AFRL-VA-WP-TP-2004-309

MULTI-VEHICLE COOPERATIVE SEARCH WITH UNCERTAIN PRIOR INFORMATION

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FEBRUARY 2004

20040324 065

AIR VEHICLES DIRECTORATE
AIR FORCE RESEARCH LABORATORY
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7542

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
February 2004	Conference Paper Preprint	

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
MULTI-VEHICLE COOPERATIVE SEARCH WITH UNCERTAIN PRIOR INFORMATION	F33615-01-C-3154
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 62201F

6. AUTHOR(S)	5d. PROJECT NUMBER
Chunlei Zhang and Raúl Ordóñez (University of Dayton) Corey Schumacher (AFRL/VACA)	2403
	5e. TASK NUMBER 06
	5f. WORK UNIT NUMBER 67

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Dayton Department of Electrical and Computer Engineering Dayton, OH 45469-0226	Control Theory Optimization Branch (AFRL/VACA) Control Sciences Division Air Vehicles Directorate Air Force Research Laboratory, Air Force Materiel Command Wright-Patterson AFB, OH 45433-7542	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY ACRONYM(S)
Air Vehicles Directorate Air Force Research Laboratory Air Force Materiel Command Wright-Patterson Air Force Base, OH 45433-7542	AFRL/VACA
	11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S) AFRL-VA-WP-TP-2004-309

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
Conference paper to be presented at the American Control Conference, Boston, MA, June 30-July 2, 2004. This work has been submitted for publication in the Proceedings of the 2004 American Control Conference. One of the authors is a U.S. Government employee; therefore, the U.S. Government is joint owner of the work. If published, the American Automatic Control Council (AACC) may assert copyright. If so, the United States has for itself and others acting on its behalf an unlimited, nonexclusive, irrevocable, paid-up, royalty-free worldwide license to use for its purposes.

14. ABSTRACT

We present a possible solution for the multi-vehicle cooperative search problem via the surrogate optimization method. We establish a discrete mathematical model for the multi-vehicle cooperative search stationary target with uncertain prior information. We reformulate the Past, Present and Predicted Future (PPP) algorithm we introduced in previous work based on the new model and propose an implementation to adapt to the new scenario. Monte Carlo simulations indicate that the new implementation achieves better performance. Moreover, the scalability of the PPP algorithm is addressed.

15. SUBJECT TERMS
Surrogate optimization, cooperative search, autonomous control

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified	SAR	18	Corey Schumacher 19b. TELEPHONE NUMBER (Include Area Code) (937) 255-8494

Multi-vehicle Cooperative Search with Uncertain Prior Information

Chunlei Zhang *   Raúl Ordóñez *   Corey Schumacher †

Abstract

We present a possible solution for the multi-vehicle cooperative search problem via the surrogate optimization method. We establish a discrete mathematical model for the multi-vehicle cooperative search stationary target with uncertain prior information. We reformulate the Past, Present and Predicted Future (PPP) algorithm we introduced in previous work based on the new model and propose an implementation to adapt to the new scenario. Monte Carlo simulations indicate that the new implementation achieves better performance. Moreover, the scalability of the PPP algorithm is addressed.

1   Introduction

The idea of multiple uninhabited autonomous vehicles (UAVs) able to adaptively react to their environment and learn about their surroundings while following either an individual or a communal agenda is an intriguing issue. Achieving such a degree of control and producing such sophisticated behavior remains an elusive goal that presents considerable challenges due to the inherent complexity of the task, and also because it may be approached from a variety of different angles. The problem of multi-vehicle coordination and control has been receiving an extraordinary amount of attention during the past few years due to its critical importance for a myriad of applications.

Existing work on multi-vehicle control focuses on receding-horizon planning (that is, optimization methods) and hierarchical structures. The research reported in this paper benefits from previous work that follows the first approach. A receding horizon trajectory planner based on Mixed Integer-Linear Programming (MILP) that is capable of planning planar trajectories to a goal constrained by no-fly areas, or obstacles, and aircraft dynamics were proposed in [1, 2, 3]. A generalized multi-vehicle formation stabilization problem, free from a leader-follower architecture is defined in [4] and model predictive control (MPC) is applied.

Game theory based cooperative decision making for multi-vehicle include [5, 6] and [7]. Moreover, graph theory is also employed extensively in multi-vehicle coordination. A disjoint path algorithm for reconfiguration of multi-vehicle was proposed in [8]. A class of triangulated graphs for algebraic representation of formations are introduced to specify a mission cost for a group of vehicles [9], then the obtained optimal control problem is solved using NTG (an optimal control program developed at Caltech). A double-graph model is used in [10] to treat the string stability as a kind of performance of multi-vehicle system with acyclic formation structures. Moreover, a theoretical explanation of using nearest neighbor rules in coordinating groups of mobile autonomous agents can be found in [11].

