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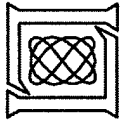
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Mean Squared Error Performance Prediction of Maximum-Likelihood Signal Parameter Estimation

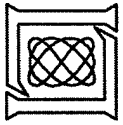
Christ D. Richmond

Adaptive Sensor Array Processing Workshop
Session III: Adaptive Detection and Estimation

12th March 2003

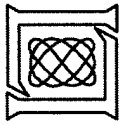
*This work was sponsored by DARPA under Air Force contract F19628-95-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the author and are not necessarily endorsed by the United States Government.

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Outline

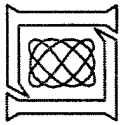
- ➔ • **Introduction**
 - **Problem**
 - **Previous Work**
- **Theory**
- **Numerical Results**
- **Conclusions**



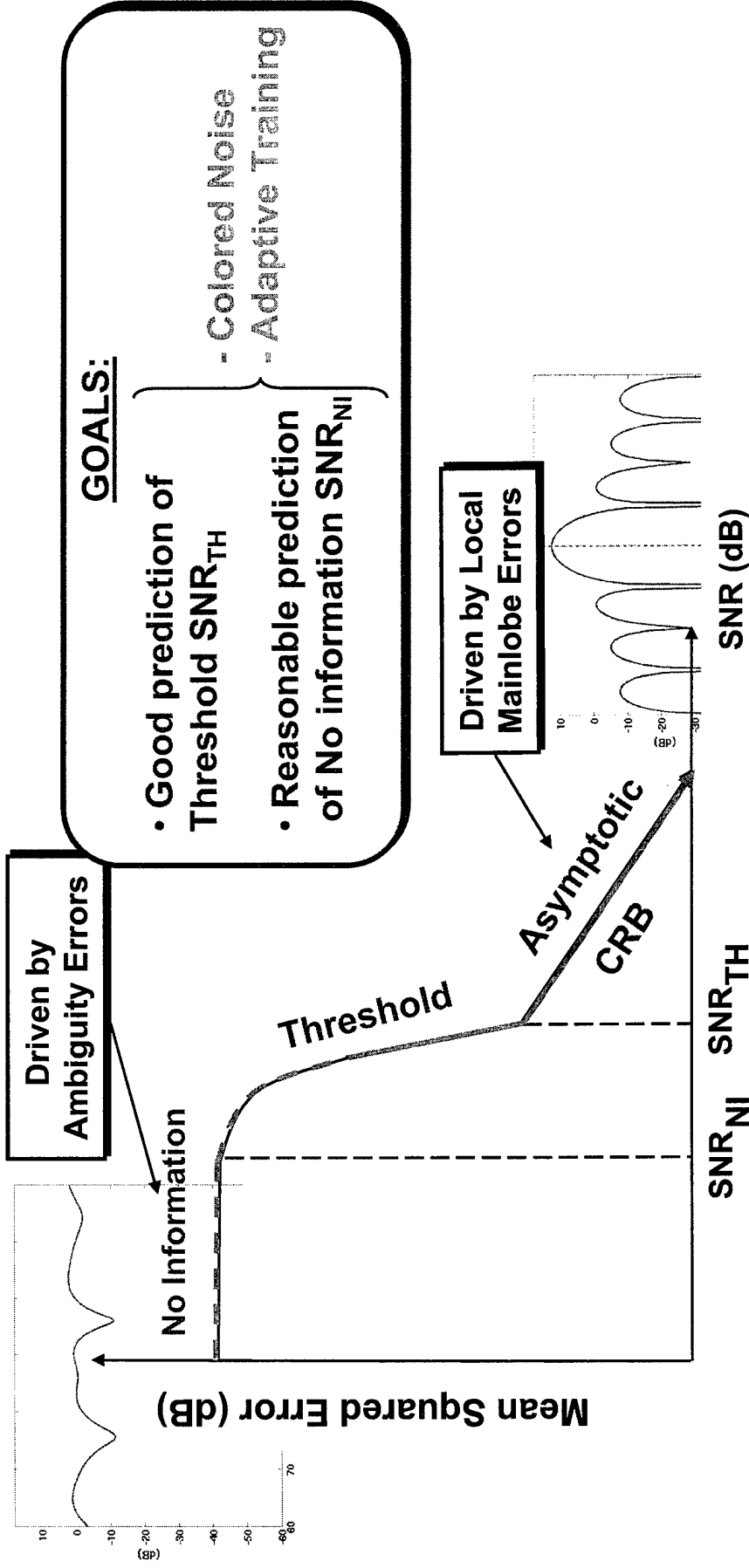
Goals of Analysis



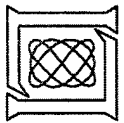
- **Problem:**
 - Mean Squared Error (MSE) performance of Maximum-Likelihood (ML) signal parameter estimation unknown
 1. Colored Noise
 2. Finite Number of Colored Noise Only Training Samples
- **Goal:**
 - Develop robust theory for prediction of ML MSE
- **Proposed Method:**
 - Use Interval Error based method proposed by Van Trees 1968
 - Must derive/approximate probability of “interval error”



Typical Composite MSE Performance



- Three definitive regions of Signal-to-Noise-Ratio (SNR)
 - No Information, Threshold, and Asymptotic (CRB)
- Recall MSE = Estimator Variance + Estimator Bias



Previous Work

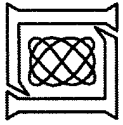


- K. Bell, Ph. D. George Mason University, 1995
- K. Bell, Y. Steinberg, Y. Ephraim, H. Van Trees, IEEE T-SP March 1997
- S. Pawlukiewicz, *4009*
- What's New ?
 - Colored Noise Allowed
