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# Space-Times Codes for an Invariant Detector of Frequency-Hopped MIMO Communications

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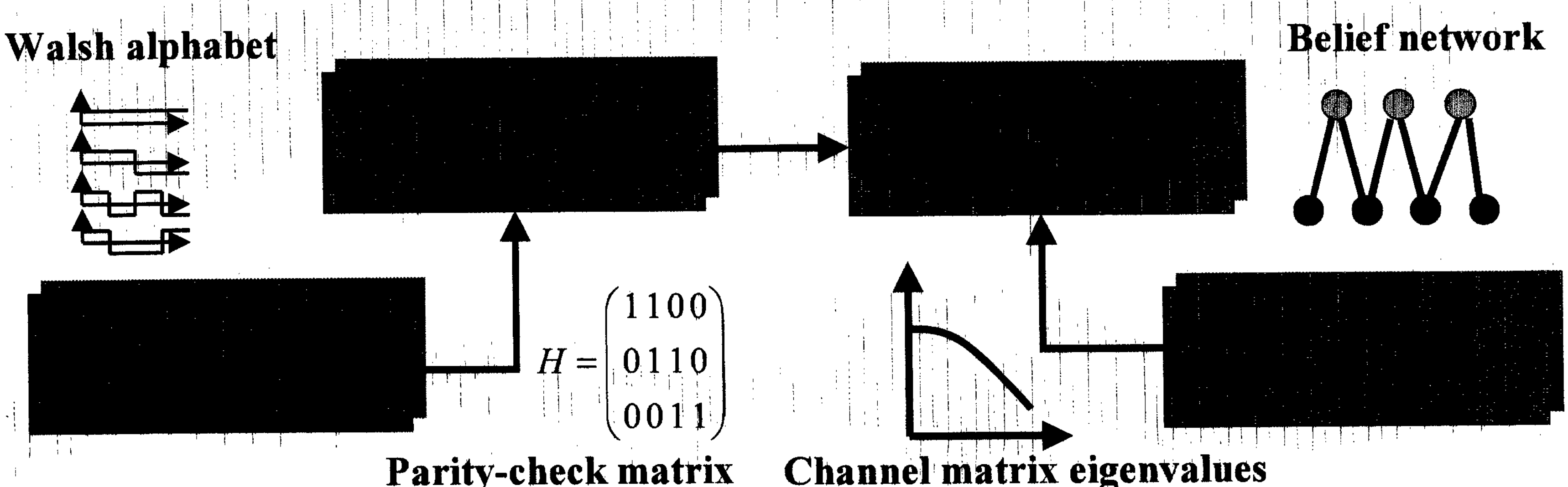
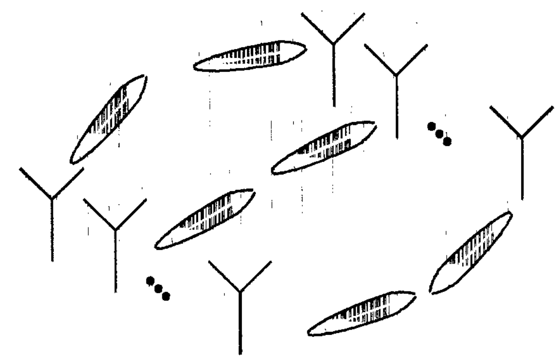
# Codec Architecture for the Metachannel of an Invariant MIMO Detector

- Multiple input multiple output (MIMO) communications
  - Multiple transmitters coordinate channel coding by introducing space-time redundancy
  - Multiple receivers separate propagation modes in process of decoding
- Frequency-hopped MIMO
  - Channel transfer function (channel matrix) varies randomly hop-to-hop
  - Space-time coding occurs over hops and provides additional fading immunity and AJ
- Invariant detector
  - Short hops and low SNR can complicate channel estimation
  - Imposed detector invariances create metachannel robust to jamming and unknown channel



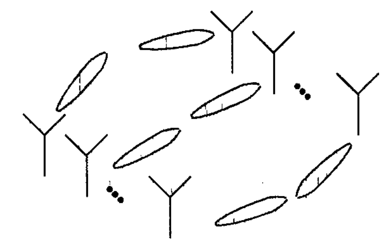
# Topics

- Introduction
- Signals in space
  - Signal model
  - Channel
  - Receiver
- Theoretical capacity
- Coding
  - Space-time inner codes
  - Low density parity-check outer codes
- Performance
  - Predictions
  - Simulations
- Summary and Conclusions



# Subspace Codes

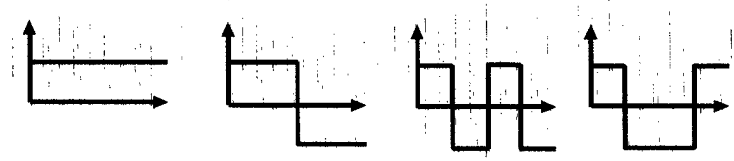
- Signal in additive noise (special case: # Rx = # Tx)



$$\overbrace{Z}^{n\times l} = \underbrace{V}_{n\times n}\overbrace{S}^{n\times l} + \underbrace{N}_{n\times l}$$

