# Space-Time Adaptive Processing Using Sparse Arrays

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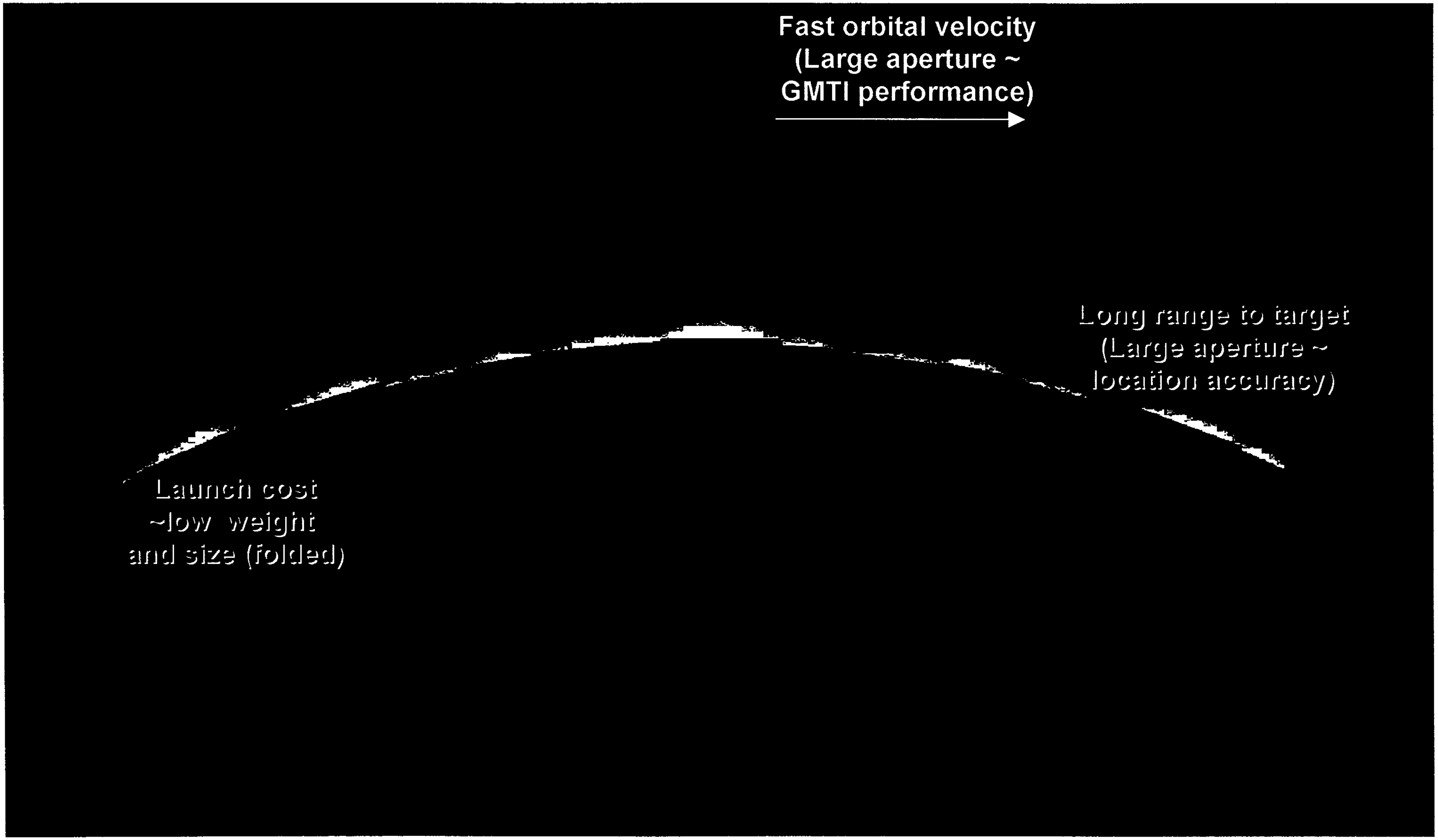
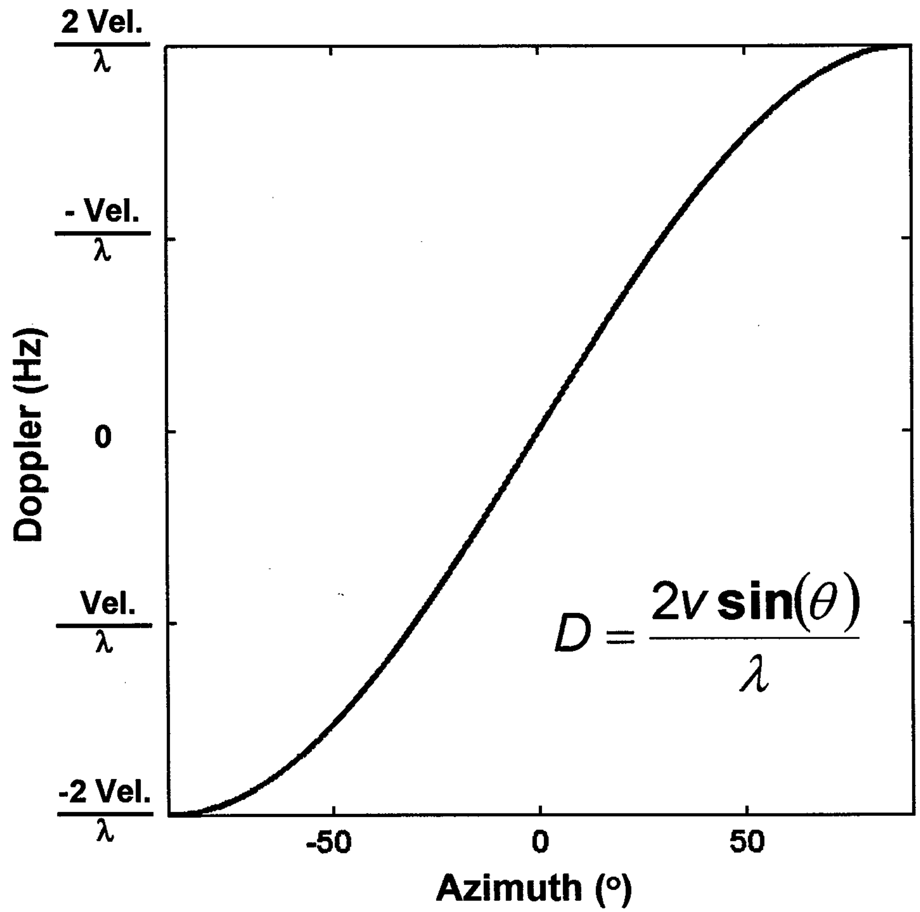
# Application: Space Based Radar



# Outline

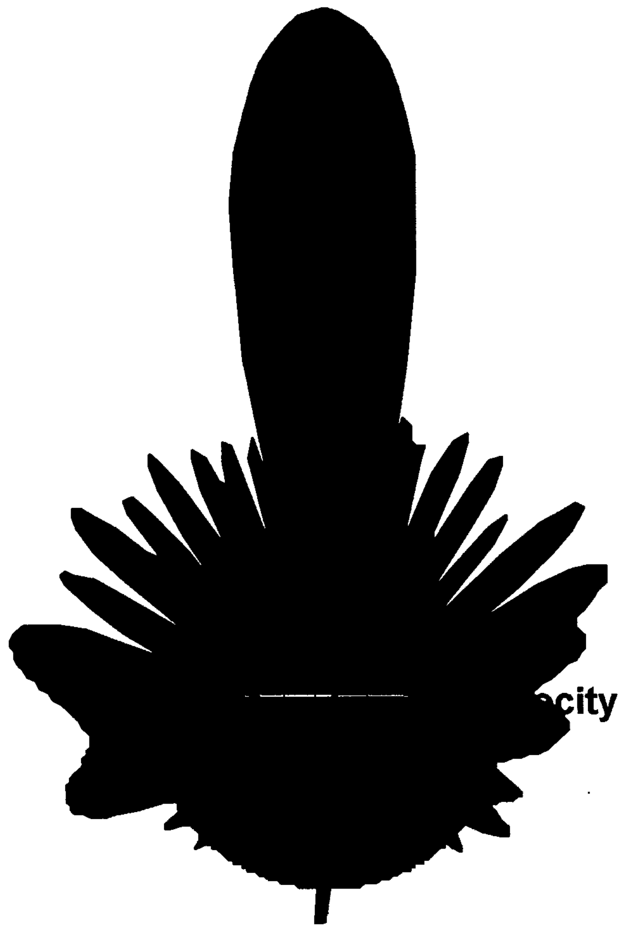
- Introduction
- Theory
- Performance
- Summary

