing is now usually applied on digital displays targeting a subset of the dynamic range in an image before clinical reading [2]. With the existence of multiple tissues of interest, multiple windowing of images can be provided sequentially. Often radiologists need to perform three such readings per case. The process of selecting windows and interpreting resultant windowed/leveled mammograms is time consuming, even with the advent of digital imaging and display technology which allow window settings to be set rather quickly at an interactive console. The dream of having an automatic single window display with simultaneous enhancement of all tissues of interest would significantly improve diagnostic efficiency.

Besides linear windowing, histogram equalization is also widely utilized as a global image enhancement technique [4, 5]. These two approaches provide simple and efficient ways of global enhancement. Nevertheless, they do not take into account local properties of the image, and therefore not all of the useful information can be enhanced simultaneously. In addition, histogram equalization has the undesired effect of overemphasizing noise [6]. Histogram equalization can be performed as a point operator [7]:

$$\tilde{x}(i, j) = CDF(x(i, j)) \quad (1)$$

where $x(i, j)$ and $\tilde{x}(i, j)$ are input and output pixel value at location (i, j) respectively (normalized to the range of [0,1]). And $CDF(x)$ is the cumulative distribution function of the input image:

$$CDF(t) = \frac{1}{A} \int_0^t H(u) du \quad (2)$$

where A is the total number of pixels in the input image and $H(u)$ is the histogram of the input image. The computation of $H(u)$ usually needs to discretize the whole dynamic range into a certain number of bins and collect the number of pixels belonging to each bin.

In order to take into account local information, achieve contrast enhancement in regions exhibiting different dynamic ranges, an adaptive histogram equalization (AHE) method was proposed [8, 9]. In this method, for each pixel, the histogram of a local window centered on the pixel is computed, and the resultant value of the pixel is assigned by the histogram equalization operator in (1) using the local histogram. The size of the local window is the only parameter for the adaptive histogram equalization enhancement. To facilitate the visualization of local details, the window size needs to be small. However, a small window size brings instability to computing the local histogram due to the inherent discretization of intensity values. On the other hand, a larger window size increases the computational load. Since one a local histogram is needed for each pixel in the image, this method required intensive computations. A bi-linear interpolation technique was introduced into the implementation to reduce the

computational cost. It first divides images into blocks, and then a histogram equalization mapping function (1) is computed for each block.

For each pixel in an image, four mapping values from mapping functions of its four nearest blocks are obtained. And a bi-linear interpolation based on those four values and the distance from the target pixel to the center of the blocks was applied to generate an enhanced value of the pixel. One disadvantage of AHE is that the amount of contrast enhancement that it performs is so great that in relatively homogeneous areas or areas of low signal the noise component becomes very prominent. The resulting appearance is unattractive and distracting to physicians reading images for diagnosis [2]. Furthermore, in certain images, the amount of enhancement produced by AHE may cause ring artifacts along strong edges [10]. A sample image processed with this method is shown in Figure 1 for its original application to CT imaging of the chest.

Other local enhancement methods as well as variations of the original AHE algorithm have since been investigated. However, these methods were performed in spatial domain, and a lack of feature selectivity mechanism to distinguish between signal and noise (mass and background). Despite the resulting noise amplification and artifact problems, the AHE method is still a remarkable local enhancement method, and therefore there is a lot of active research on improving the AHE algorithm appearing in the literature [11-14].

Wavelet transforms, on the other hand, provide a natural framework of feature analysis based on spatial-frequency properties [15-17]. Multi-scale enhancement has been applied successfully to many medical images including mammography [18-20]. Linear and non-linear threshold and enhancement operators were used in these early multi-scale enhancement methods [21]. By combining feature selectivity of multi-scale analysis and the local enhancement power of adaptive histogram equalization, we introduce in this paper a multi-scale adaptive histogram equalization (MSAHE), which theoretically, provides the local enhancement in both spatial domain and frequency domain.

METHODOLOGY

A. Implementation of Adaptive Histogram Equalization

Figure 2 (a) shows a phantom image with linearly varied background, and six blended disks of different size and brightness. Enhanced by a bi-linear interpolation version of AHE, the resultant images successfully improve the visible presence of some of the objects but create unfavorable strip-like artifacts over the background, as shown in Figures 2(b) and 2(c). The explanation of why this occurs is related to bi-linear interpolation and can be seen more clearly in a smaller version of the background demonstrated in Figure 3.

