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# **The Relationship Between Detection Algorithms for Hyperspectral and Radar Applications**

**Nirmal Keshava, Stephen M. Kogon, Dimitris Manolakis**

**March 14, 2001**

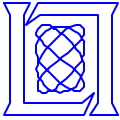
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Lexington, MA 02420**

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# Objective

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- **Overview of hyperspectral sensing**
- **Demonstrate how and why detection algorithms for hyperspectral imagery are related to detection algorithms for MTI radar**
  - **Similar physical assumptions**
  - **Common signal model**
- **Illustrate detection in hyperspectral imagery with real data and familiar detectors**



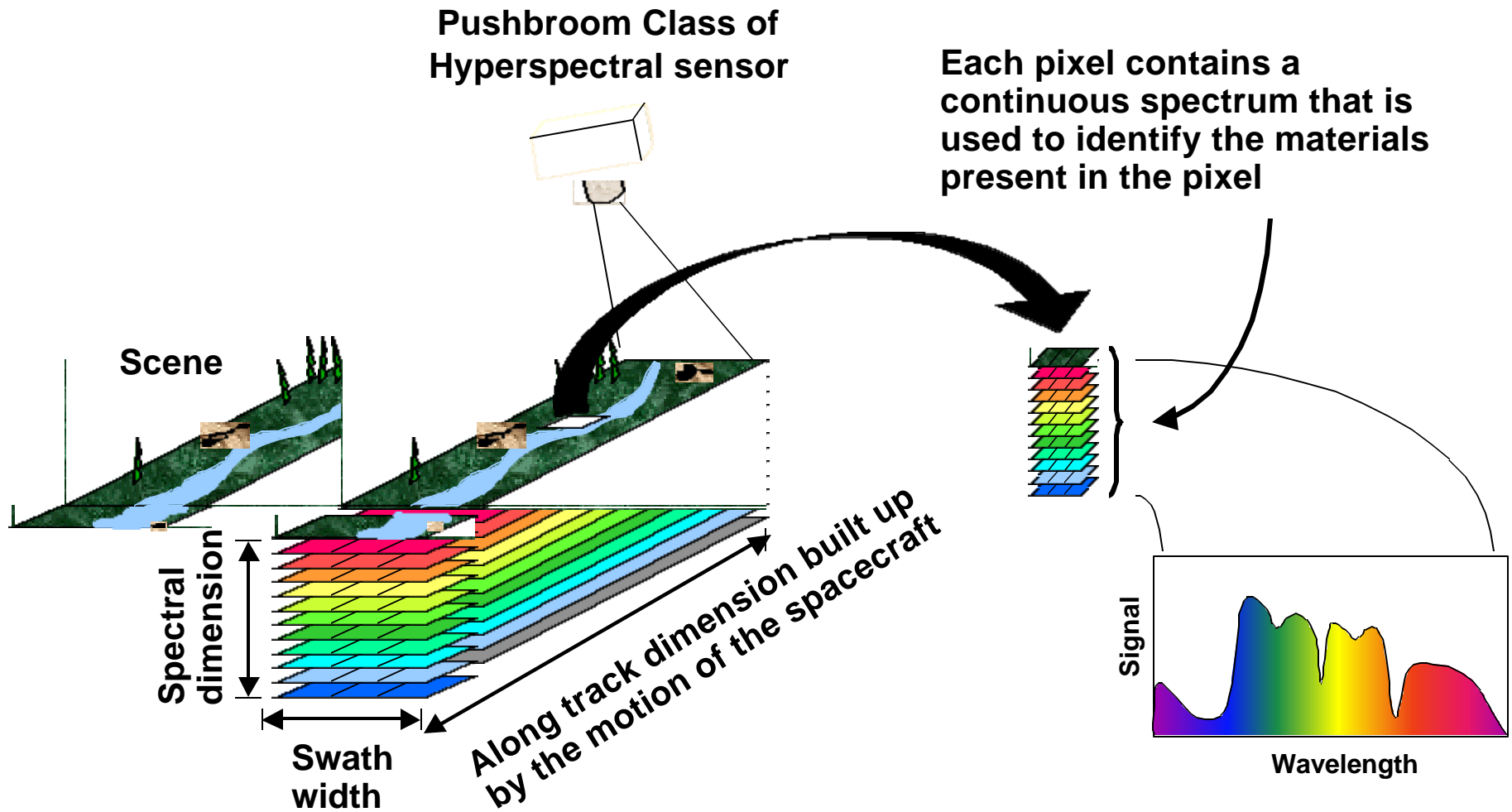
# Outline

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- **Introduction to hyperspectral sensing**
- **Signal models**
- **Detection models**
- **Hyperspectral detection results**
- **Conclusion**