# STAP Units

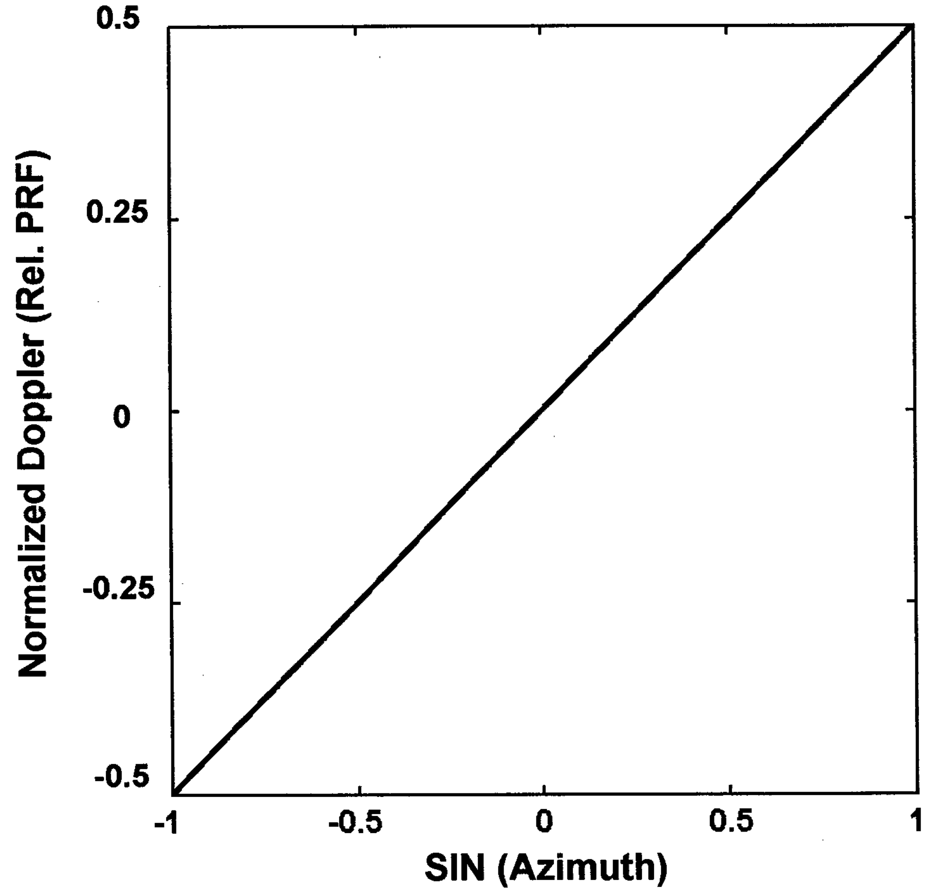


2 Vel./λ
- Vel./λ
0
Vel./λ
-2 Vel./λ
Doppler (Hz)
-50
0
50
Azimuth (°)
$D = \frac{2v\sin(\theta)}{\lambda}$

# STAP Units

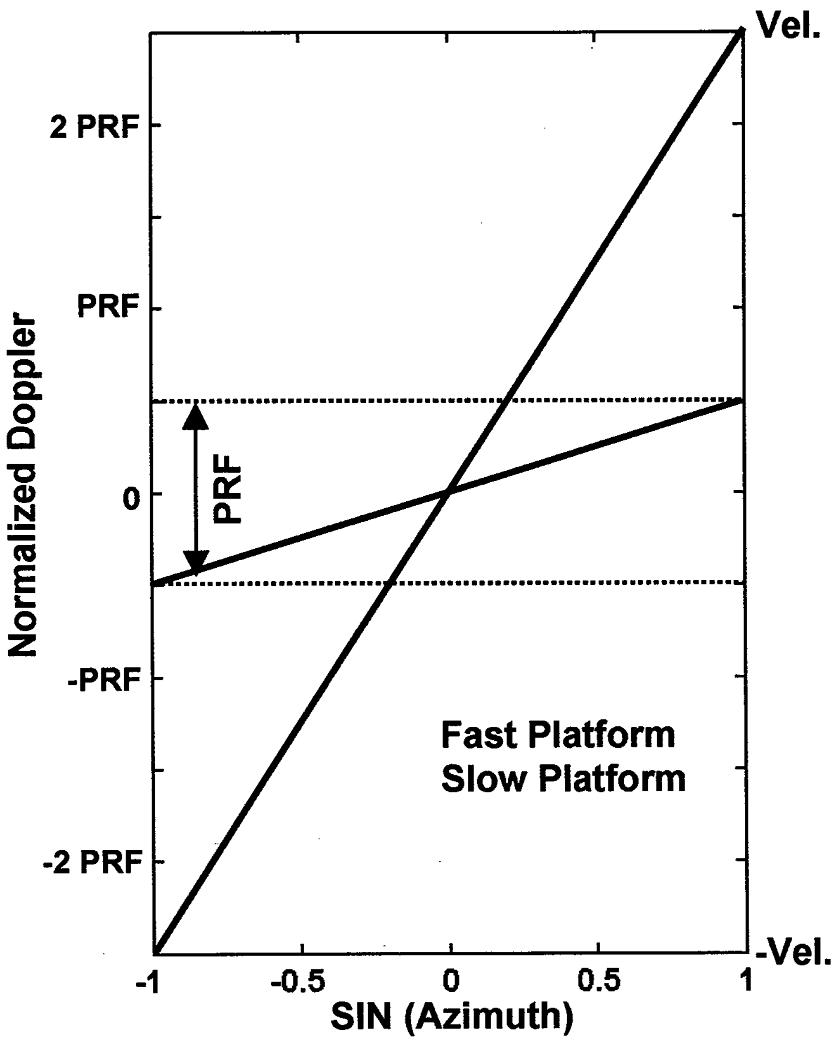


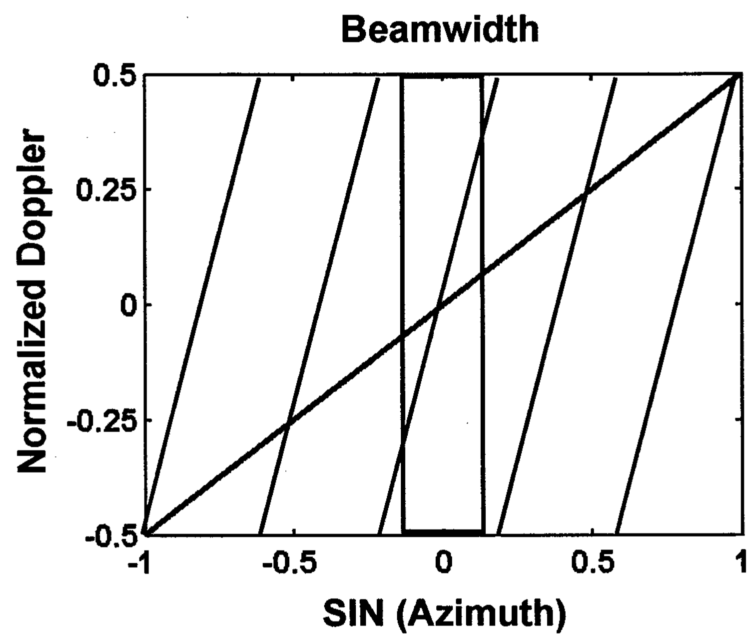
city

0.5
0.25
0
-0.25
-0.5
Normalized Doppler (Rel. PRF)
-1
-0.5
0
0.5
1
SIN (Azimuth)

# Doppler Ambiguous Clutter

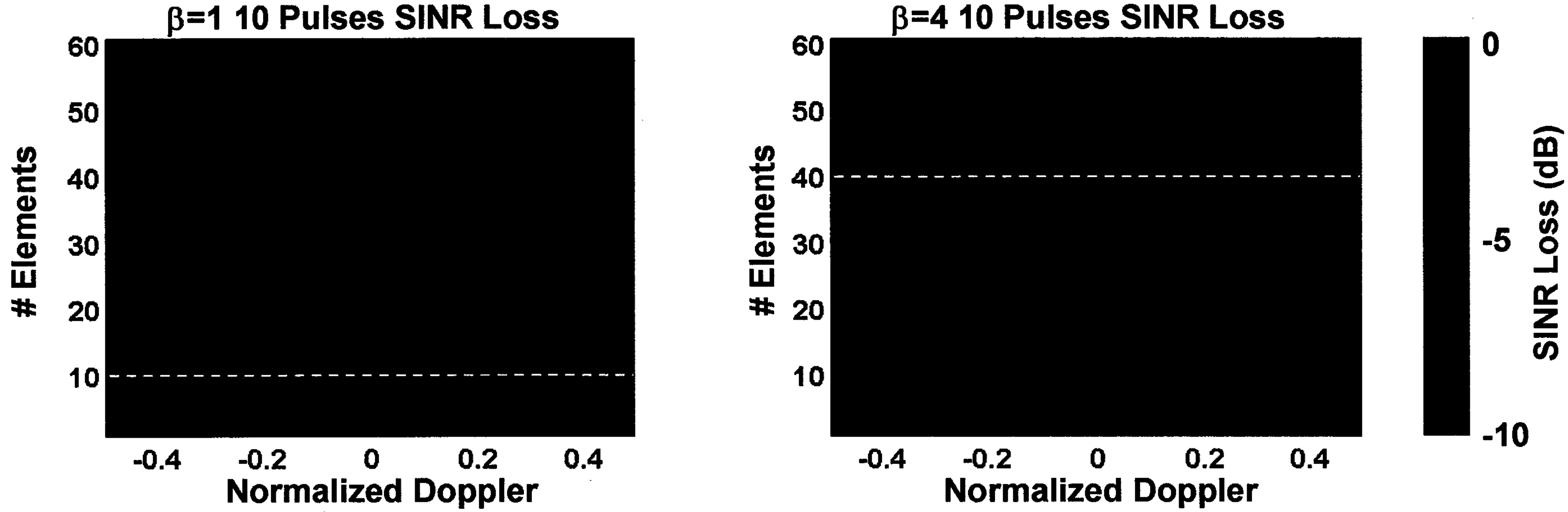



$$\beta = \frac{4v}{\lambda PRF}$$

**Main Beam Clutter Width** $= \frac{\lambda 2v}{L}(\text{m/s}) = \frac{4v}{L}(\text{Hz})$

# Aperture and Doppler Limited Performance

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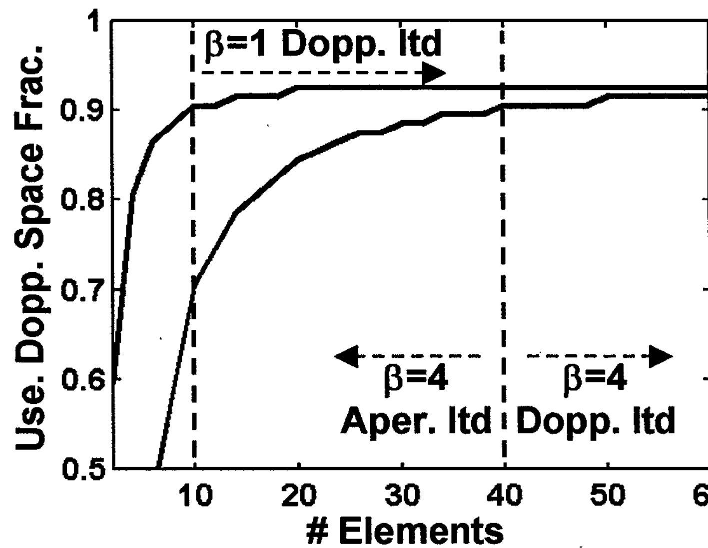
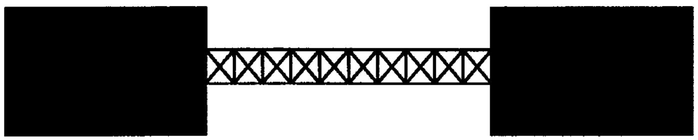
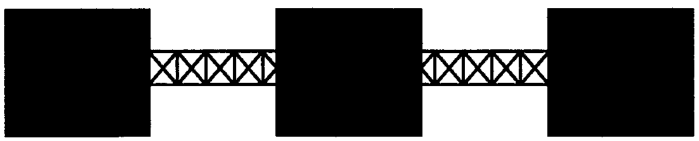


# Some Sparse Array Concepts

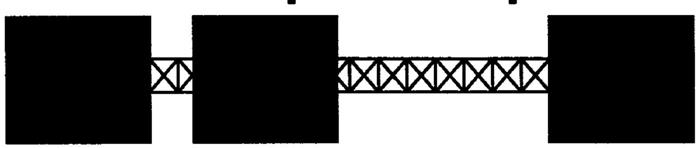
## Interferometer



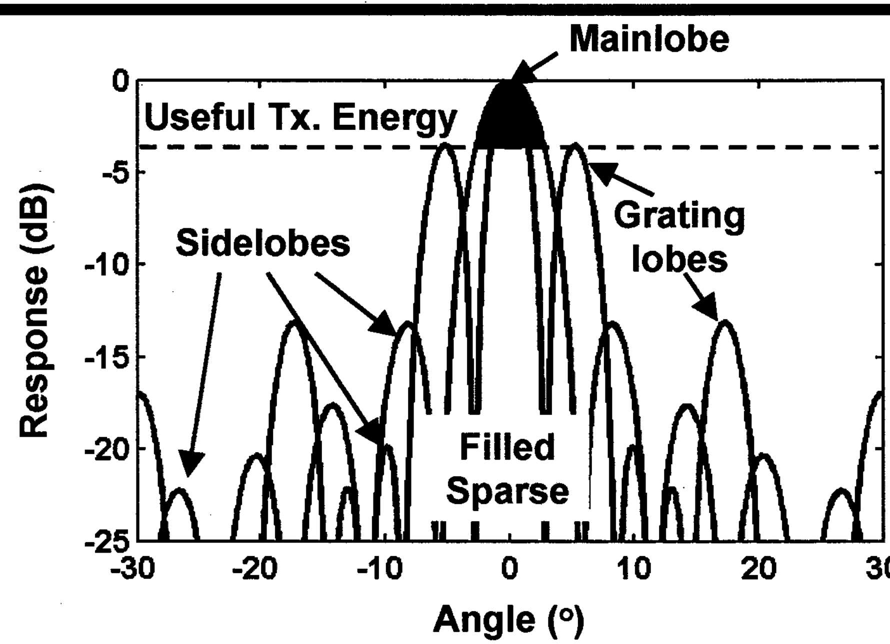

## Even Spaced Equal Size




## Uneven Spaced Equal Size




## Many Apertures


- Sparse arrays trade mainlobe width against grating lobe height to find the optimum sparseness
- Energy transferred from the mainlobe to the grating lobes is useless for Tx.
  - Use a filled section of the sparse array for Tx. And form multiple Rx. beams

