

# Space-time Beamforming with Knowledge-Aided Constraints

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# Outline

- **Background**
- **Knowledge-aided Constraints**
- **Performance Results**
- **Summary**



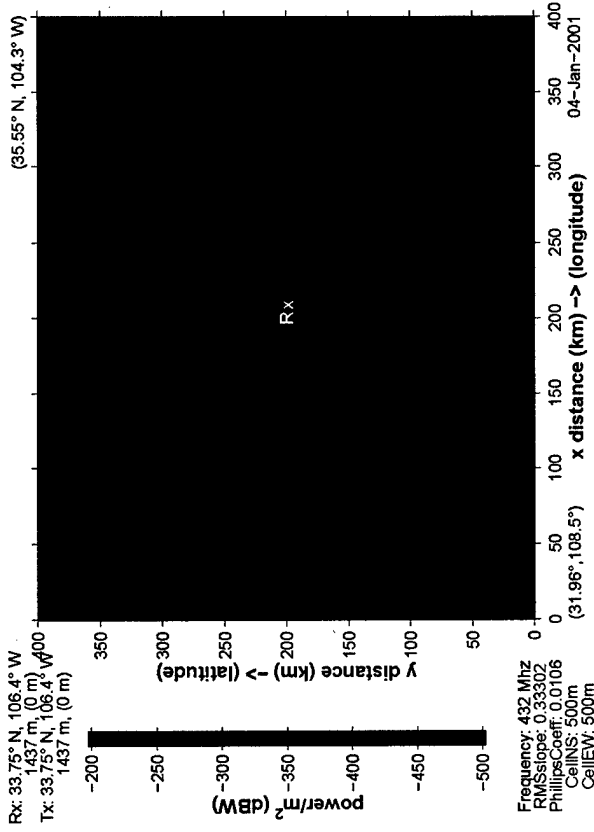
# Background

- **Real-world radar clutter environments depend on site-specific factors including:**
  - Terrain
  - Ground cover type
- **Site-specific clutter modeling is fundamental to understanding STAP performance in real-world settings**
- **This has led to the development of site-specific performance bound techniques**
  - Thermal noise limited performance is optimistic for systems operating in real-world environments
  - Theory is based on ideal site-specific clutter covariance modeling
- **It is logical that the models used in site-specific performance analyses could also be used when processing the radar data to potentially improve radar performance**
- **This is a major goal of DARPA's Knowledge-Aided Sensor Signal Processing and Expert Reasoning (KASSPER) Program**

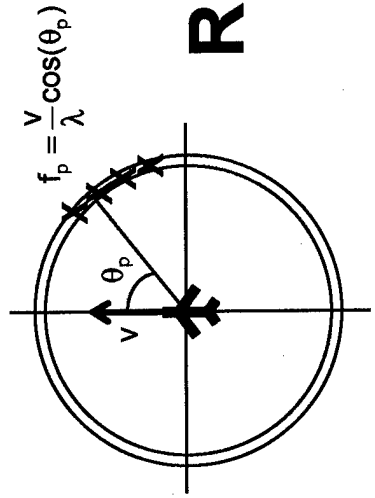
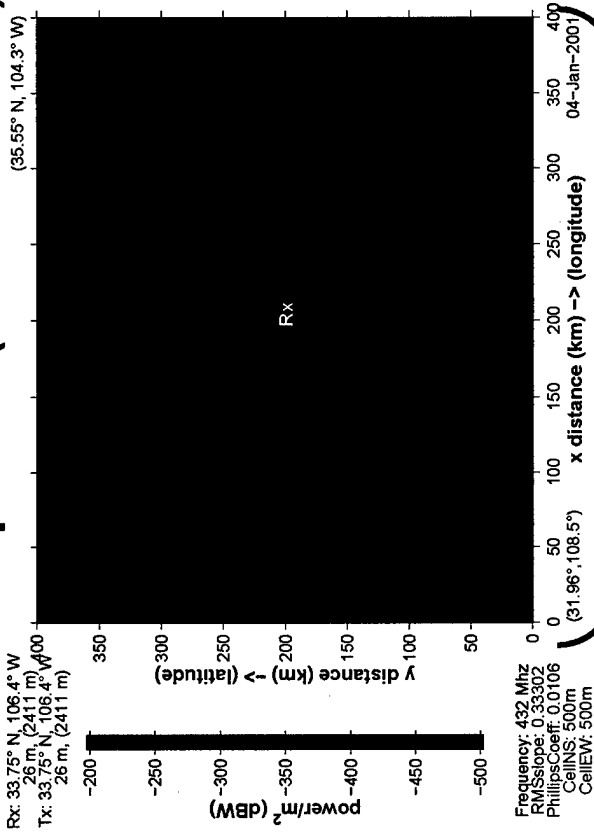


# Site-specific Clutter Modeling

**bald earth**



**site-specific (SCATS/DTED)**



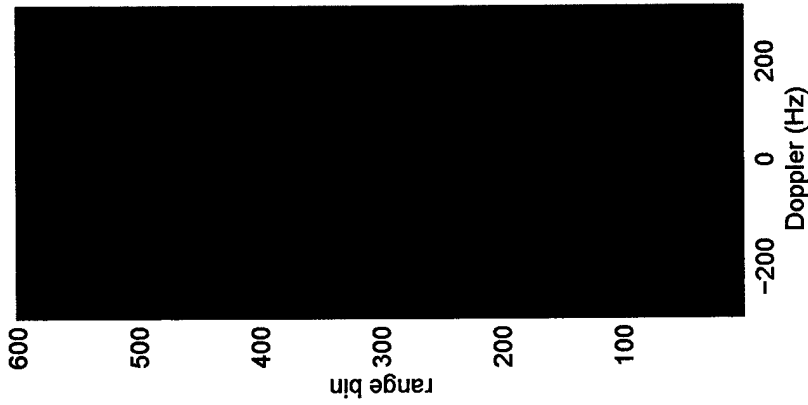
$$R_c = \sum_{p=1}^{P_c} |\alpha_p|^2 v(\theta_p, f_p) v^H(\theta_p, f_p) \circ T_p$$

