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# Resolution of Mainlobe and Sidelobe Detections using Model Order Determination

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Adaptive Sensor Array Processing Workshop ( ASAP)

March 13-14, 2001

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# Report Documentation Page

*Form Approved*  
*OMB No. 0704-0188*

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1. REPORT DATE <b>14 MAR 2001</b>	2. REPORT TYPE <b>N/A</b>	3. DATES COVERED <b>-</b>	
4. TITLE AND SUBTITLE <b>Resolution of Mainlobe and Sidelobe Detections using Model Order Determination</b>		5a. CONTRACT NUMBER	
		5b. GRANT NUMBER	
		5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) <b>Amin G. Jaffer; Joe C. Chen; Thomas W. Miller</b>		5d. PROJECT NUMBER	
		5e. TASK NUMBER	
		5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Raytheon</b>		8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) <b>Defense Advanced Research Projects Agency 3701 North Fairfax Drive Arlington, VA 22203-1714</b>		10. SPONSOR/MONITOR'S ACRONYM(S)	
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release, distribution unlimited</b>			
13. SUPPLEMENTARY NOTES <b>See ADM001263 for entire Adaptive Sensor Array Processing Workshop., The original document contains color images.</b>			
14. ABSTRACT <b>See briefing slides.</b>			
15. SUBJECT TERMS			
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>	<b>UU</b>
			18. NUMBER OF PAGES <b>24</b>
			19a. NAME OF RESPONSIBLE PERSON

# False Sidelobe and Multiple Target Resolution Problem

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- **Adaptive beamforming/filtering and STAP methods can result in high sidelobe levels, especially with limited sample support for estimating interference covariance matrix**
- **This can cause excessive “false” sidelobe detections arising from targets or undernulled interferences**
- **Sidelobe rejection capabilities of adaptive matched filter (AMF), generalized likelihood ratio test (GLRT) and AMF/ACE have been previously analyzed**

# Multiple Target and Sidelobe Detection using Weighted Least-Squares Fit to AMF and GLRT Data

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- For simplicity of exposition, we will consider the spatial domain only although the method is directly applicable to the angle-Doppler domain
- The proposed method uses the output detection test statistic computed over the entire angular extent of interest
- In any given range cell the question is whether the totality of output test statistic values computed over the beam directions of interest that exceed a preset threshold represent one target, two targets or up to a maximum  $M$  targets, i.e., the question is one of model order determination

# J-amf and J-glrt Functions - Normalized Adaptive Array Outputs

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- J-amf( $q$ ) represents the normalized adaptive array power output as a function of the angle  $q$

$$J_{AMF}(q) = \frac{|d^H(q)\hat{R}^{-1}x|^2}{d^H(q)\hat{R}^{-1}d(q)} = \frac{|w^H(q)x|^2}{w^H(q)\hat{R}w(q)}$$

- J-glrt( $q$ ) is J-amf( $q$ ) further normalized by data power term

$$J_{GLRT}(q) = \frac{J_{AMF}(q)}{1 + x^H \hat{R}^{-1} x / K}$$

- $\hat{R} = \frac{1}{K} \sum_{k=1}^K \mathbf{a} x_k x_k^H$   $x_k$ 's are secondary set of signal - free data

**K** - - Number of snapshots

# Normalized Adaptive Array Outputs

- - - continued

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- **Diagonal Loading and shaded (tapered) steering vector:**

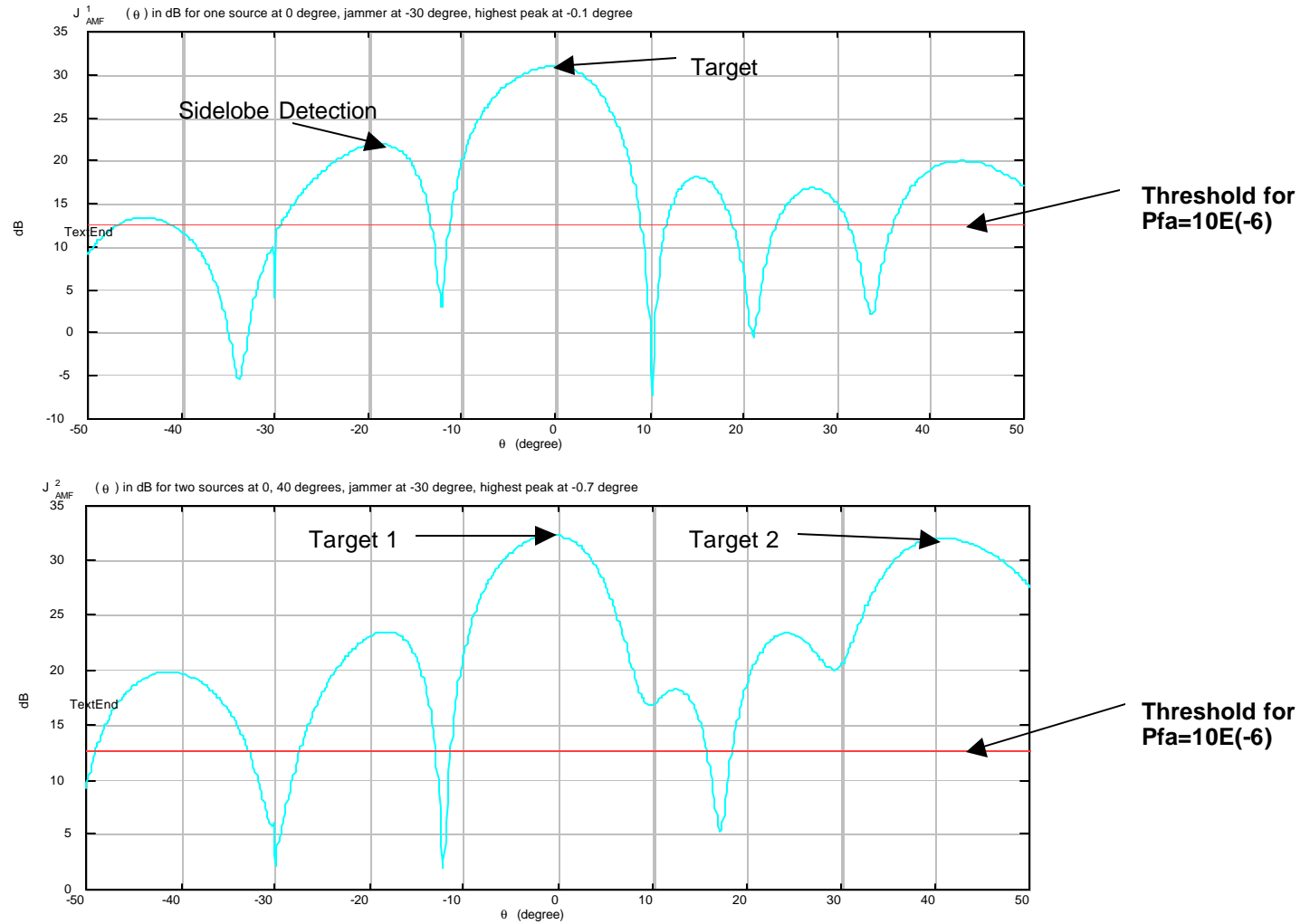
$$\hat{R}_{DL} = \hat{R} + aI$$

$d_{sh} = w_{sh} \cdot d$ ,  $\cdot$  represents the Schur product,  $w_{sh}$  is a shading or taper function