From the original definition of AHE [9], for each pixel, a local window centered at itself was extracted from the image, and the mapping function (2) was constructed, then an enhanced value was computed by (1). Note that we only need the value of the CDF function at x (the original pixel value), so there is no need to compute the complete CDF function. In (2), A is a simple counting of how many pixels are in the local window, this is a known value. The second term $\int_0^x H(u)du$ actually equals the number of pixels in the local window that have the intensity value smaller than x . Therefore the resultant value is

$$\tilde{x} = \frac{\text{Number_of_Pixels : gray_value} < x}{\text{Total_Number_of_Pixels}} \quad (3)$$

This can be implemented very efficiently, and has the advantage of:

(1) No interpolation is required; therefore we avoid the “strip” artifacts in Figure 2(c) and 2(d). The slow gradient background sometimes appears due to the acquisition or digital scanning system, so the ability to remove this artifact is important; see Figure 2(e) and 2(f).

(2). No need of quantizing the dynamic range for computing a discrete histogram. Therefore we avoid the quantization errors from the continuous histogram when the number of pixels are not sufficient. This provides the possibility of using relatively small window sizes.

Notice that (3) only works if the pixel intensity values are normalized to the range of $[0,1]$, so a minor modification is needed for the cases that pixel intensities take on arbitrary values.

$$\tilde{x} = \left[\frac{\text{Number_of_Pixels : gray_value} < x}{\text{Total_Number_of_Pixels}} \right] * [\max(x) - \min(x)] + \min(x) \quad (3a)$$

where $\max(x)$ and $\min(x)$ are the maximum and minimum intensity value over the whole image.

The size of a local window used for adaptive histogram equalization determines the enhancement result. Ideally, the smaller the analysis window, the higher local contrast in the resultant images will be expected [22]. But in the discrete implementation, smaller windows mean there are fewer pixels for analysis. Since histogram equalization is a statistical operator, and only a discrete approximation is possible too few pixels will cause numerical instability and significant deviation from the theoretical continuous case. Also, enhancement in adaptive histogram equalization is always accompanied by noise amplification. Therefore a suitable choice of the size of analysis window is essential for optimizing enhancement.

B. Multi-scale Enhancement

In this section we first briefly review the dyadic wavelet transform, and its implementation. For a one dimensional signal $f(x)$, the continuous wavelet transform is defined by [17]:

$$Wf(u, s) = f * \psi_{u,s} = \int_{-\infty}^{+\infty} f(x) \frac{1}{\sqrt{s}} \psi^* \left(\frac{x-u}{s} \right) dx \quad (4)$$

where the basis functions $\psi_{u,s}$ are generated by translating and scaling a zero averaged function $\psi(x)$ called mother wavelet,

$$\psi_{u,s} = \frac{1}{\sqrt{s}} \psi \left(\frac{x-u}{s} \right) \quad (5)$$

To allow a fast numerical implementation of a discrete wavelet transform, Mallat and Zhong [23] introduced the dyadic wavelet transform, where the scale parameter s varies only along the dyadic sequence $\{2^j\}$ with $j \in Z$. An extension of this approach to two dimensions by using a tensor product of separable wavelet basis yields the 2-D dyadic wavelet transform that partitions the hyper-plane into orientation sub-bands. Two channels of analysis along orthogonal directions x and y are generated for each dyadic scale. These two components of the 2-D signal $f(x, y)$ at scale 2^j are given by:

$$W_{2^j}^1 f(x, y) = f * \psi_{2^j}^1(x, y) \quad \text{and} \quad W_{2^j}^2 f(x, y) = f * \psi_{2^j}^2(x, y) \quad (6)$$

$$\text{with} \quad \psi_{2^j}^d(x, y) = \frac{1}{\sqrt{2^j}} \psi^d \left(\frac{x}{2^j}, \frac{y}{2^j} \right), \quad (d=1,2) \quad (7)$$

The corresponding wavelet functions $\psi_{2^j}^d(x, y)$ are defined as

$$\psi^1(x, y) = \frac{\partial \theta(x, y)}{\partial x} \quad \text{and} \quad \psi^2(x, y) = \frac{\partial \theta(x, y)}{\partial y} \quad (8)$$

where $\theta(x, y)$ is a 2-D smoothing function, whose integral over x and y is equal to 1 and converges to 0 at infinity.