sum over all scattering patches in a given range bin

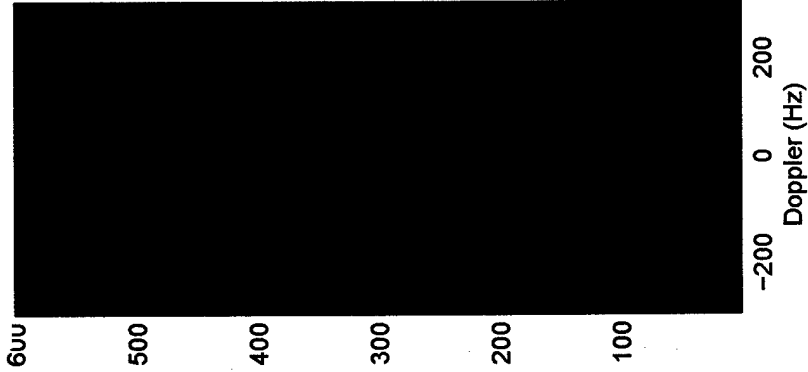
ICM  
calibration errors  
channel mismatch

# Mountain Top Monostatic Clutter

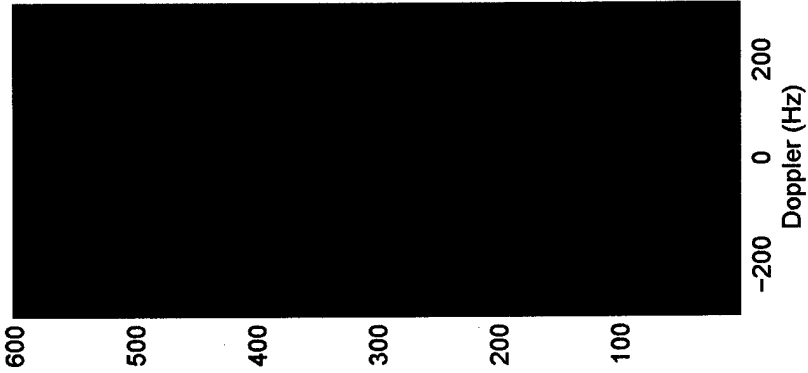
**RSTER**



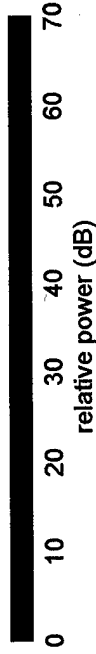
**SCATS/DTED**



**bald earth**



- Range-Doppler clutter maps shown for RSTER and simulations
- Simulation results shown both with and without DTED
- Simulation w/ DTED results in a significantly better match to the experimental data
- Site-specific models capture a majority of the clutter features



# Knowledge-Aided Signal Processing

- The *a priori* knowledge will typically be used in two ways
  - Indirect: exploit knowledge sources to segment training data, etc.
  - Direct: exploit knowledge sources to place nulls in the beamformer pattern
- This presentation develops a methodology for using *a priori* knowledge *directly* in the space-time beamforming solution
  - notch width varies little
  - significant notch width variation
- Clutter cancellation based on *a priori* knowledge alone will typically not result in adequate performance
- Focus will be on techniques that combine or “blend” adaptive and deterministic filtering
- The performance of these filtering techniques will be a function of how well the system is calibrated



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# Interference Modeling

- Assume the clutter signal plus thermal noise model

$$\mathbf{X} = \mathbf{X}_C \circ \mathbf{t} + \mathbf{n}$$

○ - Hadamard product  
(element-wise product)

- The modulation will typically be small

$$\mathbf{t} = \mathbf{1} + \mathbf{d}$$

$\mathbf{d}$  is zero-mean, variance  $\ll 1$

- Clutter signal with small modulation

$$\mathbf{X} = \mathbf{X}_C + \mathbf{X}_C \circ \mathbf{d} + \mathbf{n}$$

- Clutter correlation matrix

$$\begin{aligned} E\{\mathbf{X}\mathbf{X}^H\} &= \mathbf{R}_{xx} = E\{\mathbf{X}_C\mathbf{X}_C^H\} + E\{\mathbf{X}_C\mathbf{X}_C^H\} \circ E\{\mathbf{d}\mathbf{d}^H\} + \sigma^2 \mathbf{I} \\ &= \mathbf{R}_C + \underbrace{\mathbf{R}_C \circ \mathbf{T} + \sigma^2 \mathbf{I}}_{\text{unknown component}} \end{aligned}$$

“known” component

# Knowledge-Aided Quadratic Constraints

- The usual optimization problem:

$$\min_{\mathbf{w}} E\{|\mathbf{w}^H \mathbf{x}|^2\} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{v} = 1 \quad \rightarrow \quad \mathbf{w} = \frac{\mathbf{R}_{xx}^{-1} \mathbf{v}}{\mathbf{v}^H \mathbf{R}_{xx}^{-1} \mathbf{v}}$$

- Incorporate covariance model as a quadratic constraint

$$\min_{\mathbf{w}} E\{|\mathbf{w}^H \mathbf{x}|^2\} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{v} = 1$$

want weights to be orthogonal to a priori clutter model

$$\mathbf{w}^H \mathbf{R}_c \mathbf{w} = 0$$

this is the KA part

$$\mathbf{w}^H \mathbf{w} = \delta$$

- Gives:

$$\mathbf{w} = \frac{(\mathbf{R}_{xx} + \beta_d \mathbf{R}_c + \beta_L \mathbf{I})^{-1} \mathbf{v}}{\mathbf{v}^H (\mathbf{R}_{xx} + \beta_d \mathbf{R}_c + \beta_L \mathbf{I})^{-1} \mathbf{v}} = \frac{(\mathbf{R}_{xx} + \mathbf{Q})^{-1} \mathbf{v}}{\mathbf{v}^H (\mathbf{R}_{xx} + \mathbf{Q})^{-1} \mathbf{v}}$$