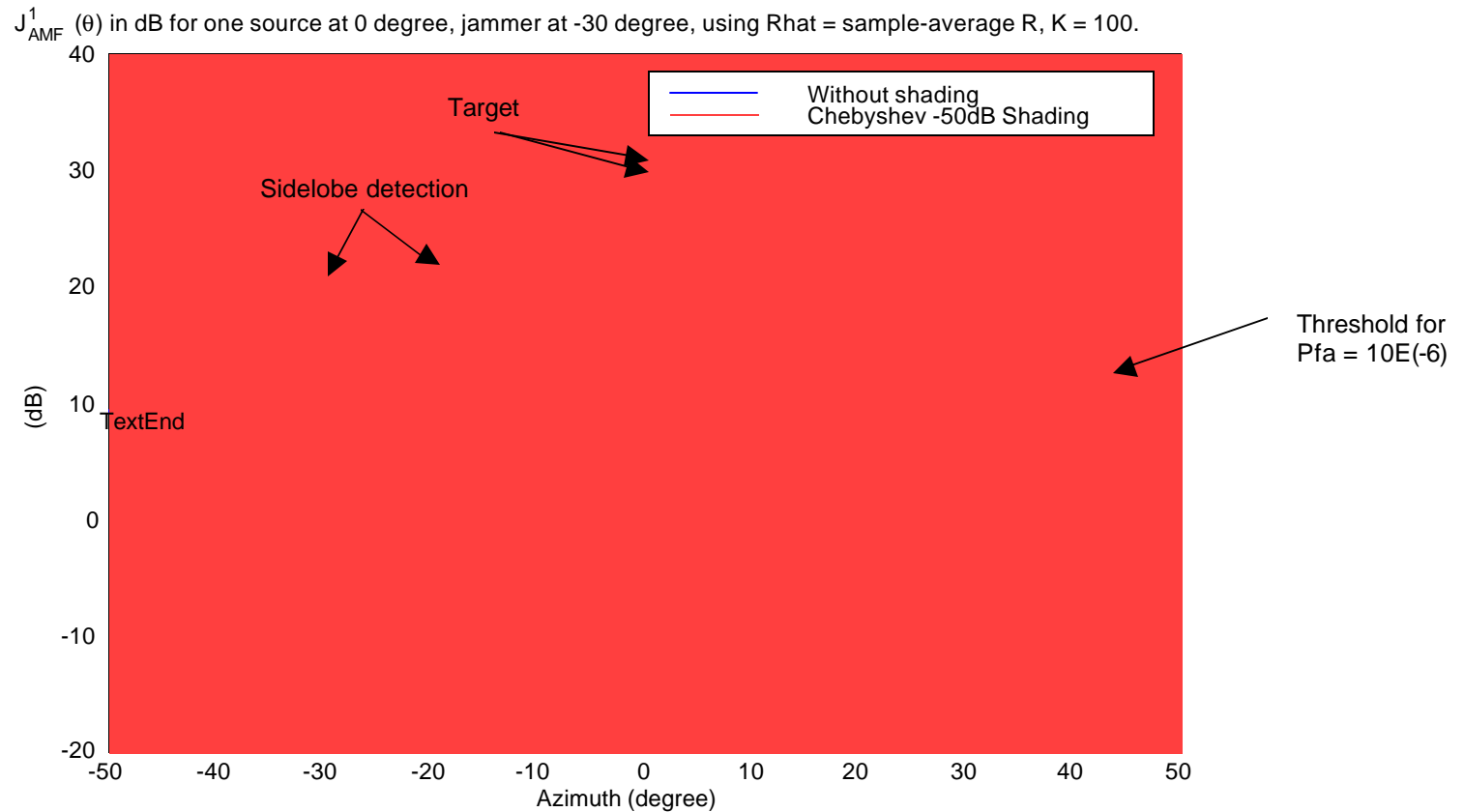
- $w(q) = \hat{R}_{DL}^{-1} d_{sh}$  is also used in the J-amf( $q$ ) and J-glrt( $q$ ) functions

# J-amf Function for One and Two Targets - - - No Taper, K=100

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# J-amf function -- unshaded steering vector and 50 dB Chebyshev taper



- False sidelobe detections can occur in adaptive arrays even with 50 dB Chebyshev taper

# Weighted Least-Squares and Model Order Determination Method

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- Determine  $L$  peaks ( or values adjacent to peaks) of  $J_{AMF}(q)$  at  $q_1, q_2, \dots, q_L$  which exceed preset threshold for given  $P_{FA}$

- Test data vector  $x$  is modeled as

$$x = D_s a + n$$

where  $D_s = [d(q_{s1}), d(q_{s2}), \dots, d(q_{sM})]$  an  $N$  by  $M$  matrix

$a$  and  $n$  are the complex signal amplitude vector and interference plus noise vectors

- The application of the weight vector  $w(q)$  to the data yields

$$y(q) = w(q)^H x = w(q)^H D_s a + v$$

# WLS Method .....continued

- where  $v(q) = w(q)^H n$

$y(q)$  is evaluated at L distinct points

- The transformed signal model is fit to the  $y(q)$  data in a weighted least-squares sense and the residual is evaluated

$$\begin{matrix}
 \hat{y}_1 \\
 \hat{y}_2 \\
 \hat{y}_3 \\
 \hat{y}_4 \\
 \hat{y}_L
 \end{matrix}
 \hat{u} =
 \begin{matrix}
 w(q_1)^H \\
 w(q_2)^H \\
 w(q_3)^H \\
 w(q_4)^H \\
 w(q_L)^H
 \end{matrix}
 D_s a \hat{u} +
 \begin{matrix}
 v(q_1) \\
 v(q_2) \\
 v(q_3) \\
 v(q_4) \\
 v(q_L)
 \end{matrix}
 \hat{u}
 \quad \text{or} \quad Y = Ha + V$$

- The residual is computed as

$$J_{res}(q_{s1}, \dots, q_{sM}) = \|I - P(q_{s1}, \dots, q_{sM})\|^2$$

## WLS Method .....continued2

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- $P(q_{s1}, \dots, q_{sM})$  is an orthogonal projection operator

For details, see paper

- $J_{res}(q_{s1}, \dots, q_{sM})$  is a function of  $q_{s1}, \dots, q_{sM}$  and needs to be minimized over those angles
- For computer simulation purposes, we limit ourselves to  $M=2$
- As a first approximation, we take  $q_{s1}$  and  $q_{s2}$  to be the two highest peaks --- computationally efficient
- Better approximation -- fix one angle at global peak and search over other to minimize  $J_{res}$  - - - more computation

# Akaike Information Criterion (AIC) for Model Order Determination

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- Compute AIC(m) for model order  $m=1,2,\dots,M$  and choose the minimum

$$\begin{aligned} \text{AIC}(m) &= J_{res} + \text{Number of free parameters in model} \\ &= J_{res} + 3m \end{aligned}$$

