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Matched Subspace Detectors for Stochastic Signals*

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*This work supported by ONR contracts N00014-00-C-00145 and N00014-01-1-1019-P0001

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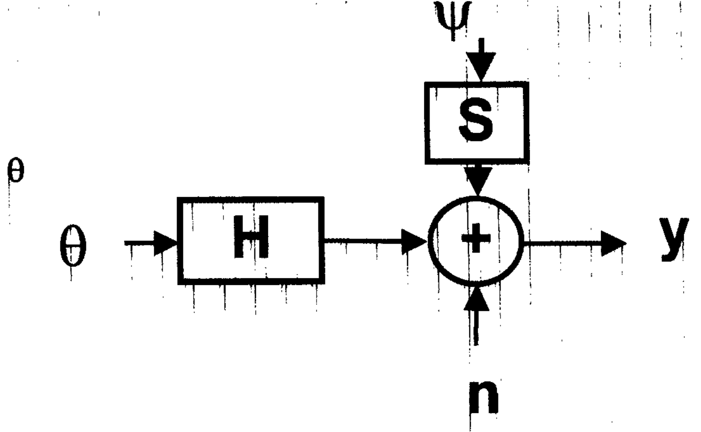
Problem Statement

- The goal is to design detectors for stochastic signals or second-order signals.
- Extension of the first-order matched subspace detectors of Scharf and Friedlander.
- It is assumed that interference is nulled prior to processing by projecting the data into the space orthogonal to the interference subspace.
- We assume various states of knowledge about the parameters σ^2 and β.

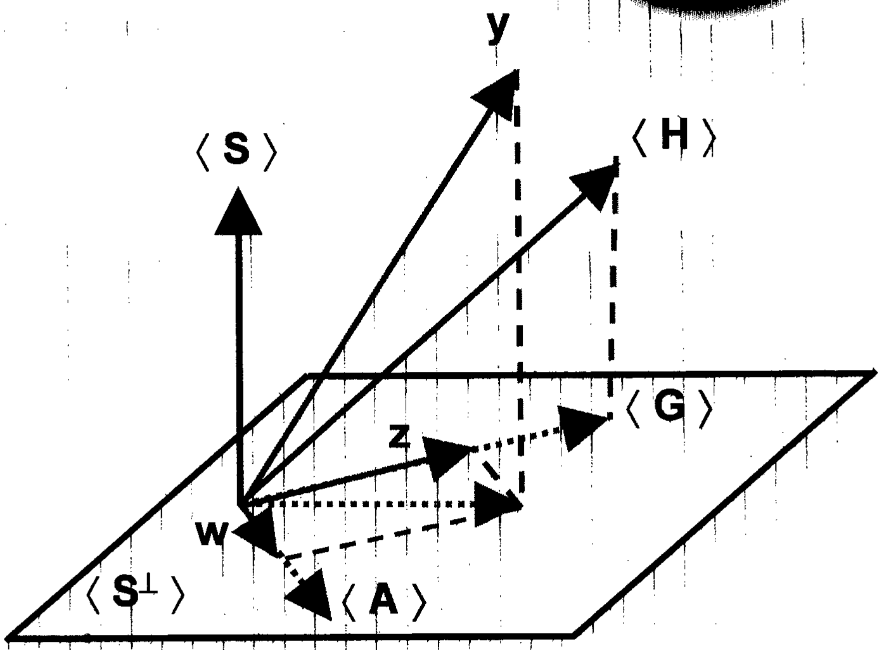


$$n \sim CN(0, \sigma^2 I)$$

$$\theta : f(\theta;\beta) \quad \text{ex. } \theta \sim CN(0, R_{\theta\theta})$$

Pre-Processing

- In order to be invariant to the interference statistics, the data are projected into the space orthogonal to the interference.
- The data are then decomposed into their signal and noise components.
- The signal component is denoted by the vector z and the noise component is denoted by the vector w.