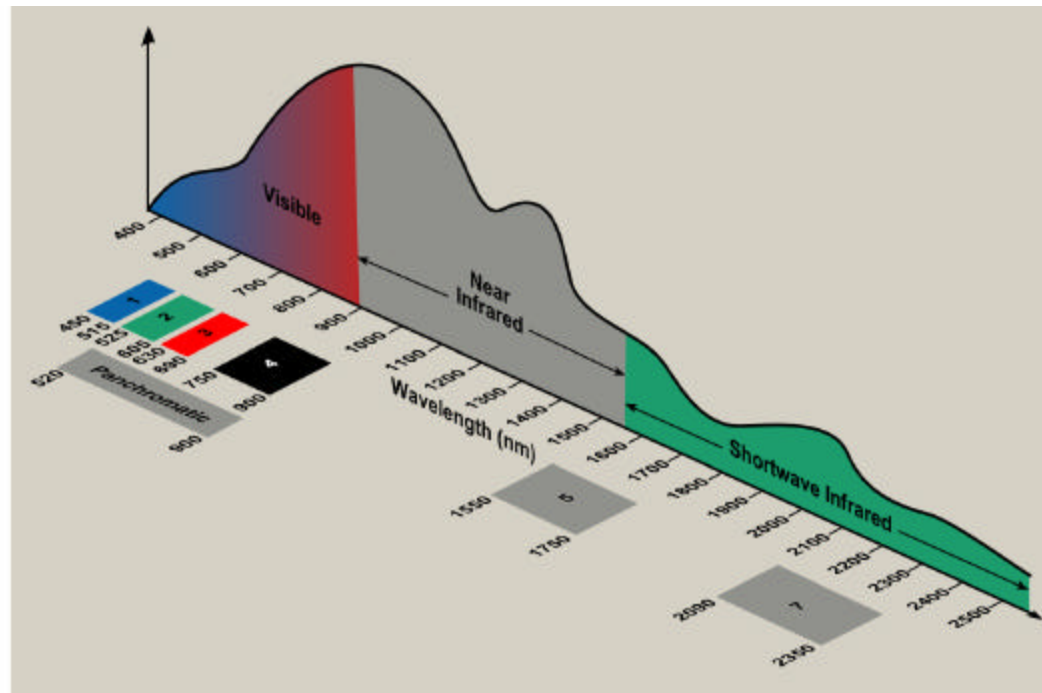
# Hyperspectral Imaging (HSI) Concept





# Hyperspectral Sensing

- **Hyperspectral imaging (HSI) is a form of *passive* imaging**
  - Extension of multispectral sensing (e.g., Landsat)
  - Hundreds of contiguous, real-valued spectral bands
  - Spatial resolution is a function of Instantaneous Field of View (IFOV) and altitude





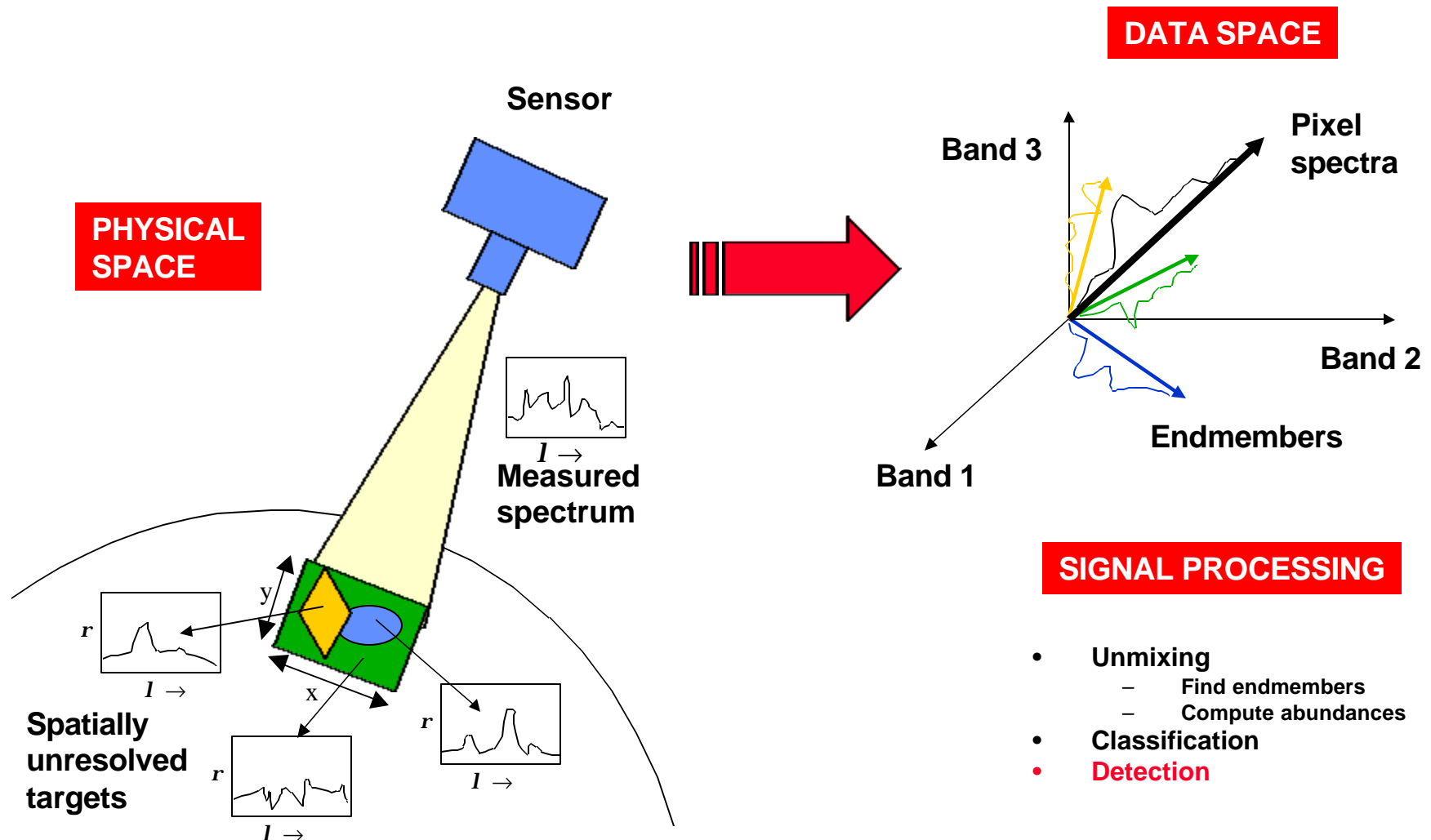
# Outline

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- Introduction to hyperspectral sensing
- **Signal models**
  - Hyperspectral sensing
  - MTI radar
- Detection models
- Hyperspectral detection results
- Conclusion