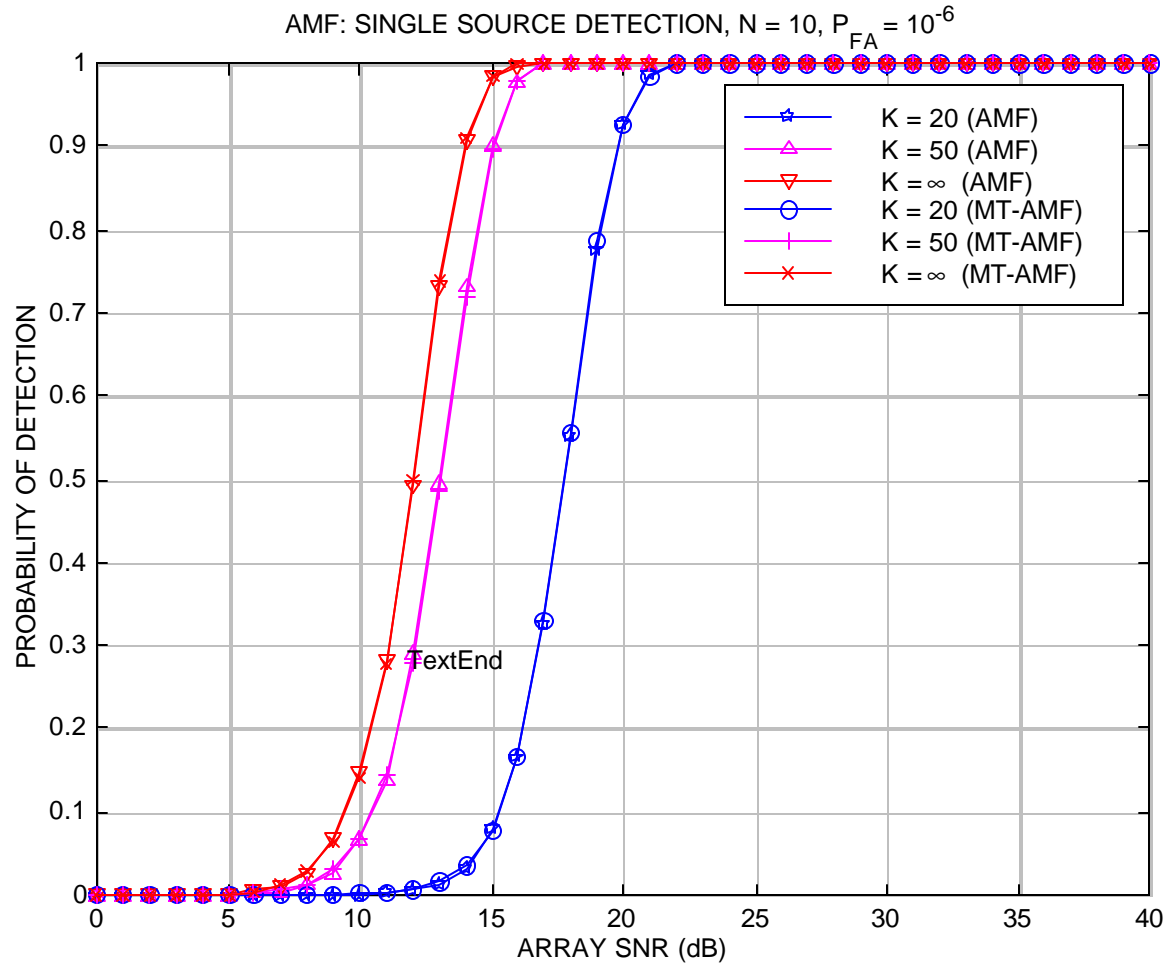
- Alternate Minimum Description Length (MDL) criterion is not applicable here because the second term ( the “penalty” term) becomes zero for single target signal snapshot

# Performance Evaluation by Monte-Carlo Computer Simulation

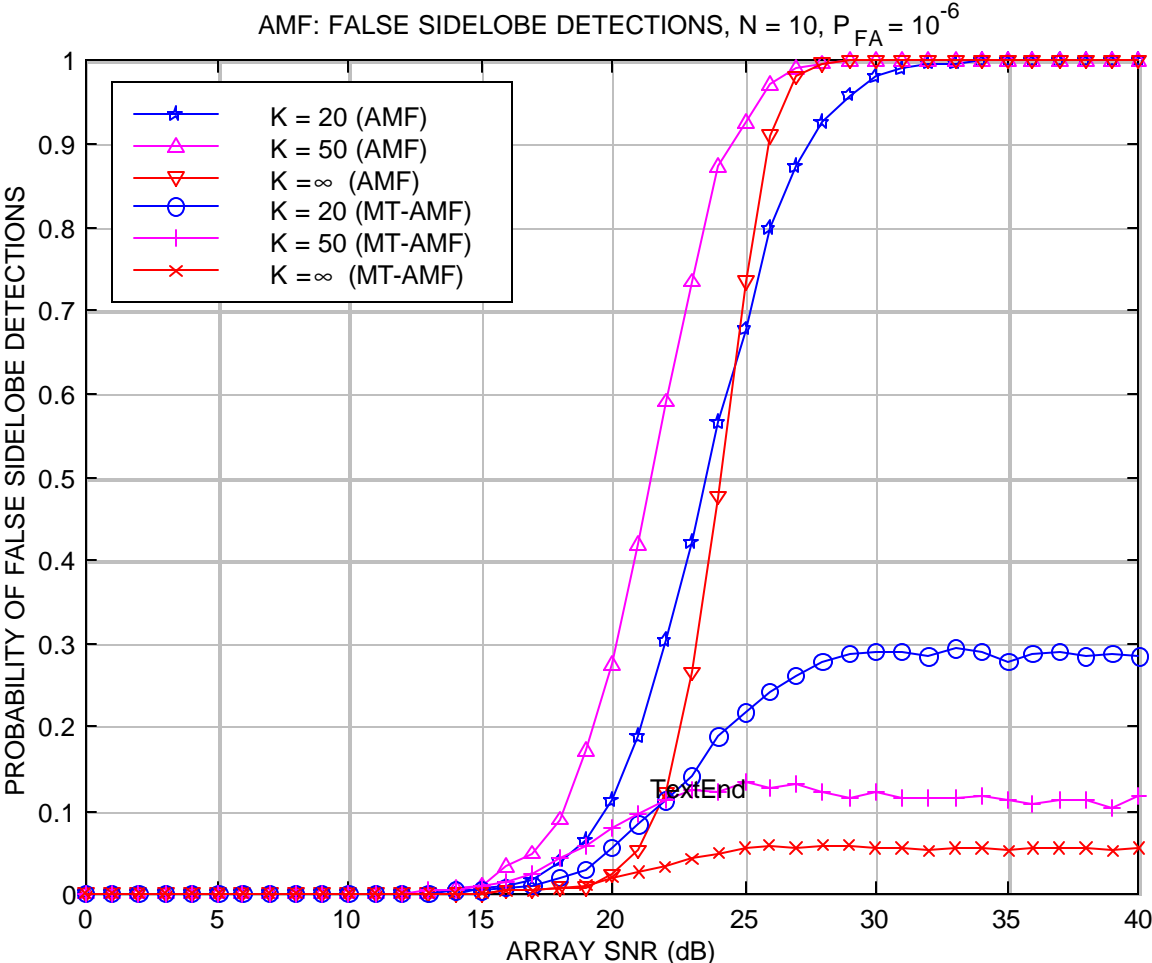
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- 10 element line array with half-wavelength inter-element spacing
- Single source placed at 0 degrees (broadside)  
Two sources at 0 and 45 degrees  
Noise jammer placed at -30 degrees, JNR=40dB
- Thresholds for AMF and GLRT methods to yield a specified  $P_{FA}$  computed in accordance with paper by F.C.Robey et.al.,  
IEEE Trans. on AES, March 1992
- The probability of detection was based on 5000 trials for each point on the curve
- A target detection was considered valid if it fell within plus or minus 3 dB beamwidth of the true target angle