Assume $l \geq 2n$

- Motivation
  - In absence of noise, rowspace$(Z)$ = rowspace$(S)$ for nonsingular $V$
  - Encode information bits in subspaces rowspace$(S)$ and use only subspace of observations
  - Decision invariant to whitening transformations $Z \leftarrow R^{-1/2}Z$
- Use scaled orthonormal signals ($SS^H \propto I_n$) to realize codes



# Invariant Detectors

- **Decision statistic** $D(Z, S)$
- **Invariances**
  - **Subspace invariance**

    $D(Z, S) = D(AZ, BS)$ **for nonsingular** $A$, $B$
  - **Independence, with Gaussian samples**

    $D(Z, S) = D(ZU, SU)$ **for unitary** $U$
- **Example:**

$$p(Z|R, V, S) = \pi^{-nl}|R|^{-l}\exp\{-\mathrm{tr}[(Z - VS)^H R^{-1}(Z - VS)]\}$$

$$p(AZ|R, V, BS) = |AA^H|^{-l}p(Z|A^{-1}RA^{-H}, AVB, T)$$

$$p(ZU|R, V, SU) = p(Z|R, V, S)$$

$D(Z, S) \triangleq |ZZ^H|^l \cdot \max_{R,V} p(Z|R, V, S)$ **has appropriate invariances**

- **Maximal invariant** $D(Z, S)$ **depends only on principal angles between subspaces rowspace**$(Z)$ **and rowspace**$(S)$
  - **Other examples:** $\mathrm{tr}(P_Z P_S)$, $|P_Z P_S|$, $\frac{|Z(I_l - P_S)Z^H|}{|ZZ^H|}$

# Hopper Metachannel

- $V$ varies randomly hop to hop
  - Prior on $V$: mean zero, complex, unity variance Gaussian i.i.d. entries
- Channel model
  - Transmit rowspace $(S)$
  - Receive rowspace $(Z)$, with $Z = aVS + N$
- Maximum likelihood detector ($p \triangleq |a|^2$)

$$D(Z, S) = \left| I_n - \frac{p}{1+p} P_Z P_S \right|^{-l}$$

- Channel capacity

$$\mathrm{E}[\log_2((1+p)^{-l(l+1)/2} |I_n - \tfrac{p}{1+p} P_Z P_S|^{-l})/l]$$

- Suboptimal detector

$$D(Z, S) = e^{l \operatorname{tr} P_Z P_S}$$

# Signal-to-Noise Ratios
## Random Channel Matrices

- For $m$ transmitters, $n$ receivers, (average) data rate $R$, average element-to-element SNR, and bandwidth $B$, define $\frac{E_b}{N_0}$ to satisfy

$$m\,n\,\text{SNR} = \frac{R}{B}\frac{E_b}{N_0}$$

- Motivating properties

$$\frac{E_b}{N_0} \to \log(2) \text{ as } B \uparrow \infty$$

$$\log 2 \leq \frac{E_b}{N_0} \text{ using average rate } R$$

$$m, n \to \infty,\ \frac{m}{n} \text{ fixed } \Rightarrow \log 2 = \frac{E_b}{N_0} \text{ for fixed rate } R$$

- Transmitted power proportional to $\frac{1}{n}\frac{E_b}{N_0}$
  - MIMO $\frac{E_b}{N_0}$ is $n$ times MISO $\frac{E_b}{N_0}$

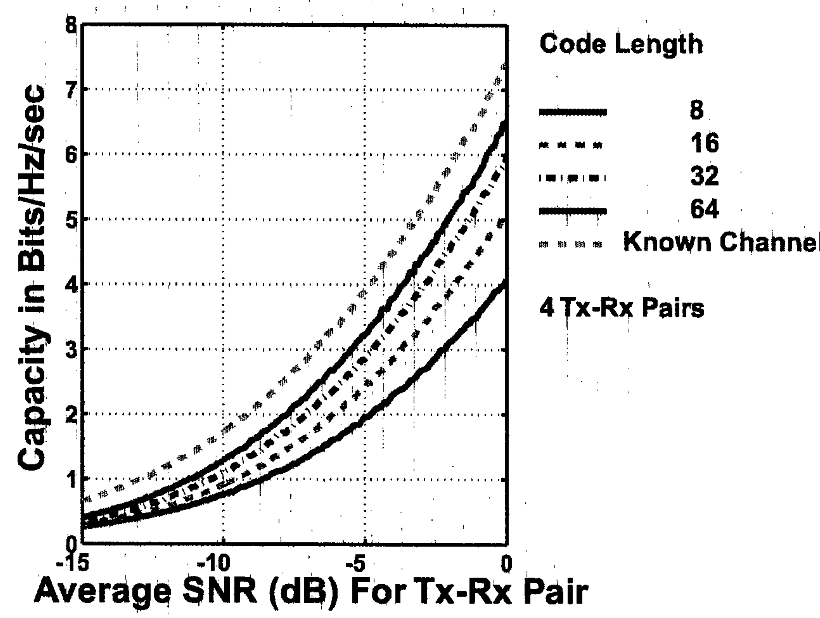
# Capacity of the Metachannel

- Upper bound on capacity
  - Capacity when channel is tracked (known channel)

