

Short Vector SIMD Code Generation for DSP Algorithms

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Report Documentation Page

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Outline

- Short vector extensions
- Digital signal processing (DSP) transforms
- SPIRAL
- Vectorization of SPL formulas
- Experimental results



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SIMD Short Vector Extensions

vector length = 4
(4-way)



- Extension to instruction set architecture
- Available on most current architectures
- Originally for multimedia (like MMX for integers)
- Requires fine grain parallelism
- **Large potential speed-up**

Name	<i>n</i> -way	Precision	Processors
SSE	4-way	float	Intel Pentium III and 4, AMD AthlonXP
SSE2	2-way	double	Intel Pentium 4
3DNow!	2-way	float	AMD K6, K7, AthlonXP
AltiVec	4-way	float	Motorola G4
IPF	2-way	Float	Intel Itanium, Itanium 2



Problems

- SIMD instructions are architecture specific
- No common API (usually assembly hand coding)
- Performance **very sensitive** to memory access
- Automatic vectorization (by compilers) **very limited**

➔ **Requires expert programmers**

Our Goal: **Automation for digital signal processing (DSP) transforms**



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DSP (digital signal processing) transforms

sampled signal (a vector)

transform (a matrix)

$$x \mapsto Mx$$

Example: Discrete Fourier Transform (DFT) size 4

$$DFT_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & i \end{bmatrix} \begin{bmatrix} 1 & 1 & & \\ & 1 & -1 & \\ & & 1 & 1 \\ & & & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$DFT_4 = (DFT_2 \otimes I_2) D (I_2 \otimes DFT_2) P$$

- Fast algorithm = product of structured sparse matrices
- Represented as **formula** using few constructs (e.g., \otimes) and primitives (diagonal, permutation)
- Captures a large class of transforms (DFT, DCT, wavelets, ...)



Tensor (Kronecker) Product of Matrices

$$A \otimes B = [a_{kl} B]_{k,l} \quad \text{for } A = [a_{kl}]_{k,l}$$

coarse structure fine structure

Examples:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes I_2 = \begin{bmatrix} 1 & 2 & & \\ & 1 & 2 & \\ 3 & 4 & & \\ & 3 & 4 & \end{bmatrix}$$

identity matrix

$$I_2 \otimes \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & & \\ 3 & 4 & & \\ & & 1 & 2 \\ & & 3 & 4 \end{bmatrix}$$



key construct in many DSP transform algorithms (DFT, WHT, all multidimensional)



SPIRAL: A Library Generator for Platform-Adapted DSP Transform

www.ece.cmu.edu/~spiral

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Observation:

- For a given transform there are **maaaany** different algorithms (equal in arithmetic cost, differ in data flow)
- The best algorithm and its implementation is **platform-dependent**
- It is **not clear** what the best algorithm/implementation is

SPIRAL:

Automatic algorithm generation
+ Automatic translation into code
+ Intelligent search for “best”

= **generated** platform-adapted implementation



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SPIRAL'S Mathematical Framework

Transform DFT_n parameterized matrix

Rule $DFT_{nm} \rightarrow (DFT_n \otimes I_m) \cdot D \cdot (I_n \otimes DFT_m) \cdot P$

- a breakdown strategy
- product of sparse matrices

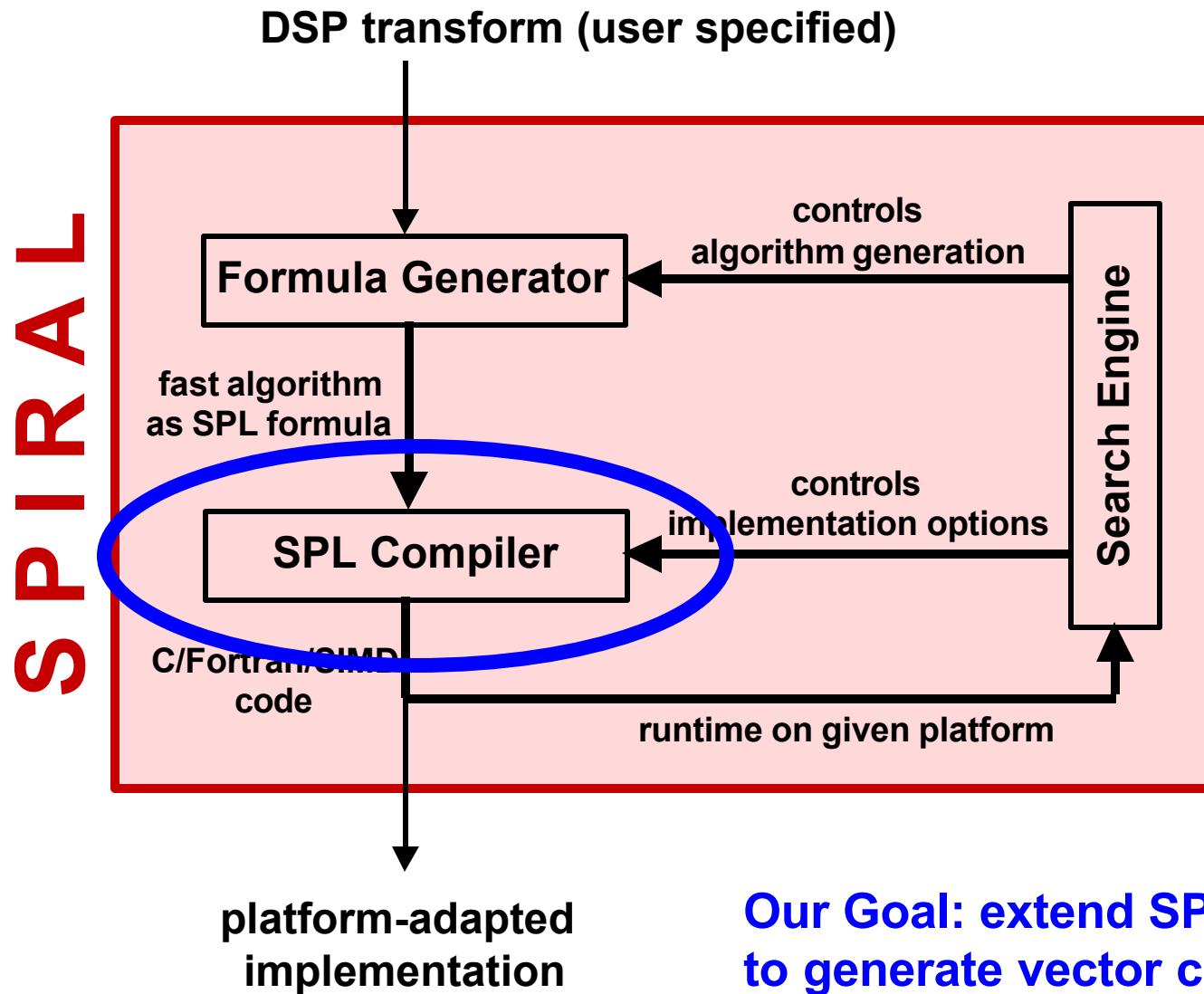
Formula $DFT_{16} = (DFT_4 \otimes I_4) \cdot T_4^{16} \cdot (I_4 \otimes DFT_4) \cdot L_4^{16}$