# Modeling of Spatially Unresolved (Mixed) Pixels





# Linear Mixing Model (LMM)

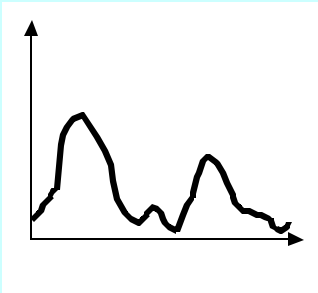
## Target and Background Modeling

$$\text{Test pixel } \mathbf{x} = \sum_{k=1}^{P_T} a_k \mathbf{s}_k + \sum_{k=P_T+1}^{P_T+P_B} a_k \mathbf{s}_k + \mathbf{n}$$

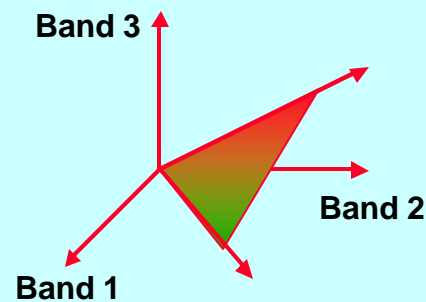
*abundance* ↓  
↑ *end member*

**LINEAR  
MIXING  
MODEL**

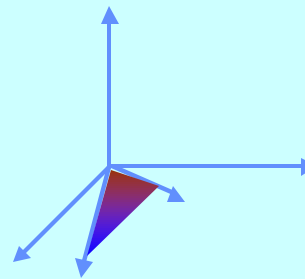
Measured spectrum



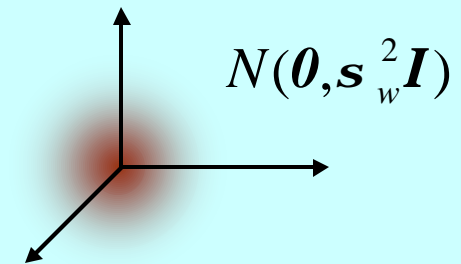
Target subspace

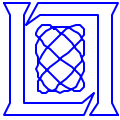


Background subspace

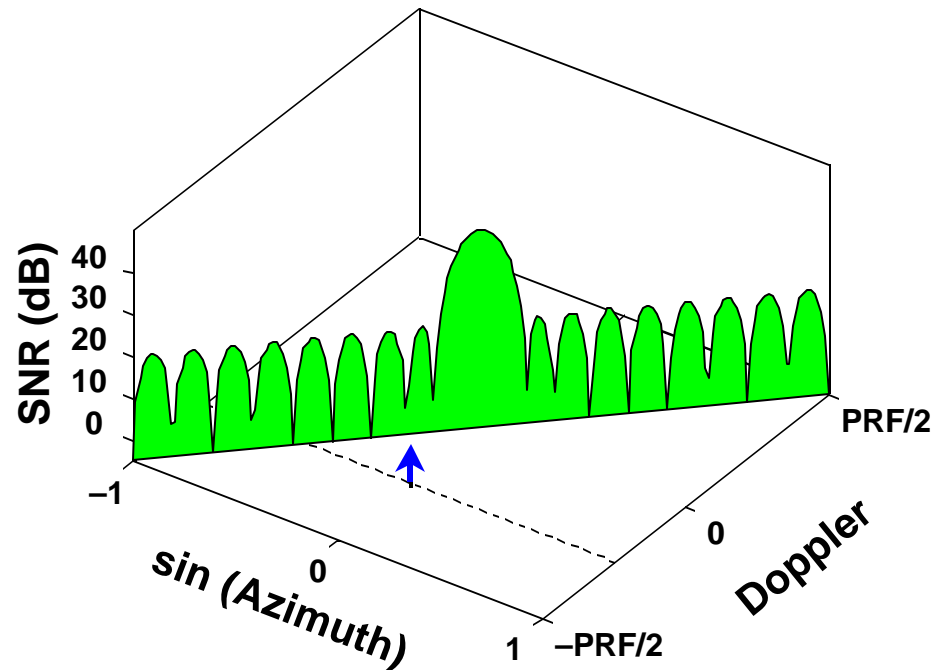
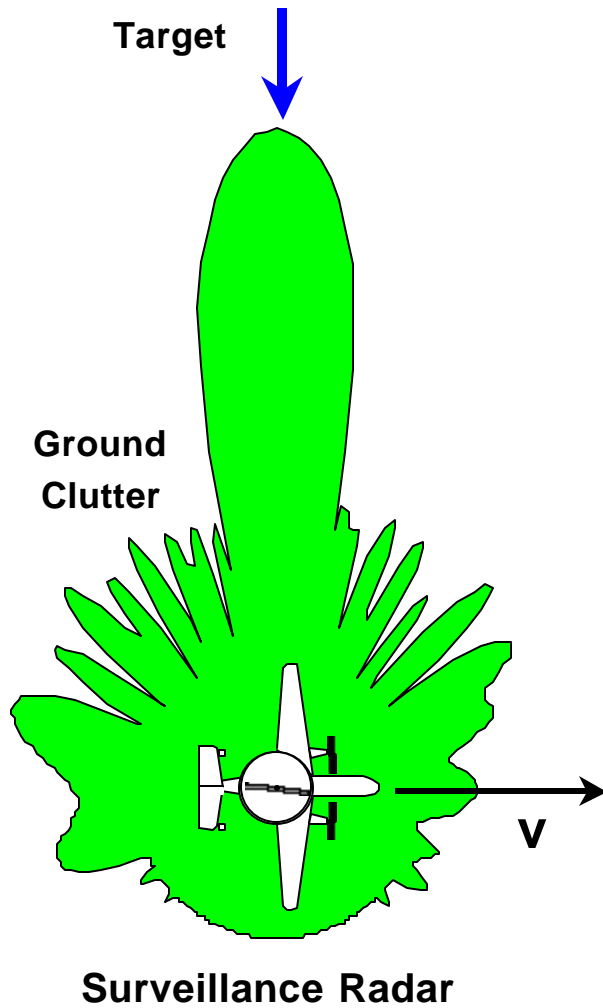