$$E_V[\log_2(|I_n + |a|^2 VV^\dagger|)]$$

- Performance
  - As symbol length increases, capacity approaches that of tracked channel
  - Scaling all dimensions (number of receivers/transmitters and symbol length), channel behaves like infinite bandwidth channel but with added loss due to channel estimation.

**Capacity of 4X4 MIMO As a Function of Symbol Length**

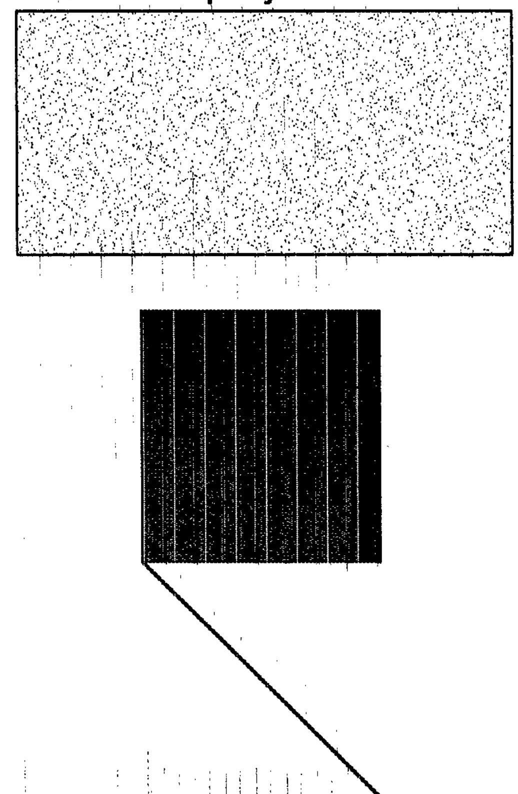


# Space-Time Codes for the FH/PN Channel
## Concatenated Coding

- Construct short space-time inner codes for each hop
  - Invariant to channel matrix
  - Matrix symbols with $2^m$ values
- Code over hops with low density parity-check (LDPC) outer code
  - Length 1024, rate ½
  - 4 nonzero entries per column, 8 per row, totaling .8% of all entries
  - Symbols over GF($2^m$)
- Utilize invariant detector with probability vectors built from (quasi)-likelihoods