Related to the work on coordination of multi-vehicle are swarms and formation control. In [12, 13, 14, 15], the authors considered a swarm which moves in an attractant/repellent profile (i.e., a profile of nutrients

---

or toxic substances) and showed collective convergence to (divergence from) more favorable (unfavorable) regions of the profile. The inter-individual interactions and the interactions with the environment in the model in these articles used *artificial potential functions*, a concept that has been used extensively for robot navigation and control [16, 17]. The results in [18, 19, 20, 21, 22] are based on using virtual leaders and artificial potentials for vehicle interactions in a group to maintain the group geometry. They use the system kinetic energy and the artificial potential energy as a Lyapunov function to prove closed loop stability and employ a dissipative term to achieve asymptotic stability of the formation. Other results using artificial potential functions include [23], where the authors consider a distributed control approach for groups of robots, called the *social potential fields* method, which is based on artificial force laws between individual robots and robot groups. The force laws are inverse-power or spring force laws incorporating both attraction and repulsion. In [24], the dynamics of the vehicle group is decomposed into average system and shape system. The internal formation control is achieved by controlling the shape system, while the maneuvering control is achieved by the controlling of the average system. Furthermore, the role of information flow in formation stability was studied in [25], similar work can be found in [26]. A more recent overview of formation control can be found in [27].

This paper is based on previous research [28] and organized as follows. The next section begins with the formulation of our research problem, a discrete mathematical model of multi-vehicle cooperative search for stationary targets with uncertain prior information is introduced, and surrogate optimization [29, 30] is applied to the cooperative search problem. We reformulate the PPP algorithm introduced in [28] in Section 3. A specific implementation of the PPP algorithm to fully use the prior information is proposed in Section 4. There, Monte Carlo simulations are used to compare these proposed schemes with exhaustive search and the scalability of PPP algorithm is discussed. Finally, Section 5 concludes the paper and discusses some future works.

2   Problem Formulation

2.1   Model

For convenience, we concentrate on discrete time models within a two-dimensional plane, since the setup fits nicely within the simulation environment. We assume whatever constraints exist for vehicles to be not dynamic, but rather kinematic in favor of focusing on high-level mechanisms originating from the group of vehicles. We will assume there are m vehicles and that the ith one obeys a discrete time kinematic model given by

$$
\begin{aligned}
x_{v_1}^i(k+1) &= x_{v_1}^i(k) + d\cos(\theta_v^i(k)) \\
x_{v_2}^i(k+1) &= x_{v_2}^i(k) + d\sin(\theta_v^i(k)) \\
\theta_v^i(k+1) &= \theta_v^i(k) + f_{\theta_v}(u^i(k))
\end{aligned}
\tag{1}
$$

where k is the discrete time index taking values in the nonnegative integers $\{0, 1, 2, \ldots\}$ (k also denotes the number of search steps); $x_{v_1}^i$ and $x_{v_2}^i$ are, respectively, the two Cartesian coordinates of ith vehicle; d is a constant step size; θ_v^i is the orientation of the ith vehicle; f_{θ_v} can be a nonlinear function encoding kinematic restrictions on the vehicles; and u^i is the local controller corresponding to the ith vehicle. For convenience, let $x_{v_p}^i = [x_{v_1}^i, x_{v_2}^i]^\top$, and $x_v^i = [(x_{v_p}^i)^\top, \theta_v^i]^\top$.

In some applications of cooperative search, we may have access to prior information (e.g., where the targets are) but with uncertainty. The environment is modeled as a two-dimensional plane, the upper right quadrant of a Cartesian coordinate system with axes (x_1, x_2). We may set up a Gaussian profile map which is known to all vehicles. The Gaussian profile encodes the *possible* target locations $x_s^i = [x_{s_1}^i, x_{s_2}^i]^\top, i = 1, \ldots, n$ offered by the prior information as centers of the Gaussian peaks, where we assume to know the number n of targets,

$$
M_p(x_1, x_2, k = 0) = \sum_{i=1}^{n} c_i \exp\left[-\frac{(x_1 - x_{s_1}^i)^2 + (x_2 - x_{s_2}^i)^2}{v_i^2}\right]
\tag{2}
$$