“colored loading”



## Pre-Filter Interpretation

- Colored loading beamformer can be expressed as:

$$\mathbf{W} = \frac{\mathbf{Q}^{-1/2} (\mathbf{Q}^{-1/2} \mathbf{R}_{xx} \mathbf{Q}^{-1/2} + \mathbf{I})^{-1} \mathbf{Q}^{-1/2} \mathbf{v}}{\mathbf{v}^H \mathbf{Q}^{-1/2} (\mathbf{Q}^{-1/2} \mathbf{R}_{xx} \mathbf{Q}^{-1/2} + \mathbf{I})^{-1} \mathbf{Q}^{-1/2} \mathbf{v}}$$

- Notice that,

$$\mathbf{W}^H \mathbf{X} = \tilde{\mathbf{W}}^H \tilde{\mathbf{X}}$$

- Where,

$$\tilde{\mathbf{X}} = \mathbf{Q}^{-1/2} \mathbf{X} \quad \text{pre-filtered or whitened data}$$

$$\tilde{\mathbf{W}} = \frac{(\mathbf{R}_{\tilde{xx}} + \mathbf{I})^{-1} \tilde{\mathbf{v}}}{\tilde{\mathbf{v}}^H (\mathbf{R}_{\tilde{xx}} + \mathbf{I})^{-1} \tilde{\mathbf{v}}} \quad \text{optimal weights for whitened data}$$

$$\tilde{\mathbf{v}} = \mathbf{Q}^{-1/2} \mathbf{v} \quad \text{whitened constraint}$$

it will generally be easier to estimate the covariance of the pre-filtered data than the original data because it is likely to have a lower effective rank

# Knowledge-Aided Linear Constraints

- Re-write quadratic constraint using the eigen-decomposition of the *a priori* clutter model,  $R_c = U^H D U$  (dominant subspace)
 
$$W^H R_c W = 0 \Rightarrow W^H (U D U^H) W = 0 \Rightarrow (W^H U) D (U^H W) = 0$$

$\Rightarrow W^H U = 0 \quad \because D$  has strictly positive diagonal elements (also has dimensions  $\ll R_c$  e.g., Brennan's Rule)

- A set of linear constraints

$$\min_w E\{|W^H X|^2\} \quad \text{s.t.} \quad W^H V = 1$$

$W^H U = 0$  } desire weights to be orthogonal to a priori clutter model

$$W^H W = \delta$$

this is the KA part

- Gives:

$$W = \frac{\bar{R}_{xx}^{-1} (I - U (U^H \bar{R}_{xx}^{-1} U)^{-1} U^H \bar{R}_{xx}^{-1}) V}{V^H \bar{R}_{xx}^{-1} (I - U (U^H \bar{R}_{xx}^{-1} U)^{-1} U^H \bar{R}_{xx}^{-1}) V} = \frac{\bar{R}_{xx}^{-1} P V}{V^H \bar{R}_{xx}^{-1} P V}$$

$$\bar{R}_{xx} = R_{xx} + \beta_L I$$



# Quadratic vs. Linear Constraints

- The two solutions:

$$W = \frac{(\bar{R}_{xx} + \beta_d R_c)^{-1} v}{v^H (\bar{R}_{xx} + \beta_d R_c)^{-1} v}$$

$$W = \frac{\bar{R}_{xx}^{-1} P v}{v^H \bar{R}_{xx}^{-1} P v}$$

- Manipulation of the quadratic constraint solution using the matrix inversion lemma and eigen-decomposition highlights the difference between the two solutions

$$\bar{R}_{xx}^{-1} P = \bar{R}_{xx}^{-1} (I - U(U^H \bar{R}_{xx}^{-1} U)^{-1} U^H \bar{R}_{xx}^{-1})$$

$$(\bar{R}_{xx} + \beta_d R_c)^{-1} = \bar{R}_{xx}^{-1} (I - U(U^H \bar{R}_{xx}^{-1} U + \frac{1}{\beta_d} D^{-1})^{-1} U^H \bar{R}_{xx}^{-1})$$

- The two solutions are equivalent in the limit of infinite loading and/or large clutter model eigenvalues
- Since the linear constraints cause the weights to be precisely orthogonal to the known covariance the quadratic constraint achieves this constraint only approximately

# **Advantages of Quadratic Constraints**

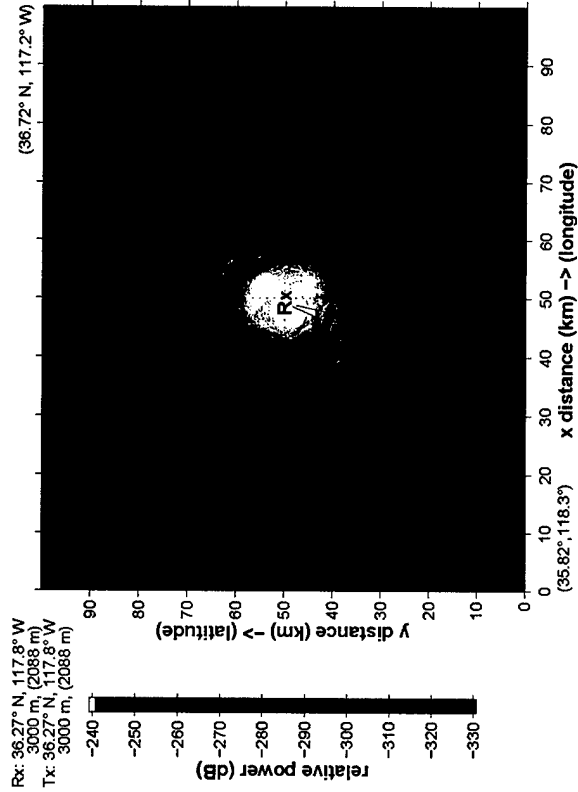
- **Quadratic constraints can be implemented more efficiently in both the covariance and the data domain**
- **Quadratic constraints offer a “blending” mechanism between the adaptive and deterministic beamformers**