# Sparse Array Issues

[redacted]

- Angle estimation performance
  - Improved accuracy due to narrower beamwidth (CRB)
  - Non-local errors due to grating lobes (WWB, ZZB, AB, …)
- SAR performance
  - Multiple spatial samples per pulse
  - Tight PRF constraints
- Hardware and cost
  - Sparse arrays require less hardware
  - Cheaper & lighter
- …

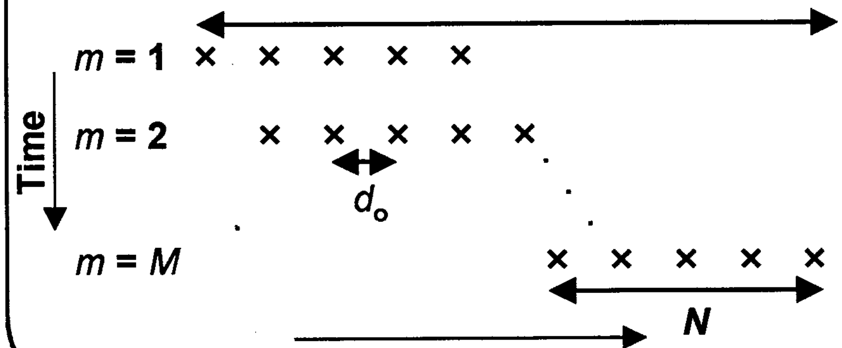
# Outline

- Introduction
- Theory
  - Clutter Rank
  - Waveforms
  - SINR Loss
- Performance
- Summary

# Brennan's Rule & Ward's Rules*

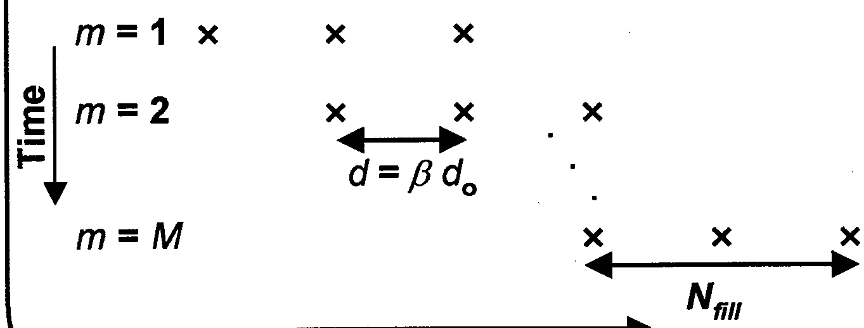
## Brennan's Rule

Rank = Total Synthetic Aperture




## Ward's Min Rank Rule

Min Rank = $N + M - 1$

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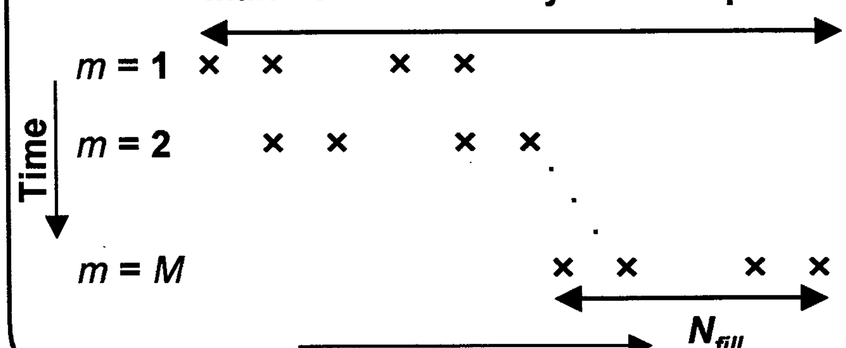
## Ward's Max Rank Rule

Max Rank = Total Synthetic Aperture

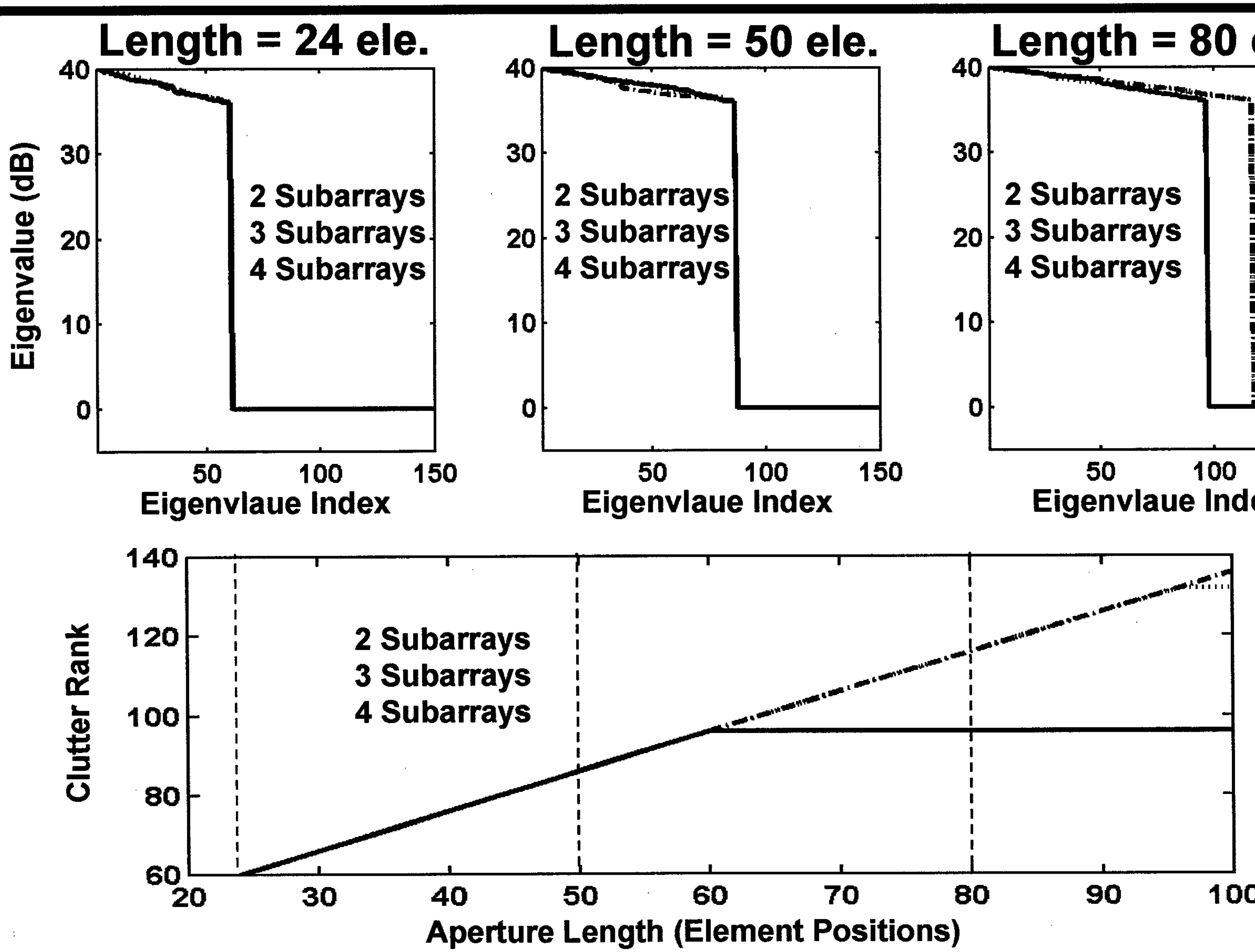



*J. Ward, Asilomar 1998

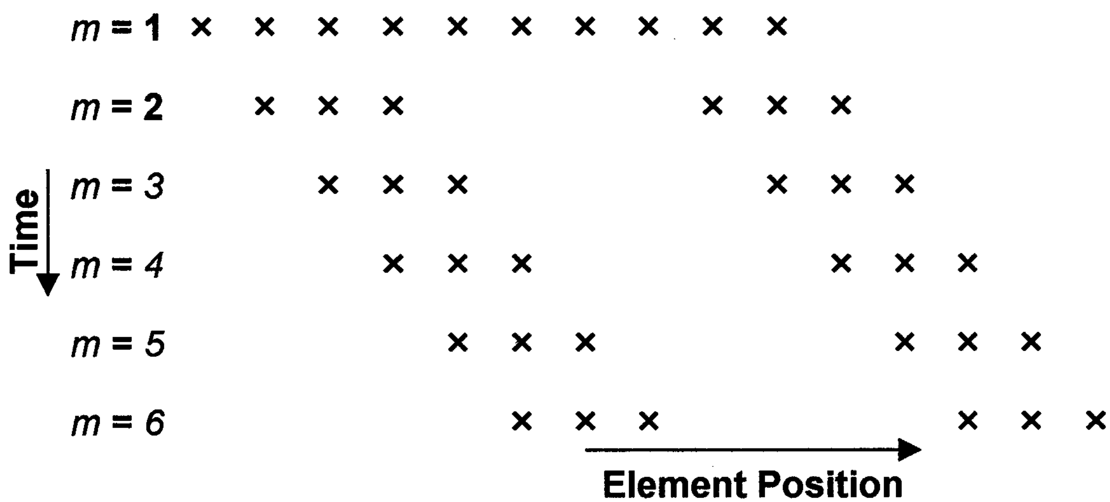
$N$ = Number of elements, $M$ = Number of pulses, $\beta = 2\, v\, T\, d_0^{-1}$, $N_{fill}$ = Number of elements in filled array

# Additional Sparse Array Behavior

## $N = 24$, $M = 10$, $\beta = 4$ Example



# New (?) Rules for Sparse Arrays




Rank=min[6+1+4,6+2*1]=min[11,8]=8

Rank=min[6+2+4,6+2*2]=min[12,10]=10

Rank=min[6+3+4,6+2*3]=min[13,12]=12

Rank=min[6+4+4,6+2*4]=min[14,14]=14

Rank=min[6+5+4,6+2*5]=min[15,16]=15

- **For arrays which move less than the smallest subarray aperture during a pulse the rank is given by :**

$$\min\left[\underbrace{N+\beta(M-1)+G}_{\text{Jim Ward's } r_{max}},\ \underbrace{N+S\beta(M-1)}_{\text{Using each sub array independently}}\right]$$

- **For equal size subarrays a sparse array is no better than a single subarray if**