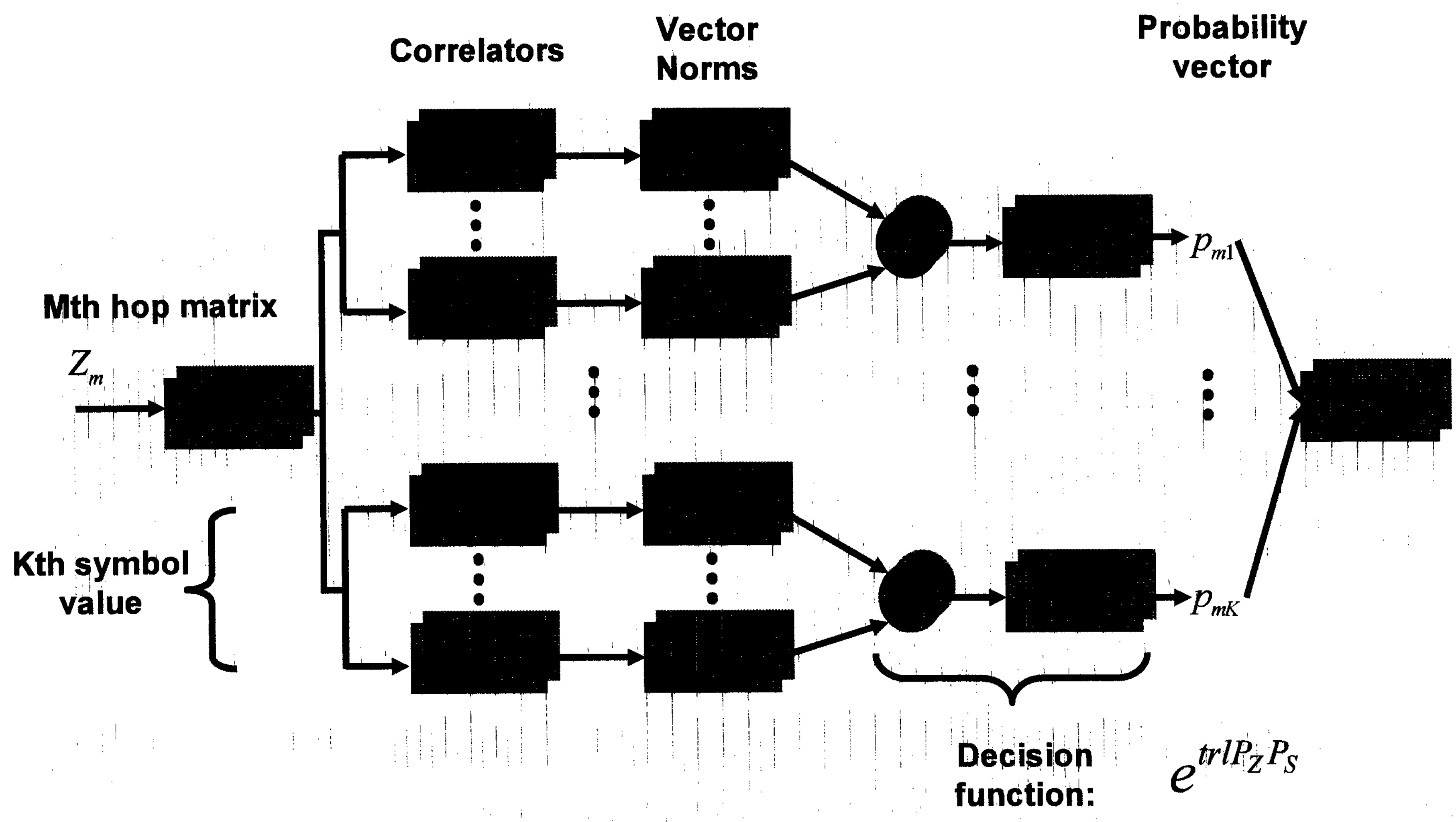
$$\left| I_n - \frac{p}{1+p} P_Z P_S \right|^{-l} \qquad e^{trlP_Z P_S}$$

Locations of 4096 nonzero entries of 512 X 1024 paritycheck matrix



Nonzero entries of 1024 X 512 generator matrix

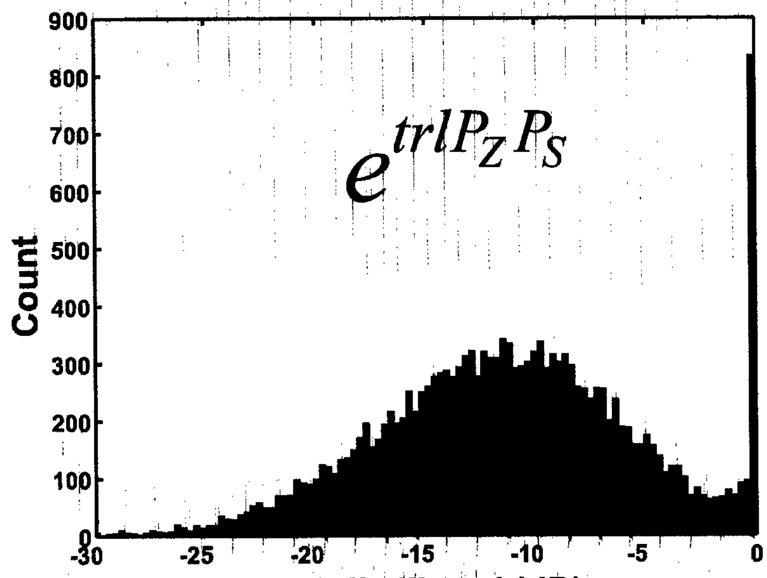
# Demodulating Matrix symbols



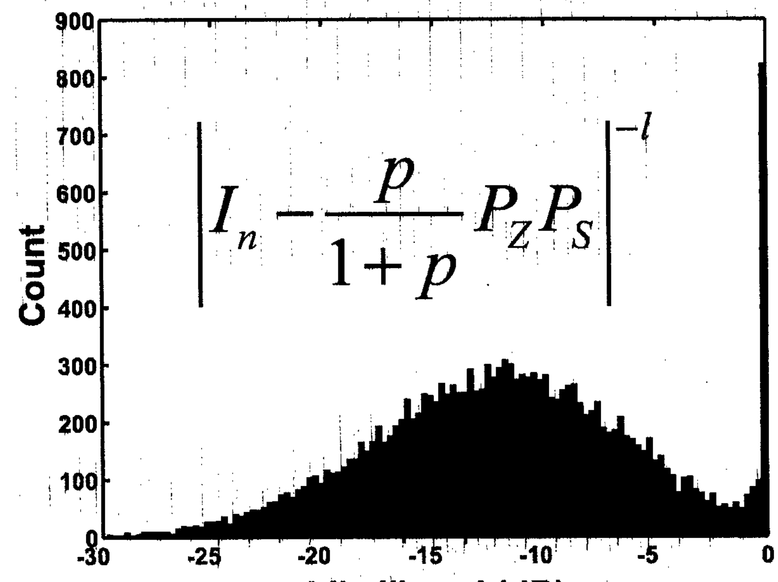
# Decision Statistics For Matrix Symbols