For our enhancement algorithm we used the quadratic spline wavelet function defined by Mallat and Zhong [23] of compact support and continuously differentiable. Its Fourier transform can be derived as

$$\hat{\psi}(\omega) = (j\omega) \left[\frac{\sin(\omega/4)}{\omega/4} \right]^4 \quad (9)$$

where $\psi(x)$ is the first derivative of a cubic spline function $\theta(x)$. These functions are displayed for the one-dimensional case in Figure 4. The quadratic spline wavelet has been used successfully for edge detection and feature enhancement, and it approximates the first derivatives of a Gaussian function [24, 25]:

(1) By the uncertainty principle. The Gaussian probability density function is optimally concentrated in both time and frequency domain, and thus suitable for time-frequency analysis.

(2) Derivatives of Gaussian functions can be used for rotation-invariant processing.

(3) The Gaussian function generates a causal (in a sense that a coarse scale depends exclusively on the previous finer level) scale space. This makes possible scale-space “tracking” of emergent features.

Unlike traditional non-redundant discrete wavelet transforms, which lack translation invariance and include aliasing after the analysis stage due to down-sampling [20], dyadic wavelet transforms provide an over-complete representation without decimation, and can be implemented by fast hierarchy filtering procedures [19,21]. Figure 5 illustrates a schematic framework of a filter bank implementation of the over-completed multi-scale enhancement [21] used in this study. The redundancy is exploited for image enhancement by first modifying transform coefficients and then reconstructing. Notice that since the DC-cap contains a large amount of energy, it is usually kept untouched during the enhancement procedure. As shown above, the enhancement function can be implemented independently from the particular set of filters and is easily incorporated into a filter bank to provide the benefits of multi-scale enhancement [18,26].

For each sub-band image (wavelet coefficients) y_i , the enhancement function (10) was applied:

$$\hat{y}_i = f_i(y_i) \quad (10)$$

where $f_i(x)$ are a set of user-defined functions designed to emphasize features of interest within sub-band i . The enhanced sub-band coefficients \hat{y}_i are then used to reconstruct the output image. In general, by defining the function $f_i(x)$, we can denote specific enhancement schemes for modifying sub-band coefficients within distinct levels of scale space.

The enhancement function $f_i(x)$ must be single-valued, monotonically increasing in order to preserve the order of intensity information (X-ray attenuation and absorption) in the original image and to avoid artifacts [19]. Figure 6 displays some of the traditionally used enhancement functions. These functions are typically used for both enhancement and denoising [27, 28].

C. Multi-scale Adaptive Histogram Equalization

Given the framework in Figure 5, it is quite straightforward to integrate the adaptive histogram equalization operator as an enhancement module. As discussed in the previous sub-section, the size of the analysis window in adaptive histogram equalization algorithm is a key parameter for enhancement, in a multi-scale framework. The following factors determine our choice of this parameter for distinct sub-band coefficients.

- (1) *The frequency properties of the sub-band.* The coarser levels include more general shape information and should not be changed too aggressively. Therefore a larger size of analysis window was chosen. On the other hand, the finer levels include more local information, which might benefit from enhancement. Therefore a relatively smaller analysis window was chosen. There is also a solid theoretical basis for choosing the window size. The dyadic wavelet expansion determines the frequency property across levels and also changes in a dyadic (power of two) ratio. Therefore we choose the analysis window to roughly match the dyadic ratio across the levels of expansion so that and the spatial-frequency properties of the sub-bands matched the support of each decomposition filter used in the expansion.
- (2) *The location of the feature of interest in spatial-frequency domain.* For the sub-bands that contain information of the features we want to enhance, we may want to choose the size of the analysis window to be adaptive. In this case, there will be a trade-off between enhancement and noise reduction.
- (3) *The size of the image.* As we discussed before, size of the analysis windows determines the number of pixels for each analyzing histogram, and therefore a larger window gives a better approximation to the continuous case. However, the choice of window size should also take into account of size of the image matrix, so that even for sub-bands that have the smallest analysis windows, there are adequate pixels in each local window to approximate the theoretical continuous case.

For a typical high resolution digital mammogram image with size of 4K by 5K pixels, a selected ROI of 512 by 512 pixels, a 5 level dyadic expansion is usually used, and analysis windows vary from 128 by 128 pixel to 16 by 16 pixels in matrix size.