# Outline

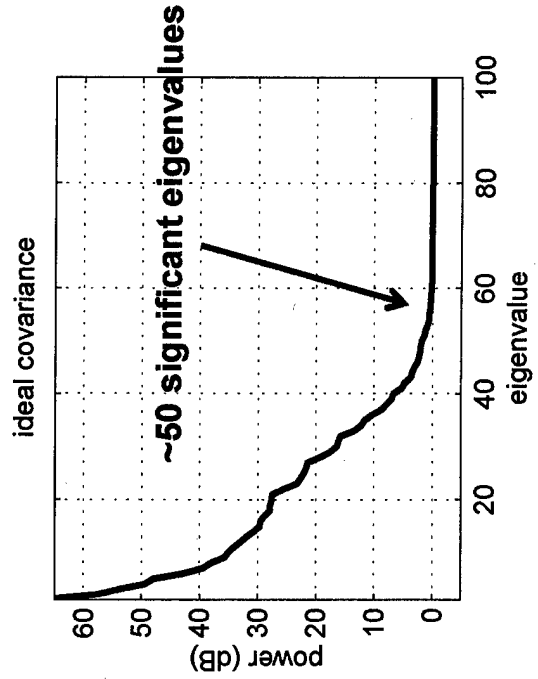
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# KASSPER Simulated Data Cube



Parameter	Value
RF frequency	1240 MHz
Bandwidth	10 MHz
PRF	1984 Hz
Peak Power	15 kW
Duty factor	10%
Noise figure	5 dB
System losses	9 dB
Platform speed	100 m/s
Platform altitude	3 km
Transmit aperture	8 vertical x 11 horizontal
Receive aperture*	8 vertical x 1 horizontal
Horizontal antenna spacing	10.9 cm
Vertical antenna spacing	14.07 cm
Number of receive sub-apertures	11
Front-to-back ratio	25 dB

- Site-specific data set generated under KASSPER program
- Heterogeneous clutter, ground vehicles, ICM, calibration errors
- We will focus on the problem of detecting slow moving targets in heterogeneous clutter → work with clutter-only data



# Colored Loading Matrix

- **Assume a ring of scatterers every 0.2° around the platform at the desired range bin**
  - No knowledge about terrain included
  - No knowledge about calibration errors (~5°-10° phase errors)
  - No knowledge about ICM included
  - No knowledge about backlobe level or Tx pattern included
  - Only platform heading, speed, and PRF are assumed known
- **Compute a matrix that represents the ground clutter (subspace):**

$$\mathbf{R}_c = \sum_{p=1}^{N_c} \mathbf{v}(\theta_p, f_p) \mathbf{v}(\theta_p, f_p)^H$$

- **Scale this matrix and add to the diagonally-loaded sample covariance matrix:**

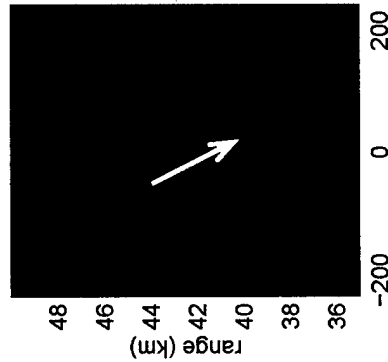
$$\mathbf{W} = \kappa(\mathbf{R}_s + \beta_L \mathbf{I} + \beta_d \mathbf{R}_c)^{-1} \mathbf{s}$$

- **Note: there are more efficient methods for computing this form of  $\mathbf{R}_c$**

# SINR Loss Surfaces

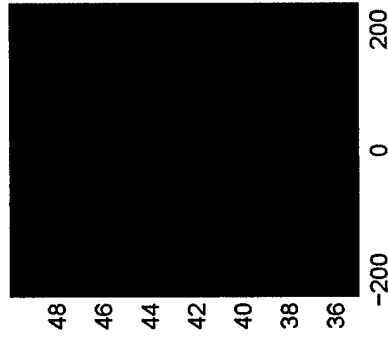
200 samples

DL, K=200



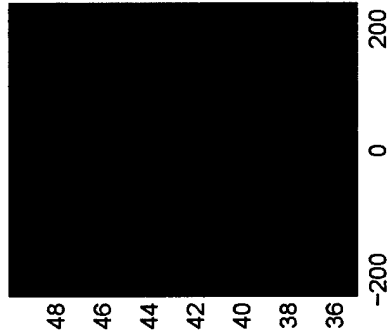
100 samples

DL, K=100



50 samples

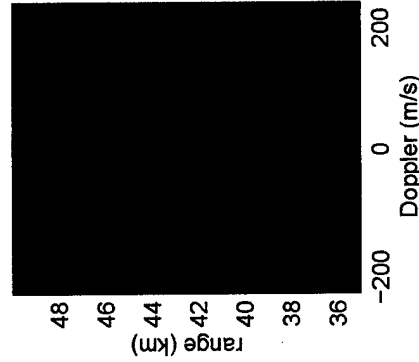
DL, K=50



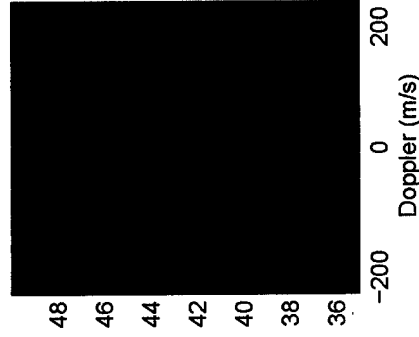
diagonal loading-only

colored loading

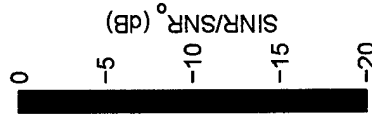
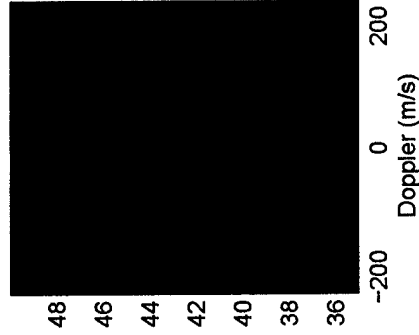
CL, K=200