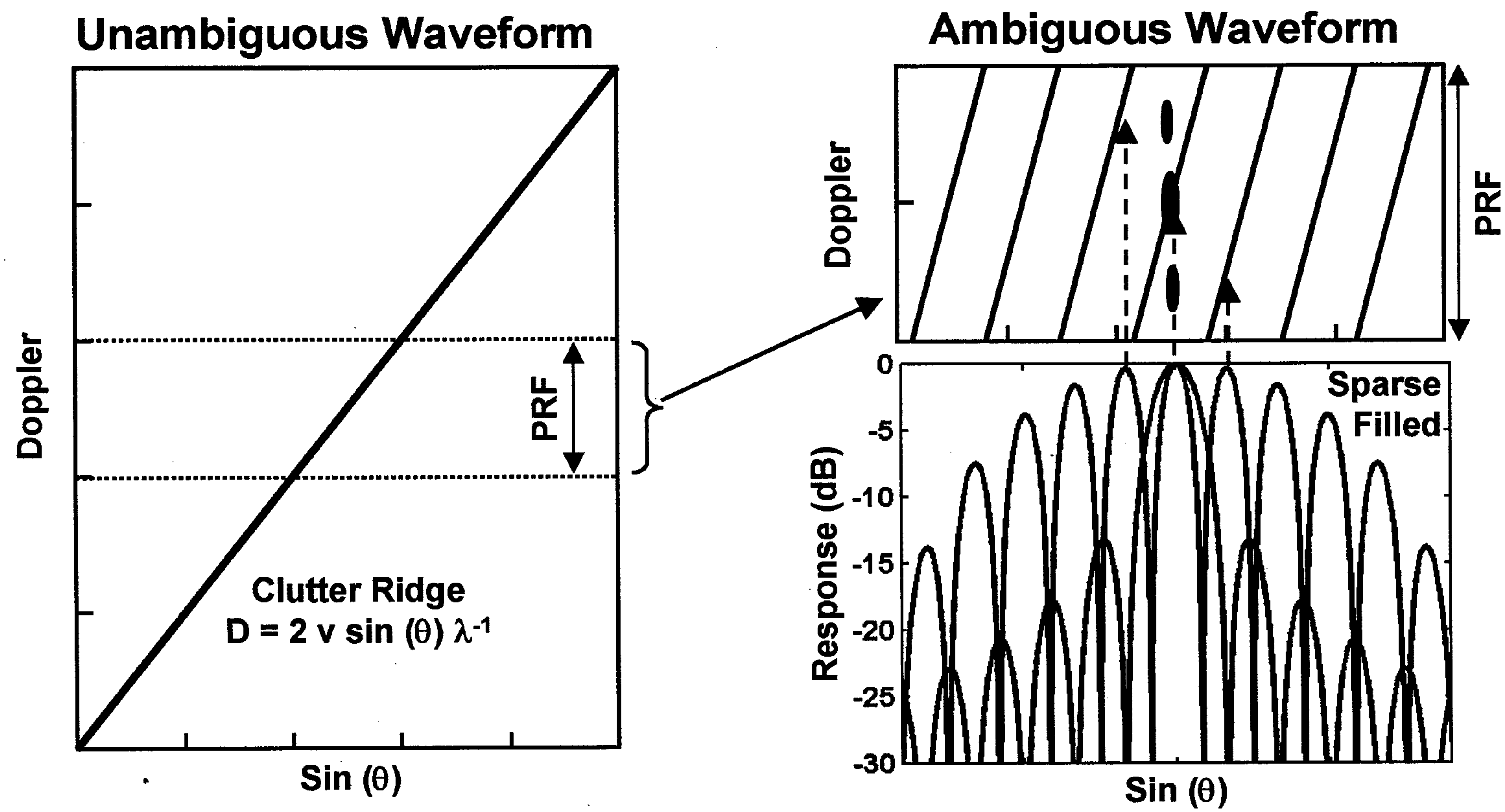
$$G > \beta(S-1)(M-1)$$

- **I.e., The array is so sparse that there is no redundancy**



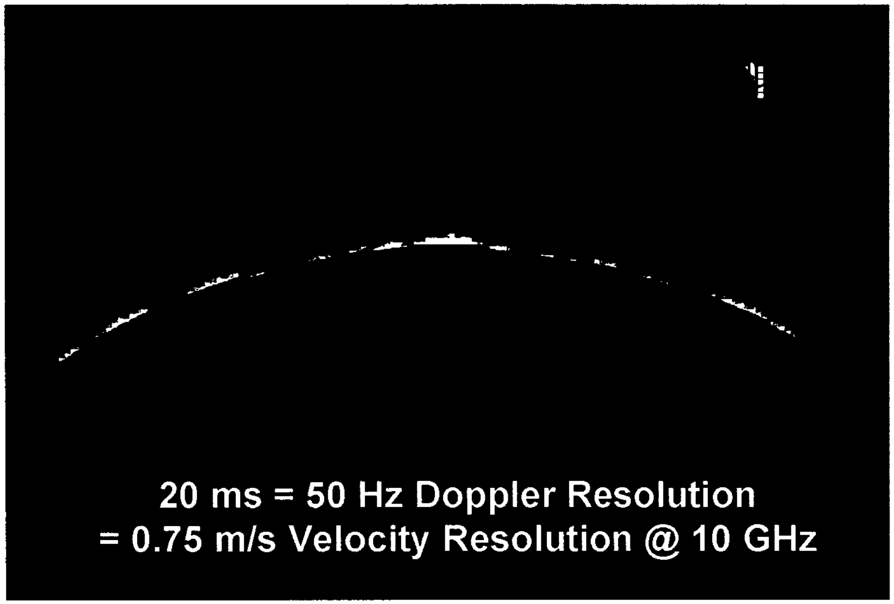
$G$ = Sum gap sizes (element positions) $S$ = Number of subarrays

# Sparse Aperture Waveforms

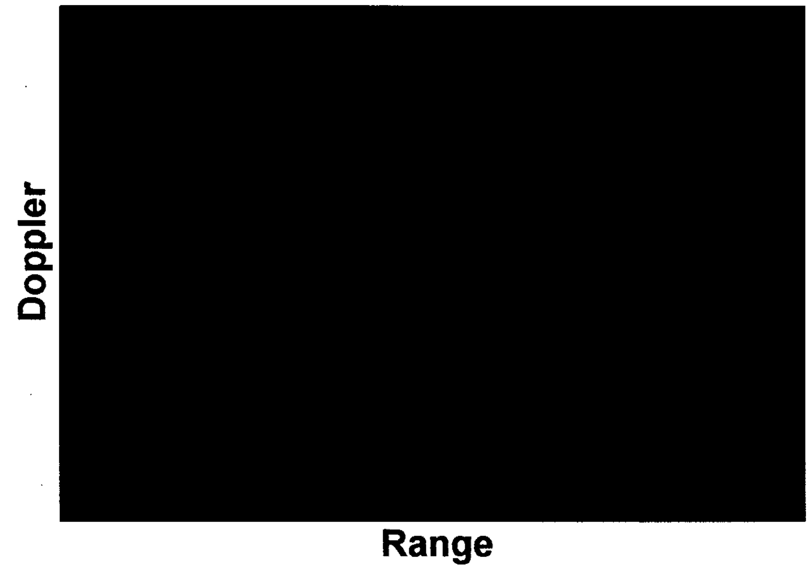



- Ambiguous waveforms (e.g., pulse-Doppler) and sparse (ambiguous) apertures lead to multiple clutter nulls
- Unambiguous waveforms preferable

# Long Single Pulse Waveforms

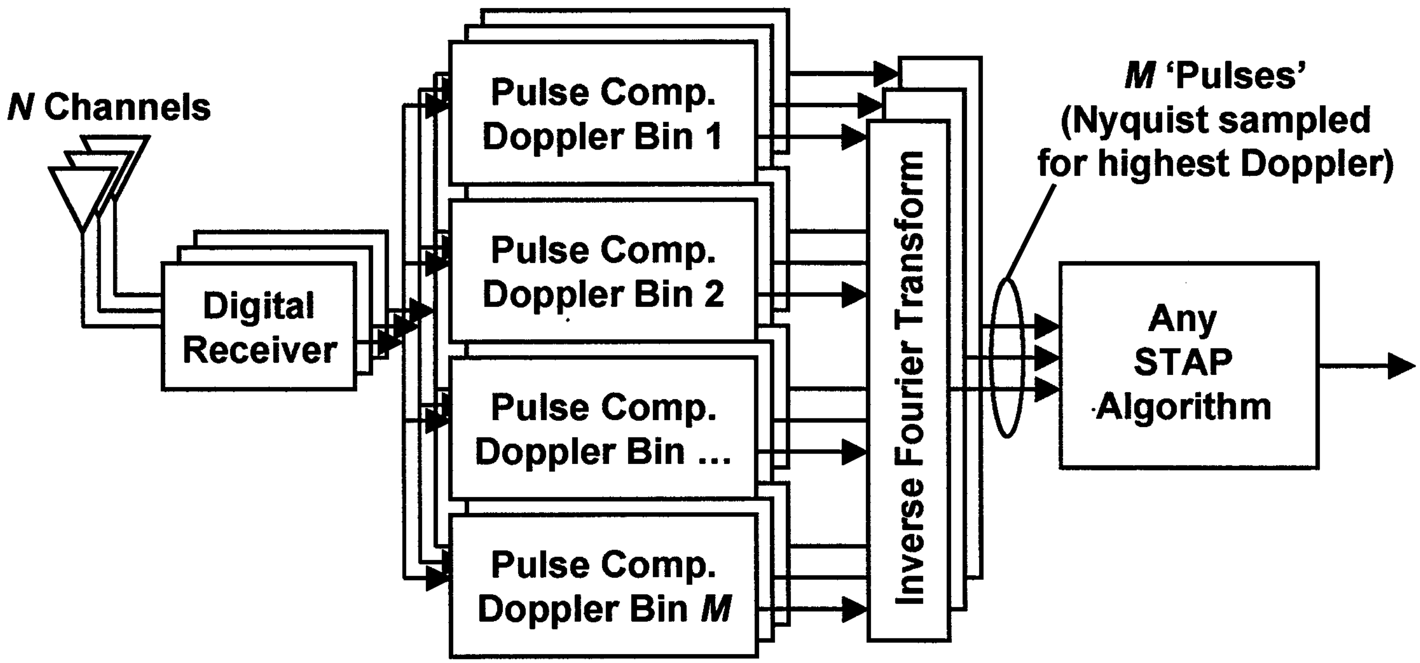



**Phase Encoded Waveform**



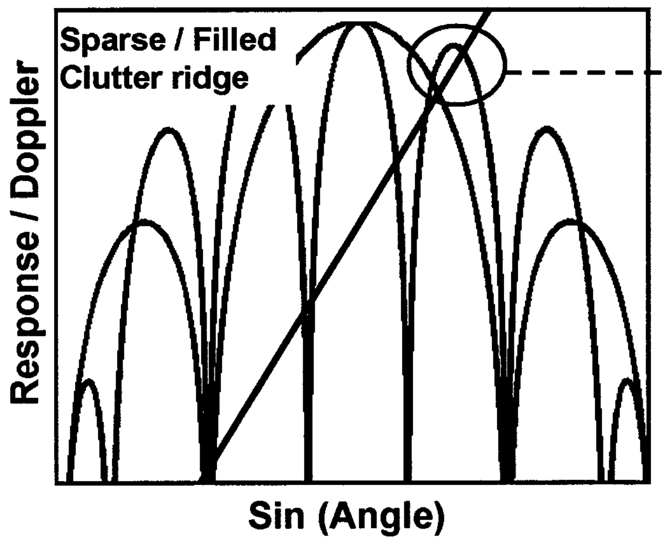

- Single pulse means no range or Doppler ambiguities
  - High chip rate sets Doppler ambiguities
- Must pulse compress each Doppler bin separately
  - More computation than pulse-Doppler waveforms
- Concern about strong sidelobe clutter > noise floor
  - Wide bandwidth & narrow antenna beampatterns

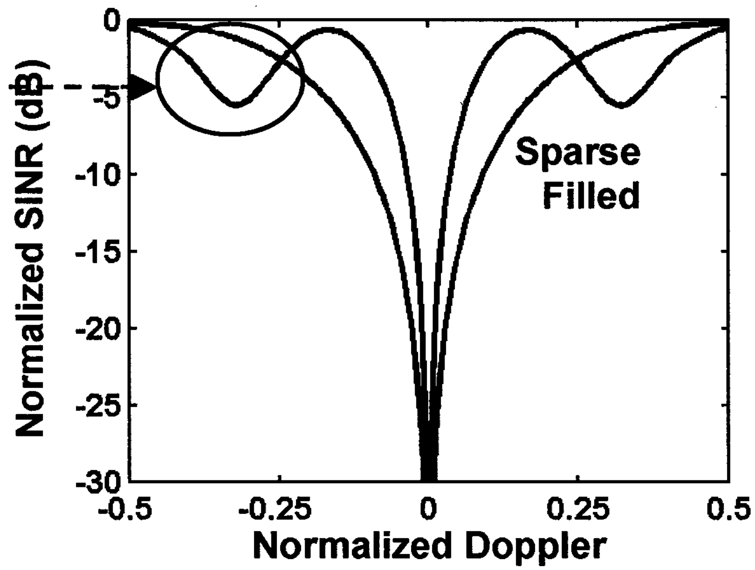
# Processing Long Single Pulse Waveform




- Long single pulse radar can be made to 'appear' like a regular pulse-Doppler radar
- Looks like high PRF radar without the range ambiguities