- Quasi-likelihood and likelihood decision statistics provide similar performance
- Examples chosen from cases with about 5% symbol error probability
  - Histogram of components from length 16 probability vectors formed by (quasi)-likelihoods

**Density of Suboptimal Quasi-likelihoods**

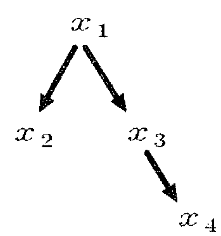



**Density of Likelihoods**



# Graphical Decoding of Low Density Parity-Check Codes Using Bayesian Belief Networks

**Variable dependencies**

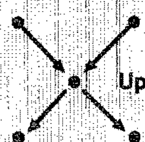



**Loopless directed acyclic graph (DAG)**
**Directed Markov field**
**Bayesian belief network**

$$p(x_1, x_2, x_3, x_4) = p(x_4|x_3)p(x_3|x_1)p(x_2|x_1)p(x_1)$$

## Message passing protocol

Parent nodes (alphabets $u_k \in B_k$)



Update node (alphabet $x \in A$)

Child nodes

| Messages passed | Received | Sent |
|---|---|---|
| likelihoods | $\lambda^C$ | $\lambda^P$ |
| priors | $\pi^P$ | $\pi^C$ |

## Node updates

Node calculations and messages

$$\pi(x) = \sum_u p(x|u_1, \ldots, u_n) \prod_{k=1} \pi_k^P(u_k)$$

$$\lambda(x) = \prod_k \lambda_k^C(x)$$

$$\pi_j^C(x) = \pi(x) \prod_{k \neq j} \lambda_k^C(x)$$

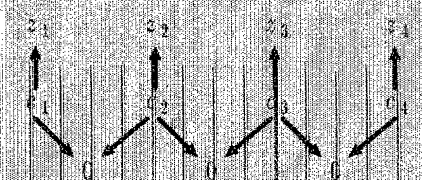
$$\lambda_j^P(u_j) = \sum_{x, u_k : k \neq j} \lambda(x) p(x|u_1, \ldots, u_n) \prod_{k \neq j} \pi_k^P(u_k)$$

Belief

$\lambda(x)\pi(x)$

## Network for a parity-check code

Bayesian Belief Network




Evidenciary nodes (observations)

Codeword components

Parity checks

Node firing order: $z\ 0\ c\ z\ 0\ c \cdots$

Stopping rule: parity check satisfied

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Parity check matrix

# Constructions of Space-Time Inner Codes

## Linear Block Codes

- Sets of orthonormal waveforms of length $l$: $\{c_k\}$: $c_j \perp c_k$
- Matrix symbols $S(c)$

$$\phi_k : GF(2^k) \rightarrow c_k, \text{ 1-1}$$

$$c \in GF(2^k)^n$$

$$S(c) \triangleq \begin{pmatrix} \phi_1(c_1) \\ \vdots \\ \phi_n(c_n) \end{pmatrix}$$

- Spectral efficiencies ($r_s$, $r_t$ inner and outer code rates)

$$\frac{R}{B} = r_t r_s \frac{k}{2^k}$$

# Examples of Space-Time Inner Codes

| Code | Parity Check Matrix | Field |
|---|---|---|
| (4,4,1) | $\mathbf{0}$ | $GF(2)$ |
| (4,2,3) | $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & \alpha \end{pmatrix}$ | $GF(4)$ |
| (4,3,2) | $(1, 1, \ldots, 1)$ | $GF(2)$ |
| (4,1,4) | $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ | $GF(2)$ |

| Code | Parity Check Matrix | Field |
|---|---|---|
| (8,8,1) | $\mathbf{0}$ | $GF(2)$ |
| (8,7,2) | $(1, 1, \ldots, 1)$ | $GF(2)$ |
| (8,6,3) | $\begin{pmatrix} 1 & 0 & 1 & 1 & \cdots & 1 \\ 0 & 1 & \alpha & \alpha^2 & \cdots & \alpha^6 \end{pmatrix}$ | $GF(8)$ |
| (8,4,4) | $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$ | $GF(2)$ |
| (8,3,6) | $\begin{pmatrix} 1 & & 0 & 1 & 1 & 1 \\ 0 & & 0 & \alpha & \alpha^2 & \alpha^3 \\ 0 & \cdots & 0 & \alpha^2 & \alpha^4 & \alpha^6 \\ 0 & & 0 & \alpha^3 & \alpha^6 & \alpha^9 \\ 0 & & 1 & \alpha^4 & \alpha^8 & \alpha^{12} \end{pmatrix}$ | $GF(8)$ |
| (8,2,7) | $\begin{pmatrix} 1 & & 0 & 1 & 1 \\ 0 & & 0 & \alpha & \alpha^2 \\ 0 & \cdots & 0 & \alpha^2 & \alpha^4 \\ 0 & & 0 & \alpha^3 & \alpha^6 \\ 0 & & 0 & \alpha^4 & \alpha^8 \\ 0 & & 1 & \alpha^5 & \alpha^{10} \end{pmatrix}$ | $GF(8)$ |
| (8,1,8) | $\begin{pmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 \\ & & & \vdots & & \\ 0 & 0 & 0 & \cdots & 1 & 1 \end{pmatrix}$ | $GF(2)$ |