 - Colored Noise Only Finite training effects
 - Exact two point error probabilities used
- F. Athley, IEEE

Ziv-Zakai Bounds

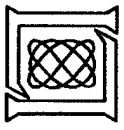
Method of Interval Error

- All previous work considered non-adaptive and white noise only case
- Error probabilities approximated via Chernoff Bounds



Outline

- Introduction
- Theory
 - Maximum-Likelihood Estimation (MLE)
 - Interval Error Based Method of MSE Prediction
- Numerical Results
- Conclusions



Maximum-Likelihood Signal Parameter Estimation

Data Model: $\pi^{-N} |\mathbf{R}|^{-1} \exp\{-[\mathbf{x} - \mathbf{S}\mathbf{v}(\theta)]^H \mathbf{R}^{-1} [\mathbf{x} - \mathbf{S}\mathbf{v}(\theta)]\}$

\mathbf{S} unknown

ML Estimator: $\theta_{ML} = \arg \max_{\theta} t_{MF}(\theta)$ $t_{MF}(\theta) = \frac{|\mathbf{v}^H(\theta) \mathbf{R}^{-1} \mathbf{x}|^2}{\mathbf{v}^H(\theta) \mathbf{R}^{-1} \mathbf{v}(\theta)}$

**Clairvoyant
Matched Filter**

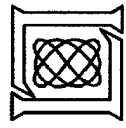
Data Model: $\pi^{-N(L+1)} |\mathbf{R}|^{-(L+1)} \exp\{-[\mathbf{x} - \mathbf{S}\mathbf{v}(\theta)]^H \mathbf{R}^{-1} [\mathbf{x} - \mathbf{S}\mathbf{v}(\theta)] - \text{tr}(\mathbf{R}^{-1} \mathbf{X}\mathbf{X}^H)\}$

\mathbf{R} unknown
 \mathbf{S} unknown

ML Estimator: $\theta_{ML} = \arg \max_{\theta} t_{AMF}(\theta)$ $t_{AMF}(\theta) = \frac{|\mathbf{v}^H(\theta) \hat{\mathbf{R}}^{-1} \mathbf{x}|^2}{\mathbf{v}^H(\theta) \hat{\mathbf{R}}^{-1} \mathbf{v}(\theta)}$ $\hat{\mathbf{R}} \equiv \frac{1}{L} \mathbf{X}\mathbf{X}^H$

**Adaptive
Matched Filter**

- **Complex Gaussian data model: All snapshots $N \times 1$**
 - Arbitrary $N \times N$ Colored Covariance
 - Deterministic Signal (“Conditional”)
- **Colored noise only training samples available**
- **Single scalar signal parameter**
 - Joint signal parameter estimation not considered