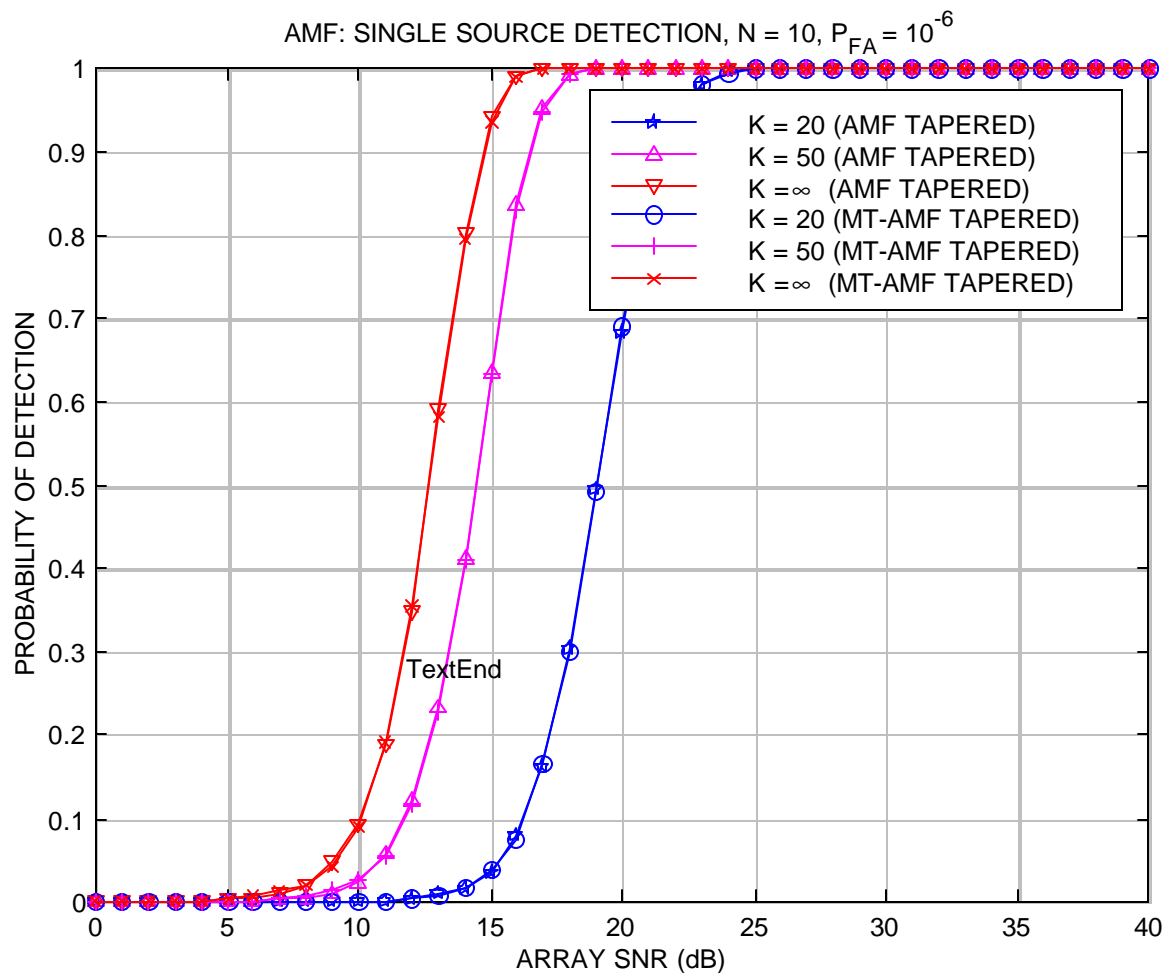
# Probability of Mainlobe Detection (AMF)