# More of Space-Time Inner Codes

## Steiner Systems

- **Orthonormal waveforms**

$$\{\vec{s}_k\},\ \vec{s}_j \perp \vec{s}_k,\ j \neq k,\ 1 \leq j, k \leq l$$

- **Matrix symbols**

$$E(c) \triangleq \begin{pmatrix} \vec{s}_{i_1} \\ \vdots \\ \vec{s}_{i_n} \end{pmatrix}$$

$\{c_{i_1}, \ldots, c_{i_n}\}$ **nonzero entries in** $c$, $\mathrm{wt}(c) = n$

- **Examples**

$$c = \begin{cases} (l = 16, 11, n = 4) & \text{140 codewords} \\ (l = 24, 12, n = 8) & \text{759 codewords} \end{cases} \quad \mathrm{wt}(c) = n$$

- **Subspace separations**

$$\dim(E(c) \cap E(c')) \leq \begin{cases} 2 & (16, 11, 4) \\ 4 & (24, 12, 8) \end{cases} \quad c \neq c'$$

**Maximally separated away from intersection**

# Theoretical Predictions
## Approximate Error Exponents

- **Effective SNR (interference covariance $R_I$ as r.v. hop to hop)**

$$\frac{nd}{4}\left(\frac{\mathrm{tr}(\mathrm{E}[R_I^{-1} V V^H])}{n^2}\right)^2 \cdot (l\,\mathrm{SNR})^2$$