- by recursive application of rules
- few constructs and primitives
- can be translated into code

**Used as mathematical high-level
representation of algorithms
(SPL = signal processing language)**



SPIRAL system



Our Goal: extend SPL compiler to generate vector code

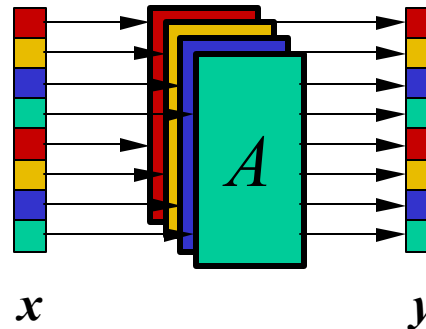


Generating SIMD Code from SPL Formulas

Example:

$$y := (A \otimes I_4)x$$

vector length



naturally represents vector operation

(Current) generic construct **completely vectorizable**:

$$\prod_{i=1}^k P_i D_i (A_i \otimes I_v) E_i Q_i$$

P_i, Q_i permutations
 D_i, E_i diagonals
 A_i arbitrary formulas
 v SIMD vector length

- Formulas contain **all structural information** for vectorization
- Construct above captures DFT, WHT, all multi-dimensional



The Approach

- Use macro layer as *API* to hide machine specifics
- Vector code generation in two steps
 1. Symbolic vectorization (formula manipulation)
 2. Code generation



Symbolic Vectorization

$$DFT_{16} = (DFT_4 \otimes I_4) \cdot T_4^{16} \cdot (I_4 \otimes DFT_4) \cdot L_4^{16}$$



Formula manipulation
(automatic using manipulation rules)

$$\overline{DFT}_{16} = \left((I_4 \otimes L_4^8) \cdot (\overline{DFT}_4 \otimes I_4) \cdot \overline{T}_4^{16} \right)$$

$$\left((I_4 \otimes L_2^8) (L_4^{16} \otimes I_2) (I_4 \otimes L_4^8) \cdot (\overline{DFT}_4 \otimes I_4) \cdot (I_4 \otimes L_2^8) \right)$$



Pattern matching

$$\prod_{i=1}^k P_i D_i (A_i \otimes I_v) E_i Q_i$$

- Manipulate to match vectorizable construct
- Separate vectorizable parts and scalar parts



Formula Manipulation

Normalizing formulas

$$(I_n \otimes L_v^{2v})(I_n \otimes L_n^{2v}) = I_{2nv}$$

$$A \otimes B = (A \otimes I_m)(I_n \otimes B)$$

$$I_v \otimes A = L_v^{nv} (A \otimes I_v) L_n^{nv}$$

$$I_{nv+l} = I_{nv} \oplus I_l$$

$$I_{mn} = I_m \otimes I_n$$

$$PD = D'P$$

Converting complex to real arithmetic

$$\overline{A \cdot B} = \overline{A} \cdot \overline{B}$$

$$\overline{A} = A \otimes I_2, \quad A \text{ real}$$

$$\overline{D} = (I_{n/v} \otimes L_v^{2v}) \overline{D'} (I_{n/v} \otimes L_2^{2v}), \quad v | n$$

$$\overline{A \otimes I_v} = (I_n \otimes L_v^{2v}) (\overline{A} \otimes I_v) (I_n \otimes L_2^{2v})$$



Vector Code Generation

$$\prod_{i=1}^k P_i D_i (A_i \otimes I_v) E_i Q_i$$

fuse with load/store operations

difficult part
(easy to loose performance)

$P_i Q_i$	permutations
$D_i E_i$	diagonals
A_i	arbitrary formulas
\hat{i}	SIMD vector length

arithmetic vector instructions

- use standard SPL compiler on A_i
- replace scalar with vector instructions

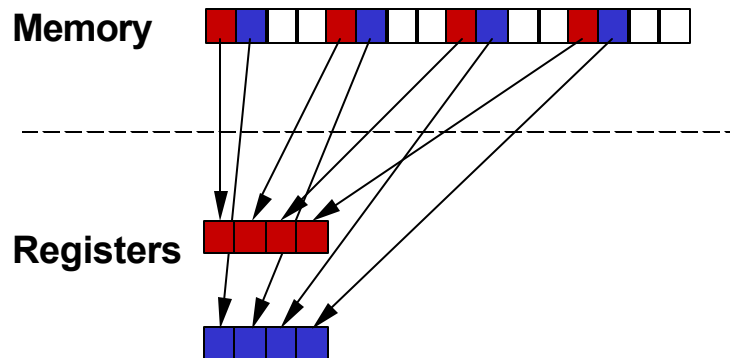
easy part
(due to existing SPL compiler)