2

A possible picture is in Figure 1 (a), where x_s^i are $[6,46]^\top, [50,6]^\top$ and $[40,40]^\top$. We encode the uncertainty of the prior information with the peak width v_i and the distance of the real (but unknown) target position to the center of the peak in terms of v_i. For example, we can say the real target is $0.5v_i, v_i, 2v_i$, etc., distance units from the center, that is, we consider the worst case scenario for each uncertainty level (e.g., given a fixed peak width $v_i = 4$ as in Figure 1 (a), the blue star represents the real target, which is at one v_i uncertainty from the center). Furthermore, we can intentionally encode the priority level of each target as the height c_i of the peak (e.g., we assign $c_i = 3$ to the peak center located at (40,40) as the most important target). Thereby, we expect the vehicles to find the most important target first, and the whole search performance should coincide with the uncertainty level about the prior information. Overall, in this kind of scenario all the vehicles share $M_p(x_1, x_2)$, meanwhile the vehicle sensor will sample a real target map $M_t(x_1, x_2)$ (Figure 1 (b)),

$$M_t(x_1, x_2) = \begin{cases} 1, & [x_1, x_2]^\top \in \{[x_{t_1}^i, x_{t_2}^i]^\top\} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and only obtain two kinds of information: 0, which means no target, and 1, means the vehicle found a target.
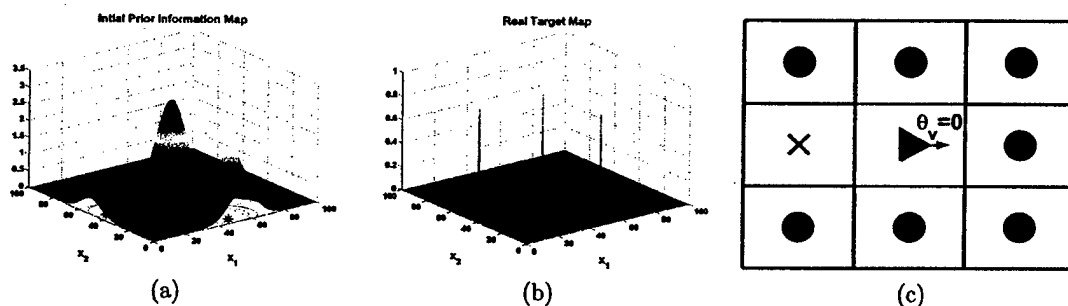

Figure 1: (a) Prior information map, (b) Real target map (c) Cell based map assumptions.

We discretize not only the vehicles' movement, but also the proposed map as shown in Figure 1 (a). We will let the vehicles move from cell to cell (i.e., the center of one cell to another) rather than move along a smooth curve, and therefore we will start with the assumption that all vehicles move at constant speed and in discrete time index. Also we assume that maximum turn angles are ±135 degrees, i.e., f_{θ_v} becomes a saturation function. Figure 1 (c) illustrates these points: the triangle in the middle cell represents the vehicle's current position with its orientation $\theta_v = 0$; with the assumptions of constant speed and discrete time index, the vehicle will only move one cell each time index; and with the assumption of maximum angles of turn, the vehicle will only have seven possible cells to go, which are denoted by the circles, and the cross denotes the cell which vehicle cannot visit since $\theta_v = 0$. Moreover, we ignore the difference between diagonal step size with vertical or horizontal step size, all denoted as d for convenience. The inter-vehicle communications are instantaneous, noiseless and have unbounded communication distance, i.e., we assume perfect communication. These assumptions together with further ones made for the Monte Carlo simulation will be used throughout this paper. Note that this is not a centralized coordination scheme, since if non-ideal communication were considered, each vehicle would have its own verison of M_p^i and would act upon it, sharing as much information as allowed by the communication channel. The level of coordination increases as the quality of the communication channel improves.

A vehicle located somewhere in the terrain is clearly unable to observe it all at once in most practical situations. In general, then, the vehicle is only able to sample a point of the discrete map (3) at its own location, to the extent that its own sensors or some external information source allow it. Therefore, we may define the system output vector as

$$y = [y^1, \dots, y^m]^\top = [M_t(x_{v_1}^1, x_{v_2}^1), \dots, M_t(x_{v_1}^m, x_{v_2}^m)]^\top \tag{4}$$

where y_i is the output corresponding to the ith vehicle sampling the proposed map function (3). Each vehicle can update the prior information map (2) with its collected sensor information at time index k,

$$M_p(x_1, x_2, k+1) = M_p(x_1, x_2, k) \cdot M_s(x_1, x_2, k+1) \tag{5}$$

3

where · means element-wise multiplication for matrices and $M_s(x_1, x_2, k+1)$ is called sensor matrix, which encodes the current time's sensor information (each time, M_s is reset to a matrix whose elements are 1 and then the places corresponding to the current vehicle locations are filled with the sensor data "0" or "1"). As available a priori, M_p could be used by the decentralized controller $u(\cdot)$ to implement output feedback regulation on system (1), in which the search and engagement tasks may be phrased as the regulation objective of "flattening out" the map in Figure 1 (b) (in other words, each time a target is destroyed, the corresponding peak would vanish). A general expression of control law under perfect communication is

$$u^i(k) = f_u^i(M_p(x_1, x_2, k), x_v(k), y(k)) \tag{6}$$

where all vehicles have the same prior information map $M_p(x_1, x_2, k)$, and the vehicle controller is able to use not only its own measurements and position, but also those from all other vehicles.