Noise hyper-sphere





# MTI Radar

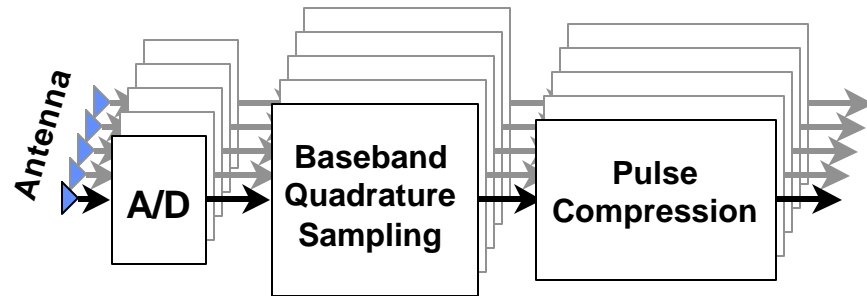
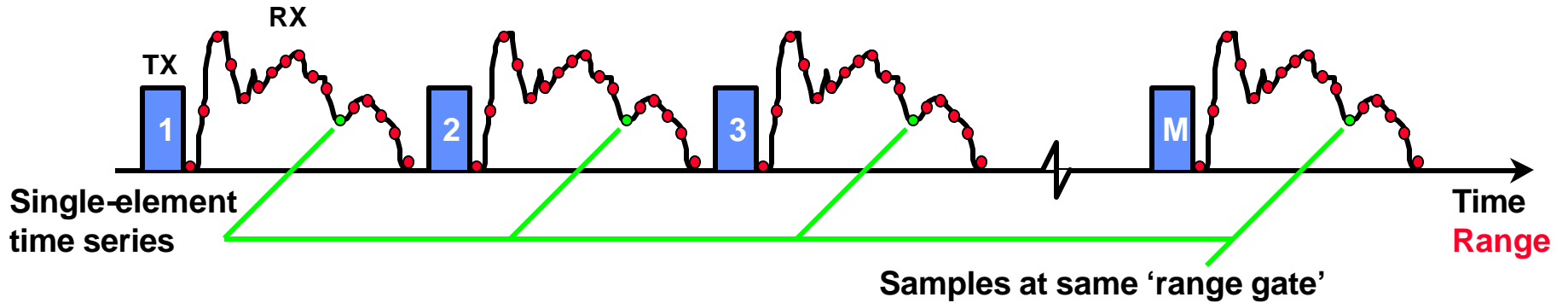


Two-dimensional filtering required to cancel interference

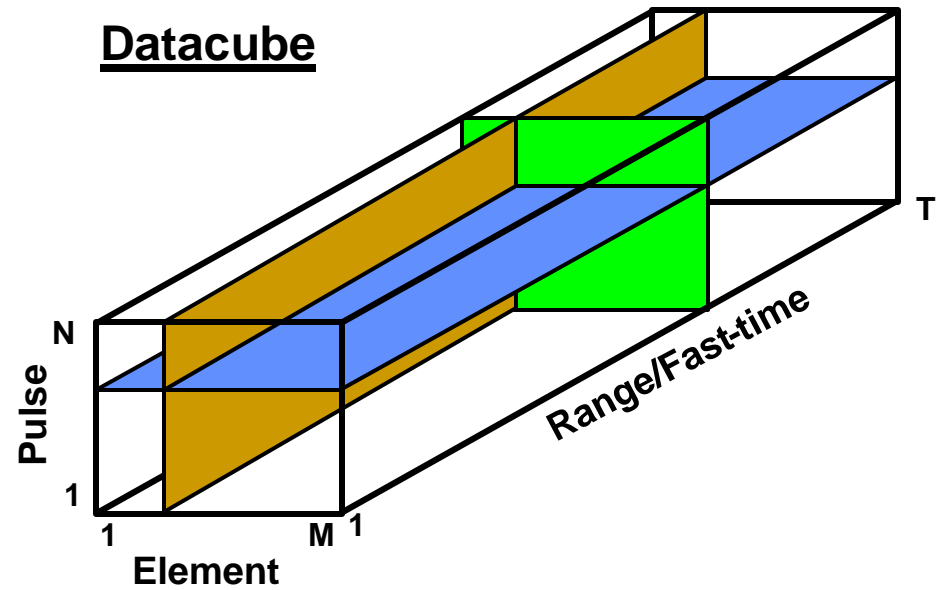
**Space-Time Adaptive Processing  
(STAP)**



# Pulsed Radar Datacube



## Datacube



### Measurement

Pulse



### Physical Quantity

Doppler (velocity)

Element

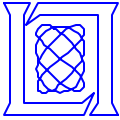


Angle

Fast-time



Range



# STAP Radar Signal Model

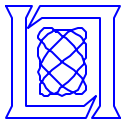
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- **Space-time snapshot for single target**

$$\mathbf{x} = \mathbf{t} + \mathbf{c} + \mathbf{n} \qquad \mathbf{t} = a \mathbf{v}(f, f)$$