Challenge: Data Access

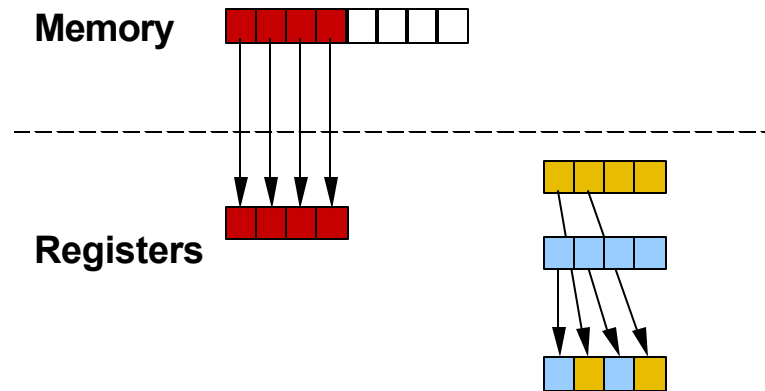
Example:

Required:



Strided load of complex numbers

Available:



Vector load plus in-register permutations

- highest performance code requires **properly aligned** data access
- permutation support differs between architectures
- performance differs between permutations (some are good, most very bad)



Solution:

- use formula manipulation to get “good” permutations
- macro layer API for efficient and machine transparent implementation

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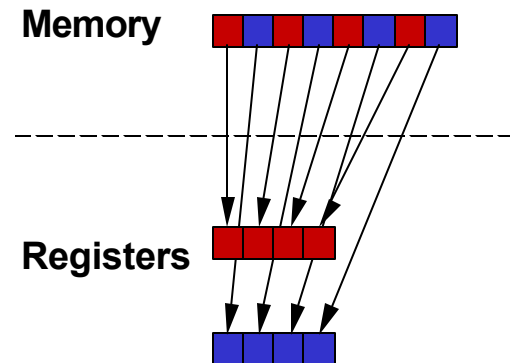
Portable High-level API

- restricted set of short vector operations
- requires C compiler with „intrinsic“-interface
- high-level operations
 - Vector arithmetic operations
 - Vector load/store operations
 - Special and arbitrary multi-vector permutations
 - Vector constant handling (declaration, usage)
 - **Implemented by C macros**

Example:

Unit-stride load of 4 complex numbers:

```
LOAD_L_8_2 (reg1, reg2, *mem)
```



Portable SIMD API: Details

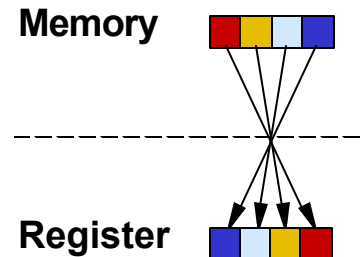
All SIMD extensions supported:

- gcc 3.0, gcc-vec
- Intel C++ Compiler, MS VisualC++ with ProcessorPack
- Various PowerPC compilers (Motorola standard)

Examples:

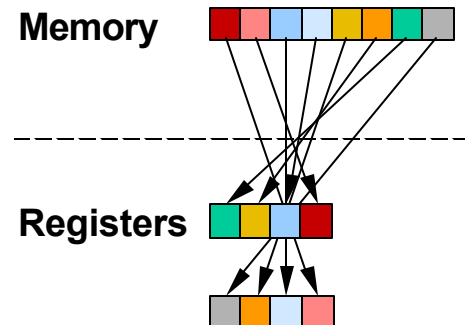
Reverse load of 4 real numbers:

```
LOAD_J_4 (reg, *mem)
```



Reverse load of 4 complex numbers:

```
LOAD_J_4_x_I_2 (r1, r2, *mem)
```



Generated Code

- Vector parts: portable SIMD API
- Scalar parts: standard C
- P_i , Q_i , D_i , E_i handled by load/store operations
- A_i handled by vector arithmetics

```
/* Example vector code: DFT_16 */
void DFT_16(vector_float *y,
           vector_float *x)
{
    vector_float x10, x11, x12;
    ...
    LOAD_VECT(x10, x + 0);
    LOAD_VECT(x14, x + 16);
    f0 = SIMD_SUB(x10, x14);
    LOAD_VECT(x11, x + 4);
    LOAD_VECT(x15, x + 20);
    f1 = SIMD_SUB(x11, x15);
    ...
    y17 = SIMD_SUB(f1, f4);
    STORE_L_8_4(y16, y17, y + 24);
    y12 = SIMD_SUB(f0, f5);
    y13 = SIMD_ADD(f1, f4);
    STORE_L_8_4(y12, y13, y + 8);
}
```

```
/* Intel SSE: portable SIMD API
   Intel C++ Compiler 5.0
*/
typedef __m128 vector_float;

#define LOAD_VECT(a, b) \
    (a) = *(b)

#define SIMD_ADD(a, b) \
    _mm_add_ps((a), (b))
#define SIMD_SUB(a, b) \
    _mm_sub_ps((a), (b))

#define STORE_L_8_4(re, im, out) \
{ \
    vector_float _sttmp1, _sttmp2; \
    _sttmp1 = _mm_unpacklo_ps(re, im); \
    _sttmp2 = _mm_unpackhi_ps(re, im); \
    _mm_store_ps(out, _sttmp1); \
    _mm_store_ps((out) + VLEN, _sttmp2); \
}
```