2.2 Cooperative Search using Surrogate Optimization

Here, we attempt to apply surrogate optimization method in multi-vehicle cooperative search for stationary targets [28, 29, 30]. In particular, the objective function will represent the unknown real target map M_t in equation (3). Vehicles actuate on the system by finding and destroying targets, thereby "flattening" M_t. As each vehicle moves about and collects data, it is able to refine M_p in Equation (2), which will play the role of the surrogate function for the vehicles. The vehicle, however, is unable to instantaneously jump between locations, and has instead to move along a trajectory towards the specified point. Along the way it continues gathering more data, thereby potentially improving M_p. Moreover, using the idea of merit function (detailed in next section), we further refine the general control law (6) to

$$u^i(k) = f_u^i(\text{merit}^i(x_1, x_2, k), x_v(k), y(k)) \tag{7}$$

where we wish to achieve coordinated behavior via a series of well constructed merit functions.

In this manner, the surrogate optimization method is modified to effectively become a control law: when a maximum's location is predicted, the controller computes the *direction* in which it needs to move from its current location in order to reach the desired point. Mathematically, the control law (7) can be expressed as

$$u^i(k) = \left[\arctan\left(\frac{x_{v_2}^i - x_{o_2}^i}{x_{v_1}^i - x_{o_1}^i}\right) \right]_q - \theta_v^i(k) \tag{8}$$

where $(x_{o_1}^i, x_{o_2}^i)$ is a maximum (predicted target) of the merit function $\text{merit}^i(x_1, x_2, k)$, i.e.,

$$\text{merit}^i(x_{o_1}^i, x_{o_2}^i, k) = \max_{(x_1, x_2)} \text{merit}^i(x_1, x_2, k) \tag{9}$$

The operator $\left[\cdot \right]_q$ is a quantization operation, which quantizes the angle $\arctan\left(\frac{x_{v_2}^i - x_{o_2}^i}{x_{v_1}^i - x_{o_1}^i}\right)$ into one of the elements from the set $\{0, \pm 45, \pm 90, \pm 135, \pm 180\}$. Finally, f_{θ_v} will be a saturation function taking care of the maximum turn angles restriction,

$$f_{\theta_v}(u^i(k)) = \begin{cases} u^i(k), & -135° \leq u^i(k) \leq 135° \\ \text{sgn}(u^i(k))135°, & \text{otherwise} \end{cases}$$

3   PPP Behavior Based Coordination

In order to realize a meaningful level of coordination and given that we have m vehicles, where $m \geq 2$, we utilize the idea of surrogate function and distance function to form certain behaviors for each vehicle. And therefore we name this coordination scheme as behavior based coordination.

If the ith vehicle uses surrogate map M_p to predict a target position, we say that the ith vehicle is in *converge* behavior (abbreviated as "C"). Moreover, each vehicle will have a predefined serpentine search path to promise high detection probability with high search effort, which we name *exhaustive search* behavior (abbreviated as "ES"). Moreover, we extend the distance function introduced in [29] into two dimensions, rewriting it as

$$D(x_1, x_2, k) = \min_{1 \leq i \leq mk} \left\{ \sqrt{(x_1 - x_{1_i})^2 + (x_2 - x_{2_i})^2} \right\} \tag{10}$$