# Space Time Adaptive Processing




- **Grating lobes lead to reduced detection performance at particular Doppler frequencies**

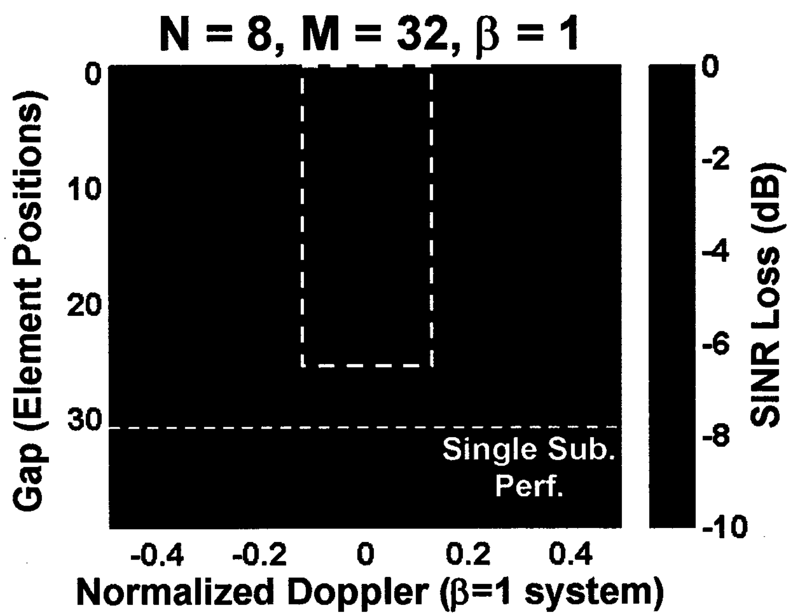
$$\text{SINR Loss} \approx \mathbf{v}^H\mathbf{v} - \left|\mathbf{v}^H\mathbf{e}\right|^2 = 1 - \frac{\text{Grating Lobe Gain}}{\text{Mainbeam Gain}}$$

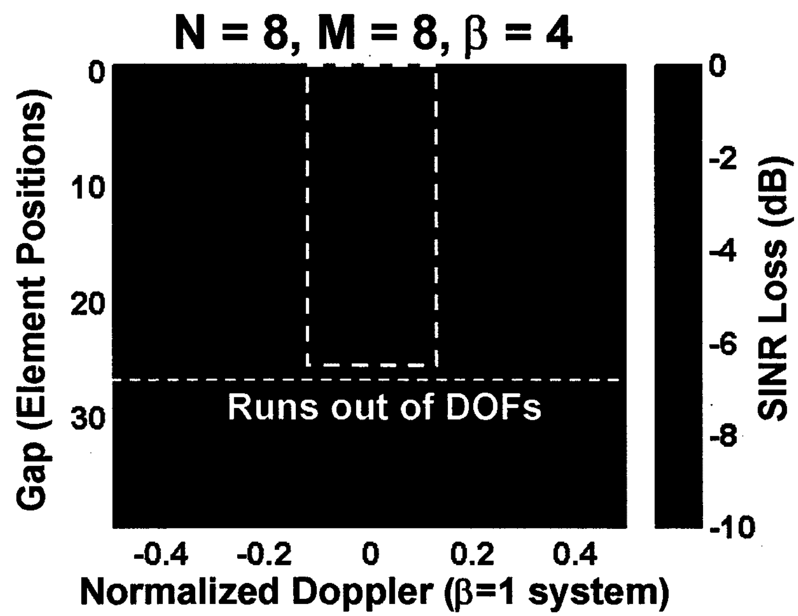
- **Should not make the array too sparse**
  - **For <3 dB SINR loss grating lobe gain must be 3 dB less than main lobe gain ($\Sigma$ grating lobes for pulse-Doppler waveforms?)**

# Outline

- Introduction
- Theory
- Performance
  - Dependence on waveform
  - SBR Design Example
- Summary

# Unambiguous vs. Ambiguous Waveforms
## Interferometer Example

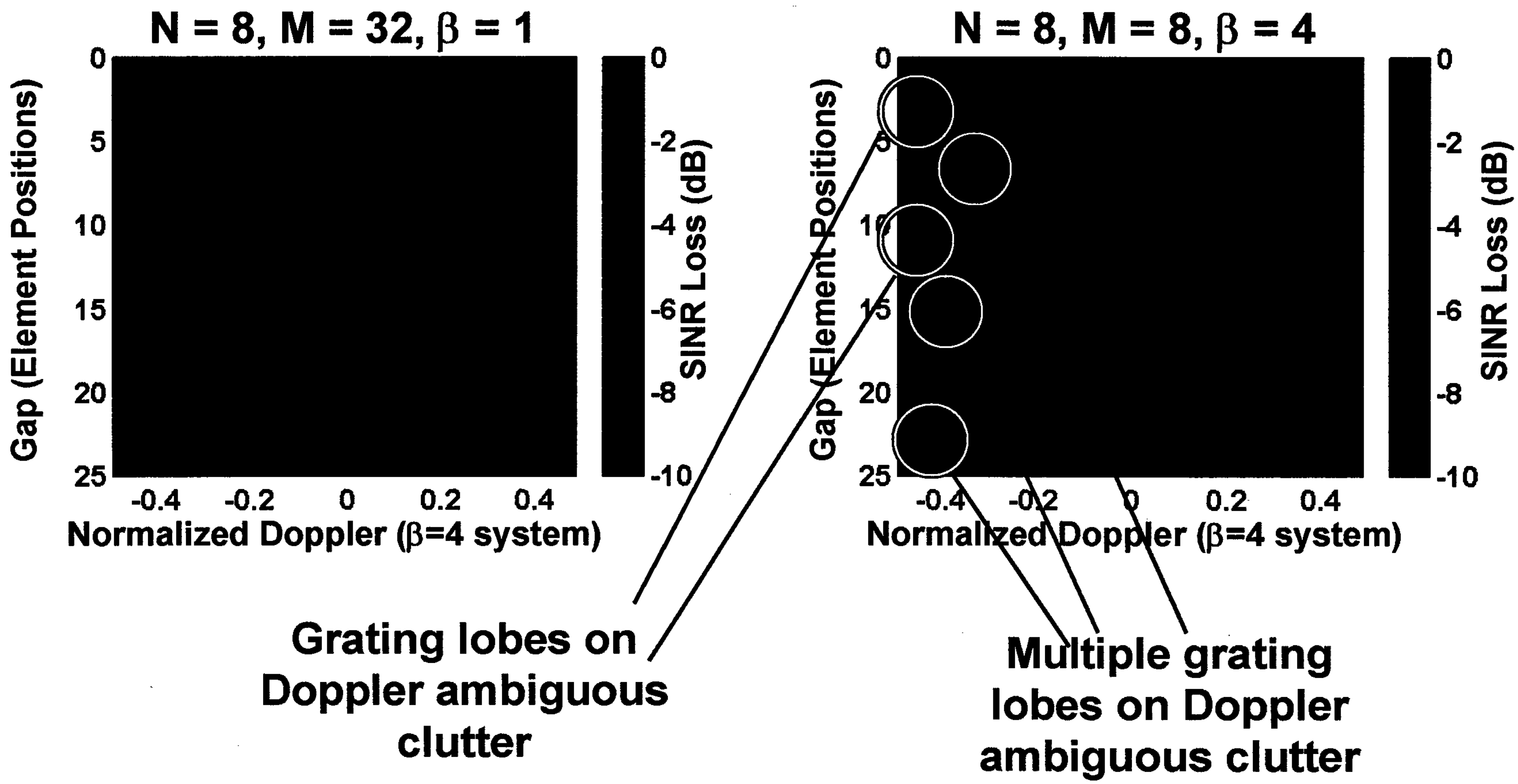



- Filled rank = 8+1(32-1) = 39
- Max. sparse rank = 8+2(32-1) = 70 (reached with a 31 element gap)
- Filled rank = 8+4(8-1) = 36
- Runs out of DOFs with a 27 element gap 8+27+2(32-1) = 63

[redacted]

# Unambiguous vs. Ambiguous Waveforms

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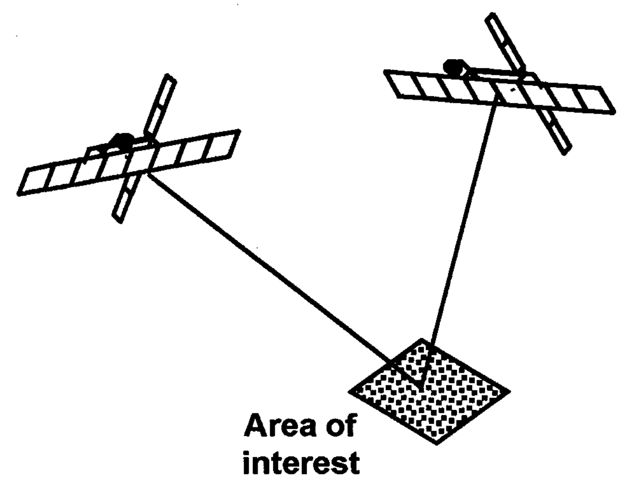
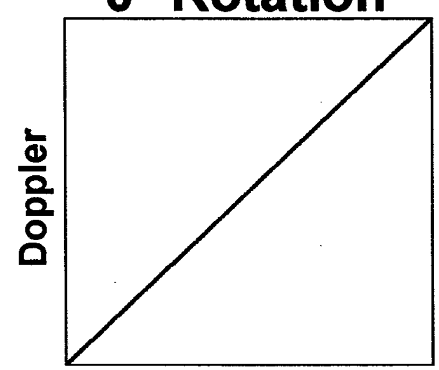
# Space Based GMTI Radar Examples

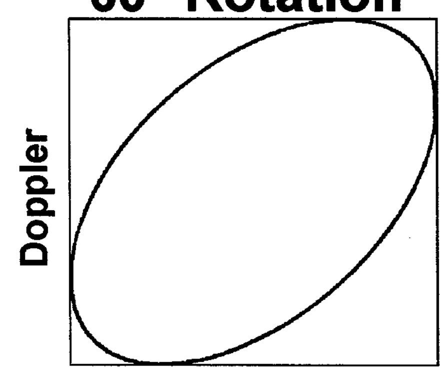
## Parameters

- 32m x 2.5m filled aperture
- 10 GHz operating frequency
- 1000 km orbit
  - 7282 m/s orbital velocity
- 1 kw peak transmit power
- 200 MHz bandwidth
- Unambiguous waveform
- -12 dB const. $\gamma$ clutter model
- 2500 km range
  - 16.67ms CPI length
  - Travel ~120m in a CPI