Experimental Results

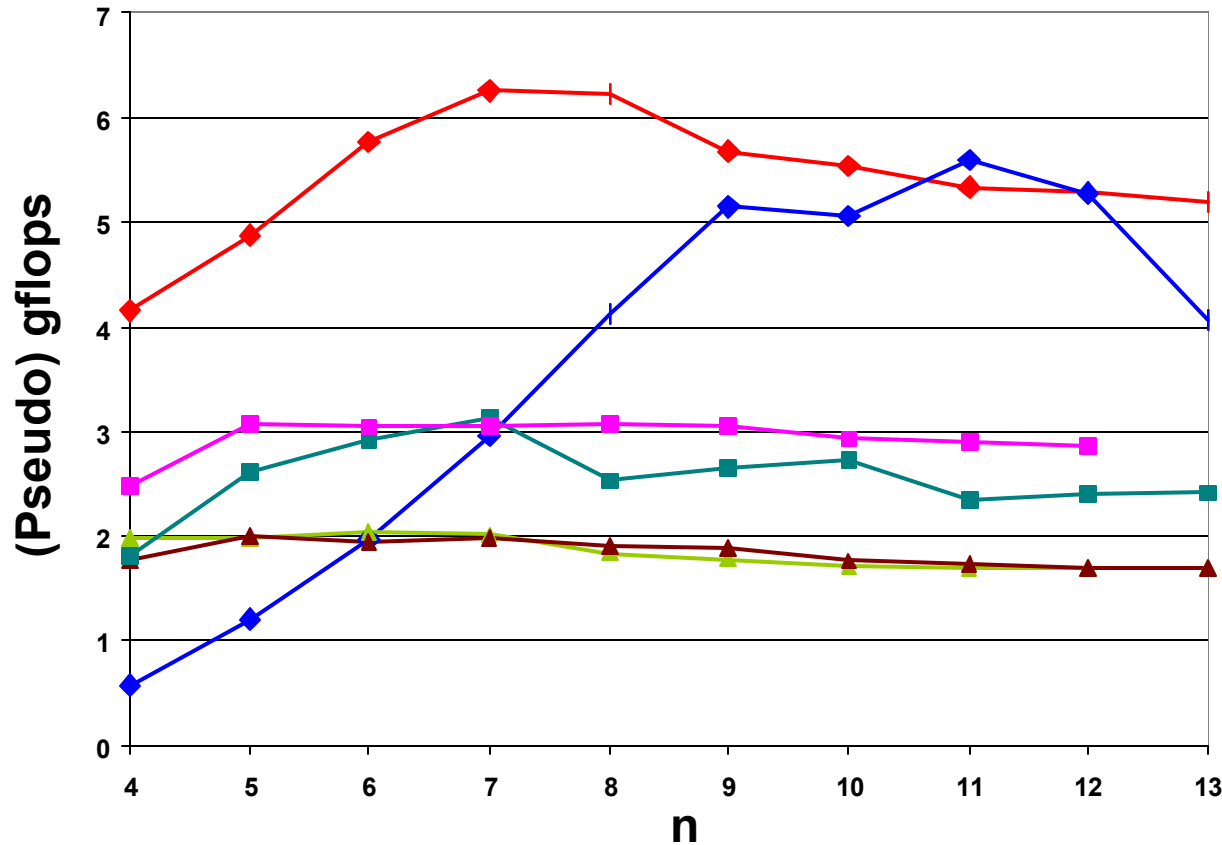
- our code is generated, found by dynamic programming search
- different searches for different types of code (scalar, vector)
- results in (Pseudo) gigaflops (higher = better)



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Generated DFT Code: Pentium 4, SSE



hand-tuned vendor assembly code

- ◆ Spiral SSE
- + Intel MKL interl.
- ▲ FFTW 2.1.3
- ▲ Spiral C
- Spiral C vect
- SIMD-FFT