which evaluates the Euclidean distance from every map cell position (x_1, x_2) to the nearest vehicles' visited site, where $[x_{1_i}, x_{2_i}]^\top \in \{x_{v_p}(1), \ldots, x_{v_p}(k)\}$ and $x_{v_p} = [x_{v_p}^1, x_{v_p}^2, \ldots, x_{v_p}^m]$ is the combination of all the vehicles' visited sites, we call this kind of distance function "Global Distance Function" (refer to Figure 2 (a)). The distance function is a measurement of uncertainty in the sense of Euclidean distance. $D(x_1, x_2, k)$ represents the uncertainty about the vehicle's knowledge of the terrain at k time index (in the sense of Euclidean distance), which is an experimental design criterion that is intended to inhibit clustering of vehicles and thereby to ensure that the vehicles' locus will spread all over the search terrain. We can consider that targets should be looked for in the most uncertain places, so *a maximum of distance function D* will represent another possible target location. Therefore, if the ith vehicle uses a global distance function to predict target position, we say that the ith vehicle is in *Drive Search* behavior (abbreviated as "DS").
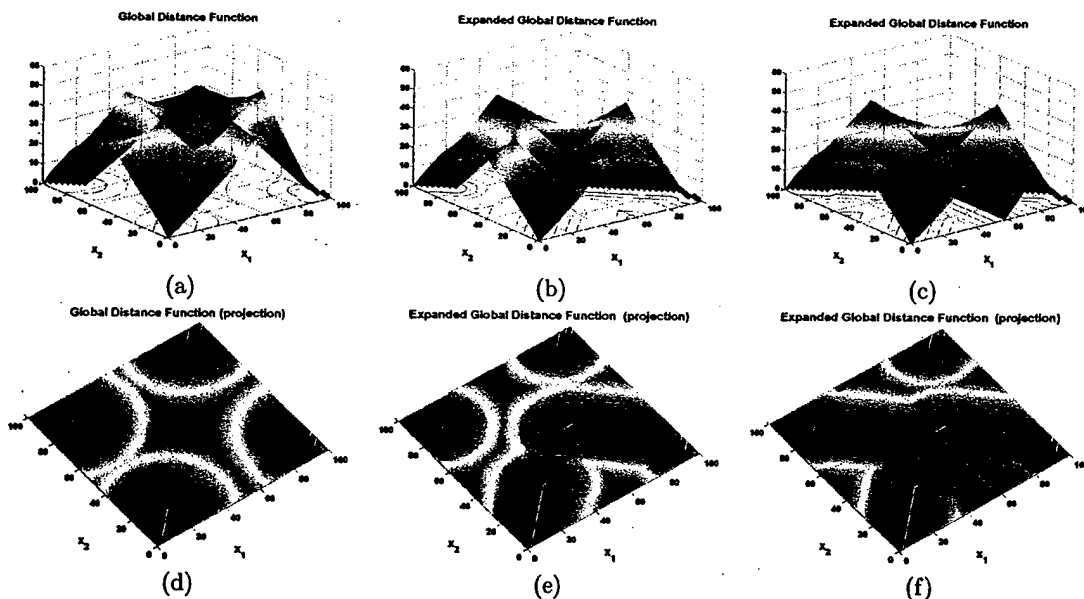

Figure 2: Global distance function: (a) Global distance function, (b) Expanded global distance function given two vehicles' planned trajectories, , (c) Expanded global distance function given three vehicles' planned trajectories, (d) Projection of (a), (e) Projection of (b), (f) Projection of (c).

Here, we reformulate the PPP algorithm as first appeared in [28]. A particular construction of the series of merit function for m vehicles can be: $\text{merit}^1(x_1, x_2, k) = M_p(x_1, x_2, k)$, $\text{merit}^2(x_1, x_2, k) = D(x_1, x_2, k)$ and $\text{merit}^i(x_1, x_2, k) = D^i(x_1, x_2, k)$ for $3 \leq i \leq m$. $D^i(x_1, x_2, k)$ is called expanded global distance function where $[x_{1_i}, x_{2_i}]^\top$ not only includes visited sites x_{v_p}, but also the *planned* trajectories at time k for the previous $i - 1$ vehicles. Examples of expanded global distance functions can be seen in Figure 2. The projection of distance functions (Figures 2 (d), (e) and (f)) to the $x_1 - x_2$ plane gives a more distinctive view of the dynamic computing of the uncertainty of the search area (notice that it makes no difference how the different merit functions are assigned to each vehicle, but we need a sequential negotiation, because the next control force depends on the former ones; the coordination is mainly displayed in the interaction among vehicles and the complexity of the algorithm also lies here). Moreover, we can see clearly the elongation of current trajectories due to the future path plan in these plots; it is the updating of the uncertainty map that solves the clustering originated from the distance function, since each vehicle is equipped with a distinct decision maker.

Figure 5 gives several snapshots of one simulation where we have four vehicles doing cooperative search for three stationary targets as demonstrated in Figures 1 (a) and (b). As we expect, one vehicle will quickly enter the Gaussian peak as directed by prior information map M_p. By incorporating the sensor information from sampling the real target map M_t, we can see an update of M_p (a dissipation of peaks). After a vehicle destroys the first target along its way to the center of the tallest peak, another vehicle rapidly heads to the second peak. The vehicle behaves like in drive search according to the uncertainty within the cone of the peak until it finds the target.
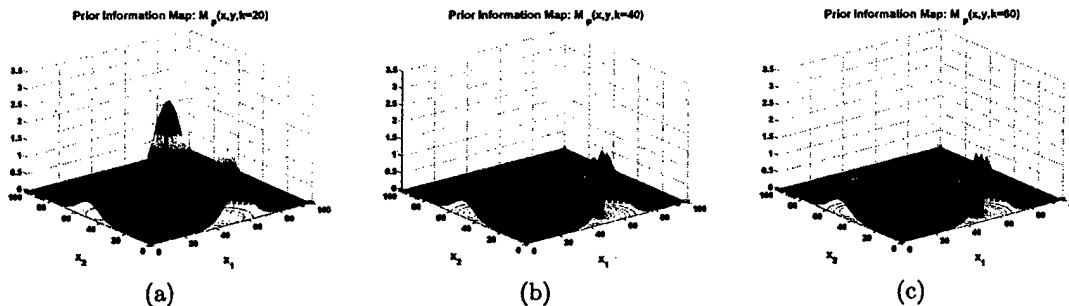