# Probability of Sidelobe Detections (AMF)

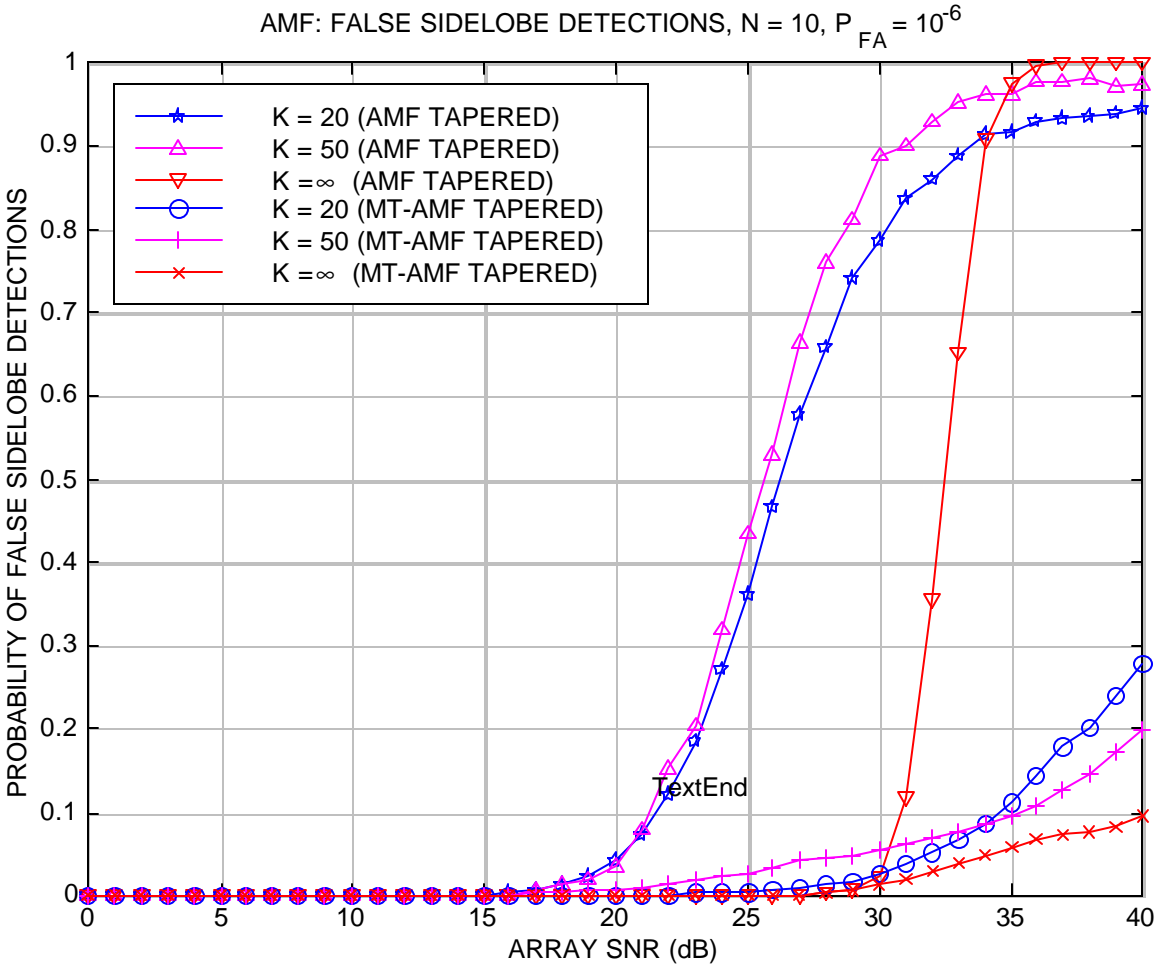


# Probability of Mainlobe Detection - - - AMF Taper



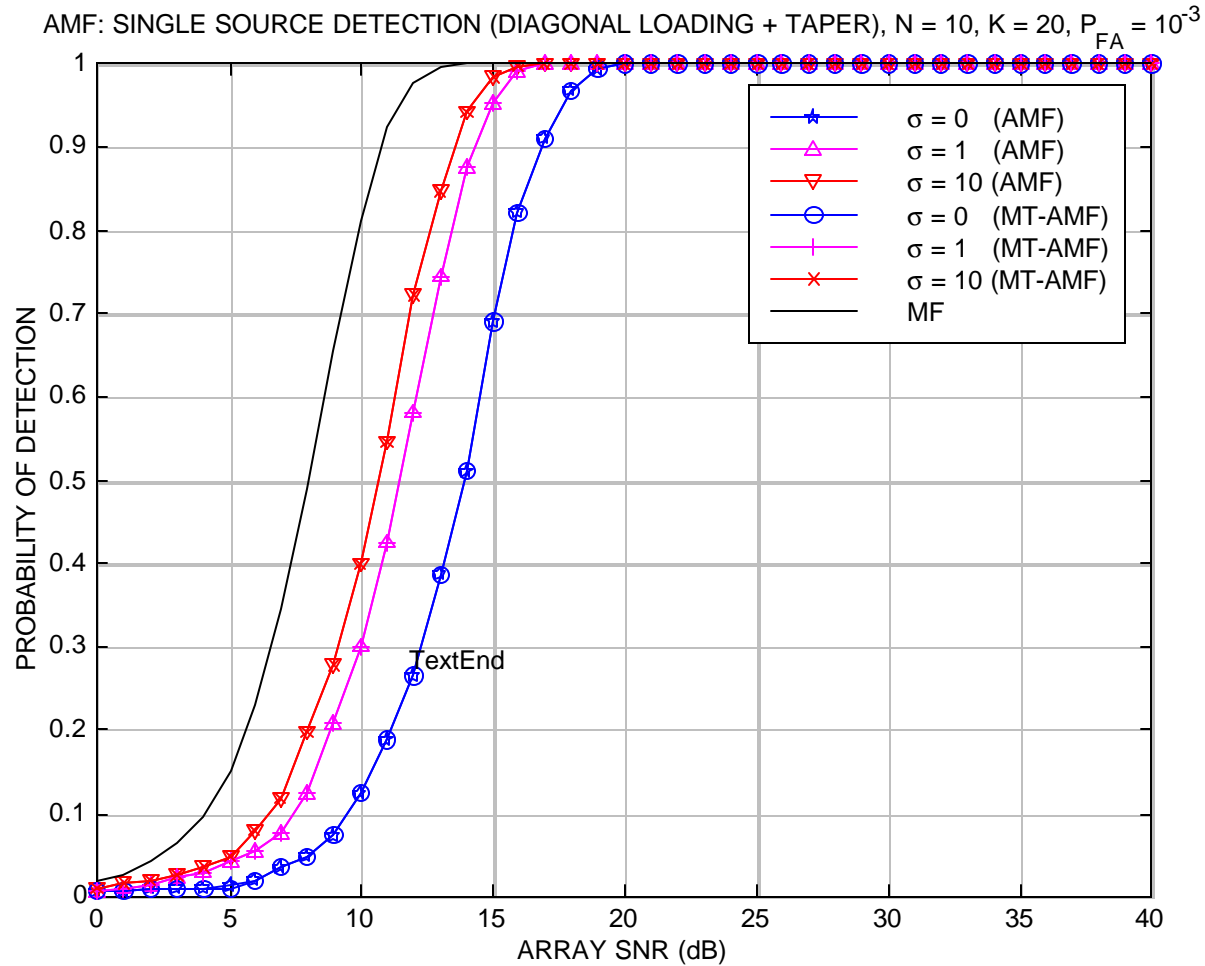
# Probability of Sidelobe Detection - - - AMF Taper