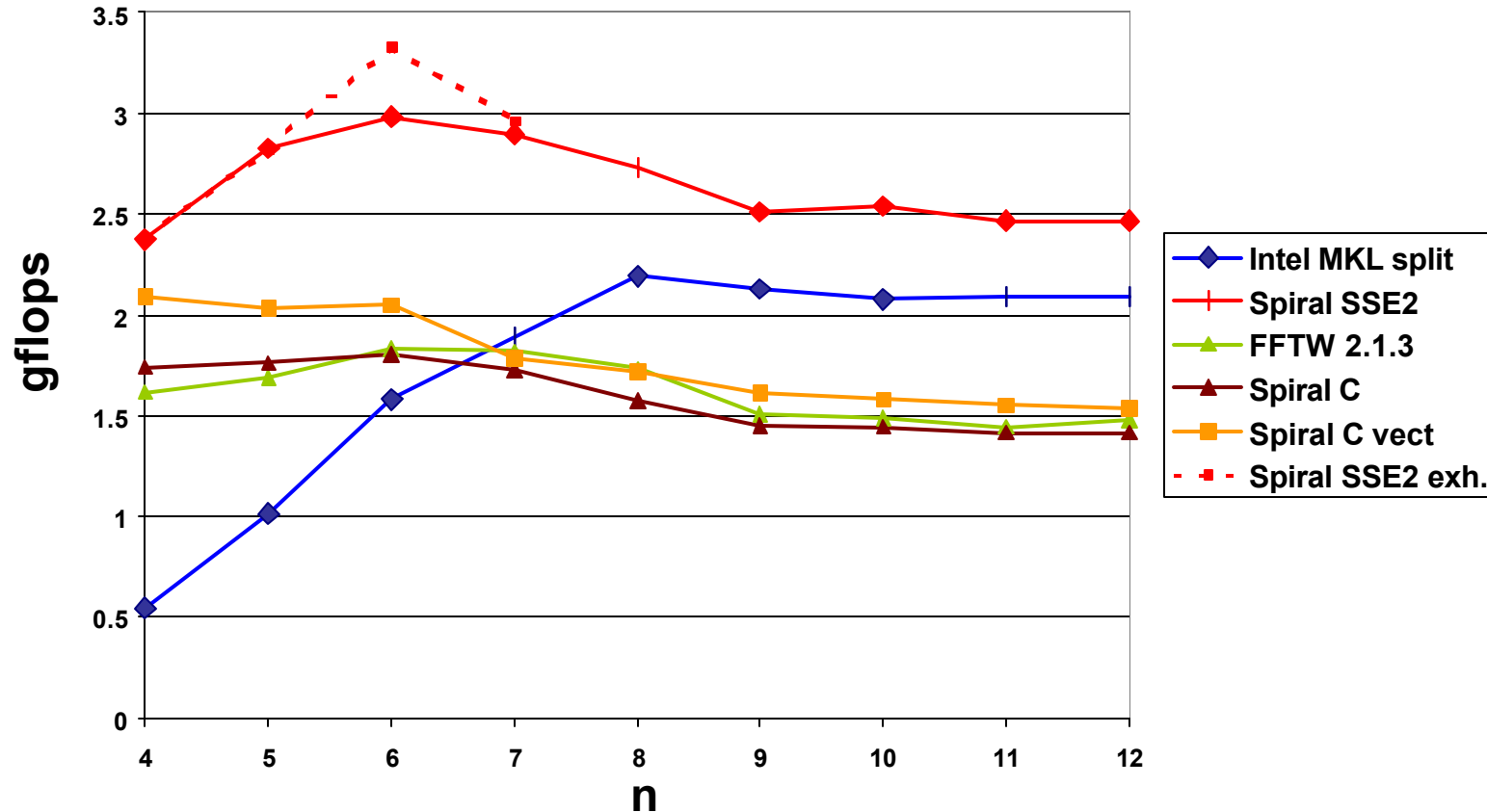
DFT 2^n single precision, Pentium 4, 2.53 GHz, using Intel C compiler 6.0



speedups (to C code) up to factor of 3.1



Generated DFT Code: Pentium 4, SSE2



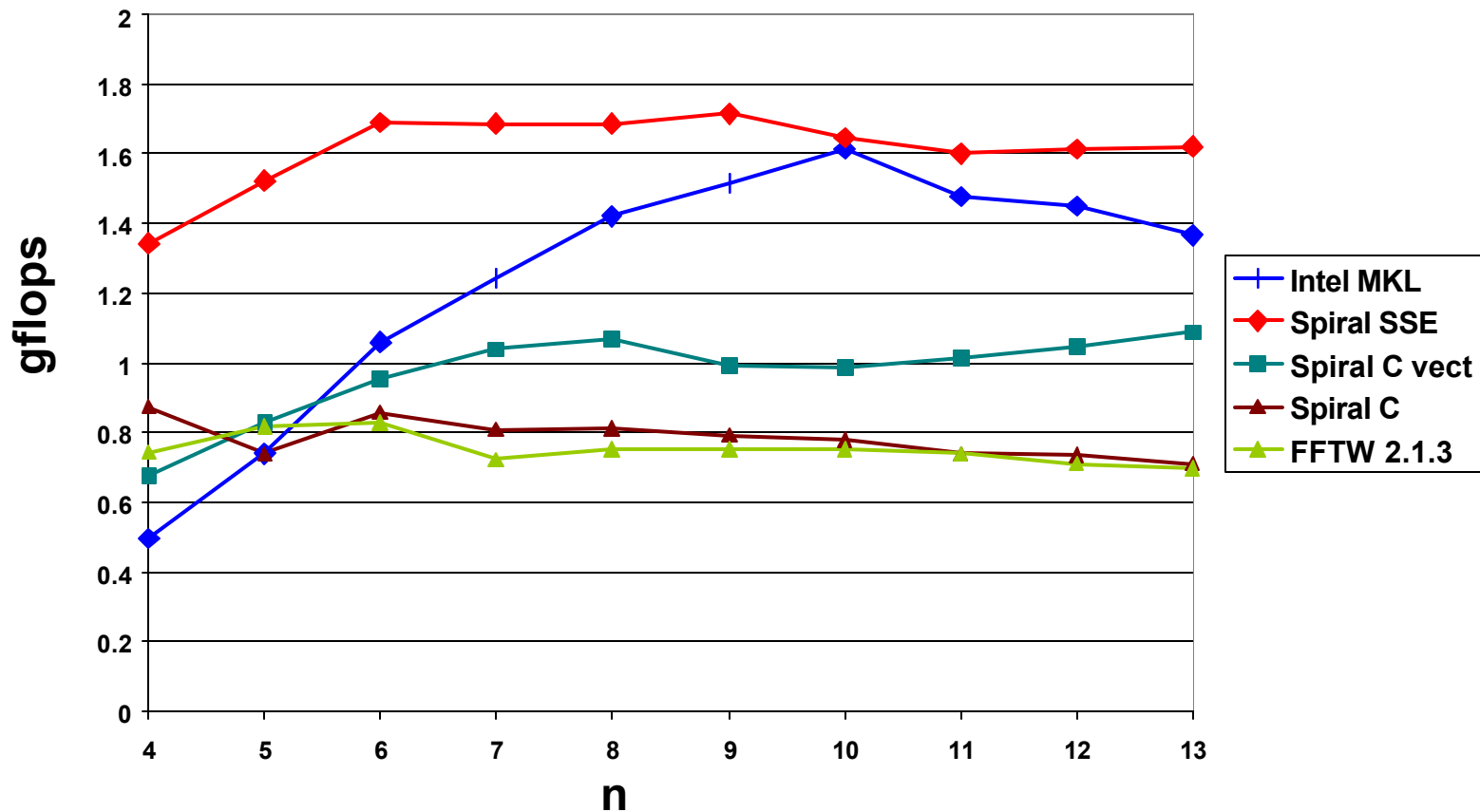
DFT 2^n *double* precision, Pentium 4, 2.53 GHz, using Intel C compiler 6.0