- $\mathbf{v}(f, f)$  is called the space-time steering vector
- **Space-time interference (clutter, noise) covariance is**

$$\mathbf{R} = E \{ (\mathbf{c} + \mathbf{n}) (\mathbf{c} + \mathbf{n})^H \} = \mathbf{R}_c + \mathbf{R}_n$$



# Hyperspectral Imaging and MTI Radar

## Summary of Properties

	Hyperspectral Imaging	MTI Radar
System	<ul style="list-style-type: none"> <li>Passive, incoherent sensing</li> <li>Resolution is a function of detector IFOV and altitude</li> </ul>	<ul style="list-style-type: none"> <li>Active, coherent sensing</li> <li>Resolution is a function of signal bandwidth and aperture length</li> </ul>
Signal Model	<ul style="list-style-type: none"> <li>LMM assumes distinct spectra mix linearly</li> <li>Real spectra are sum of <u>endmembers</u> weighted by <u>abundances</u></li> </ul> $\mathbf{x} = \mathbf{a}\mathbf{s} + \mathbf{b} + \mathbf{n}$	<ul style="list-style-type: none"> <li>Components add linearly to yield received signal</li> <li>Complex array measurements are sum of <u>steering vectors</u> weighted by <u>RCS</u> values</li> </ul> $\mathbf{x} = \mathbf{a}\mathbf{v} + \mathbf{c} + \mathbf{n}$
Data Cube		



# Outline

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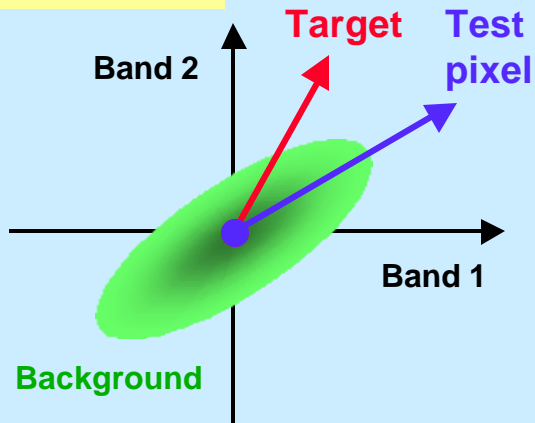
- Introduction to hyperspectral sensing
- Signal models
- **Detection models**
  - Hyperspectral sensing
  - MTI radar
- Hyperspectral detection results
- Conclusion