We note that a denoising operator can be easily added without additional computational cost. Combining (3a) and (10),

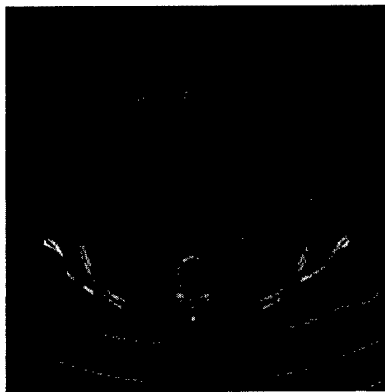
$$x_{enh} = f(\tilde{x}) \quad (11)$$

where \tilde{x} is given by (3a), where $f(x)$ can be any multi-scale denoising operator, e.g., those illustrated in Figure 6.

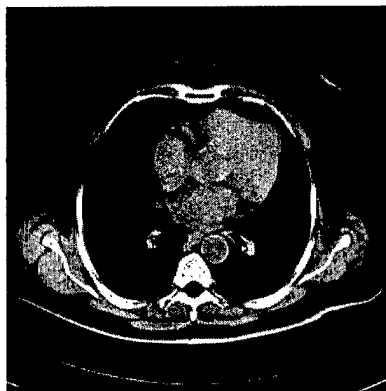
Sample results

Figures 9 and 10 illustrate results of multi-scale AHE enhancement on selected mammograms containing masses. Compared to traditional enhancement, the multi-scale enhancement provided a clear improvement in the visibility of the borders of the mass. In Figure 10, which shows the power of multi-scale AHE comparing to traditional enhancement methods, we mention that multi-scale expansion of AHE kept the overall image appearance (the linear varied background) which usually is not possible for traditional non-multiscale methods. This capability is essential for clinical use in diagnostic screening as stated previously.

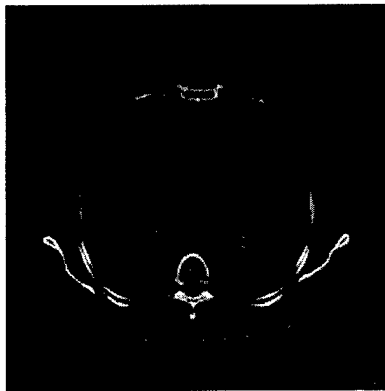
On the other hand, we notice that, like all other methods of contrast enhancement, the trade-off between detectability and over enhancement on sharp edges still exists (Figure 8). This calls for the future research of the multi-scale AHE for feature specific optimization. Thus the method could be potentially useful as an efficient tool for improving clinical diagnostic reading and mammography screening.



(a)



(b)



(c)



(d)

Figure 1: Original application of AHE processing for chest CT lung image (a), and three windowing/leveling settings specific for (b) soft tissue; (c) bone and (d) lung.

