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NON-LINEAR CONTROL ALLOCATION USING PIECEWISE LINEAR FUNCTIONS: A LINEAR PROGRAMMING APPROACH

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Non-Linear Control Allocation Using Piecewise Linear Functions: A Linear Programming Approach

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Abstract

The performance of two different approaches to solving the non-linear control allocation problem is presented. The non-linear control allocation problem is formulated using piecewise linear functions to approximate the control moments produced by a set of control effectors. When the control allocation problem is formulated as a piecewise linear program, an additional set of constraints enter into the problem formulation. One approach is to introduce a set of binary variables to enforce these constraints. The result is a mixed-integer linear programming problem that can be solved using any branch-and-bound software. A second approach is to solve the piecewise linear programming problem using a modified simplex method. The simplex algorithm is modified to enforce a subset of the decision variables to enter into the basis only if certain conditions are met. We will show that solving the optimization problem using the simplex based approach is significantly faster than solving the same problem using a mixed-integer formulation. We will then compare the closed-loop performance of a re-entry vehicle using both approaches.

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Introduction

Historically, the control allocation problem has been approached by assuming that aerodynamic control effectors produce control moments that are linear functions (at least locally) of the control surface deflection. While this approach may be valid for nearly all aircraft under nominal flight conditions when there are no failed control surfaces, the occurrence of one or more failed control effectors may result in a situation where the linear assumption is no longer valid. When a control effector fails, the resulting rolling, pitching, and yawing moments produced by the failed effector must be accommodated by reconfiguring the remaining healthy control effectors. For some failures, one or more of the remaining effectors may be driven to a position limit in order to compensate for the failed effector. The behavior of the control moment curve in regions near position limits tends be highly non-linear. Therefore, the linear model may introduce significant modelling errors that result in incorrect control surface deflections and this may ultimately lead to loss of the vehicle.

There have been recent efforts to improve upon the current control allocation approaches that assume a linear relationship between the control moments and the control effector. Doman and Oppenheimer[1] have implemented a linear control allocator that added an intercept term to an estimate of the local slope of the control moment curve in order to more accurately account for the non-linear behavior of aerodynamic control effectors. They have shown an improvement in tracking performance when there are failures present without any increase in computation time. However, the intercept correction method assumes that the control moment curves are monotonic functions of position, otherwise it is possible that the control allocator may settle in a region where the slope changes sign, resulting in gross modelling errors due to an incorrect intercept term.

Doman and Sparks[2] provided a method for the determination of the non-linear attainable moment set (AMS) for the two moment case. Their effort considered control effectors that produced a control moment that was a quadratic function of effector position about one axis and a linear function about a second axis. More recently, Bolender and Doman[3] extended the work in Reference[2] to three dimensions for the computation of the non-linear AMS volume when the control moments about the third control axis were linear functions of effector position. At the present time, this particular methods does not lend itself to be applied in a real-time control system due to the excessive computational burden required.

Most recently, Bolender and Doman[4] have shown that a piecewise linear representation of the control moment curve accounts for the non-linearities inherent in aerodynamic data. The only requirement is that the control moments generated by each effector be separable, meaning that no aerodynamic interference occurs among the effectors. The resulting control allocation problem was posed as a mixed-integer linear program and solved using a public doman branch-and-bound optimization code. The downside of this approach was the long time needed by the solver to find a solution, making implementation in a digital flight control system impracticable.

The intent of this paper is to show that the piecewise linear control allocation approach can be solved fast enough to be implemented in a digital flight-control system. The approach taken differs from the authors' initial paper on this subject in two ways. The first difference is to move away from the mixed-integer linear programming form of the optimization problem and back to a linear programming problem that imposes a set of restricted basis entry rules. The second difference is to solve a "mixed optimization" problem as opposed to the solution of the multi-branch control allocation problem. This allows us to achieve the same objectives while only having to solve one optimization problem instead of two. We compare the performance of the simplex method with restricted basis entry rules to the mixed-integer formulation and show that the two approaches give the same performance.