$G = (I-P_S)H$

$$z = (H^*(I-P_S)H)^{-1/2}H^*(I-P_S)\, y$$
$$= (G^*G)^{-1/2}G^*\, y$$

$AA^* = I - P_S - P_G$ $w = A^*\, y$

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Hypotheses

- The "noise" vector w is distributed as a white complex Gaussian vector regardless of which hypothesis is in effect.

$$f(\mathbf{w}) = \frac{1}{(\pi\sigma^2)^{N-p}} e^{-\frac{\mathbf{w}^*\mathbf{w}}{\sigma^2}}$$

- Define $\phi = (G^*G)^{1/2}\theta$.

- When signal is present the data vector z is distributed:

$$f(\mathbf{z} \mid \phi) = \frac{1}{(\pi\sigma^2)^p} e^{-\frac{1}{\sigma^2}\|\mathbf{z}-\phi\|^2}$$

- When signal is not present the data vector z is distributed:

$$f(\mathbf{z} \mid \phi = 0) = \frac{1}{(\pi\sigma^2)^p} e^{-\frac{\mathbf{z}^*\mathbf{z}}{\sigma^2}}$$

Likelihood Ratio

- For now, assume that the noise power σ^2 is known.
- In this case the vector w is common to both hypotheses and is of no use.
- The *conditional* likelihood ratio is then

$$l(\mathbf{z} \mid \phi; \sigma^2) = \frac{f(\mathbf{z}|\phi;\sigma^2)}{f(\mathbf{z}|\phi=0;\sigma^2)}$$

$$= \exp\left(\frac{\mathbf{z}^*\mathbf{z}}{\sigma^2}\right) \times \exp\left(-\frac{1}{\sigma^2}\|\mathbf{z}-\phi\|^2\right)$$

Unconditional Likelihood Ratio

- The unconditional likelihood ratio can be written as

$$l(\mathbf{z};\sigma^2,\beta) = \exp\left(\frac{\mathbf{z}^*\mathbf{z}}{\sigma^2}\right) \times \int \exp\left(-\frac{\|\mathbf{z}-\phi\|^2}{\sigma^2}\right) f_\phi(\phi;\beta)\,d\phi$$

- The log-likelihood ratio becomes

$$s(\mathbf{z};\sigma^2,\beta) = \frac{\mathbf{z}^*\mathbf{z}}{\sigma^2} + \ln \int \exp\left(-\frac{|\mathbf{z}-\phi\|^2}{\sigma^2}\right) f_\phi(\phi;\beta)\,d\phi$$

$$= \underbrace{\frac{\mathbf{y}^*\mathbf{P}_G\mathbf{y}}{\sigma^2}}_{\text{Matched Subspace Detector}} - \underbrace{p_r(\mathbf{z};\sigma,\beta)}_{\text{Resolution penalty}}$$

Resolution Penalty

- **The resolution penalty occurs because we presume to know something about the coordinate vector θ .**

- **If z is far from the "favored" orientation defined by θ then the penalty is larger than if the converse were true.**

$$p_r(\mathbf{z}; \sigma^2; \beta) = -\ln \int \exp(-\frac{\|\mathbf{z}-\phi\|^2}{\sigma^2}) f_\phi(\phi; \beta) d\phi$$

Gaussian Coordinate Vectors

- Suppose $\phi \sim CN(0, R_{\phi\phi})$.
- Write the eigenvalue decomposition of $R_{\phi\phi}$ as:

$$R_{\phi\phi} = (G^*G)^{1/2} R_{\theta\theta} (G^*G)^{1/2} = V D^2 V^*$$

$$V = [\mathrm{v}_1 \, \mathrm{v}_2 \cdots \mathrm{v}_p]; \;\; unitary$$

$$D^2 = diag[\beta_1^2, \beta_2^2, \cdots, \beta_p^2]$$

- Define the resolved signal-plus-noise to noise ratios:

$$r_i = 1 + \frac{\beta_i^2}{\sigma^2}$$

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Gaussian Penalty Term

- After some algebra the penalty term can now be written as

$$p_r(z; \sigma^2, \beta^2) = -\ln \int \exp\left(-\frac{\|z-\phi\|^2}{\sigma^2}\right) \frac{1}{\pi^p det(R_{\phi\phi})} \exp(-\phi^* R_{\phi\phi}^{-1} \phi) d\phi$$

$$= \sum_{i=1}^{p} \ln(r_i) + \sum_{i=1}^{p} \frac{(z^* P_{v_i} z / \sigma^2)}{r_i}$$

- This result implies that if the estimated signal-plus-noise to noise ratio $(z^* P_{v(i)} z / \sigma^2)$ in the resolved subspace defined by v_i greatly exceeds r_i, then the penalty is large because of this mismatch.