Figure 3: Snapshots of the update of prior information map under 1Cvs3DS PPP coordination with IMF (Ideal Map Flattening Assumption [28]).

Surrogate optimization offers us a method to organize the previous and present sensor information, which results in the updated prior information map. The essence of PPP coordination is the use of predicted future information to balance the search effort or task allocation. The expanded global distance function is just one specific implementation of this idea, which efficiently manipulates the uncertainty and theretofore inhibits clustering introduced by the distance function. The same idea can be applied to the surrogate function, or some other function the vehicles take as a guidance map. We will illustrate this concept further in Section 4. The idea of efficient utilization of past, present and predicted future information becomes the solid foundation of the various schemes throughout the paper.

4 PPP Behavior Based Coordination with Prior Information

4.1 Multiple-Attack PPP with Prior Information

Given that we have uncertain prior information about the possible target locations x_s , and assuming we know the number of targets n and number of vehicles $m > n$ (in the case $n \leq m$, we will following the same pattern in constructing merit functions until running out of vehicles), the past and present sensor information will be used to correct M_p, and the predicted future information will be used to to set up suitable maps, which offer suitable destinations for the following vehicles. We still implement control law (8) and $(x_{o_1}^i, x_{o_2}^i)$ is still the maximum (predicted target) of the ith merit function, but with a new series of merit functions: $\text{merit}^1(x_1, x_2, k) = M_p(x_1, x_2, k)$, $\text{merit}^i(x_1, x_2, k) = M_p^i(x_1, x_2, k)$ given $(2 \leq i \leq N_t)$, where N_t is the number of not-yet-found targets $(N_t \leq n)$. $M_p^i(x_1, x_2, k)$ is called expanded prior information function where a future IMF is implemented to delete peaks to be taken care of by previous planned vehicles (refer to Figure 4). We also have $\text{merit}^{N_t+1}(x_1, x_2, k) = D(x_1, x_2, k)$ and for $N_t + 2 \leq i \leq m$ $\text{merit}^i(x_1, x_2, k) = D^i(x_1, x_2, k)$, the expanded global distance function as illustrated in Figure 2. $M_p(x_1, x_2, k)$ is updated as in Equation (5). We call this new implementation multiple-attack PPP coordination scheme (abbreviated as MPPP). The illustration of the two implementations of PPP algorithm can be seen in Figure 5, where we have four vehicles doing cooperative search for three stationary targets as demonstrated in Figure 1 (a) and (b). The blue stars stand for the targets. The vehicles under new MPPP coordination spend 79 steps finding all the targets, while the old PPP coordination takes 139 steps in finding them. A more detailed analysis based on Monte Carlo simulations will be conducted in Section 4.2.
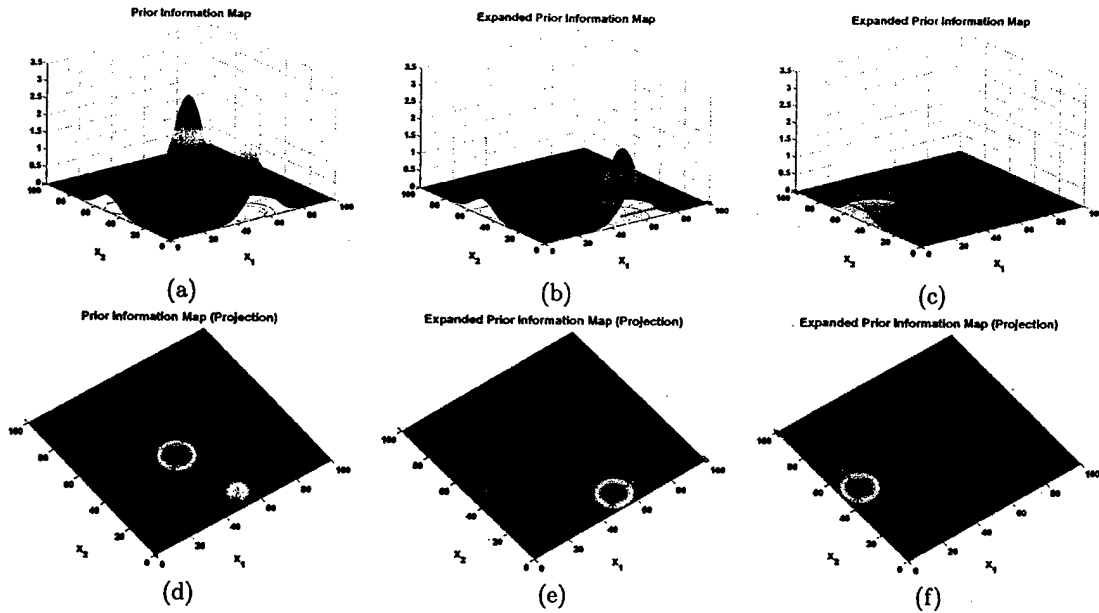
Figure 4: Prior information map, $k = 5$ (blue circle denotes previous locus, red plus sign denotes planned trajectory, blue star denotes the real target position): (a) Prior information map, (b) Expanded prior information map given one vehicle's planned trajectory, (c) Expanded prior information map given two vehicles' planned trajectories, (d) Projection of (a), (e) Projection of (b), (f) Projection of (c).
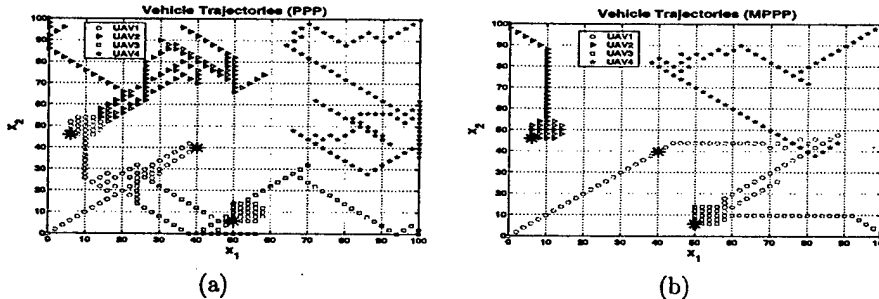