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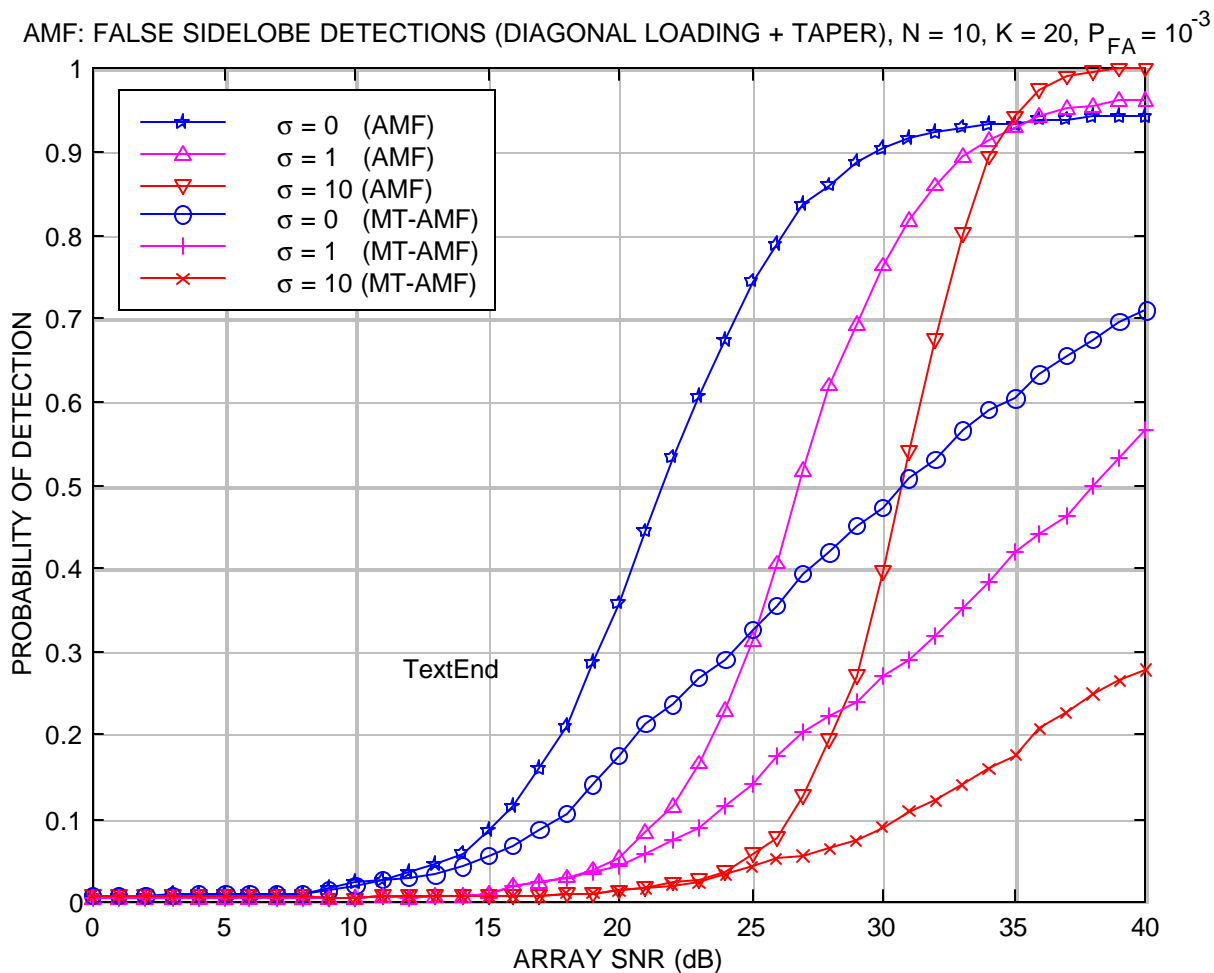
# Probability of Mainlobe Detection AMF Taper plus Diagonal Loading

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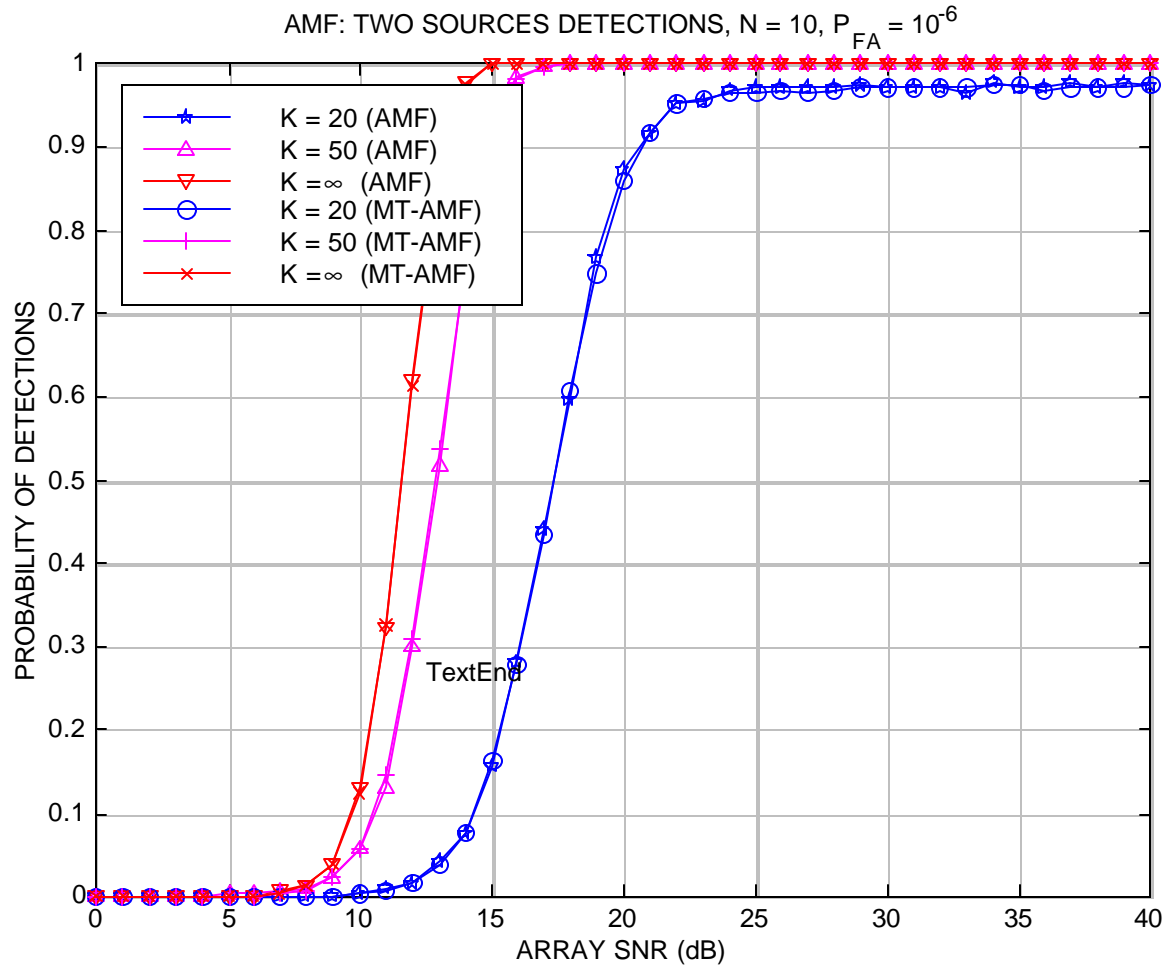
# Probability of Sidelobe Detection AMF Taper plus Diagonal Loading