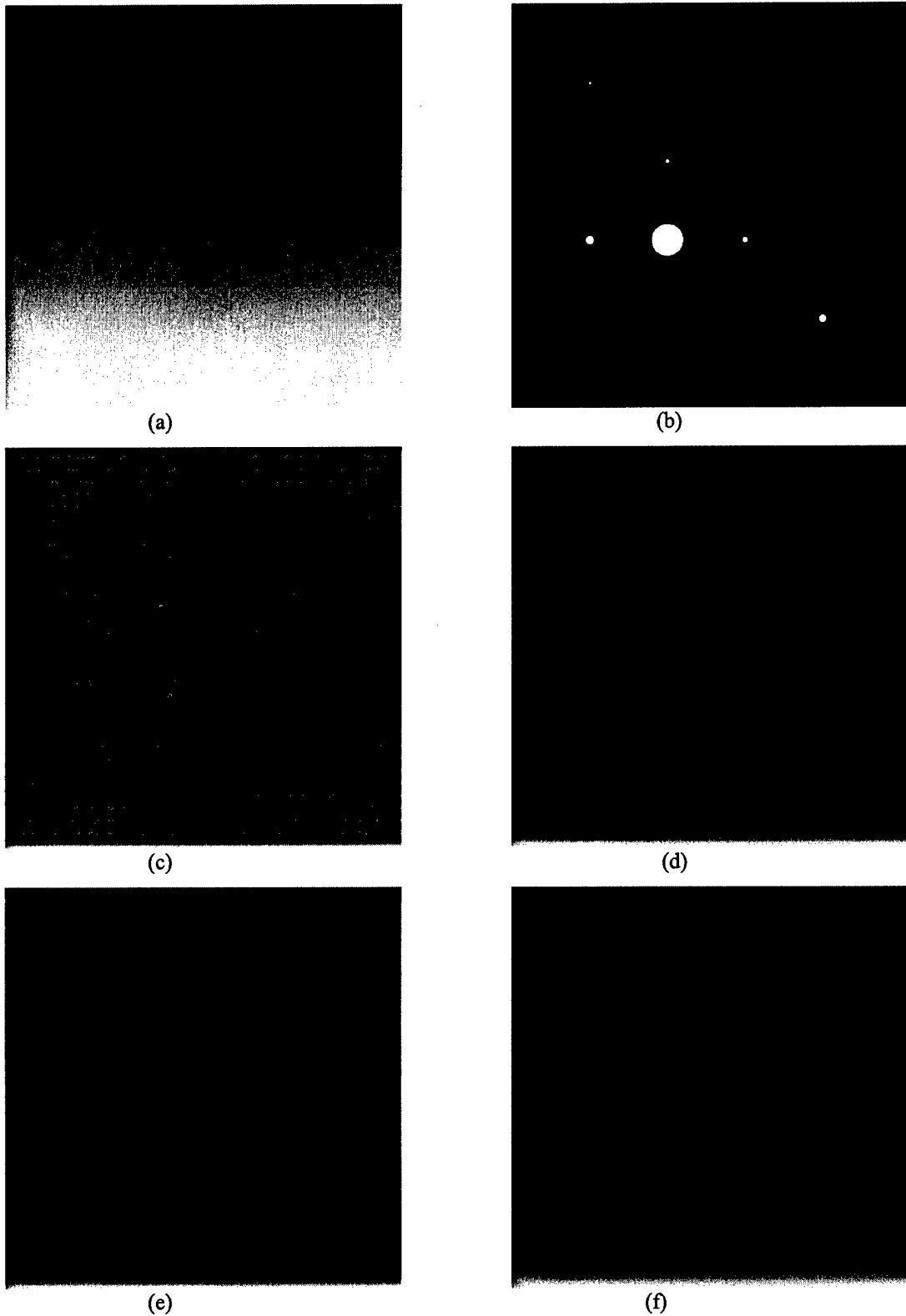


Figure 2: (a) Phantom image with linearly varied background and 6 disks with different size and brightness (b) location of the 6 disks. (c)(d) enhanced by interpolation version of AHE. (e)(f) non-interpolation version of AHE. Left column: window size 16 by 16. right column: window size 32 by 32.