Unknown Signal Power and Orientation

- Suppose that when signal is present we do not know $R_{\phi\phi}$.

- Recall the penalty term is

$$p_r(\mathbf{z};\sigma^2,\beta^2) = -\ln\int \exp\left(-\frac{\|z-\phi\|^2}{\sigma^2}\right)\frac{1}{\pi^p det(R_{\phi\phi})}\exp(-\phi^* R_{\phi\phi}^{-1}\phi)d\phi$$

$$= \sum_{i=1}^{p}\ln(r_i) + \sum_{i=1}^{p}\frac{(\mathbf{z}^* P_{\mathrm{V}_i}\mathbf{z}/\sigma^2)}{r_i}$$

- The estimates of the signal-plus-noise to noise ratios are

$$r_i = max(1, \mathbf{z}^* P_{\mathrm{V}_i}\mathbf{z}/\sigma^2)$$

- We assume that $r_i \geq 1$ in the sequel.

Estimating Orientation

- The estimates of r_i in the previous slide depend on the orientation of the vectors v_i .
- We want to minimize

$$\prod_{i=1}^{p} \frac{\mathbf{z}^* P_{\mathbf{v}_i} \mathbf{z}}{\sigma^2}$$

- We must also satisfy the constraints

$$\sum_{i=1}^{p} \frac{\mathbf{z}^* P_{\mathbf{v}_i} \mathbf{z}}{\sigma^2} = \frac{\mathbf{z}^* \mathbf{z}}{\sigma^2} \qquad r_i = \frac{\mathbf{z}^* P_{\mathbf{v}_i} \mathbf{z}}{\sigma^2} \geq 1$$

Intermediate Orientation Solution

- The solution to this optimization problem is

$$r_i = \frac{\mathbf{z}^* P_{\mathbf{v}_i} \mathbf{z}}{\sigma^2} = 1 \quad for \ i = 1, 2, \cdots, p-1$$

$$r_p = \frac{\mathbf{z}^* P_{\mathbf{v}_p} \mathbf{z}}{\sigma^2} = \frac{\mathbf{z}^* \mathbf{z}}{\sigma^2} - (p-1)$$

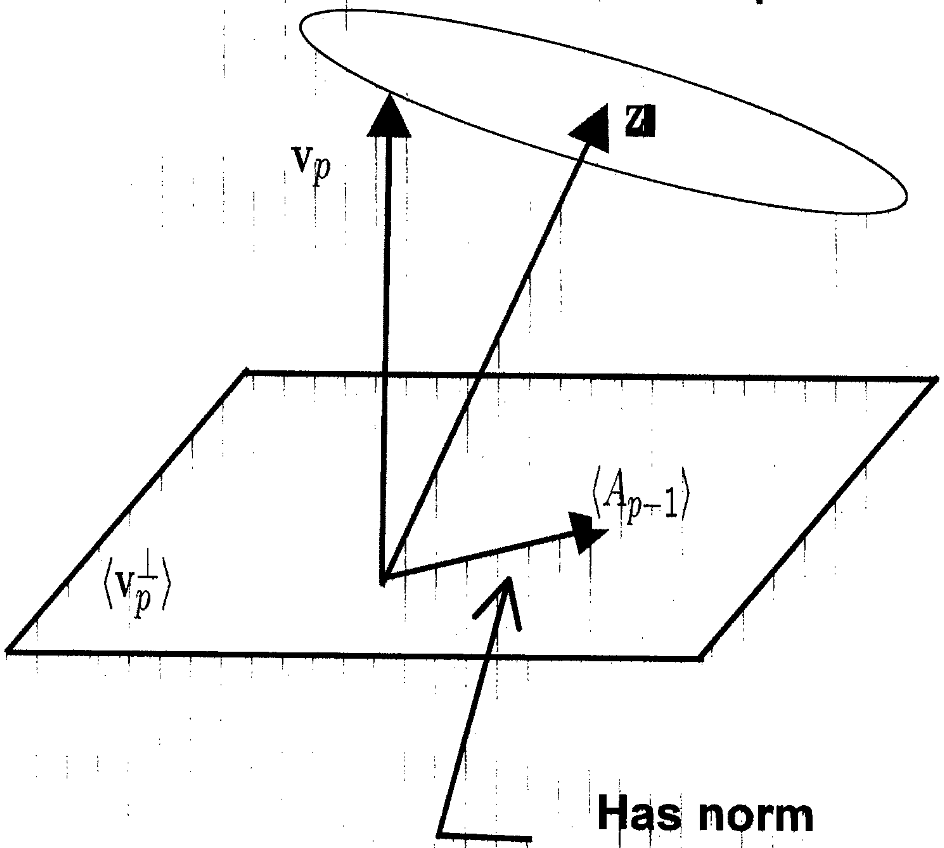
- The question remains: Is there a decomposition of ⟨ G ⟩ that has the above properties?
- The answer is yes.