- **Bounds for linear block codes ($D/N \leq 1/2$)**

  Gilbert-Varshamov (GS): $\sum_{k=0}^{D-2} (q-1)^k \binom{N-1}{k} < q^r$

  Rank: $D \leq N - K + 1$

- **Asymptotic form of Gilbert-Varshamov bound**

$$G_q(x) \triangleq \log q - x \log(q-1) - x \log x - (1-x)\log(1-x)$$

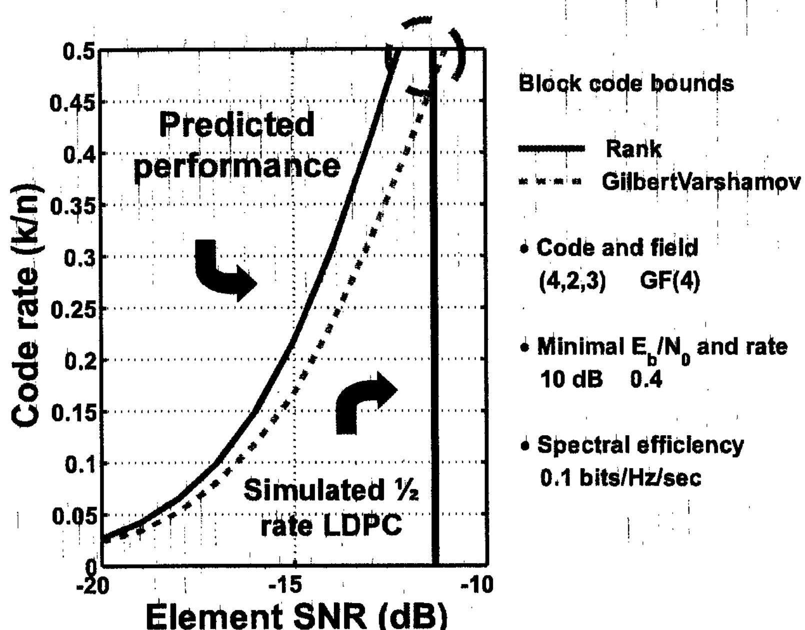
$$K/N \log q = G_q(D/N)$$

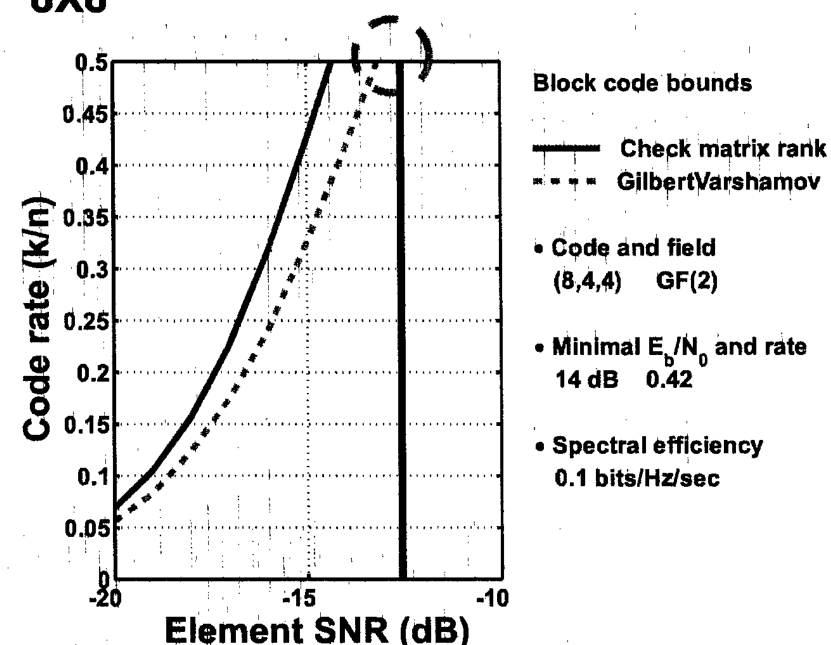
- **Error exponent (GS):** $\frac{K}{N}\log q - \mathrm{SNR}_{\mathrm{eff}} G_q^{-1}\left(\frac{K}{N}\log q\right)$

# Comparison of Theoretical and Simulated Performance

- Predicted performance expresses code rate in terms of SNR
- Minimizing $\frac{E_b}{N_0}$ over SNR results in optimal codes of rate near 1/2
- Predicted performance agrees closely with simulated ½ rate LDPC outer code concatenated with space-time inner codes

**4X4 ← MIMO → 8X8**



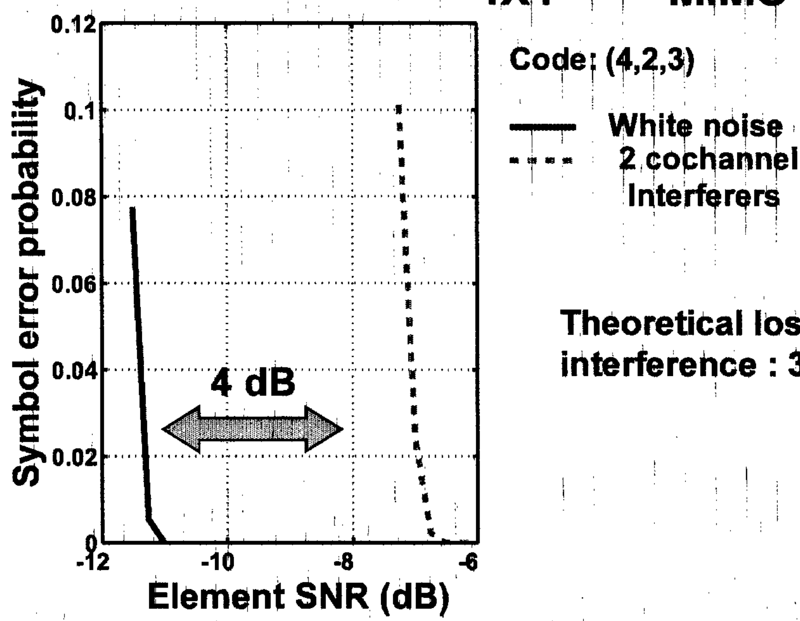



# Simulated Performance With Jamming and Nonrandom Channel Matrices

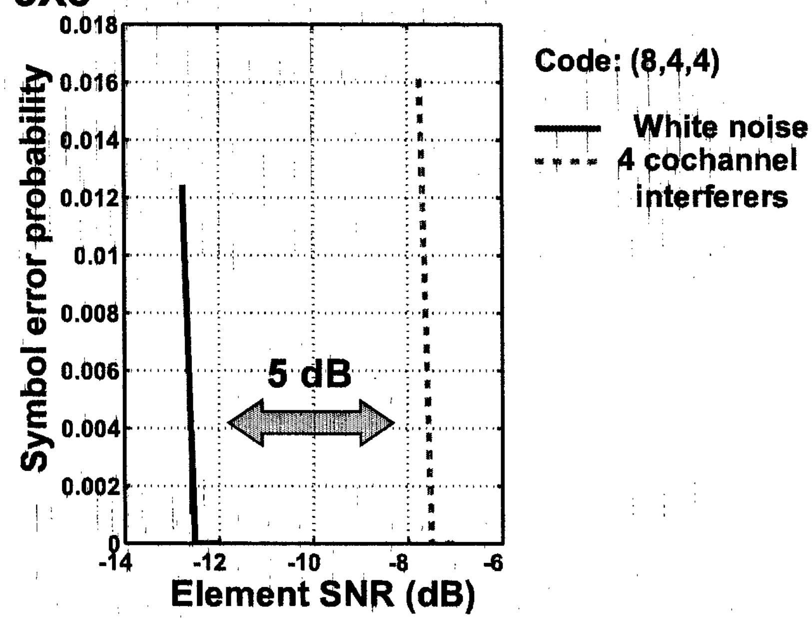
- Theoretically, K jammers result in (N-K)/N SINR loss
- Simulated results indicate losses are somewhat higher
- When channel matrix is constant over all hops, predicted performance agrees with random variation provided received power is scaled to make $tr(VV^{\dagger})/n^2$ unity