Piecewise Linear Mixed Optimization Control Allocation

The control allocation problem solved in Reference[4] was a Buffington's[5] multi-branch approach. However, instead of a linear relationship between the control effector displacements and the control moments, a non-linear relationship was used. Let the non-linear vector-valued function $\boldsymbol{G}(\boldsymbol{\delta})$ denote the relationship between the control effectors and their moments. The function $\boldsymbol{G}(\boldsymbol{\delta})$ maps $\mathbb{R}^n$ into $\mathbb{R}^m$ where $n \geq m$. Let $\boldsymbol{d}_{des}$ denote the controlled variables. In this case the controlled variables are the rolling, pitching and yawing moments. Buffington's multi-branch approach requires that two optimization problems be solved. The first optimization problem is called the control deficiency branch. The objective of the control deficiency branch is to minimize $||\boldsymbol{w}_a^T(\boldsymbol{G}(\boldsymbol{\delta}) - \boldsymbol{d}_{des})||_1$ subject to position constraints on $\boldsymbol{\delta}$. The value of the performance index for this optimization problem indicates whether or not $\boldsymbol{d}_{des}$ is feasible. If feasibility of the control allocation problem has been ascertained, a second optimization problem is then solved that minimizes some secondary objective. The objective function of this second optimization is typically taken to be $||\boldsymbol{w}_u^T(\boldsymbol{\delta} - \boldsymbol{\delta}_p)||_1$ subject to $\boldsymbol{G}(\boldsymbol{\delta}) = \boldsymbol{d}_{des}$ and position constraints on $\boldsymbol{\delta}$. This is commonly referred to as the control sufficiency branch.

The mixed optimization problem that was formulated by Bodson[6] combines the two branches of the multi-branch control allocation problem into a single optimization problem. A new parameter ϵ is introduced for the purpose of prioritizing either control deficiency or control sufficiency. The mixed optimization problem is stated as:

$$\min J = ||\boldsymbol{w}_a^T(\boldsymbol{G}(\boldsymbol{\delta}) - \boldsymbol{d}_{des})||_1 + \epsilon||\boldsymbol{w}_u^T(\boldsymbol{\delta} - \boldsymbol{\delta}_p)||_1 \quad (1)$$

subject to:

$$\boldsymbol{\delta}_{min} \leq \boldsymbol{\delta} \leq \boldsymbol{\delta}_{max} \quad (2)$$

where $\boldsymbol{w}_a$ is an $m \times 1$ vector of weights to prioritize a given control axis, $\boldsymbol{G}(\boldsymbol{\delta})$ is a non-linear, vector-valued function that maps $\mathbb{R}^n \to \mathbb{R}^m$, $\boldsymbol{\delta}$ is an $n \times 1$ vector of control effectors, $\boldsymbol{w}_u$ is an $n \times 1$ vector of weights on the control effectors, and $\boldsymbol{\delta}_p$ is an $n \times 1$ vector of "preferred" control effector displacements. Again it is assumed that $n \geq m$. For the time being we will make the assumption that $\boldsymbol{G}(\boldsymbol{\delta}) = \boldsymbol{B}\boldsymbol{\delta}$. The mixed optimization problem can then be posed as a linear programming problem. The corresponding linear program can then be solved by any readily-available linear programming software.

Bodson[6] gives one possible transformation to a linear programming problem for the optimization problem defined in Equations 1 and 2; however, we selected a transformation approach that follows that found in Bertsimas.[7] The transformation relies on the observation that $|x|$ is the smallest number x_s that satisfies both $x \leq x_s$ and $-x \leq x_s$. As a result, we are able to pose the mixed optimization problem as follows:

$$\min J = \boldsymbol{w}_a^T \boldsymbol{\delta}_s + \epsilon \boldsymbol{w}_u^T \boldsymbol{u}_s \tag{3}$$

subject to:

$$\boldsymbol{\delta}_s \geq 0 \tag{4}$$

$$\boldsymbol{u}_s \geq 0 \tag{5}$$

$$\boldsymbol{B}\boldsymbol{\delta} + \boldsymbol{\delta}_s \geq \boldsymbol{d}_{des} \tag{6}$$

$$-\boldsymbol{B}\boldsymbol{\delta} + \boldsymbol{\delta}_s \geq -\boldsymbol{d}_{des} \tag{7}$$

$$\boldsymbol{\delta} + \boldsymbol{u}_s \geq \boldsymbol{\delta}_p \tag{8}$$

$$-\boldsymbol{\delta} + \boldsymbol{u}_s \geq -\boldsymbol{\delta}_p \tag{9}$$

$$\boldsymbol{\delta}_{min} \leq \boldsymbol{\delta} \leq \boldsymbol{\delta}_{max} \tag{10}$$

The vectors $\boldsymbol{\delta}_s$ and $\boldsymbol{u}_s$ are vectors of "slack variables". The reason for selecting this particular transformation as opposed to the one in Bodson[6] is that this formulation allows us to easily implement the piecewise linear function approximation.