## Scenarios

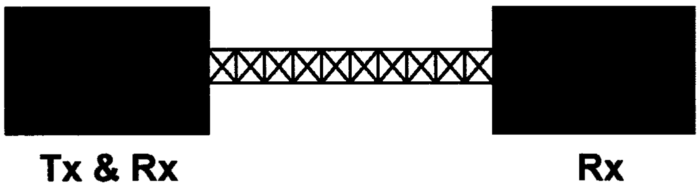


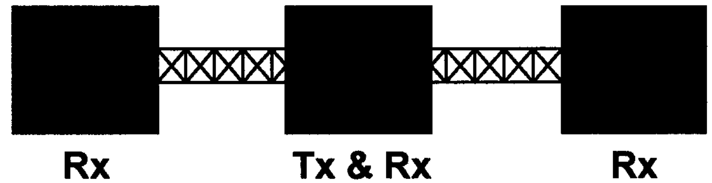



# Space Based Radar GMTI Designs

## Interferometer Array

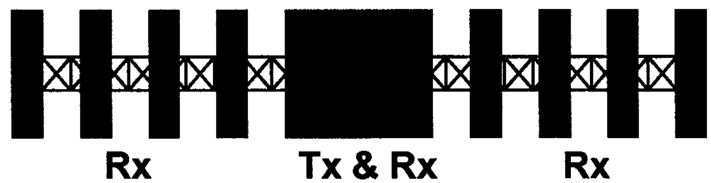



## Even Spaced Equal Size




## Uneven Spaced Equal Size


## Many Apertures

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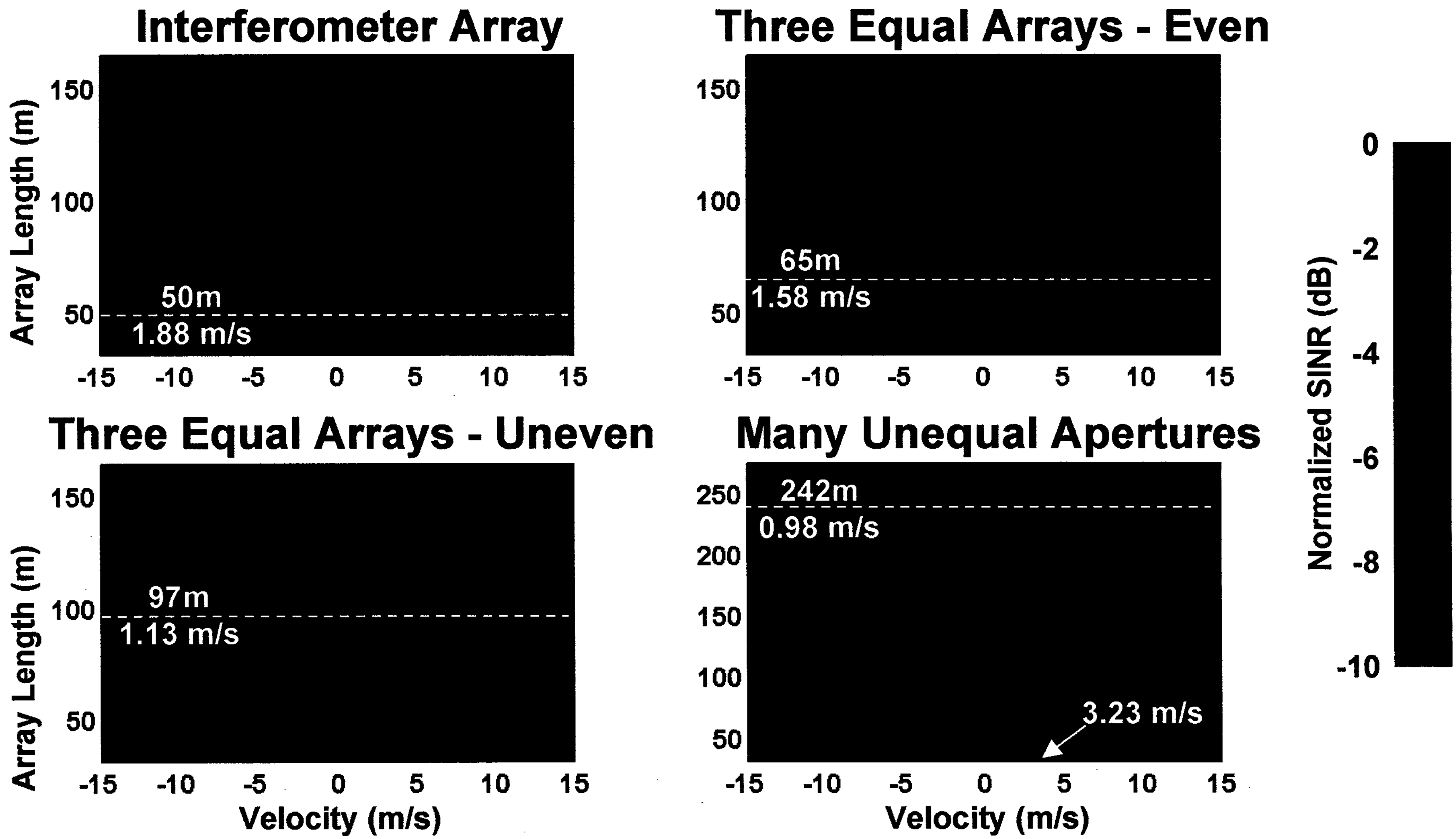
* Issues being addressed by Aerospace Corporation

# 0° Rotation Scenario

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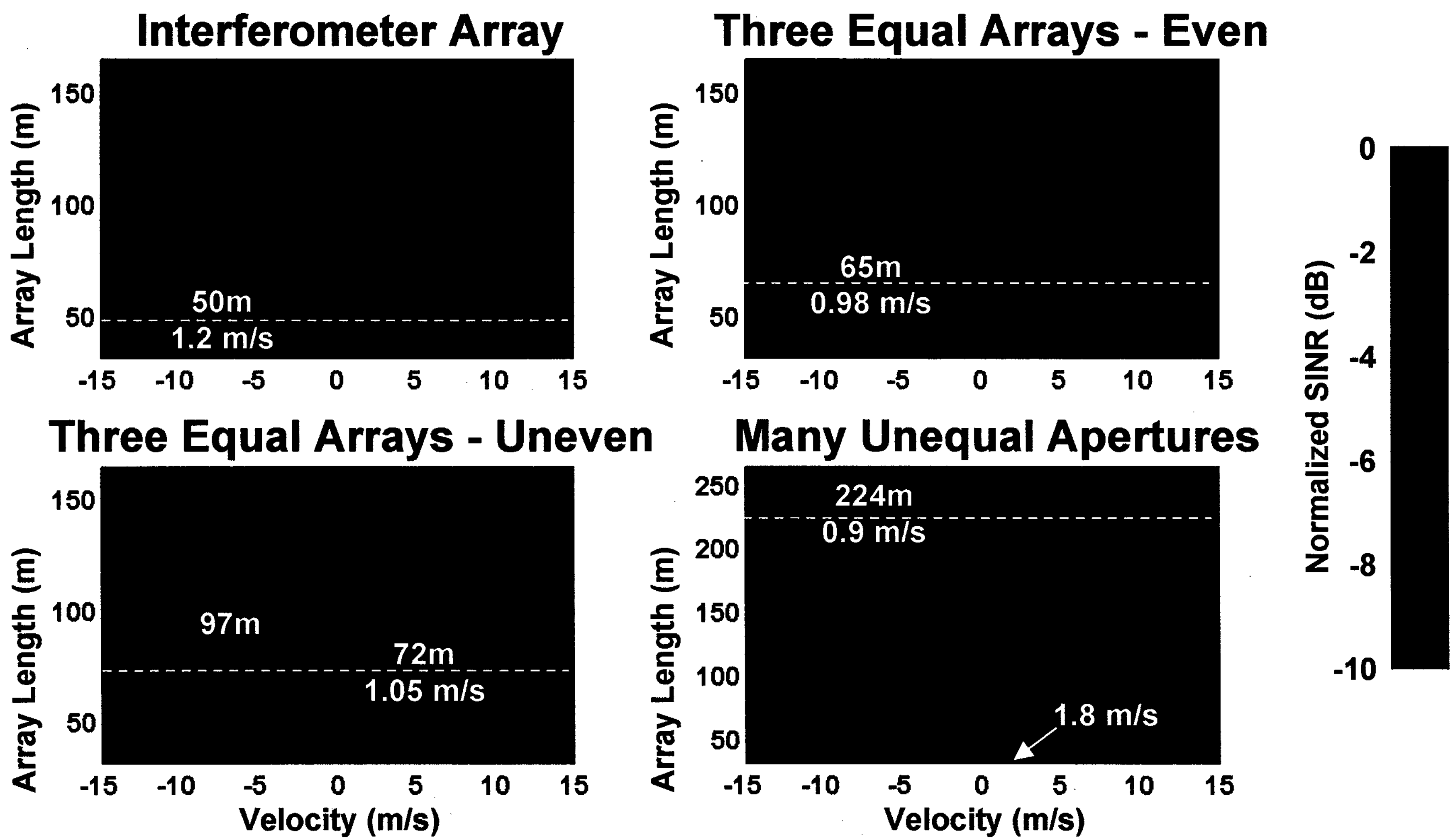
# 60° Rotation Scenario

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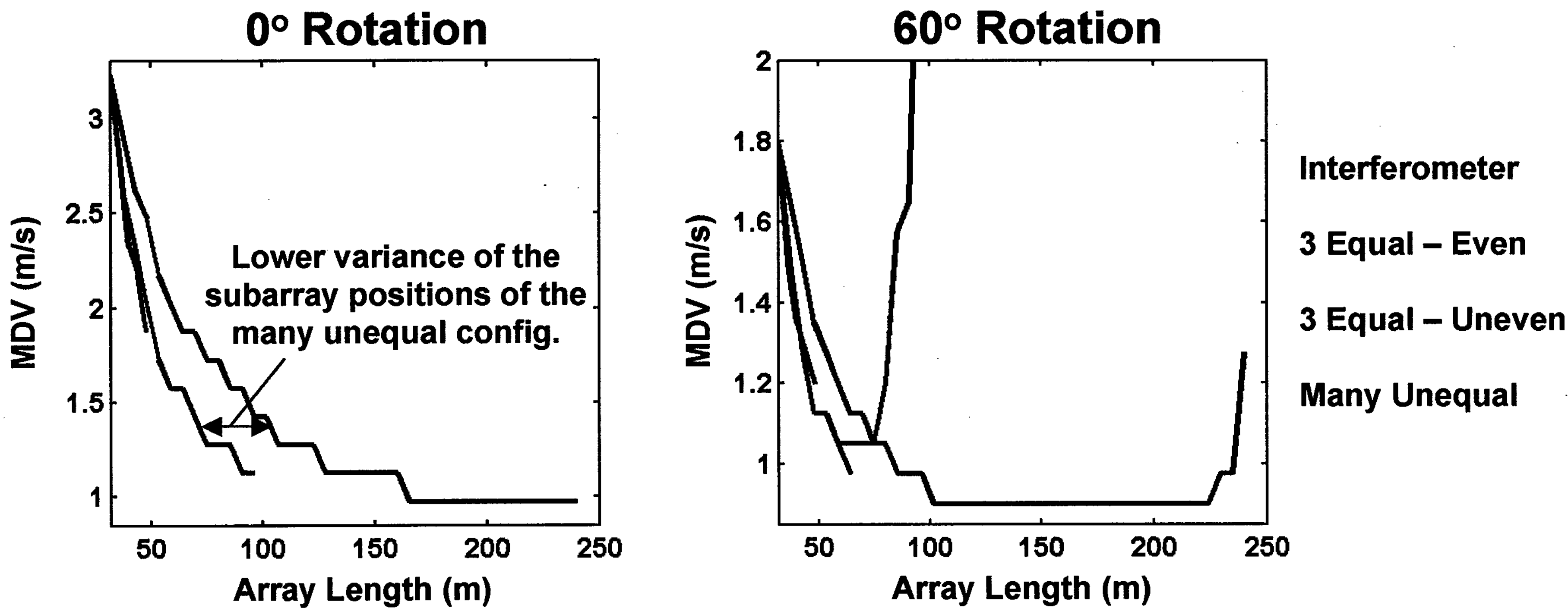
# -3 dB MDV vs. Array Length