speedups (to C code) up to factor of 1.8



Generated DFT Code: Pentium III, SSE



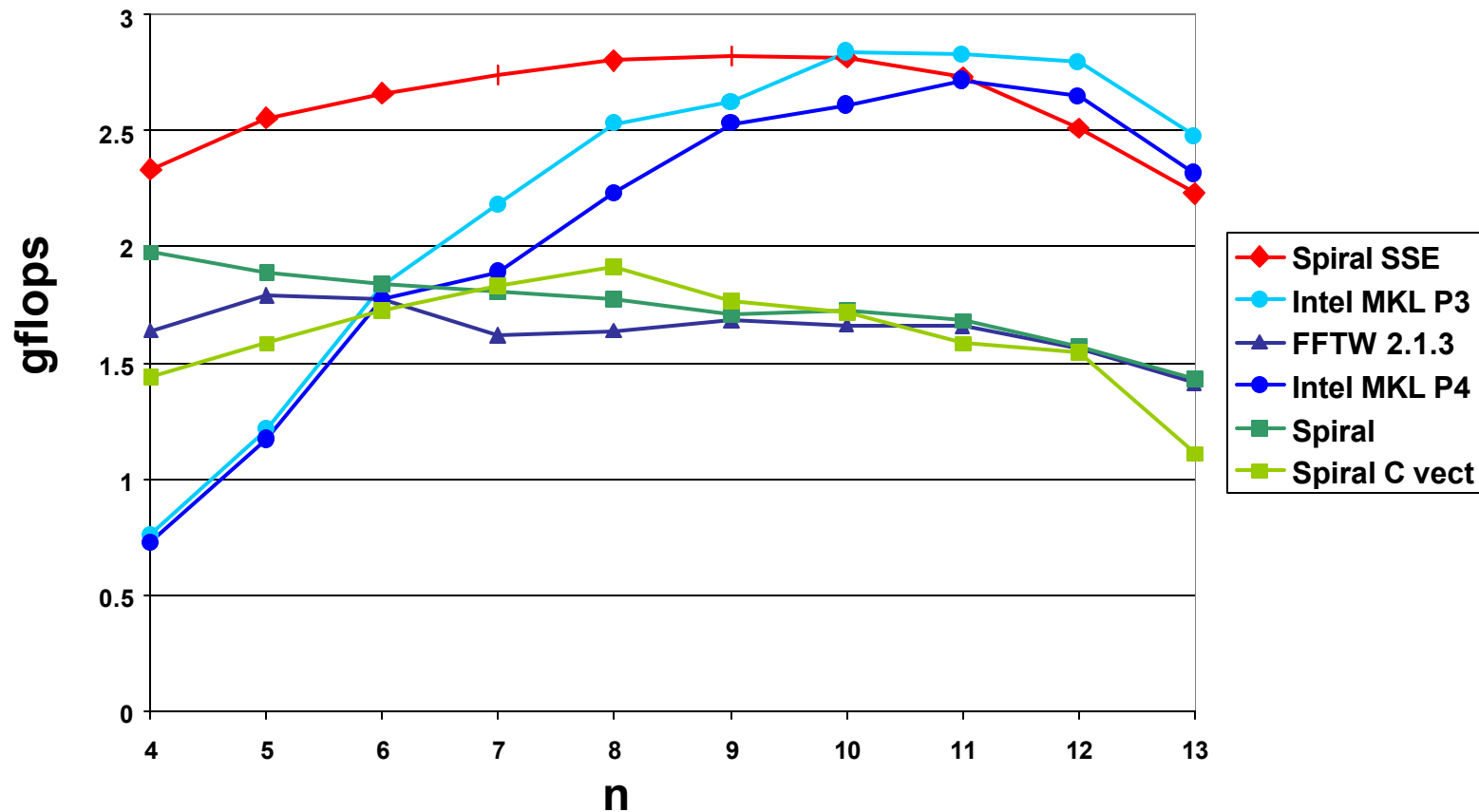
DFT 2^n single precision, Pentium III, 1 GHz, using Intel C compiler 6.0