Figure 5: Vehicle trajectories, blue star denotes the real target position (a) 1Cvs3DS, PPP, IMF, (b) MPPP, IMF.

4.2 Monte Carlo Simulation Results

A Monte Carlo simulation is performed here to evaluate PPP coordination as described Section 3 and the new MPPP coordination in dealing with the uncertain prior information. The Monte Carlo Simulation is used to present a quantified analysis. We have the following assumptions: four vehicles move within a square area divided in a 50 × 50 grid and that area contains three targets; the vehicles are initially located at the four corners of the terrain (an easy implementation of exhaustive search, but not necessary for PPP coordination); $k_{max} = 625$, which is the upper bound search time of exhaustive search started from four corners given that for simple exhaustive search we divide the map in four regions of the same size, one for each vehicle; vehicles have perfect communication and move at constant speed; the targets occupy random, fixed locations each time the simulation is run. Moreover, vehicles are assumed to be free of collision and the maximum turn angles are ±135 degrees. Monte Carlo simulation has been run with 50 random target configurations.

Since we encode the priority of the targets as the height of the Gaussian peaks, we also assign different

credits as indicators of the priority level of each target (3 credits for the target with the first priority level, 2 credits for the second one and 1 credit for the target with the least priority level), therefore a successful mission finding all the three targets will obtain 300 credits for 50 runs of different target configurations. We fix the uncertainty level v_i of M_p to be 4 grids, which is an ad-hoc selection, and is chosen with the intention of being sufficiently large with respect to the terrain size (50 × 50) to be meaningful. We assign the uncertainty of the prior information by setting the real target positions at $\frac{1}{8}v_i, \frac{1}{4}v_i, \ldots, v_i, 2v_i, 3v_i$ far from the center. Detection probability p_3 (defined as the ratio of simulations when all three targets are found to the total number of simulation), average of search step μ and the mission credits are chosen as the performance criteria. All the results refer to Figure 6.
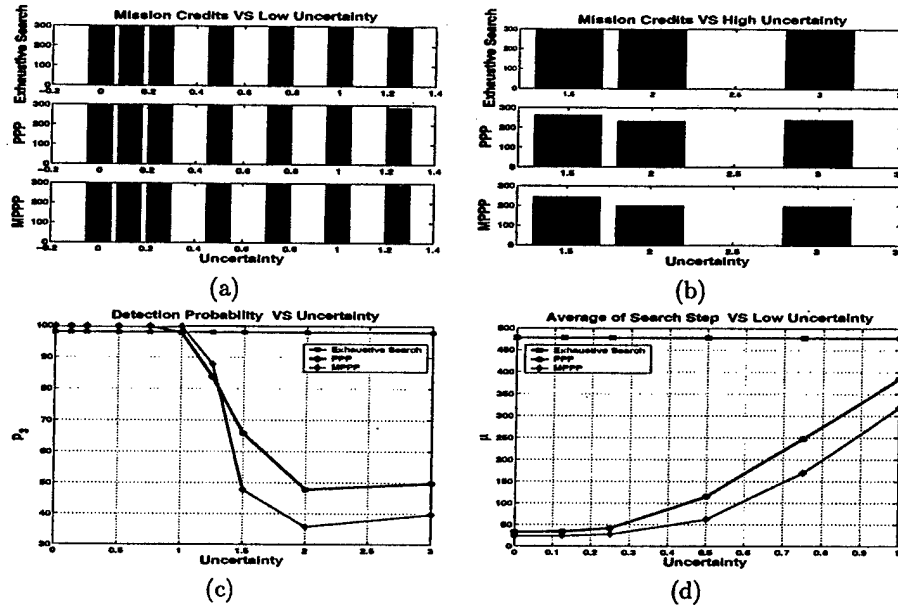