- **Many unequal subarrays configuration needs a larger baseline to obtain the same performance as the other configurations, but ultimately provides the best MDV**
  - 165m aperture optimizes MDV for 2500 km range
  - Longer apertures improve angle metrics

# Summary

- Sparse arrays potentially improve the minimum detectable performance of space-based radars
  - Approach the MDV performance of a large filled aperture much with lower size, weight and cost

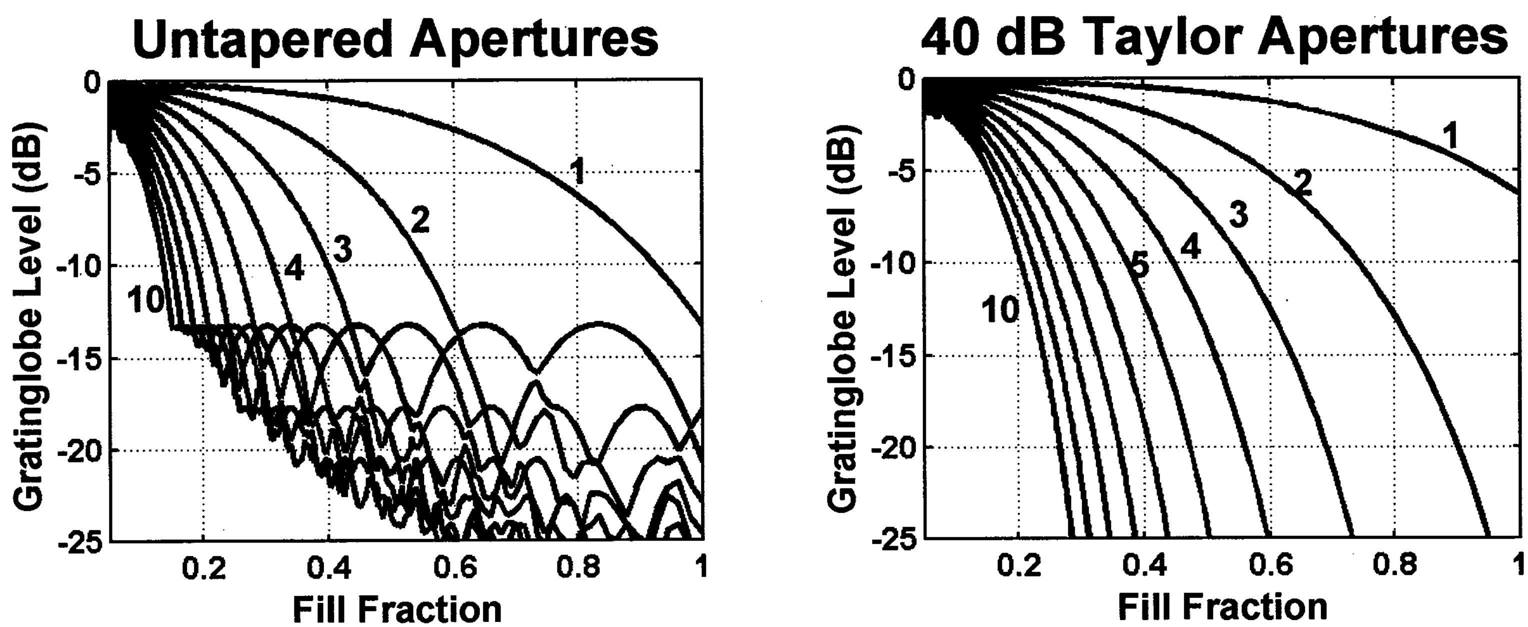
- Sparse arrays and sparse (pulse-Doppler) waveforms do not mix well
  - Sparse arrays perform well with Doppler unambiguous waveforms
  - Sparse waveforms (pulse-Doppler) perform well with filled arrays

- Long single-pulse waveforms provide range and Doppler unambiguous operation and are compatible with current STAP algorithms

- Sparse arrays with many unevenly sized unevenly spaced subarrays provide the best GMTI performance

# Backup Viewgraphs

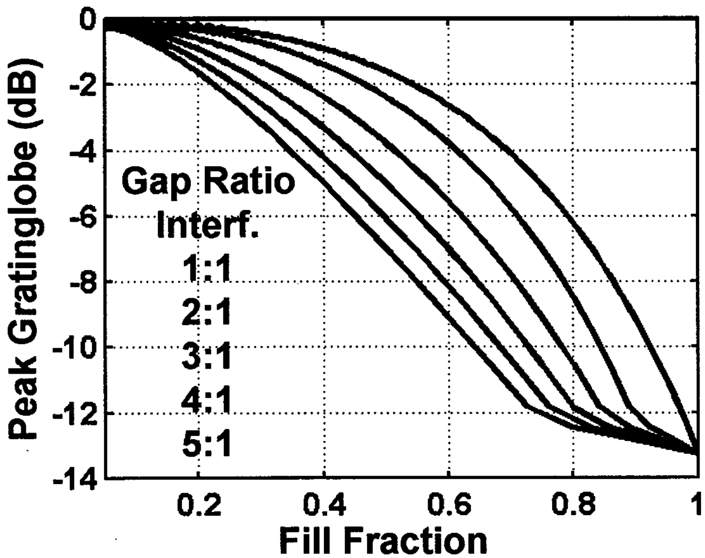
# Interferometer Array Grating Lobes

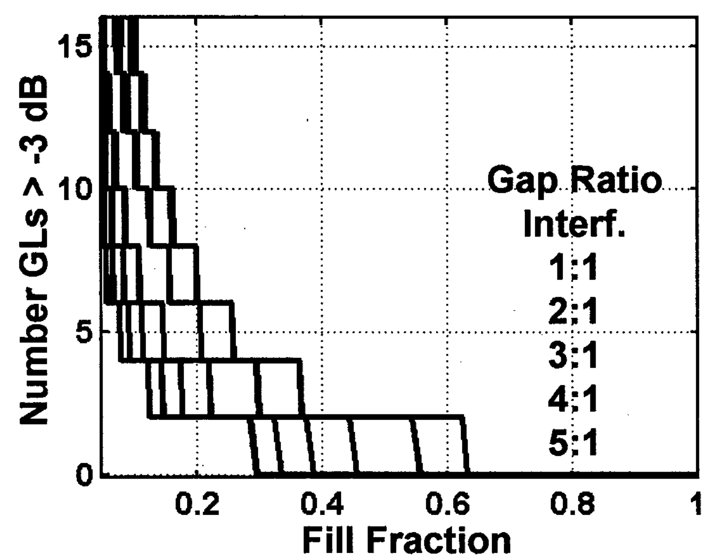


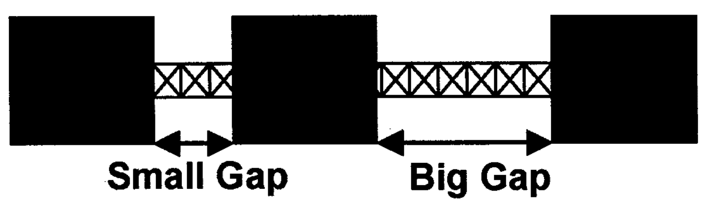

$$\text{Fill Fraction} = \frac{\text{Filled Aperture}}{\text{Total Aperture}}$$

- **Grating lobes quickly appear for interferometer array**
- **~$^2/_3$ fill fraction -3 dB grating lobes untapered apertures**

# Grating Lobe Distributions
# 3 Equal Arrays



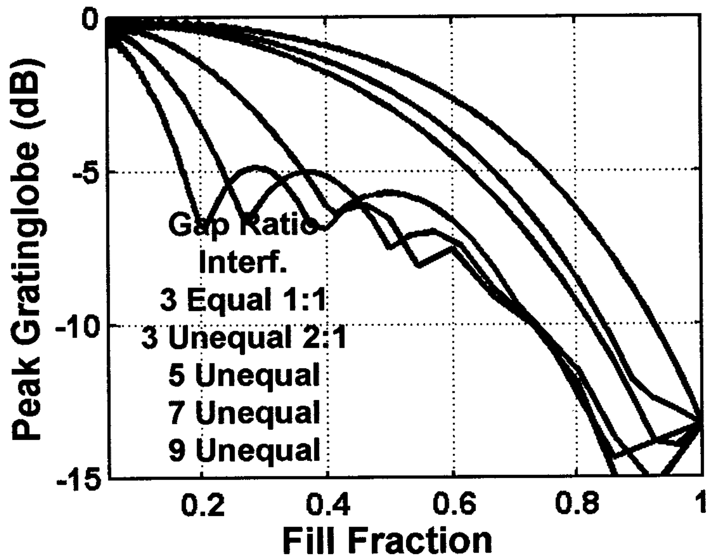



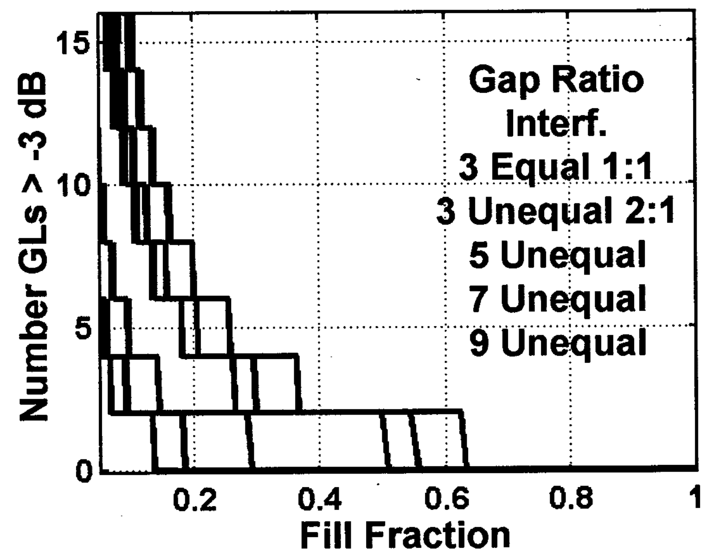




Gap Ratio = Big Gap : Small Gap

- **Lower grating lobes than interferometer**
- **Higher gap ratios lead to lower grating lobes**
  - **Also poorer MDV performance**