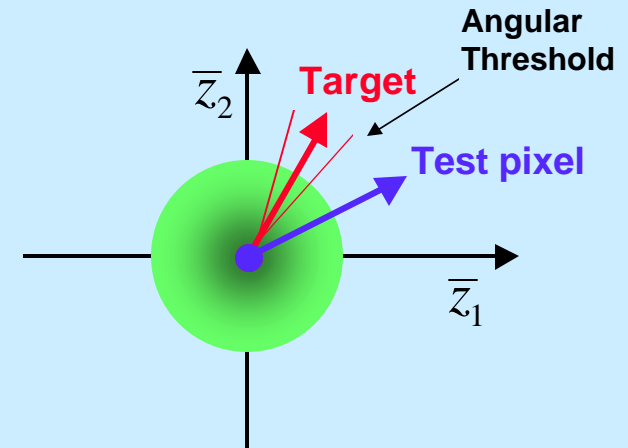
# Adaptive HSI Detection

## Known and Unknown Targets

### KNOWN TARGET

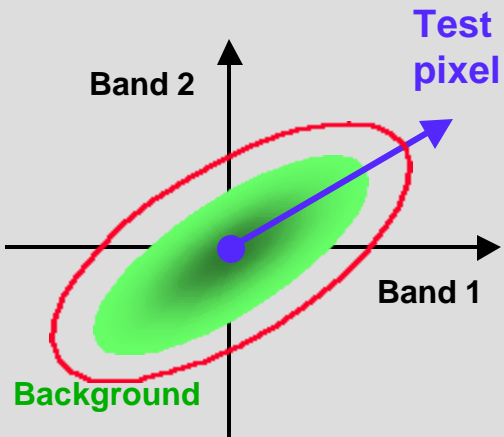


Adaptive  
Background  
Whitening

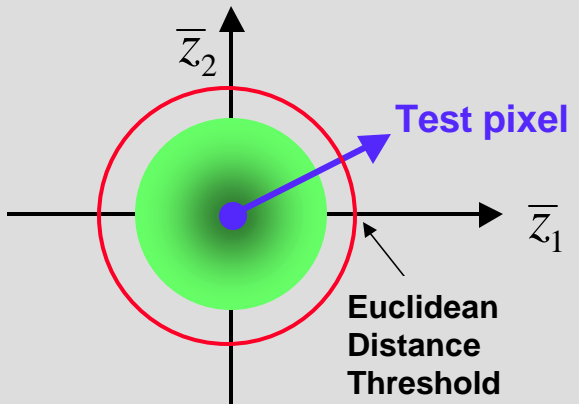


$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{x}(n)\mathbf{x}^T(n)$$

### UNKNOWN TARGET

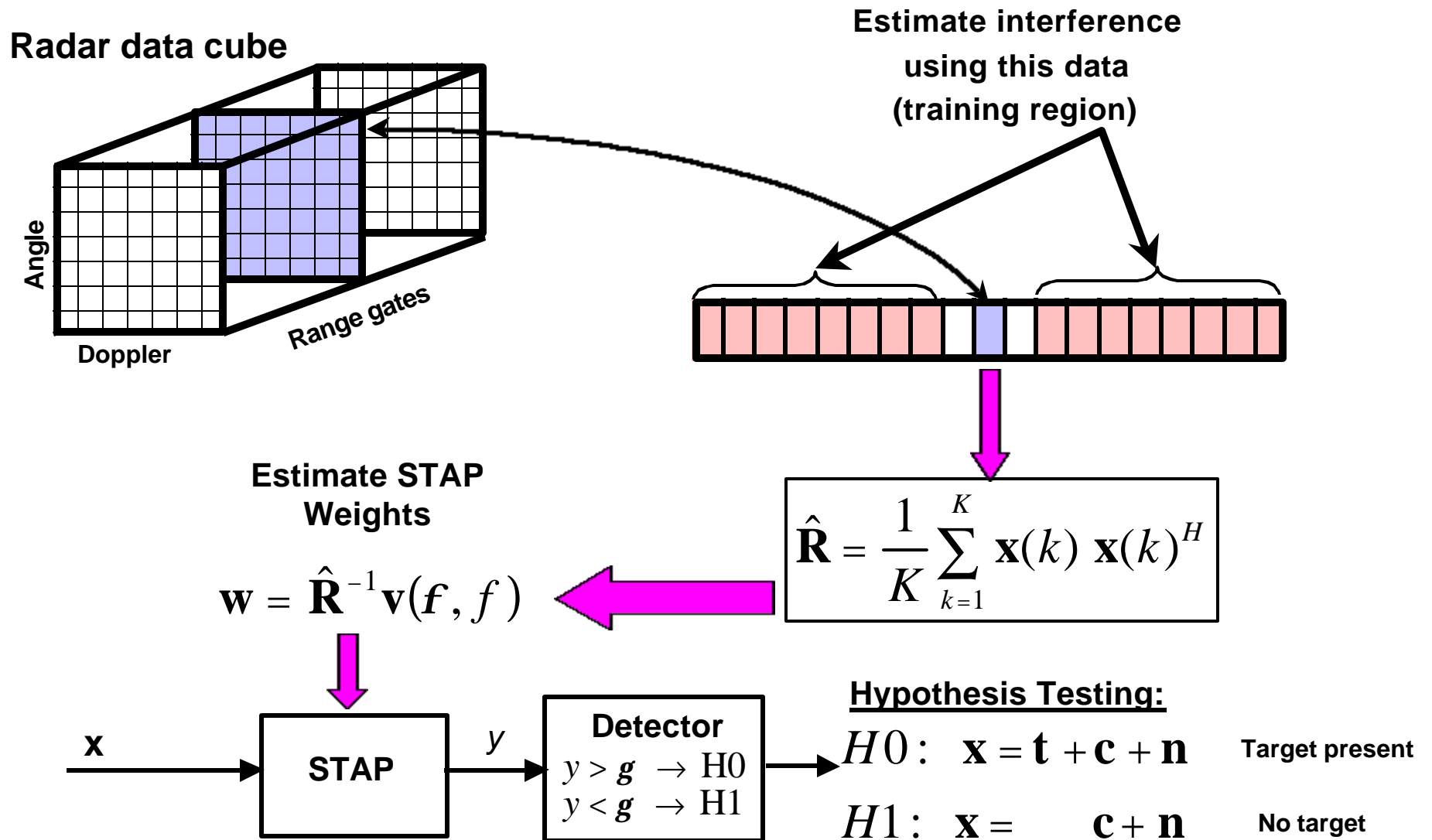


Adaptive  
Background  
Whitening





# Adaptive Detection in STAP Radar





# Replacement and Additive Target Models

- Hyperspectral detection has replacement targets

$$H_0: \mathbf{x} = \mathbf{b} + \mathbf{n}$$

$$H_1: \mathbf{x} = f\mathbf{t} + (1-f)\mathbf{b} + \mathbf{n}$$

- Interference statistics
  - Varies with  $f, 0 \leq f \leq 1$
  - Target displaces background

- Detection results
  - Insufficient target data for ROC curves
  - No theoretical models

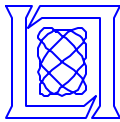
- MTI radar detection has additive targets

$$H_0: \mathbf{x} = \mathbf{c} + \mathbf{n}$$

$$H_1: \mathbf{x} = \mathbf{t} + \mathbf{c} + \mathbf{n}$$

- Interference statistics
  - Independent of target
  - Measure locally

- Detection results
  - ROC curves indicate  $P_D/P_{FA}$  values
  - Theoretical models for target



# Comparison of HSI and MTI Detection

	Hyperspectral Imaging	MTI Radar
Task	<ul style="list-style-type: none"> <li>• <b>Known target</b> <ul style="list-style-type: none"> <li>– Detect target spectrum amid background</li> </ul> </li> <li>• <b>Unknown target</b> <ul style="list-style-type: none"> <li>– Detect pixels anomalous from background</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• <b>Moving target</b> <ul style="list-style-type: none"> <li>– Detect Doppler effect at specific range and angle</li> <li>– Use data after pulse compression</li> </ul> </li> </ul>
Covariance	<ul style="list-style-type: none"> <li>• <b>Interference covariance estimated from sample pixels</b> <ul style="list-style-type: none"> <li>– Dimension equals number of bands (~ 100--200)</li> <li>– Can use subset of bands</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• <b>Interference covariance estimated from local subset of pulse/element/range measurements</b> <ul style="list-style-type: none"> <li>– Better estimate</li> <li>– Avoids non-stationarity</li> </ul> </li> </ul>
Strategy	<ul style="list-style-type: none"> <li>• <b>Replacement target model</b></li> <li>• <b>Known target</b> <ul style="list-style-type: none"> <li>– Measure spectral angle</li> </ul> </li> <li>• <b>Unknown target</b> <ul style="list-style-type: none"> <li>– Measure magnitude</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• <b>Additive target model</b></li> <li>• <b>Moving target</b> <ul style="list-style-type: none"> <li>– Exploit coherency through beamforming and Doppler filtering</li> <li>– RCS and velocity are key parameters for target visibility</li> </ul> </li> </ul>