Method of ML MSE Prediction: Based on Interval Errors

- In general MSE can be written as the sum of two terms

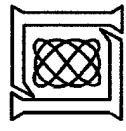
$$E\left\{\left(\hat{\theta} - \theta\right)^2\right\} = \Pr(\text{No Interval Error}) E\left\{\left(\hat{\theta} - \theta\right)^2\right\} \Big| \text{No Interval Error} \left. \vphantom{E\left\{\left(\hat{\theta} - \theta\right)^2\right\}} \right\} + \Pr(\text{Interval Error}) E\left\{\left(\hat{\theta} - \theta\right)^2\right\} \Big| \text{Interval Error} \left. \vphantom{E\left\{\left(\hat{\theta} - \theta\right)^2\right\}} \right\}$$

- MSE for Deterministic Signal Parameters **SINR Loss!!**

$$E\left\{\left(\hat{\theta} - \theta\right)^2\right\} \Big| \theta_k \left. \vphantom{E\left\{\left(\hat{\theta} - \theta\right)^2\right\}} \right\} = \left[1 - \sum_{\substack{n=1 \\ n \neq k}}^K p(\hat{\theta} = \theta_n | \theta_k) \right] \cdot \text{CRB}(\theta_k) + \sum_{\substack{n=1 \\ n \neq k}}^K p(\hat{\theta} = \theta_n | \theta_k) (\theta_n - \theta_k)^2$$

- Challenge is accurate calculation of error probabilities given by

$$p(\hat{\theta} = \theta_n | \theta_k) = ?$$



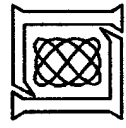
Union Bound (UB) Approximation: Interval Error Probabilities

- Recall the ML approach: $\hat{\theta} = \underset{\theta_1, \theta_2, \dots, \theta_K}{\operatorname{argmax}} t(\theta)$
- The probability of interval error is bounded by the relation

UNION BOUND

$$p(\hat{\theta} = \theta_n | \theta_k) = \Pr \left\{ \bigcup_{\substack{k=1 \\ k \neq n}}^K [t(\theta_n) > t(\theta_k)] \mid \theta = \theta_k \right\} \leq \sum_{\substack{k=1 \\ k \neq n}}^K \Pr [t(\theta_n) > t(\theta_k) | \theta = \theta_k]$$

- UB is a useful tool for computation of error probabilities in Digital Communication Schemes
 - Approximation relies on two point error probabilities
- UB often over estimates error in “No Information” region of the MSE curve



Two Point Probabilities for the Adaptive Matched Filter: R unknown

- Let $\begin{bmatrix} t_{AMF}(\theta_n) \\ t_{AMF}(\theta_k) \end{bmatrix} = \begin{bmatrix} |y_{AMF,1}|^2 \\ |y_{AMF,2}|^2 \end{bmatrix}$; the desired probability can be written

$$\Pr[t_{AMF}(\theta_n) > t_{AMF}(\theta_k) | \theta = \theta_k] = \Pr \left\{ \mathbf{y}_{AMF}^H \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{y}_{AMF} < 0 \right\}$$

- It can be shown that AMF outputs can be written equal in distribution


$$\mathbf{y}_{AMF} = \begin{bmatrix} |y_{AMF,1}|^2 \\ |y_{AMF,2}|^2 \end{bmatrix} = \begin{bmatrix} \sqrt{a_{11}} & \sqrt{a_{12}} \\ \sqrt{a_{21}} & \sqrt{a_{22}} \end{bmatrix} (\mathbf{V}^H \mathbf{R}^{-1} \mathbf{V})^{-1/2} \mathbf{x}_{AMF} \quad \text{where}$$

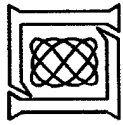
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \sim CW(L - N + 2, \mathbf{V}^H \mathbf{R}^{-1} \mathbf{V}) \quad \text{and} \quad \mathbf{x}_{AMF} \sim CN_{2 \times 1} \left(\mathbf{s} \begin{bmatrix} \sqrt{\mathbf{v}(\theta_n) \mathbf{R}^{-1} \mathbf{v}(\theta_n)} & 1 \\ 0 & \beta_{L-N+3, N-2} \end{bmatrix} \right)$$

