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Documentation of a Computer Code by HKC Research to Calculate the Transmission of a Sonic Boom through a Wavy Ocean Surface

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John R. Edwards
SMC/AXF

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14. ABSTRACT
A "sonic boom wavy surface" computer code was developed by HKC Research at the University of Southern California (USC) over the last few years. It was distributed in April 2003 via CD through PARSONS, Pasadena, to the Air Force. A final report was also distributed. The problem addressed by this code is that of a sonic boom passing over a wavy ocean surface. The pressure wave transmitted through the wavy interface is the primary output of this code. The theory upon which this code is based is presented in a USC Report authored by H. K. Cheng and C. J. Lee. The purpose of the current report is to document the code by explaining the calculations performed therein, and placing the numerical routines in correspondence with the equations which appear in the USC Report. Code results from a sample input set are also presented in order to provide a benchmark for future users and against which to compare possible future releases of this code by Cheng and Lee. The emphasis of this report is on code documentation. No assessment nor interpretation of the theory is offered. Although the Cheng and Lee code is not currently in standard use for predicting sonic boom transmission into the ocean, this document may hasten its adoption as a standard tool for this purpose.

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1 Introduction

As part of The National Environmental Policy Act of 1969, it is "the continuing responsibility of the Federal Government to use all practicable means" to study, analyze and report on the impact military operations have on the environment. This Environmental Impact Analysis Process (EIAP) is managed at the Air Force Space and Missile Systems Center (SMC) by the Acquisition Civil and Environmental Engineering Group (SMC/AXF.) For more than the past ten years, SMC/AXF has conducted a multi-phase, multi-year study of the effects of space launch vehicle (and object reentry) sonic booms have on land and sea mammals. This requires both computer models that predict the in-air sonic boom that reaches the Earth's surface from launch and reentry vehicles, as well as computer models that predict sonic boom pressure disturbances that are expected under the ocean's surface.

As part of the SMC/AXF sonic boom study, a "sonic boom wavy surface" computer code was developed by HKC Research at USC over the last few years and was distributed in April 2003 via CD (Ref. 1) through PARSONS, Pasadena, to SMC/AXF. A user's guide (Ref. 2) for the code was provided to demonstrate the steps of execution required to obtain a solution for a given set of input parameters. The theory upon which this code is based is presented in Ref. 3. Examples of and extensions to the theory are given in Ref. 4. A final report was also distributed through PARSONS, Pasadena (Ref. 5.)

The purpose of the current report is to document the code, using notation which matches (as closely as possible) that which is used in the theoretical development put forth in Ref. 3. This work thus provides a correspondence between Cheng and Lee's equations and the routines which are executed within the code. In addition, code results are plotted for a sample input set in order to provide benchmark results for future users and for possible future releases by Cheng and Lee.

The emphasis of this report is on code documentation. No assessment nor interpretation of the theory is offered. Readers interested in the theory may consult Ref. 3.

This report is intended to supplement the user's guide (Ref. 2) supplied by Cheng and Lee in that it provides a bridge between the theoretical equations in Ref. 3 and the code itself (Ref. 1.) The user's guide (Ref. 2) supplied on the CD does not provide this bridge. HKC Research has reviewed our work and approved of this supplemental documentation.

The Cheng and Lee code is not currently used as a standard tool for predicting sonic boom transmission into the ocean. The more standard approach is due to Sawyers (Ref. 6), wherein the effect of a wavy surface is neglected.

2 Problem Description

The problem attacked in Ref. 3 is that of a sonic boom, generated in air, impinging upon and interacting with a water surface. It is desired to find the overpressure (as a function of time) at some specified (fixed) location beneath the water's surface. For the case of a flat water surface (no waves), this problem was worked out some time ago by Sawyers (see Ref. 6.) The generalization considered in Ref. 3 is the situation where there is a wave train on the water surface. The water surface waves are assumed to propagate in a direction which is nearly opposite to the horizontal flight path of the supersonic body which generates the sonic boom. If the supersonic body speed is denoted by U, the water wave (phase) speed by c, and the distance between water wave crests by λ, then it is easy to see that the sonic boom "hits" the water wave crests with a frequency (in hz) of $(U+c)/\lambda$. And since the water wave phase speed is very small compared with U, the interaction frequency is closely approximated by U/λ. This interaction frequency is an important parameter in the analysis contained in Ref. 3.