To convert the linear programming problem into a piecewise linear programming problem, we simply replace $\boldsymbol{B\delta}$ above with a piecewise linear representation to each control moment curve, (i.e., $L_i(\delta_i)$, $M_i(\delta_i)$, and $N_i(\delta_i)$). We choose a set of breakpoints such that for each δ_i, $i = 1, \ldots, n$:

$$\delta_i = \sum_{k=1}^{K_i} \lambda_i^{(k)} \delta_i^{(k)} \tag{11}$$

$$\sum_{k=1}^{K_i} \lambda_i^{(k)} = 1 \tag{12}$$

$$\lambda_i^{(j)} \lambda_i^{(k)} = 0, \quad \text{if } k > j+1,\, j = 1, \ldots, K_i - 1 \tag{13}$$

where $\lambda_i^{(k)}$ is a non-negative interpolating coefficient corresponding to the i^{th} control effector at breakpoint k, and K_i denotes the number of breakpoints for the i^{th} control effector. Equation 13 is necessary in order ensure that δ_i is approximated by no more than two *adjacent* values of $\lambda_i^{(k)}$. If δ_i falls at a breakpoint, there will only be one value of $\lambda_i^{(k)}$ that is non-zero and Equation 13 is still valid.

The piecewise linear approximations for the control moments as a function of $\delta_i^{(k)}$ are written as

$$L_i \approx \sum_{k=1}^{K_i} \lambda_i^{(k)} L_i^{(k)} \tag{14}$$

$$M_i \approx \sum_{k=1}^{K_i} \lambda_i^{(k)} M_i^{(k)} \tag{15}$$

$$N_i \approx \sum_{k=1}^{K_i} \lambda_i^{(k)} N_i^{(k)} \tag{16}$$

where $L_i^{(k)}$, $M_i^{(k)}$, and $N_i^{(k)}$ are the values of the rolling, pitching, and yawing moment curves evaluated at the k^{th} breakpoint for the i^{th} control effector. We are now able to re-write the $\boldsymbol{B}$ matrix as

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} L_1^{(1)} & L_1^{(2)} & \ldots & L_i^{(k)} & \ldots & L_m^{(K_m)} \\ M_1^{(1)} & M_1^{(2)} & \ldots & M_i^{(k)} & \ldots & M_m^{(K_m)} \\ N_1^{(1)} & N_1^{(2)} & \ldots & N_i^{(k)} & \ldots & N_m^{(K_m)} \end{bmatrix} \tag{17}$$

Furthermore, we define a vector $\boldsymbol{\Lambda}$ as

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1^{(1)} \\ \lambda_1^{(2)} \\ \vdots \\ \lambda_i^{(k)} \\ \vdots \\ \lambda_n^{(K_n)} \end{bmatrix} \tag{18}$$

such that $\boldsymbol{B\delta}$ in Equations 6 and 7 is replaced by $\tilde{\boldsymbol{B}}\boldsymbol{\Lambda}$. The vector $\boldsymbol{\Lambda}$ is of length $\sum_{i=1}^{n} K_i$ and $\tilde{\boldsymbol{B}}$ is a matrix of size $n_{c_r} \times \sum_{i=1}^{n} K_i$ where n_{c_r} is the number of controlled variables. In the piecewise linear optimization problem, the constraints $\boldsymbol{\delta}_{min} \leq \boldsymbol{\delta} \leq \boldsymbol{\delta}_{max}$ are replaced by $\lambda_i^{(k)} \geq 0$. The upper and lower bounds on $\boldsymbol{\delta}$ are accounted for in the selection of the breakpoints for each δ_i. Once we obtain an optimal solution to the problem, we compute each δ_i using Equation 11. It is also necessary to include in the problem the n constraints that correspond to Equation 12.