- The necessary two point probabilities can be thus obtained

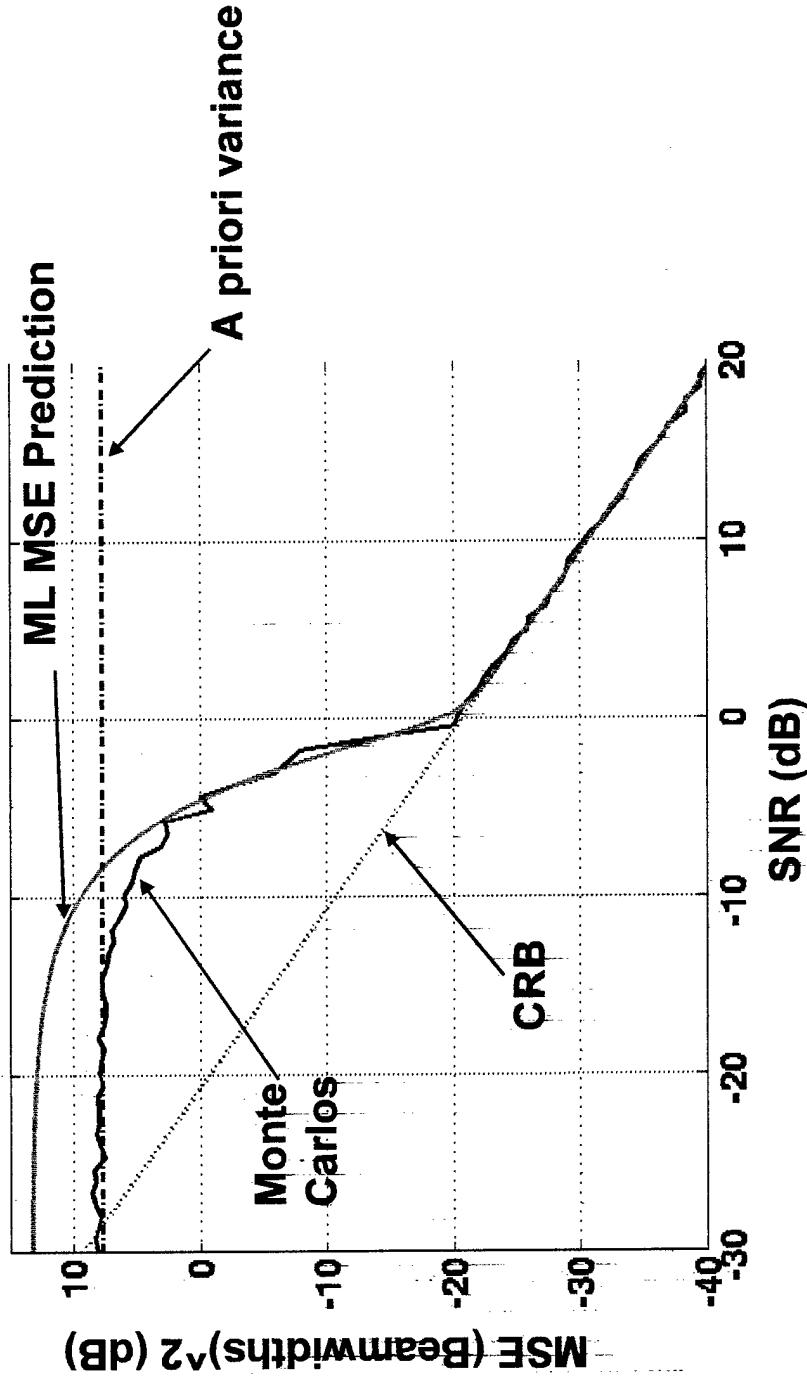


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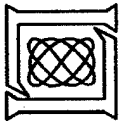
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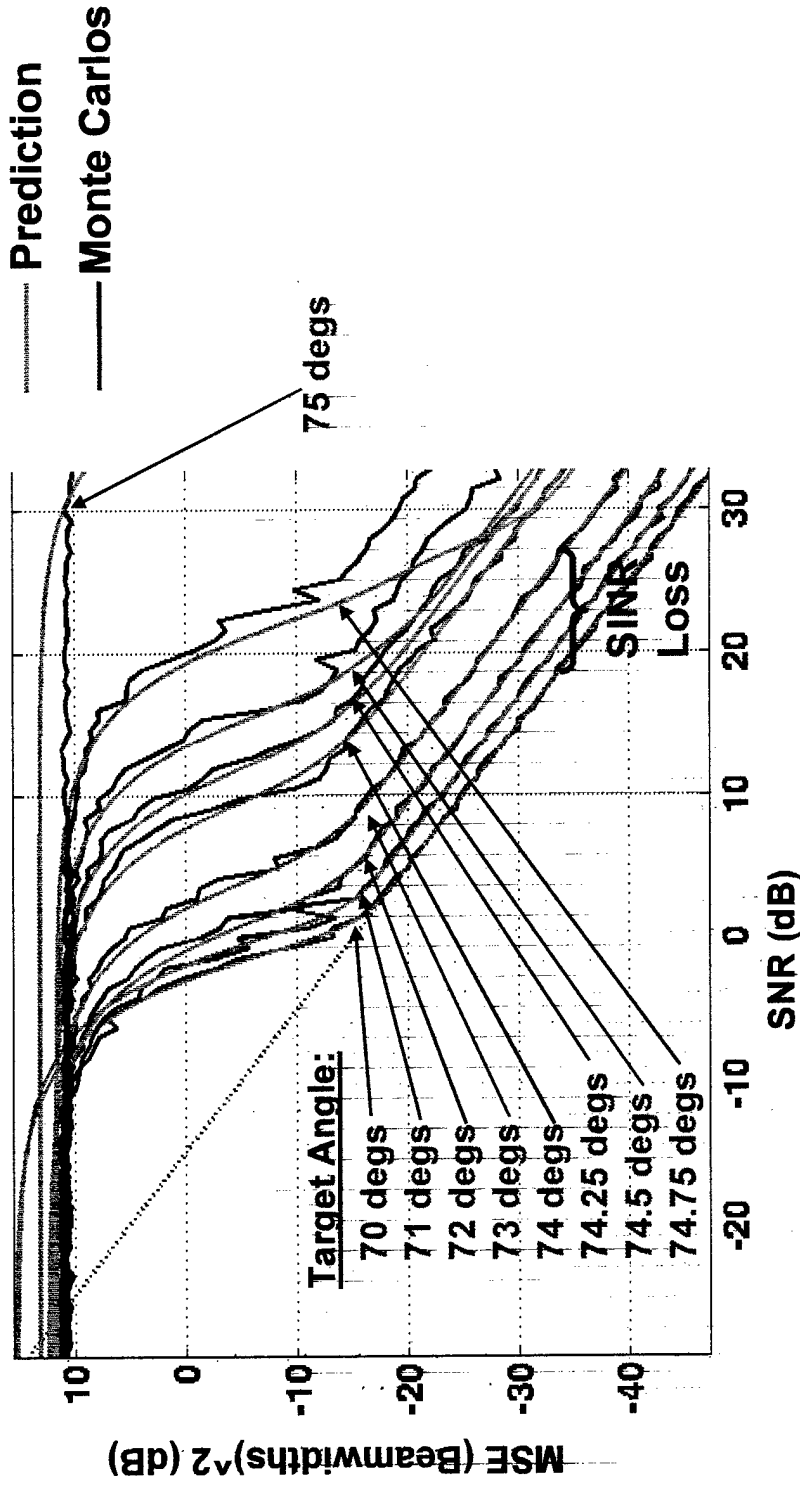
White Noise Example: R known



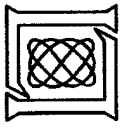
- N=18 element ULA, ($\lambda/2.25$) element spacing, broadside at 90 degs, endfire at 0 and 180 degs
- 0dB white noise



Colored Noise Example: R known



- N=18 element ULA, ($\lambda/2.25$) element spacing, broadside at 90 degs, endfire at 0 and 180 degs
- 0dB white noise plus 30dB Jammer at 75 degs

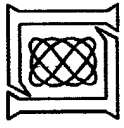


White Noise Example:

R unknown

Need Figure

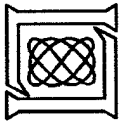
- N=18 element ULA, ($\lambda/2.25$) element spacing, broadside at 90 degs, endfire at 0 and 180 degs
- 0dB white noise
- Adaptive Training: L = 1.5N, 2N, and 3N