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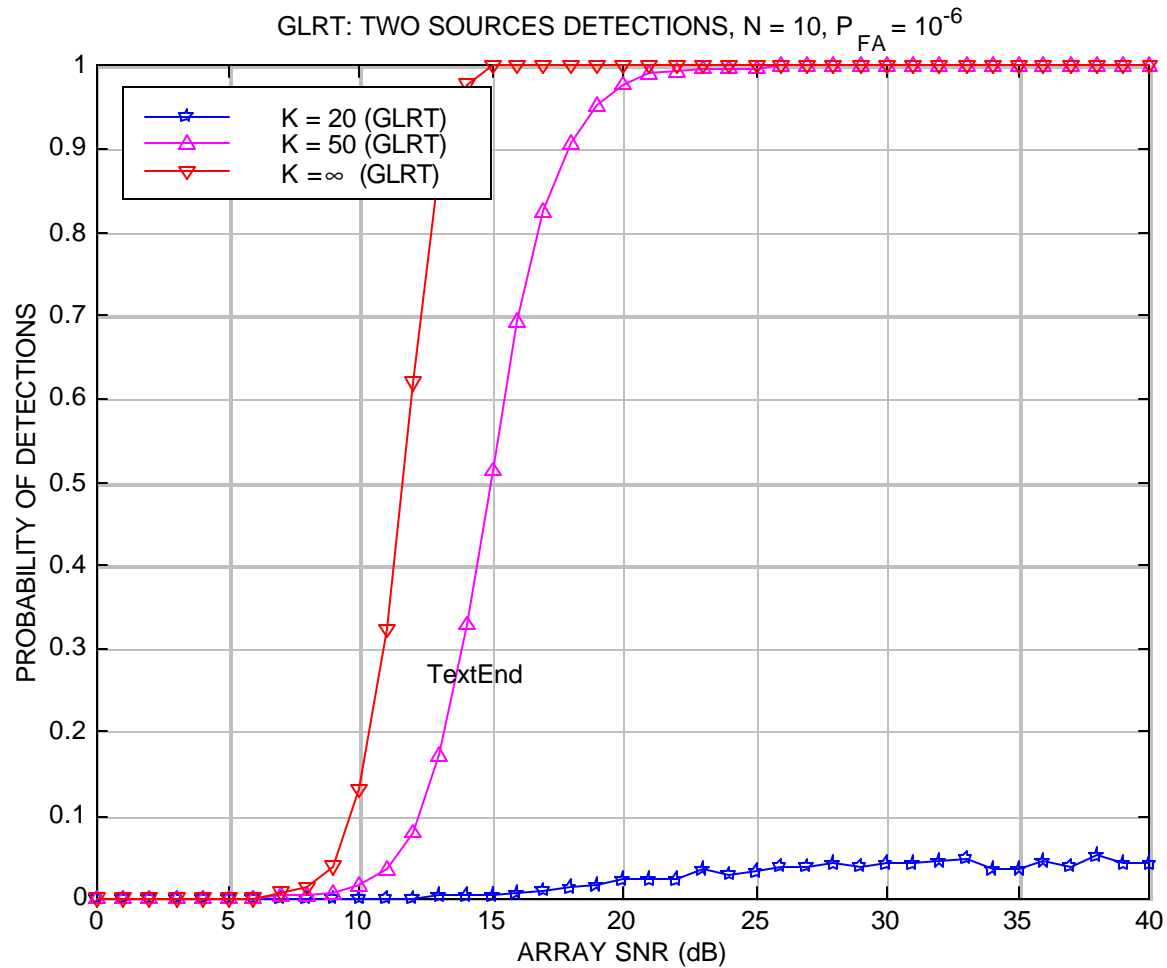
# Probability of Detecting Two Sources - AMF

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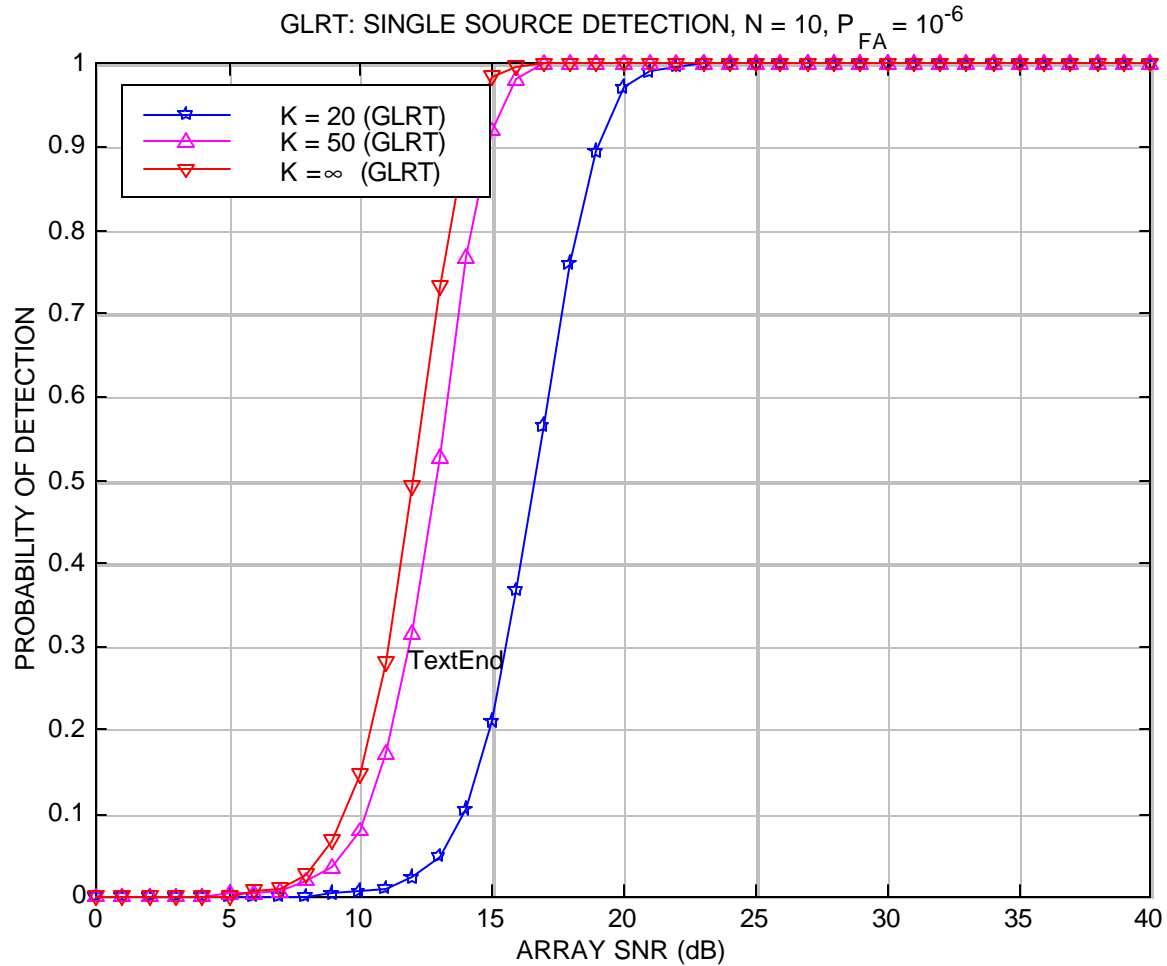
# Probability of Detecting Two sources - GLRT