The resulting optimization problem is

$$\min J = \boldsymbol{w}_a^T \boldsymbol{\delta}_s + \epsilon \boldsymbol{w}_u^T \boldsymbol{u}_s \tag{19}$$

subject to:

$$\boldsymbol{\delta}_s \geq 0 \tag{20}$$

$$\boldsymbol{u}_s \geq 0 \tag{21}$$

$$\tilde{\boldsymbol{B}}\boldsymbol{\Lambda} + \boldsymbol{\delta}_s \geq \boldsymbol{d}_{des} \tag{22}$$

$$-\tilde{\boldsymbol{B}}\boldsymbol{\Lambda} + \boldsymbol{\delta}_s \geq -\boldsymbol{d}_{des} \tag{23}$$

$$\sum_{i=1}^{K_i} \lambda_i^{(k)} \delta_i^{(k)} + u_{s,i} \geq \delta_{p,i} \tag{24}$$

$$-\sum_{i=1}^{K_i} \lambda_i^{(k)} \delta_i^{(k)} + u_{s,i} \geq -\delta_{p,i} \tag{25}$$

$$\lambda_i^{(k)} \geq 0 \tag{26}$$

$$\sum_{i=1}^{K_i} \lambda_i^{(k)} = 1 \tag{27}$$

$$\lambda_i^{(j)} \lambda_i^{(k)} = 0, \quad \text{if } k > j+1,\ j = 1, \ldots, K_i - 1 \tag{28}$$

The constraint given by Equation 28 forces us to choose one of two approaches in order to obtain a solution to the above optimization problem. The first approach is to define a set of binary variables along with a set of additional constraints that enforce Equation 28. The resulting optimization problem is then a mixed-integer linear program. The second approach is to solve the optimization problem as a linear programming problem using a modified simplex algorithm. In order to accommodate Equation 28, "restricted basis entry rules" are used to determine which $\lambda_i^{(k)}$ are allowed to enter the basis.

Mixed Integer Linear Program Formulation

Begin by considering the piecewise linear approximation shown in Figure 1. Note that if there are K breakpoints, then there are $K-1$ linear segments. We assign a variable $y^{(k)}$

that corresponds to the k^{th} linear segment of the piecewise linear approximation such that

$$y^{(k)} = \begin{cases} 1 & \text{if } \lambda^{(k)} \neq 0 \text{ and } \lambda^{(k+1)} \neq 0, \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

for $k = 1, \ldots, K-1$. Next, we make the observation that if $\lambda^{(1)} \neq 0$ and $\lambda^{(2)} \neq 0$, then

$$\lambda^{(1)} \leq y^{(1)} \tag{30}$$

$$\lambda^{(2)} \leq y^{(1)} \tag{31}$$

where $y^{(1)} = 1$. However, if we are on the segment where $\lambda^{(2)} \neq 0$ and $\lambda^{(3)} \neq 0$, such that $y^{(2)} = 1$, then

$$\lambda^{(2)} \leq y^{(2)} \tag{32}$$

$$\lambda^{(3)} \leq y^{(2)}. \tag{33}$$

If we proceed in this manner, we observe that the following restrictions can be placed on the $\lambda^{(k)}$

$$\lambda^{(1)} \leq y^{(1)}, \tag{34}$$

$$\lambda^{(k)} \leq y^{(k-1)} + y^{(k)}, \; k = 2, \ldots, K-1 \tag{35}$$

$$\lambda^{(K)} \leq y^{(K-1)}. \tag{36}$$

The rationale behind Equation 35 is as follows: the $\lambda^{(k)}$ that correspond to points that are interior to the interval (i.e., they are not the endpoints of the interval on which x is defined) can be associated with one of two line segments. A particular $\lambda^{(k)}$ is the endpoint for the line segment immediately preceding it in addition to the line segment that comes immediately after it. Only one of these two line segments may be "active" at any time; therefore, the right-hand side of Equation 35 is never greater than one. In addition to Equations 34-36, we have an additional constraint to ensure that only one of the $K-1$ line segments is active, hence only one of the $y^{(k)}$ can be equal to one:

$$\sum_{k=1}^{K-1} y^{(k)} = 1 \tag{37}$$

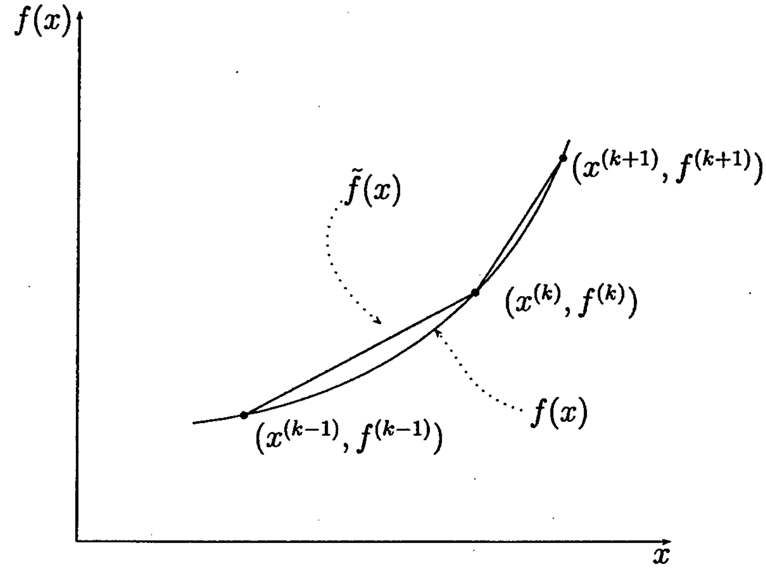

Figure 1: Piecewise Linear Function Approximation

The above can be generalized for the multi-variable case as follows.

$$\lambda_i^{(1)} \leq y_i^{(1)}, \quad i = 1, \ldots, m \tag{38}$$

$$\lambda_i^{(k)} \leq y_i^{(k-1)} + y_i^{(k)}, \quad i = 1, \ldots, m, \quad k = 2, \ldots, K_i - 1 \tag{39}$$

$$\lambda_i^{K_i} \leq y_i^{(K_i-1)}, \quad i = 1, \ldots, m \tag{40}$$

$$\sum_{k=1}^{K_i-1} y_i^{(K_i-1)} = 1, \quad i = 1, \ldots, m \tag{41}$$

$$y_i^{(k)} \in \{0, 1\} \tag{42}$$

Equations 38-42 replace Equation 28 above. As a result, we have a mixed-integer linear programming problem.

Linear Program with Restricted Basis Entry Rules

1

An alternate method of handling Equation 28 is to solve the piecewise linear program given in Equations 19-28 minus Equation 28. Without Equation 28, we are left with a linear programming problem that can be easily solved using the simplex method. However, we need an alternate approach of enforcing the adjacency constraint. Instead of defining Equation 28 explicitly and including it in the problem formulation, it is to be enforced through a set of rules in the simplex algorithm that only admit a $\lambda_i^{(k)}$ if Equation 28 is satisfied. The rules are commonly referred to as "restricted basis entry rules". More details on the simplex algorithm with restricted basis entry rules can be found in Miller[8] and Hadley.[9]

A survey of commercial linear programming solvers indicated that simplex based solvers with restricted basis entry rules were unavailable. The GNU Linear Programming Kit (GLPK), which is an open-source general linear program and mixed-integer linear program solver written in C, was modified to accommodate restricted basis entry rules.

Results

The results presented in this section are given for an unpowered re-entry vehicle. This particular vehicle utilizes the following six control effectors: left and right ruddervators, left and right flaperons, a speed brake, and a body flap. We show that the mixed-optimization problem implemented with piecewise linear approximations of the control moments and solved using a simplex method with restricted basis entry rules gives a solution that is comparable to that given in Bolender and Doman.[4] We also demonstrate that the mixed-optimization problem can be solved fast enough that it is a candidate for future use on a digital flight control computer equipped with a processor that is comparable to one found on current desktop computers. For this study, the GLPK solver was implemented as a C mex file and compiled to minimize execution speed. The computer on which this analysis was run was equipped with an Athlon 1800XP+ processor, 1.5 GB of RAM, and the Windows 2000 operating system. We compare the execution time of the simplex method with restricted basis entry rules to the time required to solve the same optimization problem using the mixed-integer linear programming formulation of the mixed-optimization problem. The mixed-integer linear programs are solved using the branch-and-bound solver supplied by the GLPK package.