CL, K=100



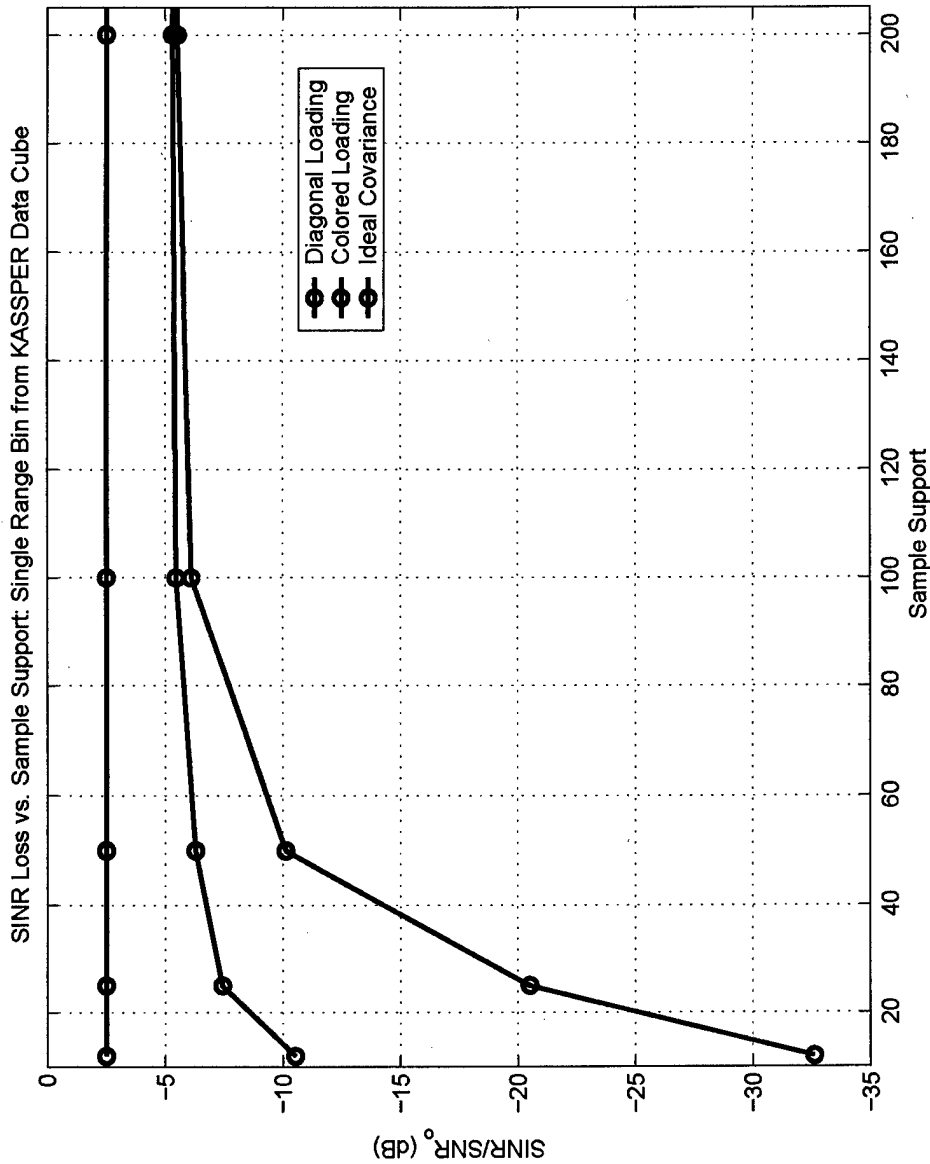
CL, K=50



- Loading levels:
  - $\beta_L = 0$  dB
  - $\beta_d = 30$  dB
- Colored loading beamformer is more robust to reductions in sample support
- Full-DoF SMI
  - 32 pulses
  - 11 elements



# Beamformer Convergence Summary



- Same loading matrix assumed
- Doppler = 20 m/s (two-way)
- Improved convergence with colored loading
- The chosen Doppler bin is relatively close to the mainbeam clutter
- Range bin 584

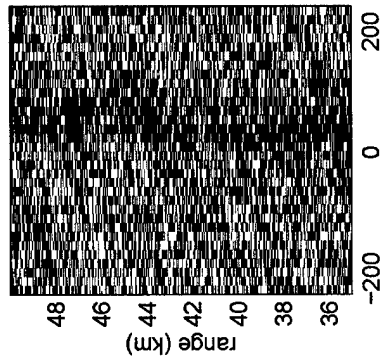
Technique works in the presence of calibration errors



# Beamformer Residue

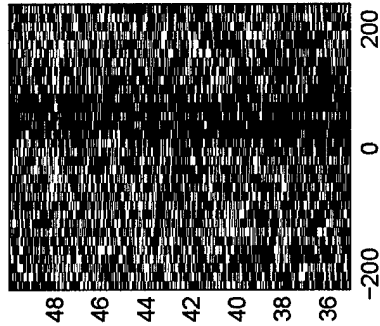
200 samples

DL, K=200



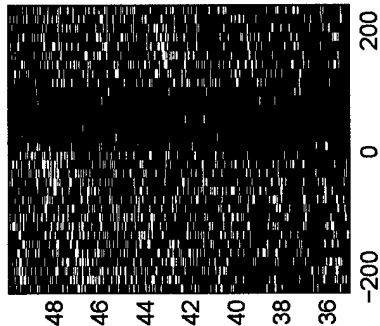
100 samples

DL, K=100



50 samples

DL, K=50



• Loading levels:

- $\beta_L = 0$  dB,
- $\beta_d = 30$  dB

• Colored loading beamformer is more robust to reductions in sample support

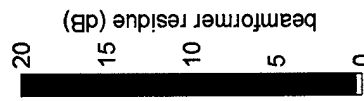
• Colored loading results in fewer false alarms

• Weights normalized s.t.  $w^H w = 1$

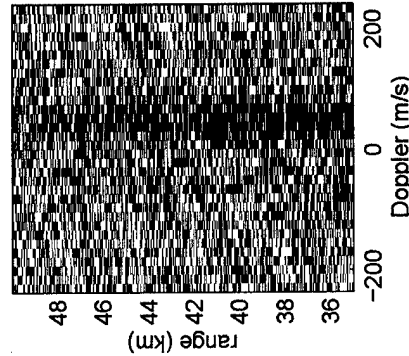
• Thermal noise power is  $\sim 0$  dB

diagonal loading-only

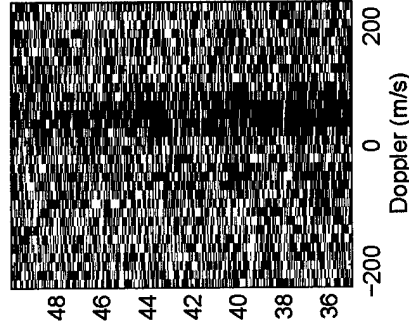
colored loading



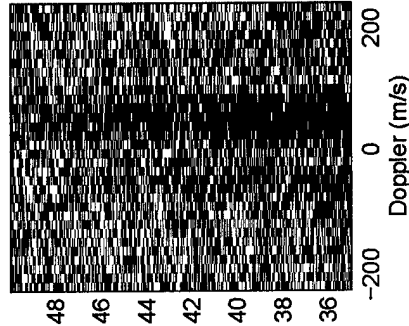
CL, K=200



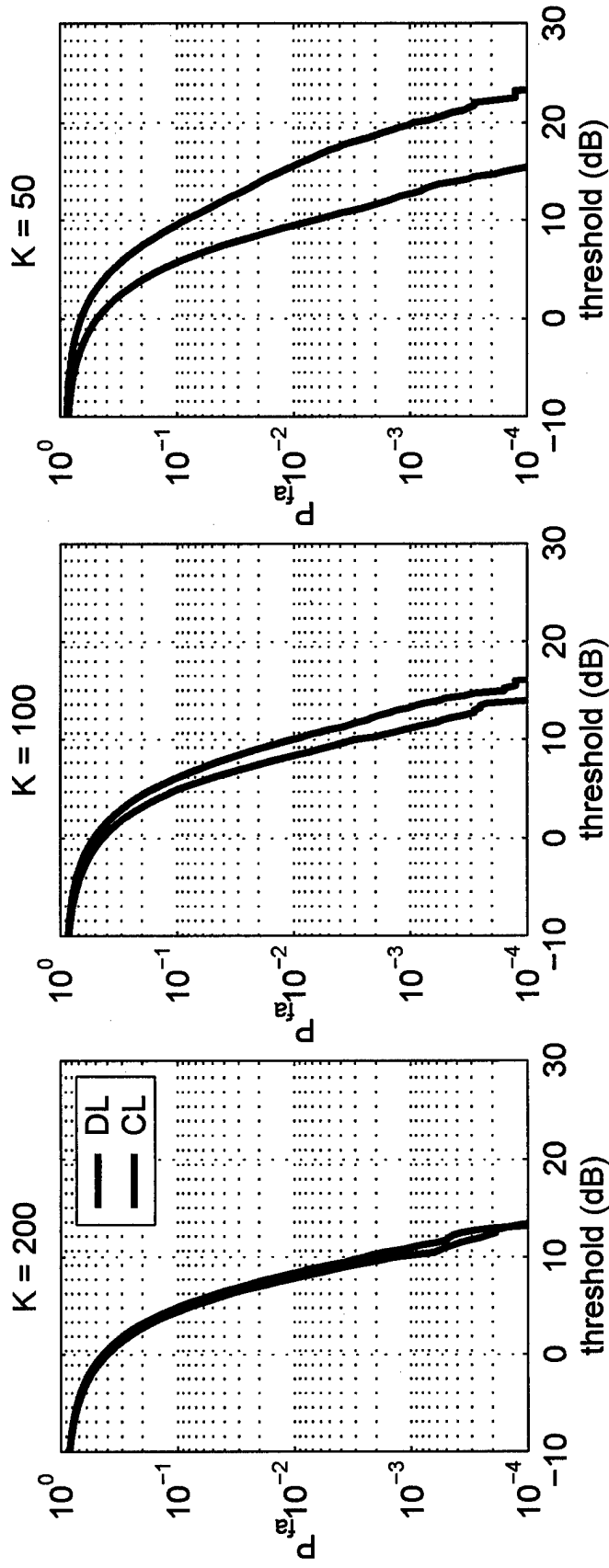
CL, K=100



CL, K=50



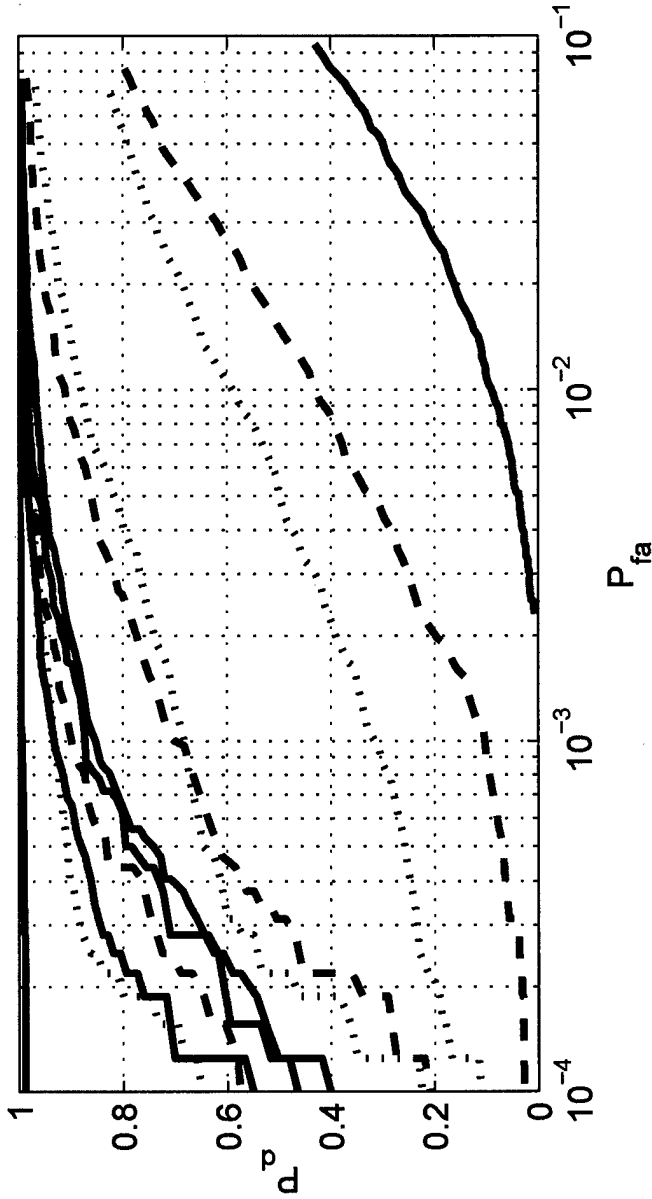
# Raw False Alarm Summary



- Fraction of pixels outside mainlobe clutter Doppler bins (3 bins) exceeding a given threshold is shown
- Colored loading beamformer maintains a similar false alarm characteristic as the number of training samples is decreased