# Grating Lobe Distributions
# Unequal Arrays




50% filled aperture in center subarray

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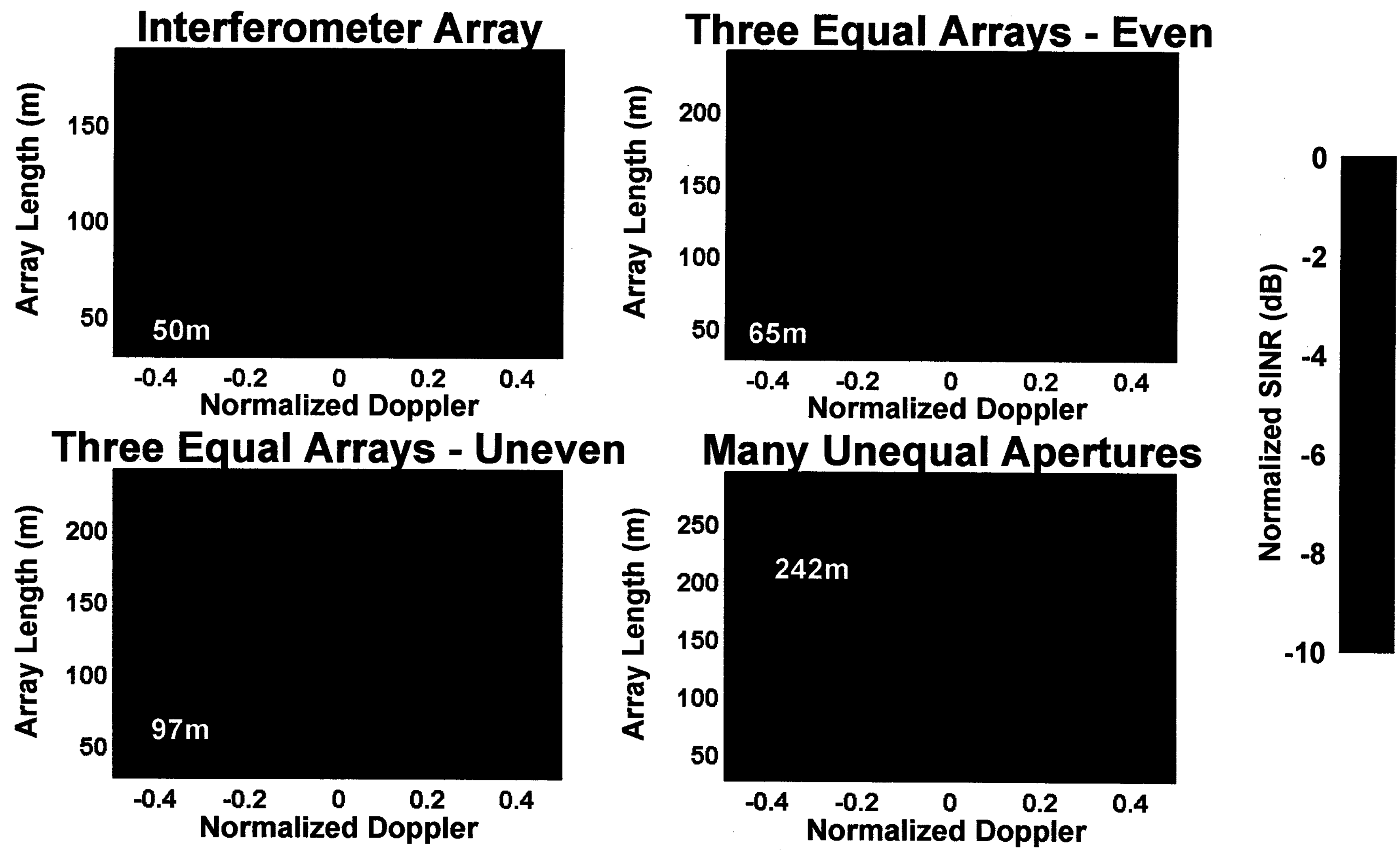
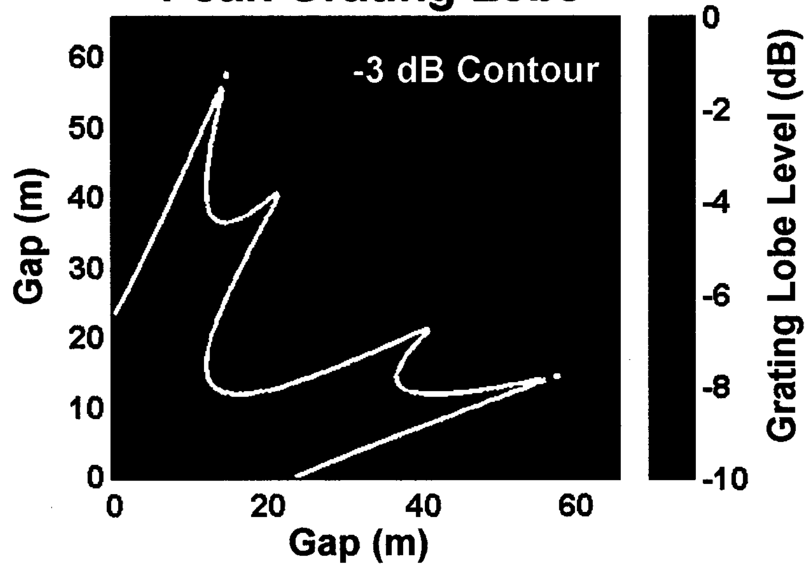
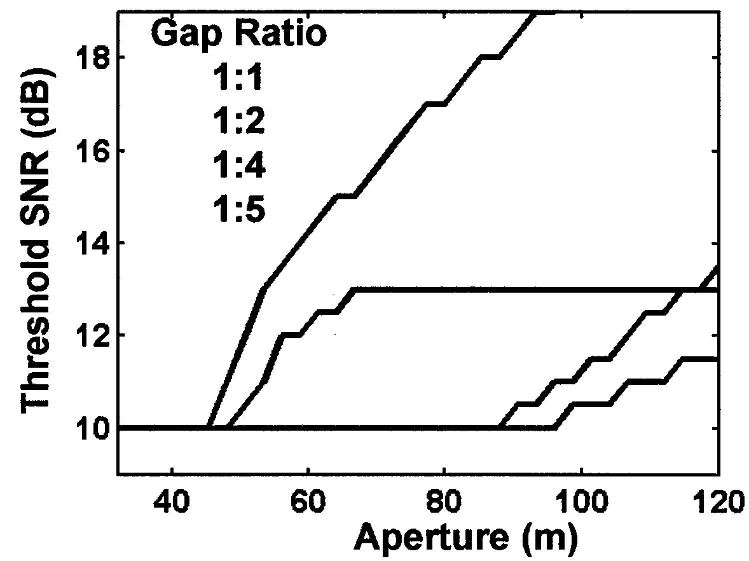
# 0° Rotation Scenario



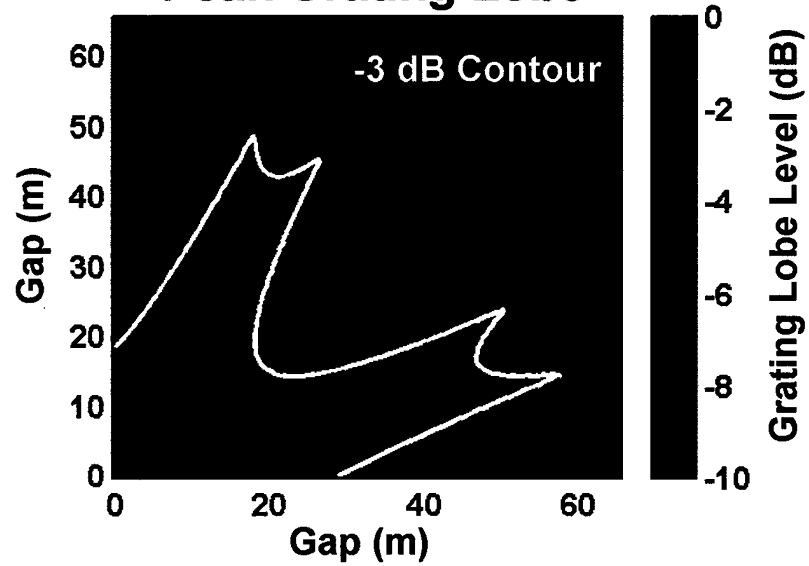
# Three Equal Apertures Target Location

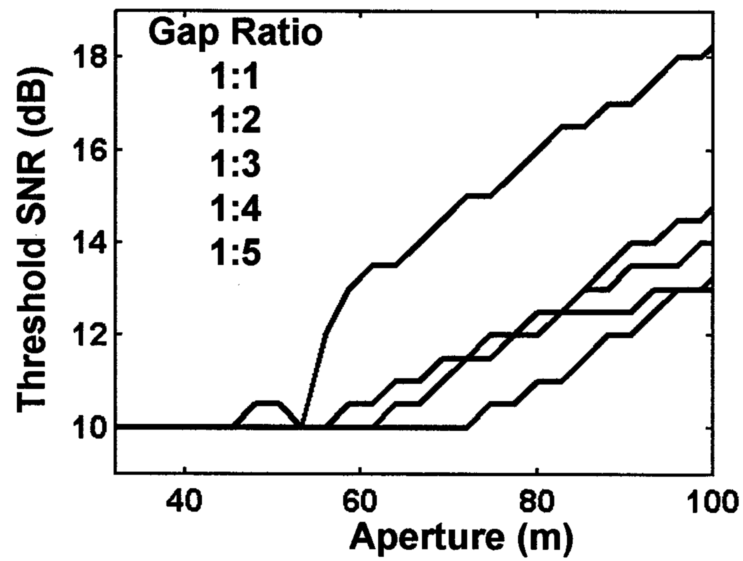



- 96 m aperture largest possible without increasing the threshold SNR
  - Provides 89 m rms error at 6° grazing
  - 82 m gives 107 m rms error

# Three Unequal Apertures Target Location

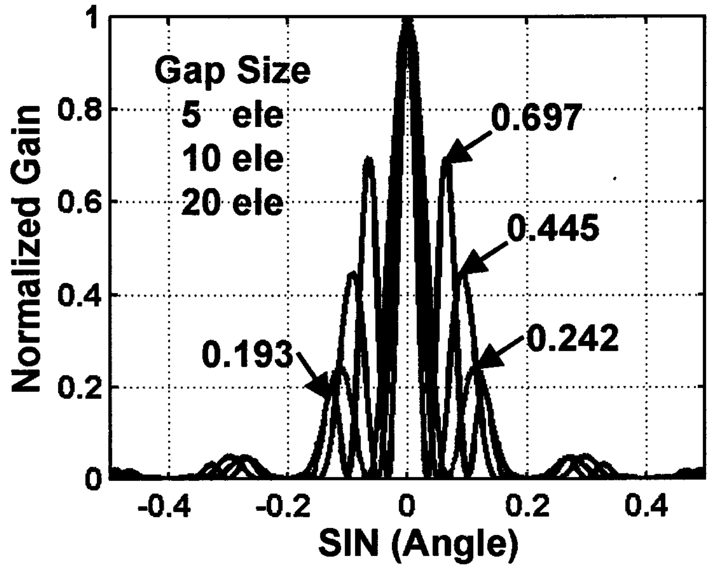


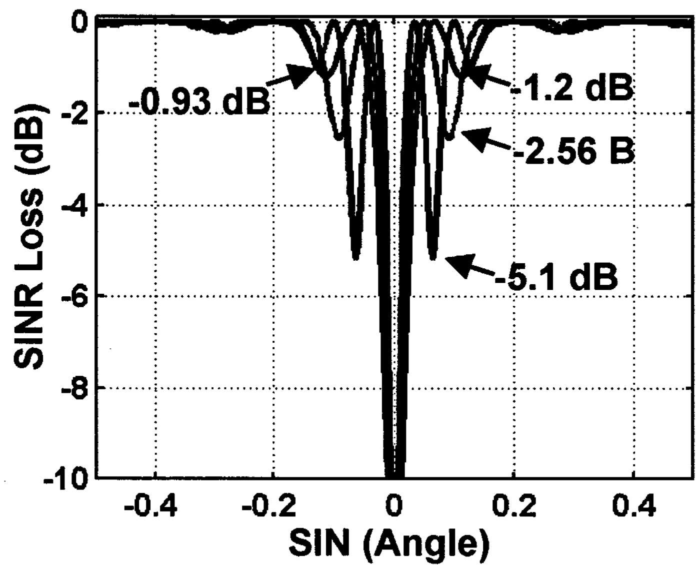




- 72 m aperture largest possible without increasing the threshold SNR
- 72m aperture Provides 119m rms error at 6° grazing

# SINR Loss Due To Grating Lobe
## (Spatial Only Example)

### 20 Element Array Example




- **Under the high INR assumption:**

$$\text{SINR Loss} \approx \boldsymbol{v}^H\boldsymbol{v} - \left|\boldsymbol{v}^H\mathbf{e}\right|^2 = 1 - \frac{\text{Grating Lobe Gain}}{\text{Mainbeam Gain}}$$

- **i.e., for 3 dB loss grating lobe gain (sum grating lobes for pulse-Doppler ?) must be 3 dB less than main lobe gain**