Simulation Results

The results that follow give the closed-loop vehicle performance when there are two failures injected into the flight control system at different times during the approach and landing phases. It is assumed that there is some type of fault detection capability on-board the aircraft to identify the failures. The failure information is immediately passed to the control allocation algorithm in order to facilitate re-configuration of the vehicle's effectors. The aircraft's trajectory begins at an altitude of about 15,000 ft above the runway and 4 miles downrange from the runway threshold. The first failure occurs 30 s into the simulation, and involves the body flap being locked at -5 deg. This failure contributes a constant pitching moment to the aircraft. A second failure, where the right rudder becomes locked at 1 deg, occurs at 40 s. This particular failure adds not only a pitching moment to the aircraft, but also rolling and yawing moments. This particular failure combination was chosen because it requires the flaperons to operate in a highly non-linear region of the control moment curve. After the failures are introduced, the aircraft tries to follow the nominal approach trajectory to the runway threshold. The aircraft extends the landing gear at about 68 s and flares immediately before touchdown. The simulation ends at touchdown when the weight-on-wheels switch is triggered.

Presented below in Figures 2 and 3 are the time histories for the control moment error and the control effector command time histories. In Figure 2 the moment error is defined by

$\log_{10} \|\tilde{\boldsymbol{B}}\Lambda - \boldsymbol{d}_{des}\|_2$ where $\tilde{\boldsymbol{B}}$ and Λ are defined above. Recall that $\tilde{\boldsymbol{B}}$ is the piecewise linear analog to the $\boldsymbol{B}$ matrix that one encounters when using a linear approximation; therefore, the product $\tilde{\boldsymbol{B}}\Lambda$ is the linear interpolation of the control moment data. We see that the modelling errors are negligible until the rudder failure is introduced at the 40s mark. Between 40s and 60s there is a control deficiency; therefore, the desired moment, $\boldsymbol{d}_{des}$ is not feasible under the failure conditions. The performance of the mixed-optimization is identical to that given in Reference [4].

The control surface time histories given in Figure 3 are the control deflections returned by the control allocator. The control surface deflections compare favorably. There are very minor differences in the right flap deflection after the rudder failure is introduced, but this discrepancy does not appear to be an issue.

Performance Results

The result given in Table 1 is for a fixed flight condition, and considers 10000 different control input vectors with each component selected randomly from a uniform distribution. For each input vector, the control allocation problem was solved using both a mixed-integer linear program and simplex with restricted basis entry rules. The preference vector was taken to be $\boldsymbol{\delta}_p = \mathbf{0}$. The execution time of both approaches was measured using the standard C library function `clock()`. It is apparent that the simplex method, on average, is an order of magnitude faster than the MILP approach. If it is assumed that the typical sample time of a digital flight control system is 0.02 sec, then the solution time given in Table 1 for the simplex method with restricted basis entry is more than adequate for practical application given a processor that is in the same class as that tested above. The mean control deflection for each method is given in Table 2. Note that for this particular set of random $\boldsymbol{d}_{des}$, the