Orientation Solution

- Solve for v_p first.
- Choose a v_p on the spherical invariance set defined by

$$\frac{z^* P_{v_p} z}{\sigma^2} = \frac{z^* z}{\sigma^2} - (p-1).$$

- Repeat this procedure in the spaces
  $\langle A_{p-1} \rangle, \langle A_{p-2} \rangle$, 6 , $\langle A_1 \rangle$



Great circle on invariance sphere

Has norm
$\sigma^2(p-1)$

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Compressed Likelihood

- Compressing the likelihood ratio with this solution gives the statistic

$$s(\mathbf{z};\sigma^2,\hat{R}_{\phi\phi}) = \frac{\mathbf{y}^* P_H \mathbf{y}}{\sigma^2} - \left[\ln\left(\frac{\mathbf{y}^* P_H \mathbf{y}}{\sigma^2}\right) - constants\right]$$

- This statistic is a monotonic function of the matched subspace detector. We can therefore use the MSD as the detection statistic

$$s = \frac{\mathbf{y}^* P_H \mathbf{y}}{\sigma^2}$$

- Then the result for 2nd-order models is the same as for 1st-order models.

Unknown Noise Power

- In the case of unknown noise power the GLRT detector can be written as a sum of the CFAR matched subspace detector and a penalty term

$$s(\mathbf{z};\hat{\sigma}^2,\hat{R}_{\phi\phi}) = \ln(1+\tilde{s}) - [\ln(\tilde{s}) - constants]$$

- We can equivalently use the statistic

$$\tilde{s} = \frac{\mathbf{y}^* P_H \mathbf{y}}{\hat{\sigma}^2}; \qquad \hat{\sigma}^2 = \frac{1}{N-p}\mathbf{y}^*(I - P_H)\mathbf{y}$$

- These detectors are identical to the 1st-order results.

Rank-One Assumptions

- Here we assume that the signal subspace is rank-one.
- The complex-valued signal amplitude is written in polar form

$$\theta = M e^{j\phi}$$

- Assume that the phase and magnitude are uncorrelated and that the phase is uniformly distributed over $[0, 2\pi)$.
- Assume that the signal magnitude has a generalized Rayleigh distribution

$$f_M(M) = \frac{2M}{\beta^2}\left(\frac{M^2}{\beta^2}\right)^L \frac{e^{-M^2/\beta^2}}{L!}$$

Detectors with Known Noise

- L=0. This is the previous results with complex Gaussian amplitudes.

- L $\neq$ 0. The penalty function is

$$p_r = (L+1)\ln(r) + \frac{(\mathrm{y}^* P_h \mathrm{y}/\sigma^2)}{r} - \ln\left[\sum_{k=0}^{L} \frac{binom(L,k)}{k!}\left(\frac{(\mathrm{y}^* P_h \mathrm{y}/\sigma^2)(r-1)}{r}\right)^k\right]$$

- Minimize the penalty term with respect to $r=1+\beta^2/\sigma^2$.
- Compress the likelihood function with this term to obtain

$$s = \frac{\mathbf{y}^* P_h \mathbf{y}}{\sigma^2} - p_r(\widehat{r}).$$