speedups (to C code) up to factor of 2.1



Generated DFT Code: Athlon XP, SSE



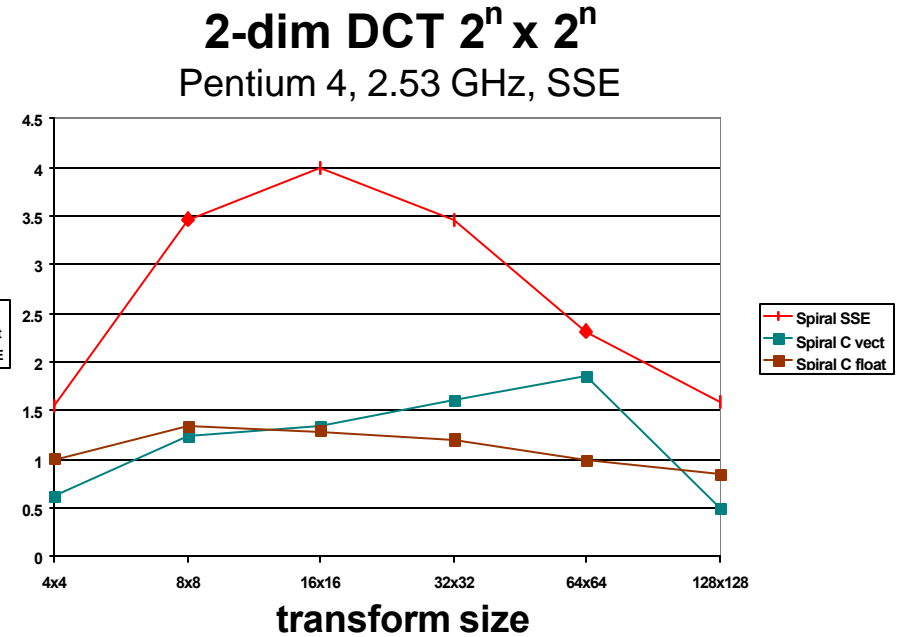
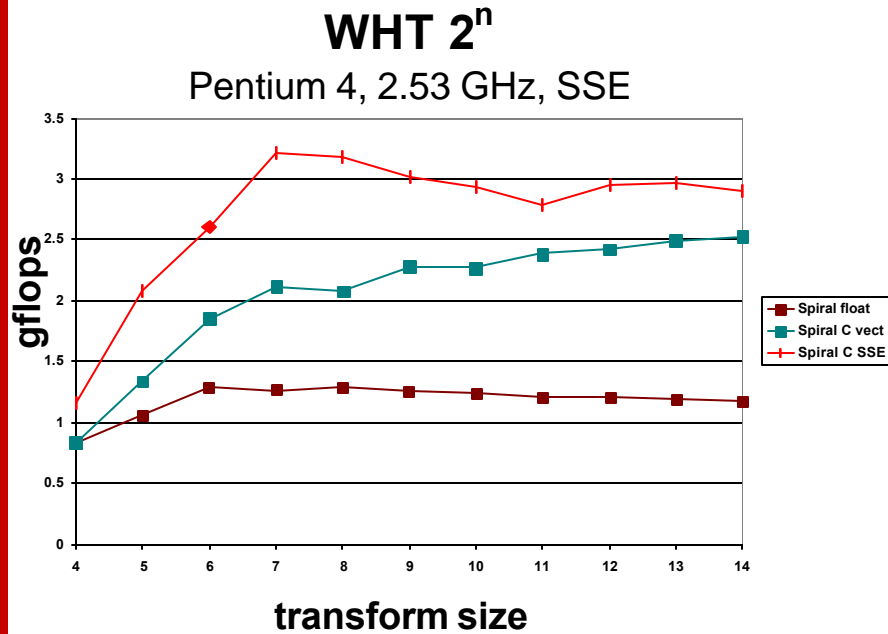
DFT 2^n single precision, Pentium III, 1 GHz, using Intel C compiler 6.0



speedups (to C code) up to factor of 1.6



Other transforms



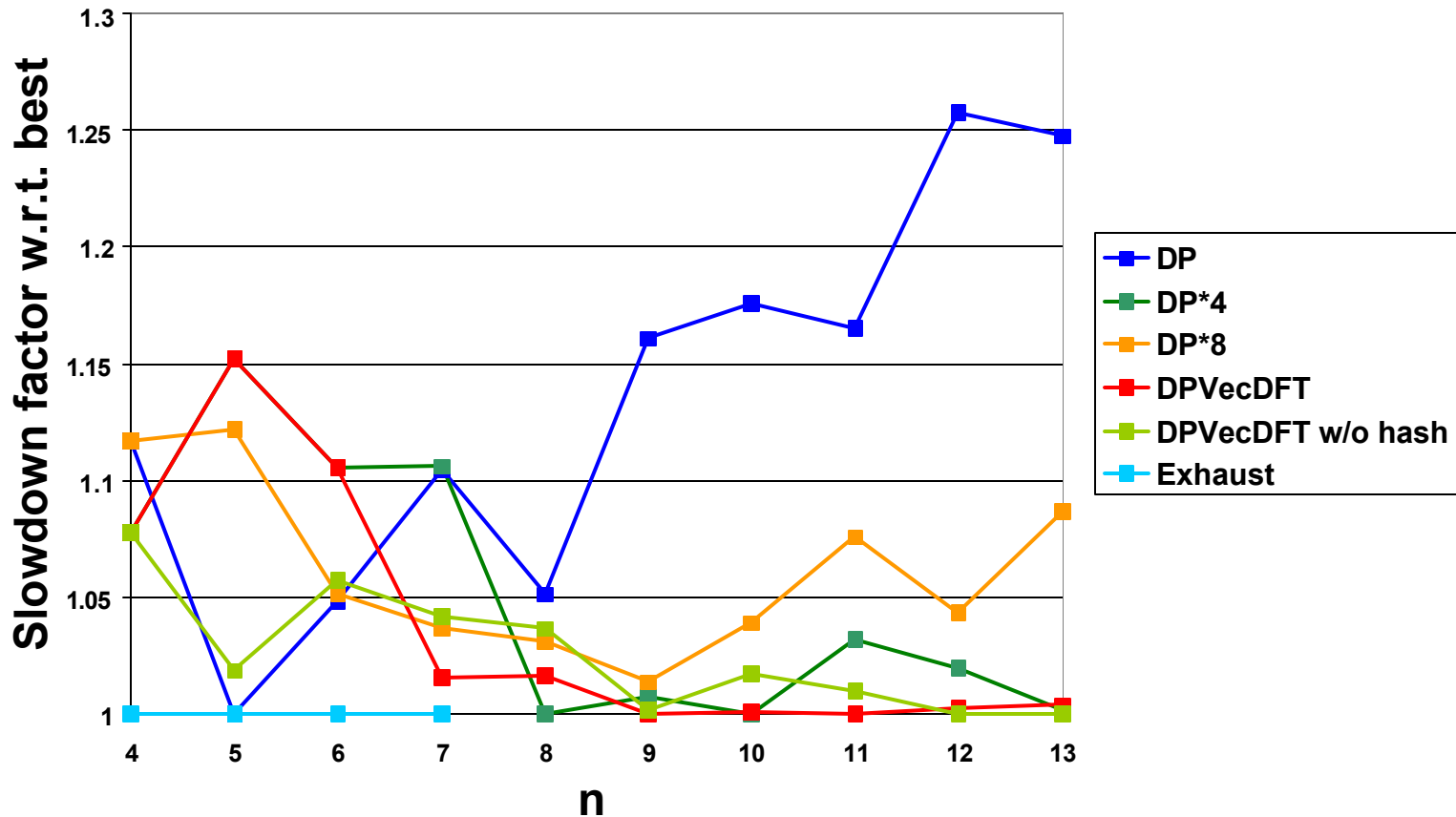
- WHT has only additions
- very simple transform