**Simulated performance with and without jamming**

**4X4 ← MIMO → 8X8**

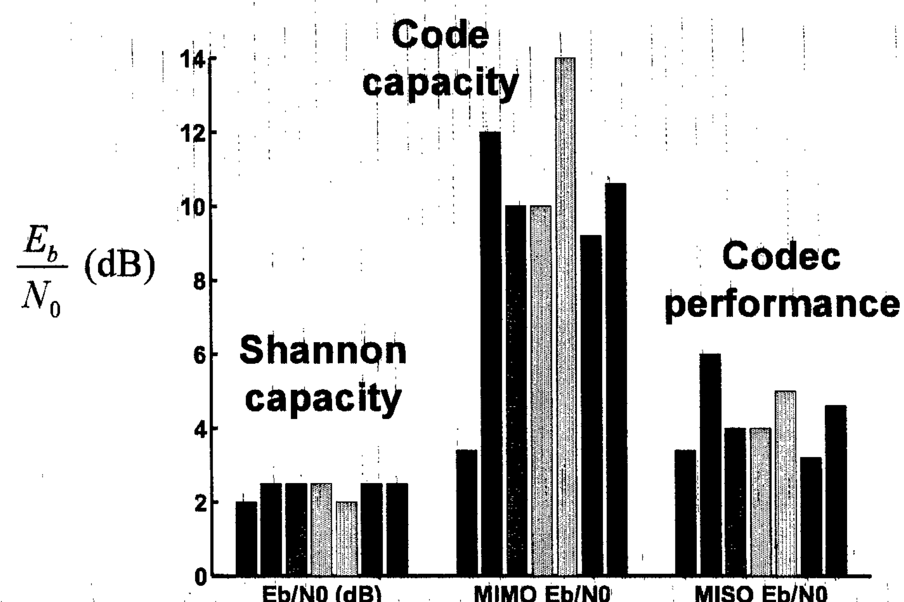


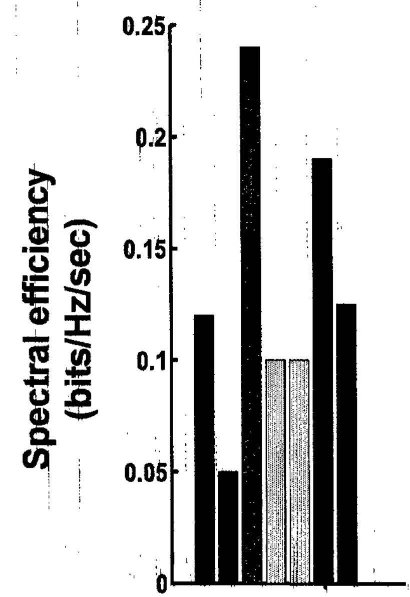

Theoretical loss in interference : 3 dB



# Summary of Performance
## Random Channel Matrices

- **Codes**
  - Inner code specified by block code parameters, Steiner system parameters or random matrix symbols (4X4 MIMO with 16 length 16 matrix symbols)
  - Outer code: (1024,512) LDPC over GF(16), GF(128), or GF(256)
- **Performance**
  - Predicted by effective SNR and Gilbert-Varshamov bounds (except random case)
  - Bounds validated by simulation (within several tenths dB)



# Summary and Conclusions

- Class of invariant detectors formulated for robust demodulation and decoding in unknown interference with unknown channels
  - Capacity evaluated for the frequency-hopped (FH) channel as received by an invariant detector
- Family of concatenated codes examined for frequency-hopped, pseudo-noise (FH/PN) channel
  - Family uses linear block codes, Steiner systems, etc. for space-time inner code matrix symbols and low density parity-check outer codes
  - Theoretical performance agrees with simulations
- Performance
  - Concatenated codes considered operate around 3 to 4 dB (MISO) $\frac{E_b}{N_0}$
  - Concatenated codes examined are 7-8 dB worse than channel capacity bound in white noise
  - Space-time codes provide $n^2$ diversity even when channel matrices remain constant hop to hop
  - Space-time codes and invariant detector handle interferers and unknown channels gracefully with little sensitivity to interference geometry