# Outline

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- Introduction to hyperspectral sensing
- Signal models
- Detection models
- **Hyperspectral detection results**
  - Detection taxonomy
  - Sub-pixel target detection
- Conclusion



# Taxonomy of Hyperspectral Detectors

Noise model	Signal model	Available data	Test statistic $T(x)$	References	Comments
$\mathbf{R}$ = completely unknown interference (unstructured)	$\mathbf{s} = a\mathbf{s}_t$ known direction	$\mathbf{x}$ = test measurement $\{\mathbf{x}_n\}_1^N$ = “signal-free” training data	$\frac{ \mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{x} ^2}{(\mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{s}_t)(1 + \mathbf{x}^T \bar{\mathbf{R}}^{-1} \mathbf{x})}$	Generalized Likelihood Ratio Test (GLRT) Kelly (1986)	
		$\bar{\mathbf{R}} = \sum_{n=1}^N \mathbf{x}_n \mathbf{x}_n^T$	$\frac{ \mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{x} ^2}{\mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{s}_t}$	Adaptive Matched Filter (AMF) Robey et al (1992) Chen and Reed (1991)	$T_{CEM}(\mathbf{x}) = \frac{\mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{x}}{\mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{s}_t}$
		$\hat{\mathbf{R}} = \frac{1}{N} \bar{\mathbf{R}}$	$\frac{ \mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{x} ^2}{(\mathbf{s}_t^T \bar{\mathbf{R}}^{-1} \mathbf{s}_t)(\mathbf{x}^T \bar{\mathbf{R}}^{-1} \mathbf{x})}$	Adaptive Coherence Estimator (ACE) Conte et al (1995) Scharf and McWhorter (1996)	$\mathbf{x} = \bar{\mathbf{R}}^{-1/2} \mathbf{x}, \mathbf{s}_t = \bar{\mathbf{R}}^{-1/2} \mathbf{s}_t$ $\cos q = \frac{ \mathbf{s}_t^T \mathbf{x} }{\ \mathbf{s}_t\  \ \mathbf{x}\ }$ $\mathbf{R} = \mathbf{s}^2 \mathbf{I} \Rightarrow \text{SAM}$
	$\mathbf{s} = \sum_{k=1}^P a_k \mathbf{s}_k = \mathbf{S}\mathbf{a}$ $1 \leq P \leq M$		$\frac{\mathbf{x}^T \bar{\mathbf{R}}^{-1} \mathbf{S} (\mathbf{S}^T \bar{\mathbf{R}}^{-1} \mathbf{S})^{-1} \mathbf{S}^T \bar{\mathbf{R}}^{-1} \mathbf{x}}{1 + \mathbf{x}^T \bar{\mathbf{R}}^{-1} \mathbf{x}}$	Kelly (1987,1989); $P = M \Rightarrow$ unknown deterministic target, Reed-Yu (1990)	$P = 1 \Rightarrow$ GLRT $P = M \Rightarrow$ $T(\mathbf{x}) = \mathbf{x}^T \bar{\mathbf{R}}^{-1} \mathbf{x}$ Simplicity $\Rightarrow \mathbf{S} \equiv \mathbf{I}_M$
structured interference $\mathbf{R} = \mathbf{s}^2 \mathbf{I} + \sum_{k=1}^Q \mathbf{z}_k \mathbf{z}_k^T$	$\mathbf{s} = a\mathbf{s}_t$	$\mathbf{x}$ = test measurement $\mathbf{S} \equiv [\mathbf{s}_1 \mathbf{s}_2 \dots \mathbf{s}_P]$ $\mathbf{Z} \equiv [\mathbf{z}_1 \mathbf{z}_2 \dots \mathbf{z}_Q]$	$\mathbf{S} = \mathbf{s}_t \Rightarrow$ $\hat{a} = \frac{\mathbf{s}_t^T \mathbf{P}_Z^\perp \mathbf{x}}{\mathbf{s}_t^T \mathbf{P}_Z^\perp \mathbf{s}_t}$	Classical F-test for linear statistical models; OSP: Harsanyi-Chang (1994)	Orthogonal subspace projection (OSP): $T(\mathbf{x}) = \mathbf{s}_t^T \mathbf{P}_Z^\perp \mathbf{x}$
			$\mathbf{s} = \sum_{k=1}^P a_k \mathbf{s}_k = \mathbf{S}\mathbf{a}$ $1 \leq P \leq M$	$T'(\mathbf{x}) = \frac{\mathbf{x}^T \mathbf{P}_Z^\perp \mathbf{P}_G \mathbf{P}_Z^\perp \mathbf{x}}{\mathbf{x}^T \mathbf{P}_Z^\perp \mathbf{P}_G^\perp \mathbf{P}_Z^\perp \mathbf{x}}$ $\mathbf{P}_G \equiv \mathbf{G} (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T$ $\mathbf{G} \equiv \mathbf{P}_Z^\perp \mathbf{S} \quad \mathbf{P}_G^\perp \equiv \mathbf{I} - \mathbf{P}_G$	Classical F-test for linear statistical models; Signal processing interpretations Matched Subspace Detector (MSD), Scharf-Friedlander (1994)