Colored Noise Example: R unknown

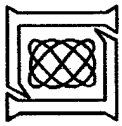
Need Figure

- **N=18 element ULA, ($\lambda/2.25$) element spacing, broadside at 90 degs, endfire at 0 and 180 degs**
- **0dB white noise plus 30dB Jammer at 75 degs**
- **Adaptive Training: L = 1.5N, 2N, and 3N**



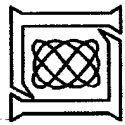
Conclusions

- Interval error method represents a viable and numerically efficient technique
 - Theory and simulation have very good match
 - UB overestimates MSE, however, in “No Information” region
- Two point probabilities have been computed in closed form
 - Colored Noise
 - Adaptive Finite Training Effects
- Established a the notion of SINR Loss for the parameter estimation problem



Future Work

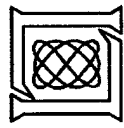
- Explore tighter bounds on probability of interval errors than that given by the Union Bound
 - Expurgating terms of Union Bound, for example
- Extend to Stochastic / Unconditional signal models
- Generalize to vector signal parameters
- Comparisons with Bayesian Bound predictions
 - Ziv-Zakai, Weiss-Weinstein, etc.



Backups

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Method of ML MSE Prediction: Based on Interval Errors

- In general MSE can be written as the sum of two terms

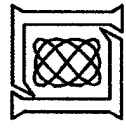
$$E\left\{(\hat{\theta} - \theta)^2\right\} = \Pr(\text{No Interval Error}) E\left\{(\hat{\theta} - \theta)^2 \mid \text{No Interval Error}\right\} \\ + \Pr(\text{Interval Error}) E\left\{(\hat{\theta} - \theta)^2 \mid \text{Interval Error}\right\}$$

- Deterministic Signal Parameters

$$E\left\{(\hat{\theta} - \theta)^2 \mid \theta_k\right\} = \Pr(\text{No Interval Error} \mid \theta_k) \cdot \text{CRB}(\theta_k) + \sum_{n=1}^K \Pr(\hat{\theta} = \theta_n \mid \theta_k) (\theta_n - \theta_k)^2$$

$$E\left\{(\hat{\theta} - \theta)^2\right\} = \int_{\hat{\Theta}} (\hat{\theta} - \theta)^2 p(\theta \mid \theta) d\hat{\theta}$$

$$E\left\{(\hat{\theta} - \theta)^2 \mid \theta\right\} = \int_{\hat{\Theta}:\text{MAINLOBE}} (\hat{\theta} - \theta)^2 p(\hat{\theta} \mid \theta) d\hat{\theta} + \int_{\hat{\Theta}:\text{AMBIGUITIES}} (\hat{\theta} - \theta)^2 p(\hat{\theta} \mid \theta) d\hat{\theta}$$



Two Point Probabilities for the Matched Filter: R known

- These probabilities are expressible in terms of the Marcum Q-function:

$$\Pr[t_{MF}(\theta_n) > t_{MF}(\theta_k) | \theta = \theta_k] = \Pr \left[\frac{\chi_1^2(m_1)}{\chi_1^2(m_2)} \leq \frac{-\lambda_{VX,2}}{\lambda_{VX,1}} \right] =$$
$$\left[\frac{\lambda_{VX,2}}{\lambda_{VX,2} - \lambda_{VX,1}} \right] \cdot \left\{ \frac{-\lambda_{VX,1}}{\lambda_{VX,2}} Q_1 \left(\sqrt{\frac{2|m_1|^2 \lambda_{VX,1}}{\lambda_{VX,1} + \lambda_{VX,2}}}, \sqrt{\frac{2|m_2|^2 \lambda_{VX,2}}{\lambda_{VX,2} - \lambda_{VX,1}}} \right) + [1 - Q_1 \left(\sqrt{\frac{2|m_2|^2 \lambda_{VX,2}}{\lambda_{VX,2} - \lambda_{VX,1}}}, \sqrt{\frac{2|m_1|^2 \lambda_{VX,1}}{\lambda_{VX,1} - \lambda_{VX,2}}} \right)] \right\}$$



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