

## Another Look at Non-Rotating Origins

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**Abstract.** Two “non-rotating origins” were defined by the IAU in 2000 for the measurement of Earth rotation: the Celestial Ephemeris Origin (CEO) in the ICRS and the Terrestrial Ephemeris Origin (TEO) in the ITRS. Universal Time (UT1) is now defined by an expression based on the angle  $\theta$  between the CEO and TEO. Many previous papers, e.g., Capitaine, Guinot, & McCarthy (2000), developed the position of the CEO in terms of a quantity  $s$ , the difference between two arcs on the celestial sphere. A similar quantity  $s'$  was defined for the TEO.

As an alternative, a simple vector differential equation for the position of a non-rotating origin on its reference sphere is presented here. The equation can be easily numerically integrated to high precision. This scheme directly yields the unit vector of the CEO in the ICRS, or that of the TEO in the ITRS, as a function of time. This simplifies the derivation of the main transformation matrix between the ITRF and the ICRS. The directness of the development may have pedagogical and practical advantages for the vast majority of astronomers who are unfamiliar with the history of this topic.

IAU resolution B1.8 from the 2000 General Assembly, which establishes the Celestial Ephemeris Origin (CEO) and the Terrestrial Ephemeris Origin (TEO), and redefines UT1 in terms of the angle  $\theta$  between them, has not received wide publicity within the general astronomical community. Among those who are aware of it, probably few appreciate its basis or implications. The CEO and TEO are specific examples of the concept of a “non-rotating origin” (NRO), first described by Guinot (1979). The concept itself is quite simple, but the mathematical details of its practical implementation, as usually presented (e.g., Capitaine et al. 2000), may discourage non-specialists. This paper describes an alternative mathematical development, which is precise enough for practical computations, but avoids some of the untidy aspects of an analytical approach based on spherical trigonometry. In the space allotted here, only an outline of the scheme can be presented.

A non-rotating origin can be most simply described as a point on the moving equator whose instantaneous motion is always orthogonal to the equator. If  $\mathbf{n}(t)$  is a unit vector toward the instantaneous pole and  $\mathbf{x}(t)$  is a unit vector toward an instantaneous non-rotating origin, a simple geometric construction based on this definition yields the following differential equation:

$$\dot{\mathbf{x}}(t) = -\left(\mathbf{x}(t) \cdot \dot{\mathbf{n}}(t)\right) \mathbf{n}(t) \quad (1)$$

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That is, if we have a model for the motion of the pole,  $\mathbf{n}(t)$ , the path of the non-rotating origin is described by  $\mathbf{x}(t)$ , once an initial point on the equator,  $\mathbf{x}(t_0)$ , is chosen. Conceptually and practically, it is simple to integrate this equation, using, for example, a standard 4th-order Runge-Kutta integrator. For the motions of the real Earth, fixed step sizes of order 0.5 day work quite well, and the integration is quite robust. Since this is really a one-dimensional problem carried out in three dimensions, two constraints can be applied at each step:  $|\mathbf{x}| = 1$  and  $\mathbf{x} \cdot \mathbf{n} = 0$ .

By numerically integrating this equation, we obtain a time series of unit vectors,  $\mathbf{x}(t)$ . Depending on our model of the motion of the pole,  $\mathbf{n}(t)$ , these vectors point toward either the celestial or terrestrial non-rotating origin. That is, the vectors define the directions of either the CEO in the ICRS or the TEO in the ITRS. If we are working in the celestial frame and the CEO is the non-rotating origin of interest, the model of the pole's motion is:

$$\mathbf{n}(t) = \mathbf{F} \mathbf{P}^T(t) \mathbf{N}^T(t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2)$$

where  $\mathbf{F}$ ,  $\mathbf{P}$ , and  $\mathbf{N}$  are the standard matrices for frame bias, precession, and nutation, respectively. The transpose symbols on the precession and nutation matrices indicate that they transform vectors from time  $t$  to J2000.0. The frame bias matrix transforms vectors from the dynamical system to the ICRS.