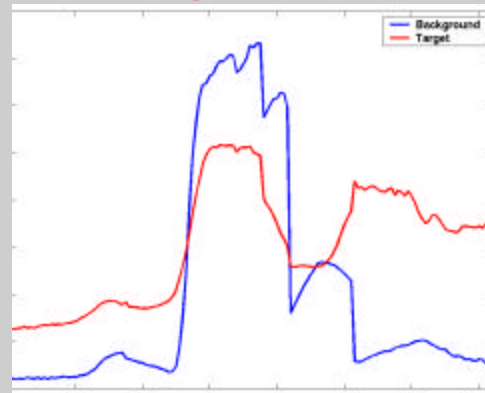


# Hyperspectral Detection Results

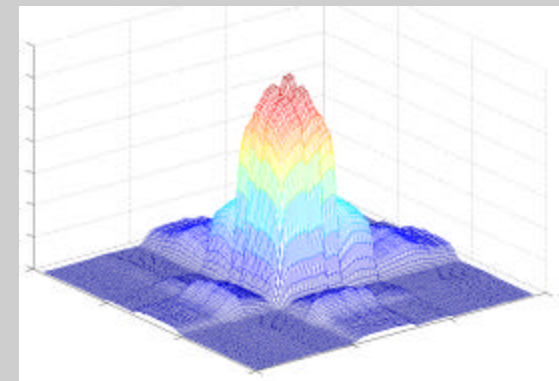


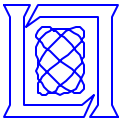
- HYDICE (HYperspectral Digital Imagery Collection Experiment)
  - Airborne sensor
- 210 spectral bands
  - 399-2501 nm
  - Channel widths ~ 3 – 11 nm
  - Spatial resolution, 1m x 1m
- Look for sub-pixel targets

Mean **Target/Tree** Spectra



Covariance for **Trees**





# Comparative Detector Performance

## Sub-pixel Targets

- 8232 tree pixels
- 8232 synthetic mixed pixels
  - 25% / 75%
  - 50% / 50%
  - 75% / 25%

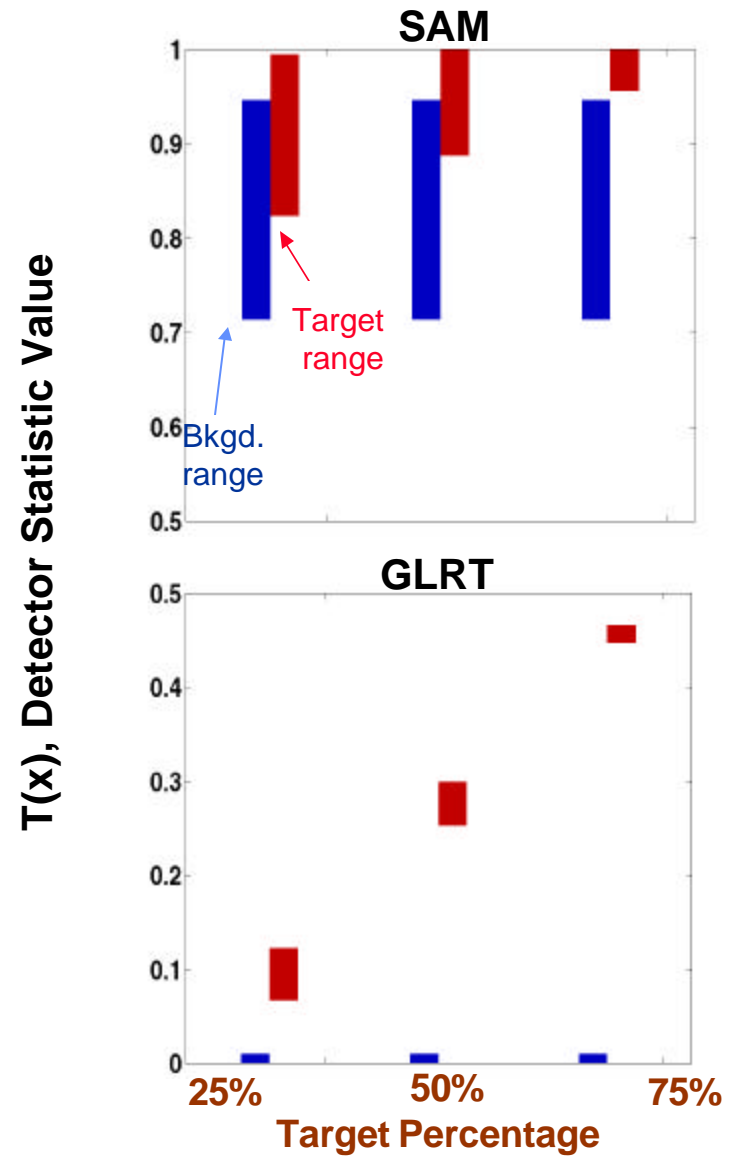
- Two detectors
  - SAM (“unwhitened”)

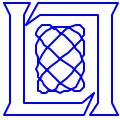
$$T_{SAM}(\mathbf{x}) = \frac{(\mathbf{s}^T \mathbf{x})}{\sqrt{(\mathbf{s}^T \mathbf{s})} \sqrt{(\mathbf{x}^T \mathbf{x})}}$$

- GLRT

$$T_{GLRT}(\mathbf{x}) = \frac{(\mathbf{s}^T \bar{R}_b^{-1} \mathbf{x})^2}{(\mathbf{s}^T \bar{R}_b^{-1} \mathbf{s})(1 + \mathbf{x}^T \bar{R}_b^{-1} \mathbf{x})}$$

- Measure range of test statistics





# Conclusions

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- **Under LMM, hyperspectral sensing shares a common signal model with MTI radar**
  - Endmembers « Steering vectors
  - Abundances « RCS
- **Hyperspectral processing has leveraged optimal detection algorithms from radar**
  - Exploit spectral differences between targets and background
- **Successful sub-pixel target detection depends upon**
  - Target/background subspace relationship
  - Fraction of target present