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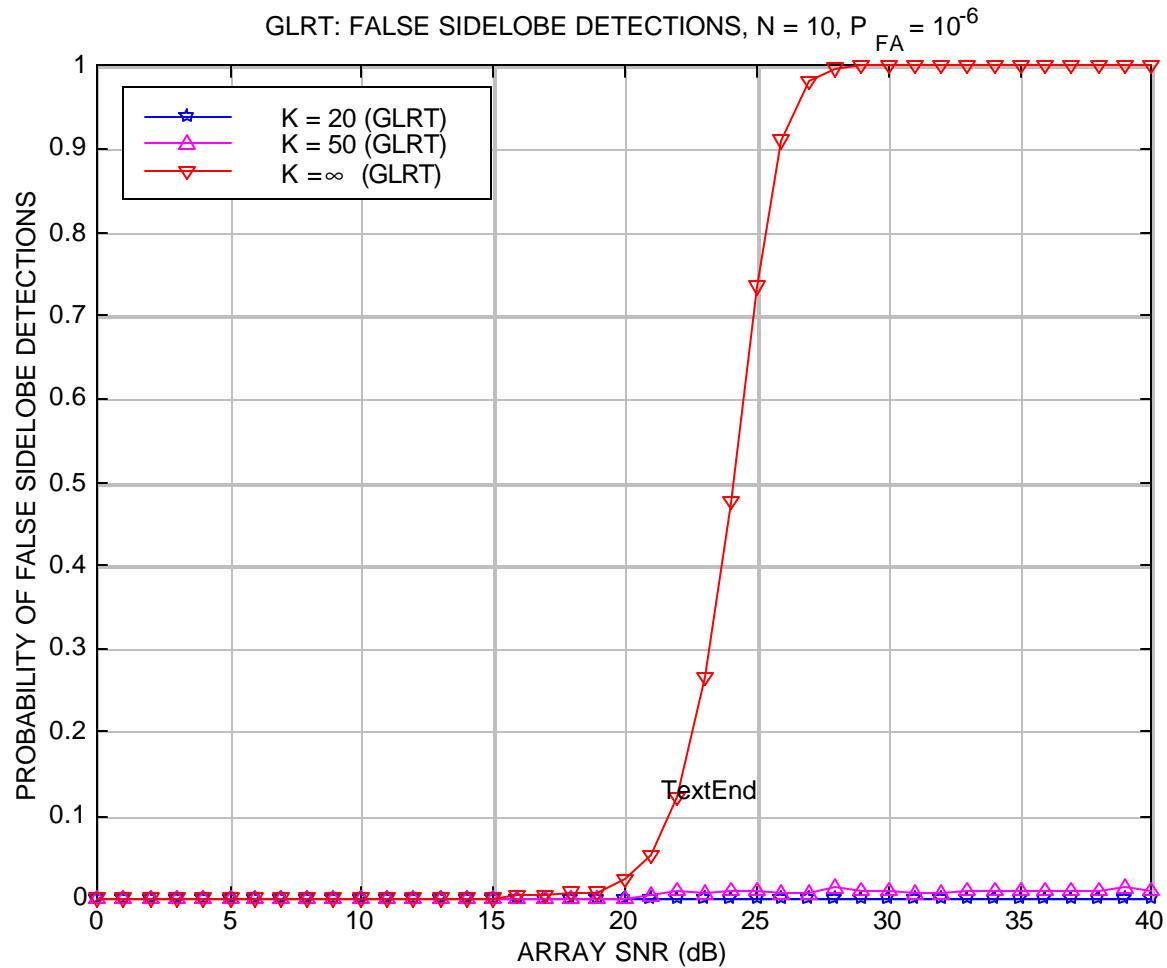
# Probability of Detection - - GLRT

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# Probability of Sidelobe Detection - - GLRT

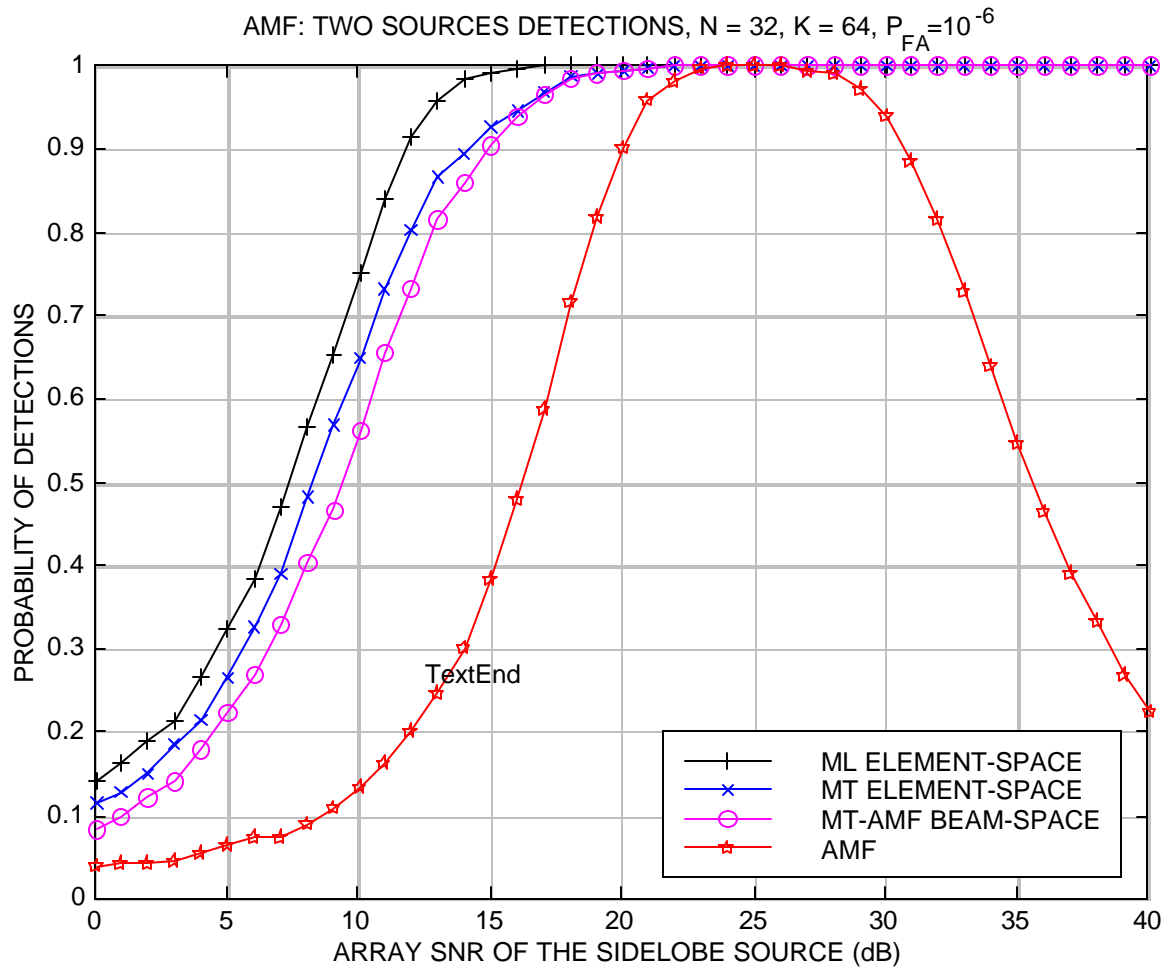
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# Probability of Detecting Two Sources

## - - - Various MT-AMF methods

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# Conclusions

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- **Proposed new method, using weighted least-squares fit to AMF or GLRT data combined with model order determination by Akaike Information Criterion, can significantly reduce false sidelobe detections**
- **This is true even when amplitude taper and diagonal loading is used**
- **The approximate method for angle estimation is computationally efficient and yields good detection and sidelobe rejection results**