Figure 6: Monte Carlo Simulation Results (a) Mission Credits VS Low Uncertainty (b) Mission Credits VS High Uncertainty (c) Detection Probability VS Uncertainty (d) Average Search Steps VS Low Uncertainty.

From Figure 6, the two PPP coordination schemes are robust in the sense that all of them achieve 100% detection probability given the uncertainty of the prior information up to $\frac{3}{4}v_i$ (Figure 6 (c)) and much better than the exhaustive search in average of search steps (Figure 6 (d)). Given an uncertainty less than $\frac{5}{4}v_i$, MPPP coordination is the best among these three schemes since it utilizes most efficiently the relatively precise prior information (Figure 6 (d)). Overall, as the uncertainty grows, the search performance of the PPP and MPPP coordination schemes degrades accordingly. As expected, the performance degrades approximately quadratically given good detection probability since a linearly growing uncertainty radius leads to a quadratically larger search area (Figure 6 (d)). As the uncertainty grows beyond $1.5v_i$, the PPP algorithm no longer achieves a good detection probability since the prior information is not anymore valuable enough to guide the search, therefore MPPP behaves worse than PPP method (Figure 6 (a) and (b)). Note that in those two figures that exhaustive search is not affected by uncertainty and so always obtain the same number of credits, in average, whereas PPP and MPPP degrade with increasing uncertainty. The advantage of these methods lies clearly in cases where prior information is relatively accurate.

4.3 Scalability

The decentralized architecture of the coordination schemes we present mainly determines the reliability of the system, and it is easy to expand it to incorporate more vehicles and targets. It is easy to incorporate more targets in the simulation since the number of targets does not affect the algorithm's complexity; however, an increase in the number of vehicles may increase computational complexity, especially in the sequential plan, i.e., the construction of the expanded global distance functions or the expanded prior information maps.

In particular, the communication network is the main issue for the scalability of the system. An efficient and reliable communication network will largely guarantee the whole performance of the group of vehicles. However, the algorithm is still functional in the case of constrained communication as long as we construct suitable merit functions (recall Equation (7)).

The current implementations of PPP algorithm require a certain amount of communications (negotiations) between the vehicles. If we assume broadcasting communication, the number of communications at time k for vehicles exchanging individual data to update the common prior information map is m, where m is the number of vehicles and the data transferred include vehicle position $x^i_{v_p}$, current sensor information $M_t(x^i_{v_1}(k), x^i_{v_2}(k))$, and target found or not information. Then, the number of communications required in the sequential plan for the m vehicles is $\frac{m^2}{2} + \frac{m}{2} - 1$, if we require link verification. Totally, we require $\frac{m^2}{2} + \frac{3m}{2} - 1$ communication interchanges at each step. This number, while not the same as computational complexity, is an indicator of it, and implies that the complexity of the entire group grows as a polynomial with respect to the number of vehicles.

5 Concluding Remarks

We present a possible solution for the multi-vehicle cooperative search problem via an adaptation of the surrogate optimization method. Monte Carlo simulations provide partial evidence of the feasibility of our schemes and yield several important guidelines for further research. The PPP algorithm is robust in the sense that it can adapt to different scenarios as long as we can smartly incorporate the predicted future information in generating the control force. Better choices of merit functions can be expected to boost the search performance. Overall, the idea of efficient utilization of past, present and predicted future information becomes the solid foundation of the various schemes throughout the paper. The scalability of our methods also illustrates the feasibility the algorithm. The merit function appears to have great potential to achieve better coordination. An expansion of cooperative search for moving target is not straightforward since the assumption of IMF is no longer practically implementable. The mathematical model and heuristic coordination schemes proposed so far are a start towards the control stability analysis of the multi-vehicle coordination problem.

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