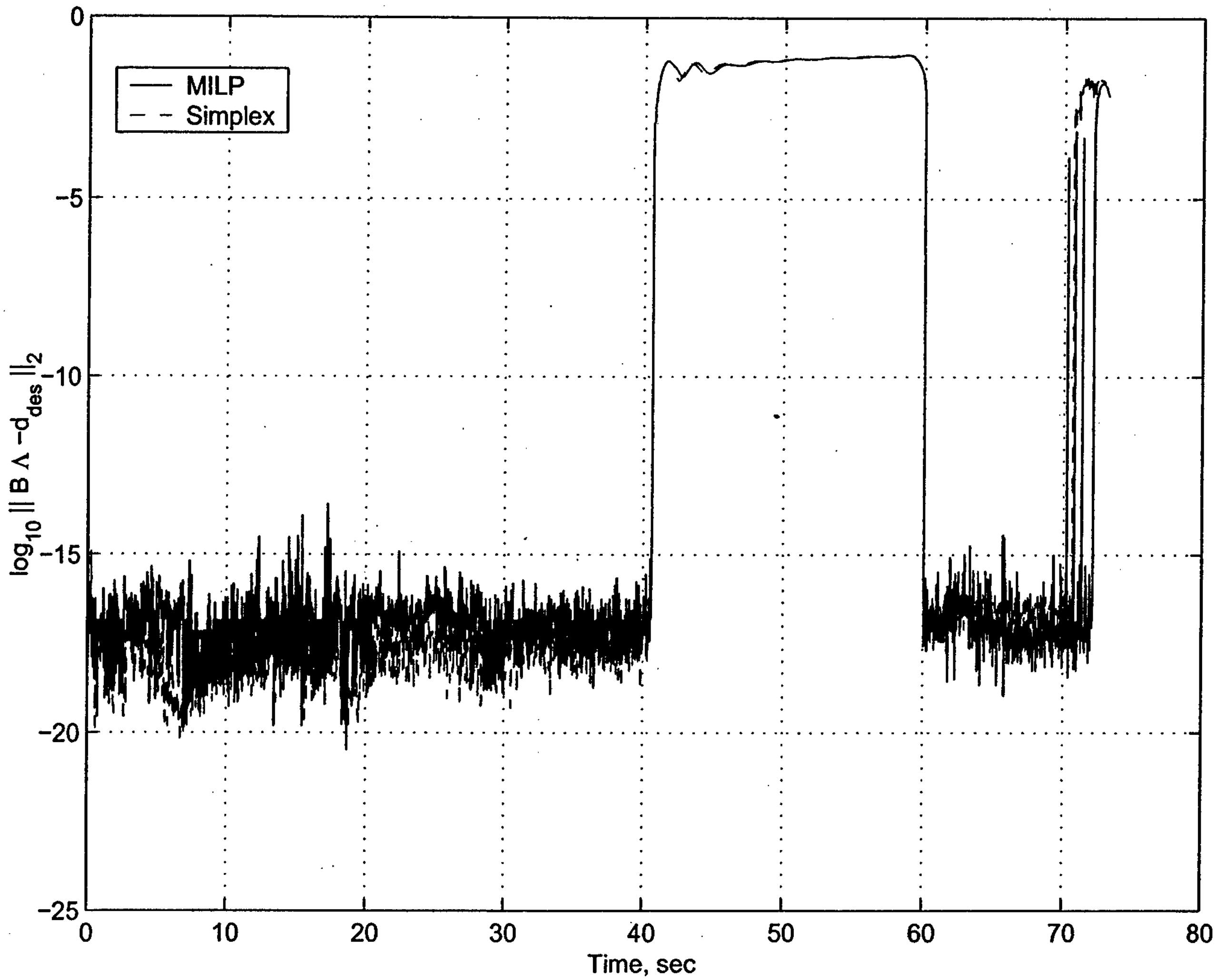

Figure 2: Difference Between Commanded and Applied Moments

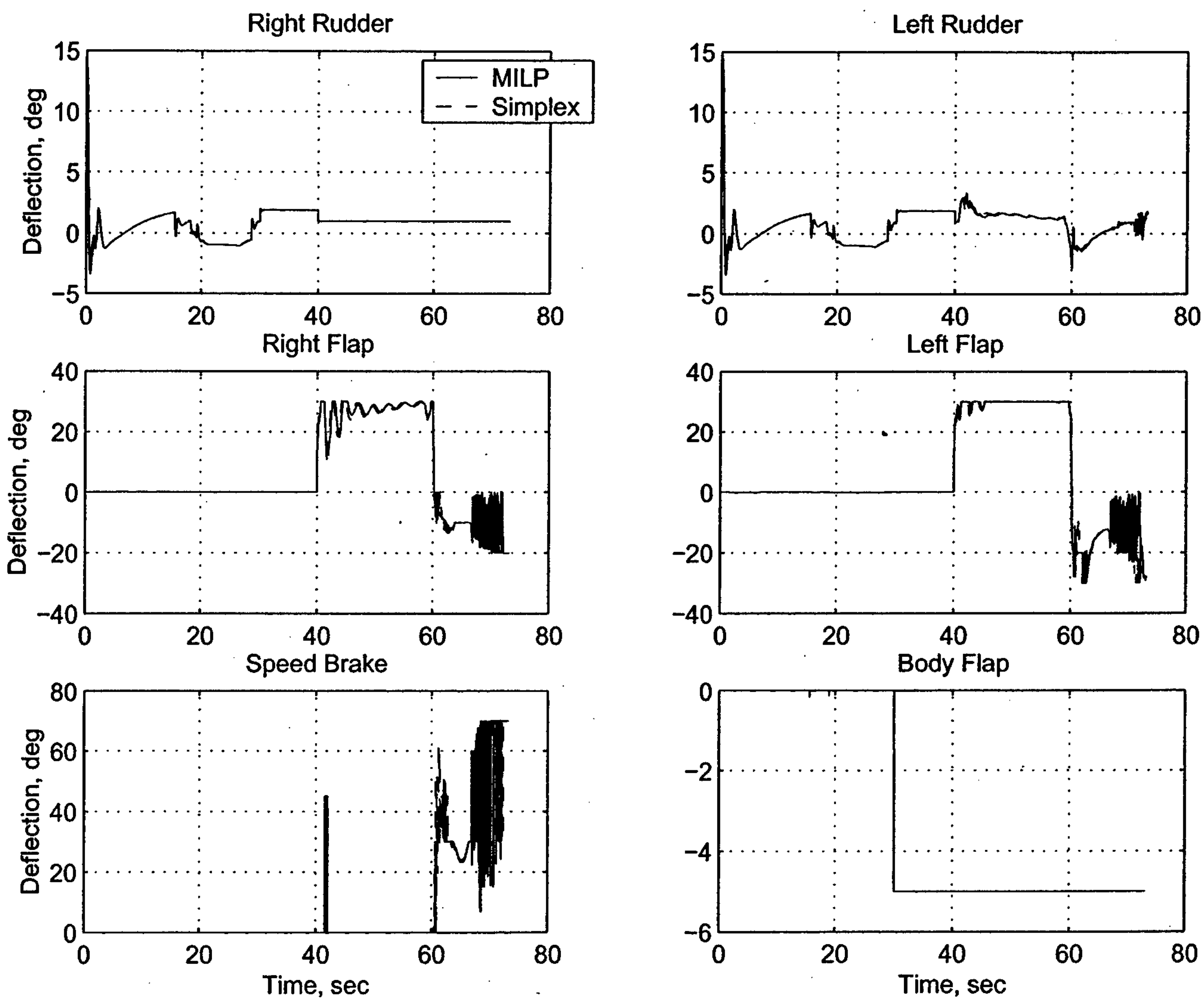

Figure 3: Commanded Control Surface Deflections

	Mean	Std. Dev.
MILP	0.0885	0.0423
RBE	0.0083	0.0042

Table 1: Solver Execution Time Statistics: Athlon XP1800+/Windows 2000

	Mean	Std. Dev.
MILP	18.2365	8.6293
RBE	18.2657	8.5736

Table 2: Mean and Standard Deviation of $\|\delta\|_2$.

mean deflections are nearly the same for both approaches. On the other hand, Table 3 shows a significant difference in the average moment error. There were a small number of $\boldsymbol{d_{des}}$ for which the Simplex Method with Restricted Basis Entry Rules failed to find a set of control deflections that would produce a feasible moment while the MILP formulation succeeded. The average error for these vectors is 0.4938 and is the cause for the poor correlation between the MILP solution and the Simplex with Restricted Basis Entry Rules. For each of these vectors, the control deficiency occurs in the yawing moment. Although it occurs in less than 0.5% of the cases that were tested, the cause of this behavior is currently under investigation.

Conclusions

The approach to non-linear control allocation that was presented assumed that control moments generated by the deflection of aerodynamic surfaces were separable functions. This assumption allows us to approximate any separable non-linear function by a piecewise

	Mean	Std. Dev.
MILP	4.4547×10^{-17}	1.0709×10^{-16}
RBE	1.4139×10^{-5}	2.6927×10^{-4}

Table 3: Mean and Standard Deviation of $\|\tilde{\boldsymbol{B}}\boldsymbol{\Lambda} - \boldsymbol{d_{des}}\|_2$.

linear function in the control allocation problem. As a result, we were able to cast a nonlinear optimization problem as a linear programming problem where a subset of the decision variables are subject to a set of restricted basis entry rules. Although this still gives an approximate solution to the control allocation problem, it is much more accurate than the traditional methods that assume linear relationships between the control moments and the control effector positions . It was subsequently shown that a simplex algorithm that enforces the restricted basis entry rules is probably fast enough, given the appropriate processor clock speed, for use in a real time flight control system.

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