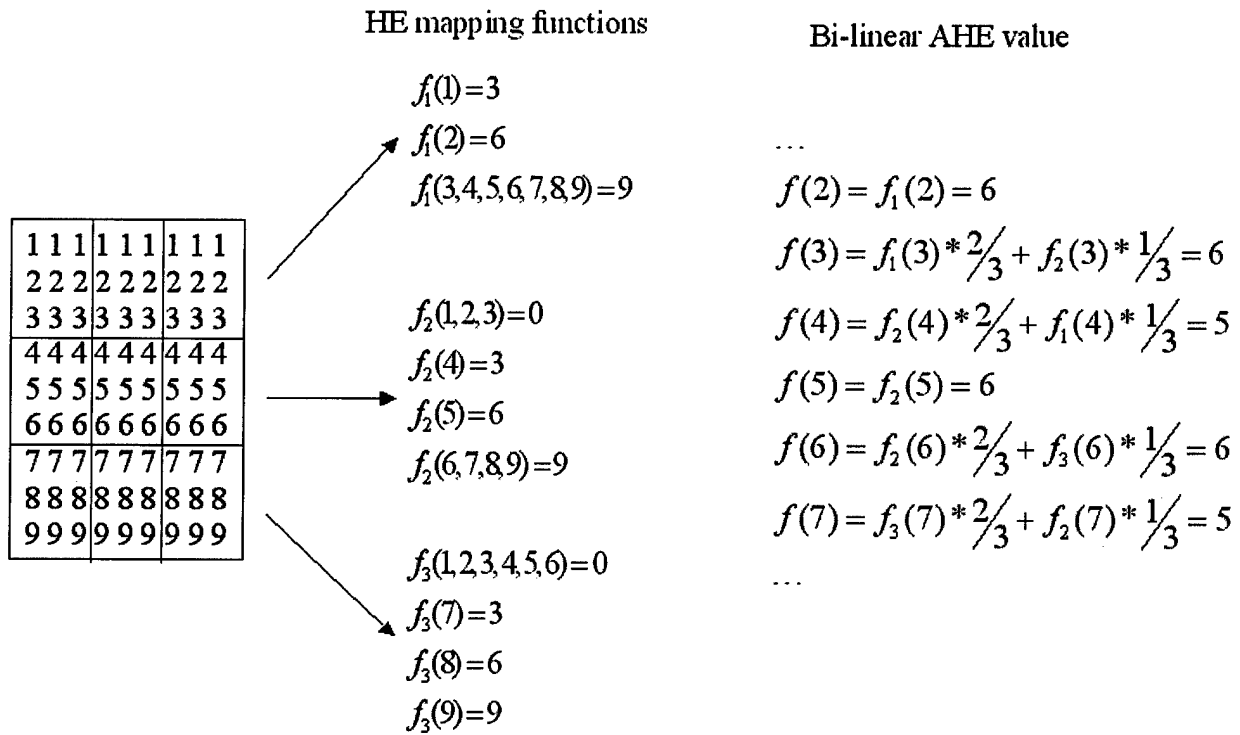


Figure 3: An illustration of how the bi-linear interpolation in the original AHE algorithm causes the strip-like artifact on a linearly varying intensity field image. For simplicity, the dynamic range was assumed to be [0, 9]. Since the intensity value in the original image is constant row-wise, the bi-linear interpolation becomes a linear interpolation along the column. Notice that the “dark” strip appears along the 4th and 7th row after the bi-linear AHE enhancement processing.

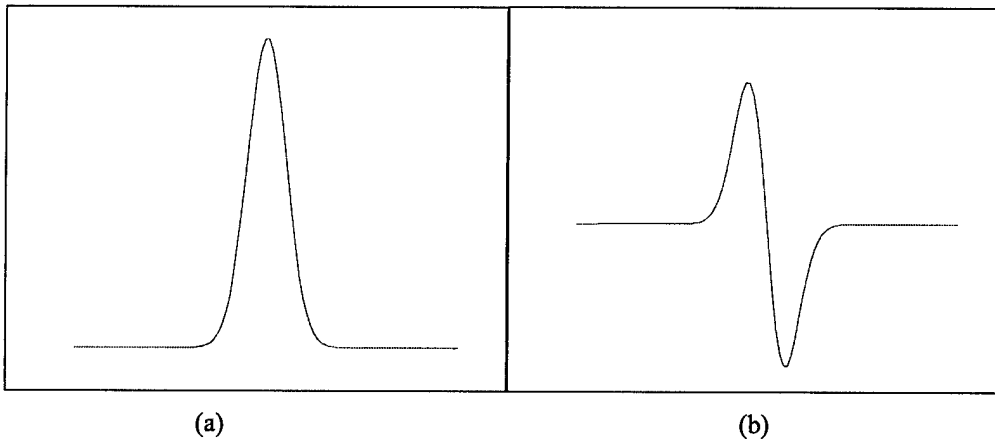
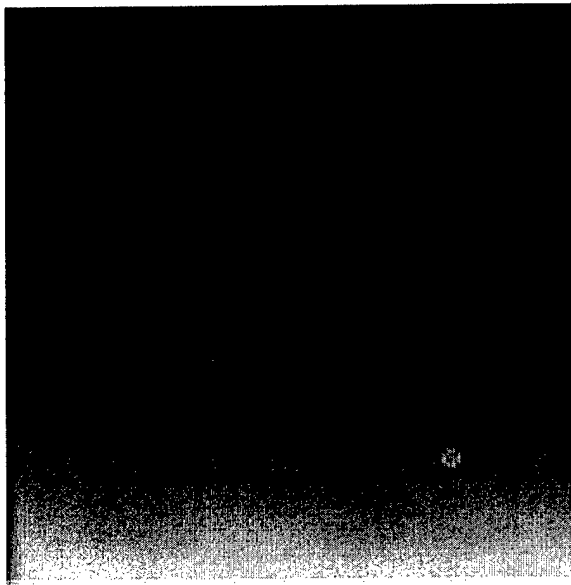
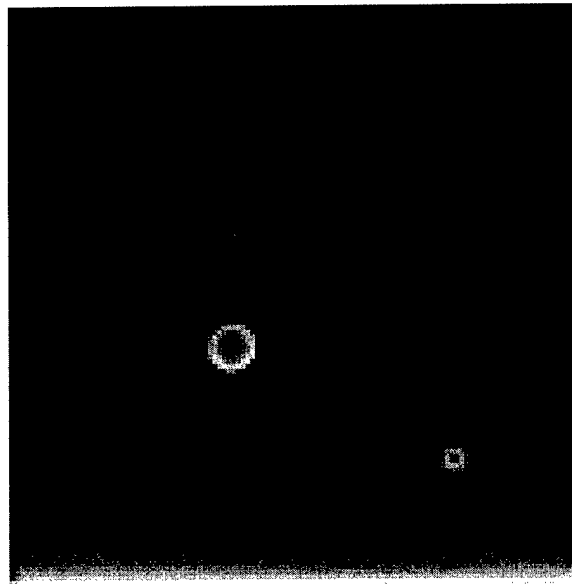