Once the integration is completed, the main part of the ITRS-to-ICRS transformation (which converts a terrestrial vector to the equivalent celestial vector) can be simply expressed, for any epoch  $t_i$ , in terms of the components of three unit vectors:  $\mathbf{x}(t_i)$ ,  $\mathbf{n}(t_i)$ , and  $\mathbf{y}(t_i) = \mathbf{n}(t_i) \times \mathbf{x}(t_i)$ . These three vectors have components expressed with respect to the ICRS axes and define the orthonormal basis for what has been called the “intermediate” coordinate system. This system has the celestial pole in the z-direction and the CEO in the x-direction, with the instantaneous equator as the xy-plane. The transformation matrix that takes a vector from the intermediate system to the ICRS is simply:

$$\mathbf{C} = \begin{pmatrix} \mathbf{x} & \mathbf{y} & \mathbf{n} \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & n_1 \\ x_2 & y_2 & n_2 \\ x_3 & y_3 & n_3 \end{pmatrix} \quad (3)$$

But to apply this matrix, we must first get the terrestrial vector into the intermediate system. A vector in the ITRS, after correction for polar motion, is transformed to the intermediate system by a simple rotation through the angle  $\theta$ , which is a linear function of UT1 (the expression is specified in IAU resolution B1.8 of 2000). So the complete transformation from the ITRS to the ICRS is

$$\mathbf{r}_c = \mathbf{C} \mathbf{R}_3(-\theta) \mathbf{W}' \mathbf{r}_t \quad (4)$$

where  $\mathbf{r}_t$  is a vector in the ITRS,  $\mathbf{r}_c$  is the corresponding vector in the ICRS,  $\mathbf{W}'$  is the polar motion matrix, and  $\mathbf{R}_3$  represents a simple rotation about the z-axis. (See Chapter 5 of the IERS Conventions (2003) for the expression for  $\theta$  and the form of  $\mathbf{W}'$ , there referred to as  $\mathbf{W}(t)$ .)

This approach also provides a very simple equation for apparent sidereal time. Greenwich apparent sidereal time (GAST) is simply the Greenwich hour

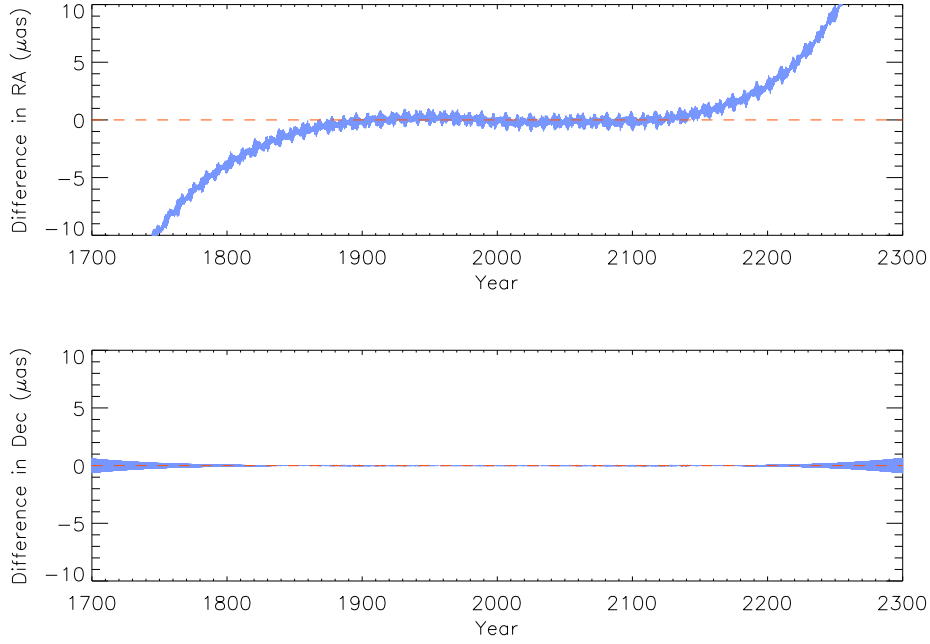


Figure 1. Differences in results of ITRS-to-ICRS transformation methods: CEO-based minus equinox-based. A numerical integration of the CEO position in the ICRS was used for the CEO-based method.