speedups (to C code) up to factor of 3



Different search strategies



DFT 2^n single precision, Pentium 4, 2.53 GHz, using Intel C compiler 6.0



standard DP loses up to 25 % performance



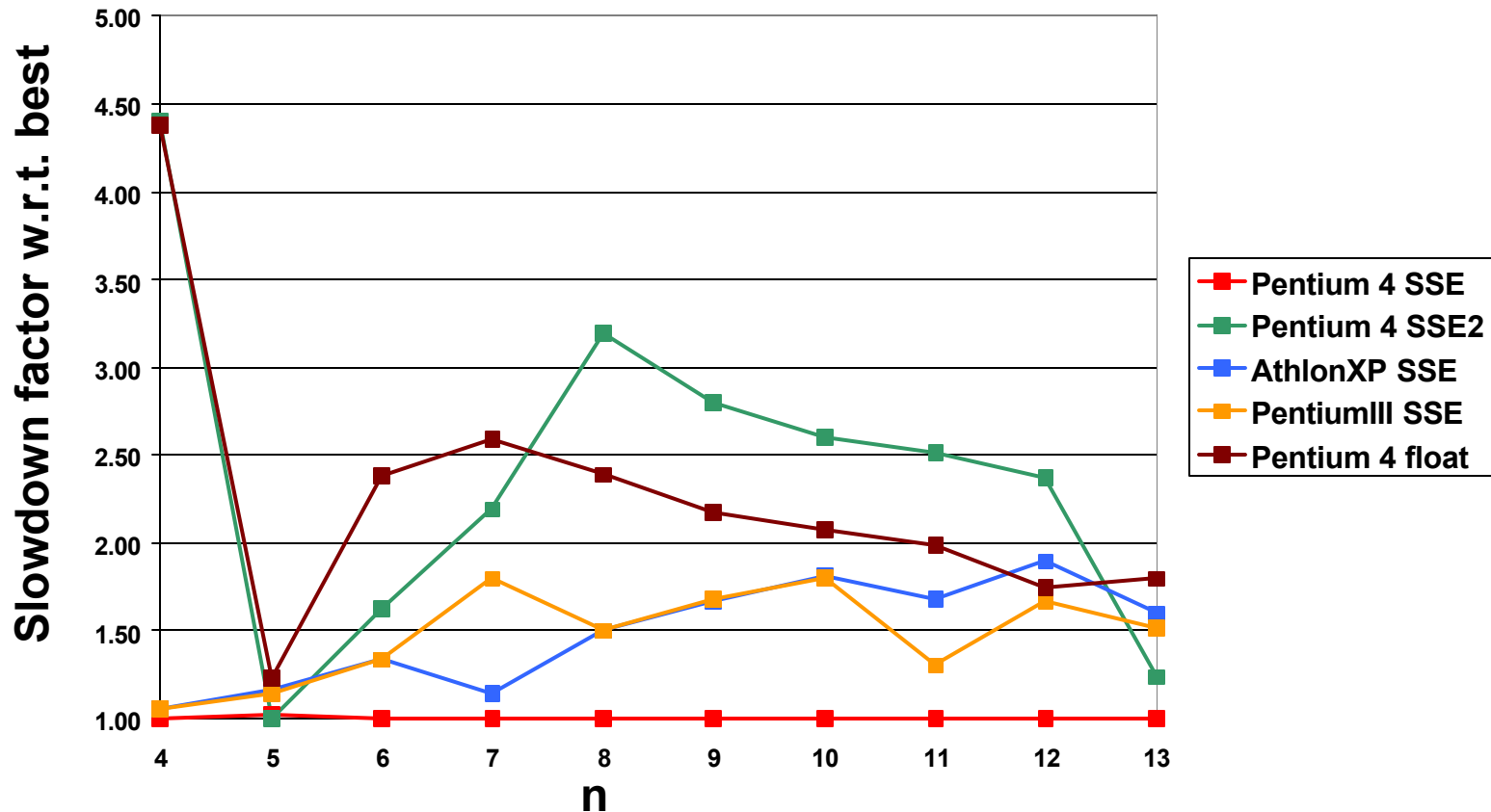
Best DFT Trees, size $2^{10} = 1024$

	Pentium 4 float	Pentium 4 double	Pentium III float	AthlonXP float
scalar				
C vect				
SIMD				



trees platform/datatype dependent

Crosstiming of best trees on Pentium 4



DFT 2^n single precision, runtime of best found of other platforms



software adaptation is necessary



Summary

- Automatically generated vectorized DSP code
- Code is platform-adapted (SPIRAL)
- We implement “constructs”, not transforms
 - tensor product, permutations, ...
 - DFT, WHT, arbitrary multi-dim supported
- Very competitive performance

Ongoing work:

- port to other SIMD architectures
- include filters and wavelets

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<http://www.math.tuwien.ac.at/~aurora>



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