Figure 4: (a) Cubic spline smoothing function $\theta(x)$. (b) Quadratic spline wavelet $\psi(x)$ of compact support defined as the derivative of the smoothing function.

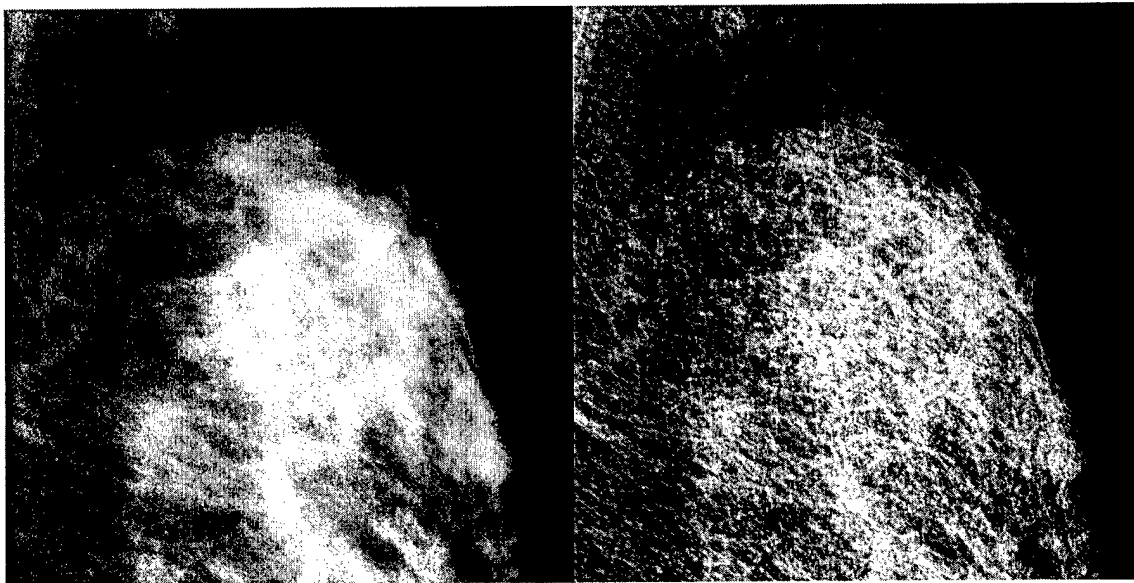


(a)



(b)

Figure 8: Enhancement of the phantom image in Figure 2(a) by multi-scale AHE using different window size combination.

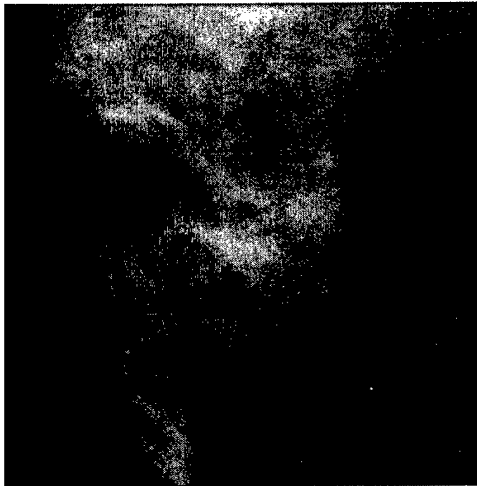


(a)

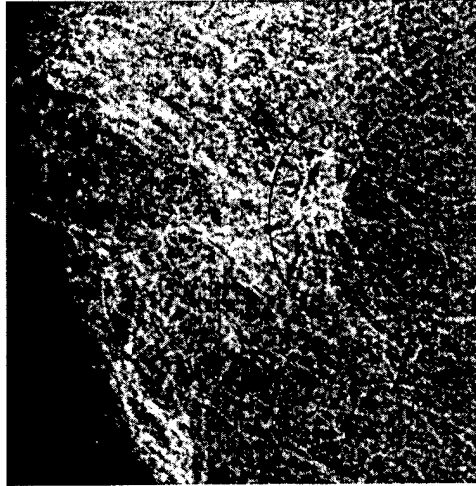


(b)