angle (GHA) of the true equinox of date. Using the form of eq. (2), the direction of the true equinox in the ICRS is given by

$$\Upsilon(t) = \mathbf{F} \mathbf{P}^T(t) \mathbf{N}^T(t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

and its direction in the intermediate system must be  $(\Upsilon \cdot \mathbf{x}, \Upsilon \cdot \mathbf{y}, 0)$ . If we define the Greenwich meridian to be the plane that passes through the TEO and the poles, then Greenwich apparent sidereal time is just

$$\text{GAST} = \text{GHA } \Upsilon = \theta - \arctan \left( \frac{\Upsilon \cdot \mathbf{y}}{\Upsilon \cdot \mathbf{x}} \right) \quad (6)$$

since  $\theta$  is the angle between the TEO and the CEO, and the latter defines the direction  $\mathbf{x}$ .

Does this whole approach work in practice? If so, its results must match those given by a conventional equinox-based development. Figure 1 shows the differences in the results of CEO-based and equinox-based methods of performing the ITRS-to-ICRS transformation on a vector in the equatorial plane. For these computations, the numerical integration of the CEO position was based on the motion of the pole defined by the IAU 2000A precession-nutation model, as given in the IERS Conventions (2003). The new IERS formula for sidereal time (including all the “complementary terms”) was used in the equinox-based

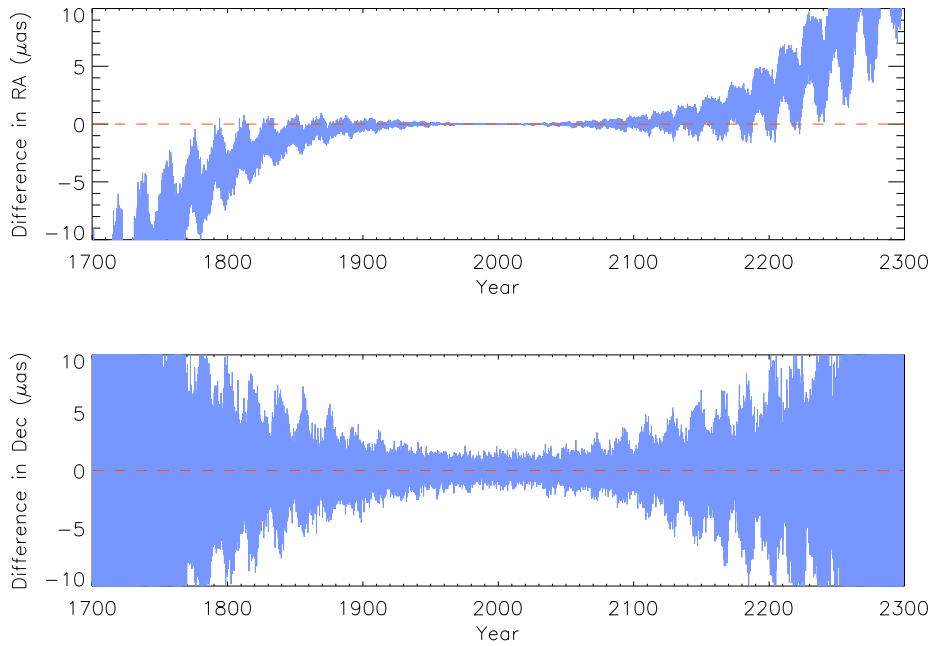


Figure 2. Same as Fig. 1, except that IERS algorithms based on analytic series were used for the CEO-based method.

(conventional) transformation. The initial CEO position used in the integration was adjusted so that the difference between the two methods was zero at 2003.0. (The ICRS right ascension of the CEO at J2000.0 turned out to be  $0^{\circ}002012$ .) Polar motion and  $\Delta T$  were assumed zero throughout. The figure shows that the two transformation schemes match to within a few microarcseconds from 1800–2200, with some divergence outside of that time frame. A similar comparison of the two kinds of sidereal time formulas yields differences that are identical by eye to the top plot of Fig. 1. The same systematic pattern (with some additional numerical noise) is seen in Fig. 2, which shows the same kind of ITRS-to-ICRS transformation comparison, but using only IERS algorithms throughout. Therefore, the CEO integration approach described here seems to be at least as precise as that provided by the IERS.

The CEO integration development and results will be described in more detail in Kaplan (2004).

## References

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