# Detection Performance Summary (“endo-clutter”)

$P_{fa}$  vs.  $P_d$ :  $SNR_0 = 25$  dB, Doppler = 24.9021 m/s



- CL, K=50
- CL, K=100
- CL, K=200
- - DL, K=50
- - DL, K=100
- - DL, K=200
- ⋯ post-Dop, K=50
- ⋯ post-Dop, K=100
- ⋯ post-Dop, K=200
- ideal cov.
- model-only

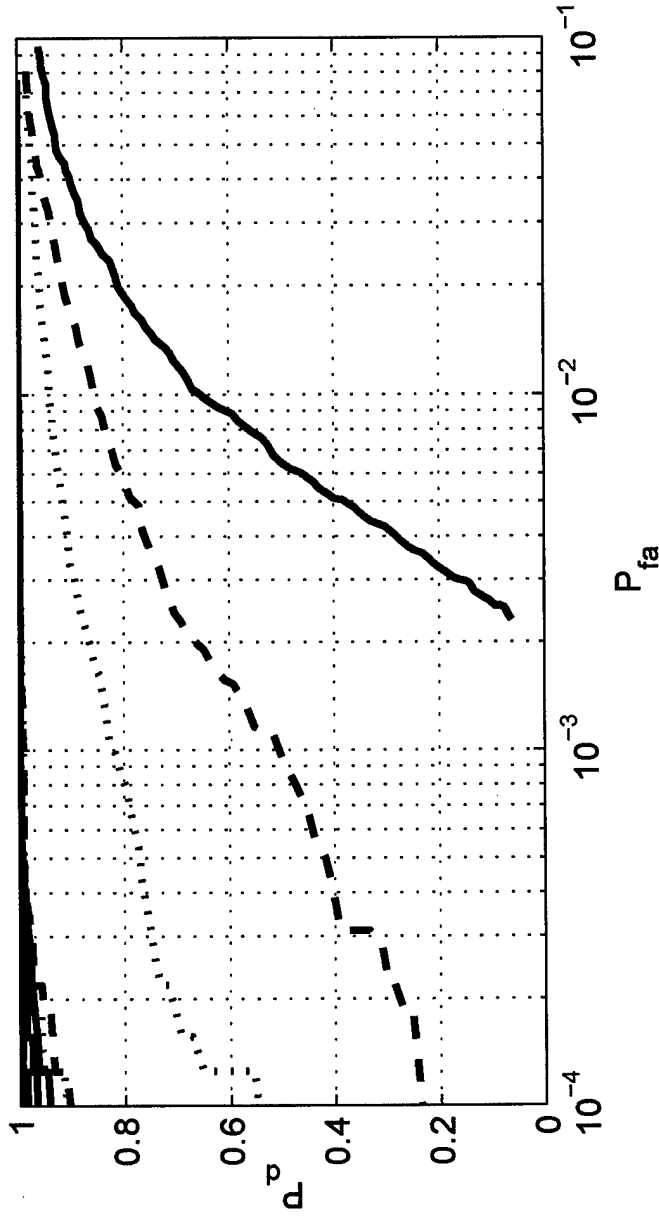
Post-Doppler uses  
5 adjacent bins  
2 bins away from  
the mainlobe clutter bin

- Detector includes median CFAR normalization of the beamformer output prior to thresholding
- No targets in the secondary beamformer or CFAR training data
- 1000 Injected test targets: all ranges, Doppler = 24.90 m/s, Target SNR is 25 dB at closest range bin (~5 dBsm)
- Colored loading beamformer is more robust as sample support is reduced