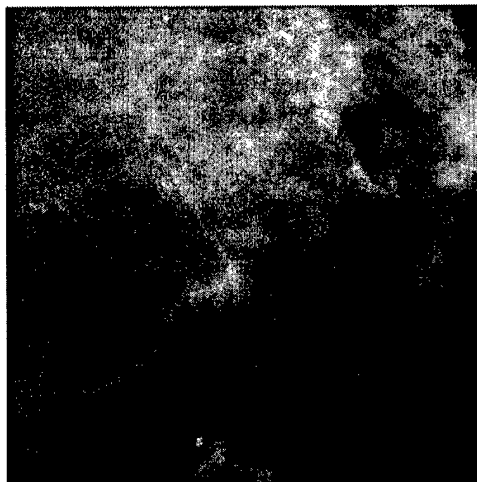
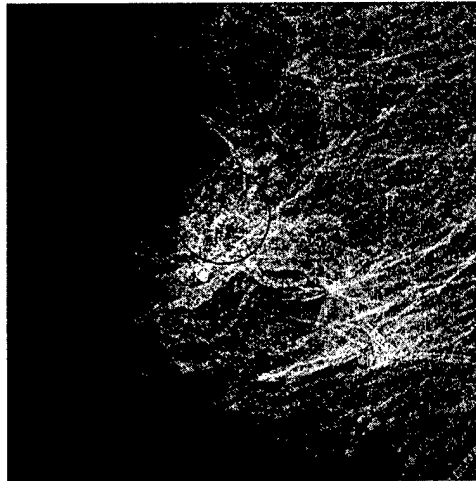
Figure 9: Enhancement of selected mammogram region. (a) original mammogram. (b) enhanced mammogram by MAHE.



subtlety: 3



subtlety: 2



subtlety: 1

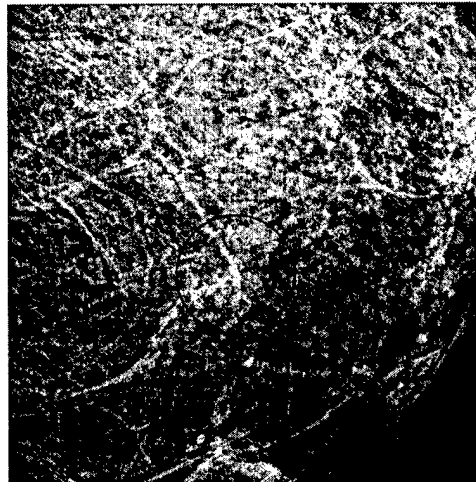


Figure 10: The subtlety value is a subjective ranking of the subtlety of a lesion by an expert radiologist. It has a value between 1 and 5, where 1 is "subtle" and 5 is "obvious." The subtlety value for a lesion may indicate how difficult it is to find the lesion.

3. KEY RESEARCH ACCOMPLISHMENTS

During this year of the project we focused on an enhancement method to be carried out prior the detection algorithm and have begun to incorporate an expansion based method of detecting masses in digital radiographs.

- We showed that the method of adaptive multi-scale histogram equalization can be integrated into our existing expansion framework as a preprocessing step to assist detection.
- The MSAHE pre-processing method was integrated into our previously described multi-scale expansion framework and will be tested using a local database of digital mammograms for validation during the final year of the project.

4. REPORTABLE OUTCOMES

1. Publications:

Results of this study will be presented at the SPIE Mathematical Imaging Symposium: Wavelets X, August 2-8, 2003, San Diego, CA. *DETECTION OF MASSES IN DIGITAL MAMMOGRAPHY*,

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5. CONCLUSIONS

We introduced a new contrast enhancement algorithm, inspired from adaptive histogram equalization and over-complete multi-scale expansions, which provide a method for automatic simultaneous enhancement of the full dynamic range of digital mammograms. Unlike adaptive histogram equalization and its variations, the noise amplification problem was handled naturally by an implicit multi-scale analysis framework. It can significantly reduce diagnostic interpretation time for screening, and can possibly bring out more anatomical details not visible by simply windowing and leveling techniques. MAHE works on a platform of a multi-resolution expansion and redundant representation. This algorithm maybe improved further by incorporating clipping levels into the adaptive equalization process to specifically target challenging diagnostic problems.

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