# Detection Performance Summary

## ("exo-clutter")

Pfa vs. Pd: SNR<sub>0</sub> = 25 dB, Doppler = 99.8502 m/s



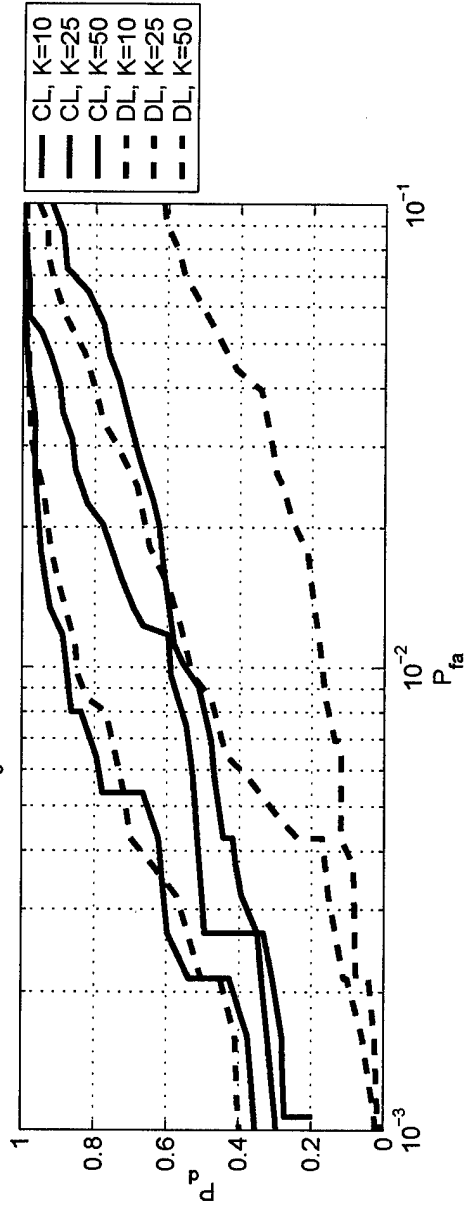
- CL, K=50
- CL, K=100
- CL, K=200
- - DL, K=50
- - DL, K=100
- - DL, K=200
- ..... post-Dop, K=50
- ..... post-Dop, K=100
- ..... post-Dop, K=200
- ideal cov.
- model-only

Post-Doppler uses  
5 adjacent bins  
3 bins away from  
the mainlobe clutter bin

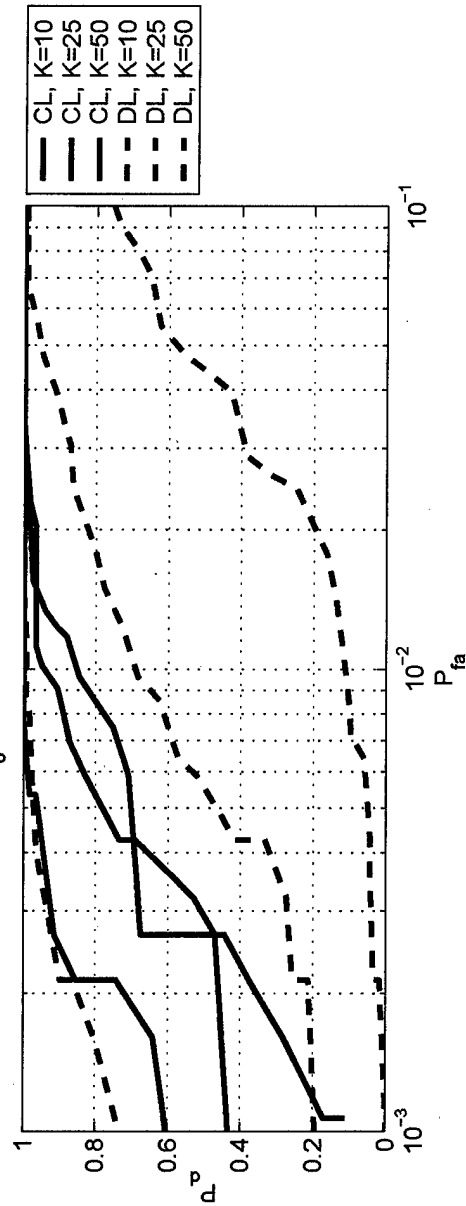
- Same result as previous slide except injected target Doppler is 99.85 m/s
- Most of the beamformers with adaptivity perform well when target is separated from the mainbeam clutter
- Use the most computationally efficient algorithm in these Doppler bins

# Mountain Top IDPCA Experimental Data

IDPCA65, SNR<sub>0</sub> = 25dB, Doppler = -130.1183 m/s



IDPCA65, SNR<sub>0</sub> = 25dB, Doppler = 72.2879 m/s



- **Data set parameters:**
  - UHF, PRF= 625 Hz
  - 178 range samples
  - 10 elems., 16 pulses

• **STAP: full-DoF SMI**

• **Boresight azimuth**

• **Two Doppler bins shown**

• **Same colored loading model used in previous results ( $\beta_L = 0$ dB,  $\beta_d = 40$  dB)**

• **Similar trend as simulated data**

• **Injected test targets are 25 dB SNR at all ranges**

**Unknown system errors: calibration, multipath, transmitter instabilities**

# Summary

- A method for incorporating *a priori* knowledge in the space-time beamformer solution using linear or quadratic constraints has been presented
- Quadratic constraint solution results in “colored” loading which can be implemented efficiently in the data domain and offers a “blending” between adaptive and deterministic filtering
- The fidelity of the colored loading matrix will depend on the available *a priori* knowledge sources and computational resources
- The technique was applied to KASSPER site-specific simulation data and shown to result in more robust performance near the mainbeam clutter → improved MDV performance
- Similar performance trends observed with experimental data
- Extension to low-DoF STAP implementations (e.g., post-Doppler) is currently under way