In the Cheng & Lee theory, the desired overpressure time history is assumed to be comprised of two contributions.

$$\overline{p}(\overline{t},\overline{x},\overline{z}) = \overline{p}_1(\overline{t},\overline{x},\overline{z}) + \overline{p}_2(\overline{t},\overline{x},\overline{z}) \tag{2.1}$$

> [In the above equation, over-bars are used to denote **dimensional** variables. The reason for this notation is that in the following analysis, we will almost always deal with dimensionless (scaled) variables, which will be denoted without the over-bar.]

The symbol $\overline{p}$ is used to represent the overpressure (psf), as a function of time $\overline{t}$ (sec), at a fixed depth of $\overline{z}$ (ft) and a fixed horizontal location, $\overline{x}$ (ft). The first term, $\overline{p}_1$, is the flat-surface (Sawyers) contribution, and the second, $\overline{p}_2$, is the contribution due to interaction of the sonic boom with the wavy water surface. As shown in Ref. 3, the flat surface term dominates at small depths. But at larger depths, the interaction term, $\overline{p}_2$, dominates. This is due to the different dependence of the two terms with depth, when depth is large. Specifically, these two terms fall off with depth as follows:

$$\overline{p}_1 \sim \overline{z}^{-2} \quad \text{whereas} \quad \overline{p}_2 \sim \overline{z}^{-1/2}$$

The geometry of the problem is explained with the aid of Fig. 1, which shows a snapshot at $\overline{t}=0$. At this instant, the surface signature of the sonic boom (which is running to $-\overline{x}$ at speed U) is non-zero only on the interval $0<\overline{x}<L$.

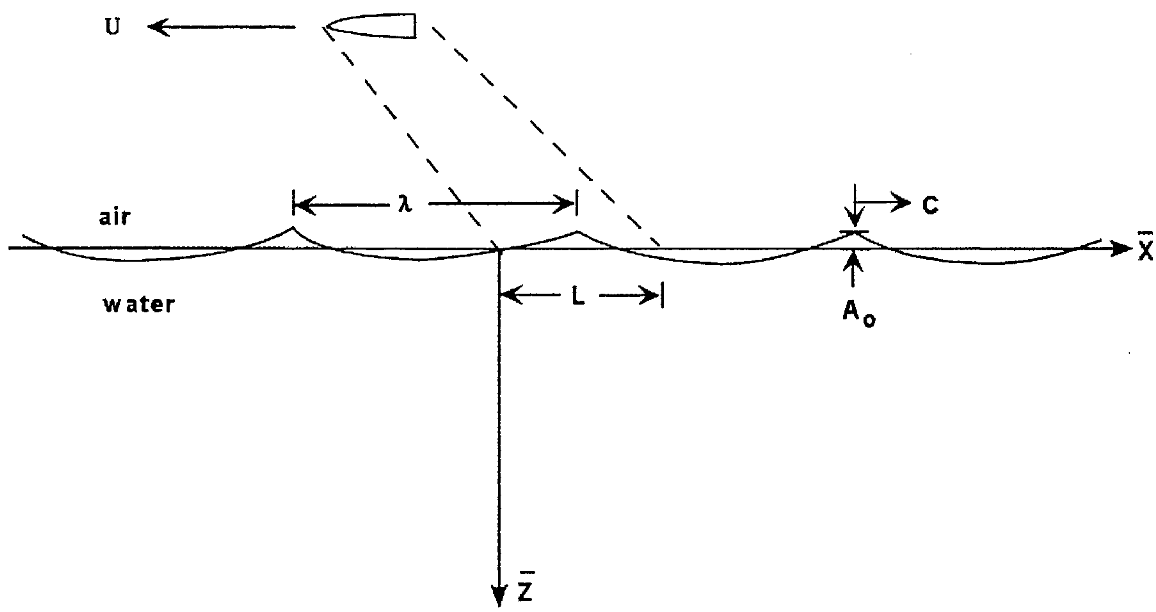

Figure 1. Elevation view.

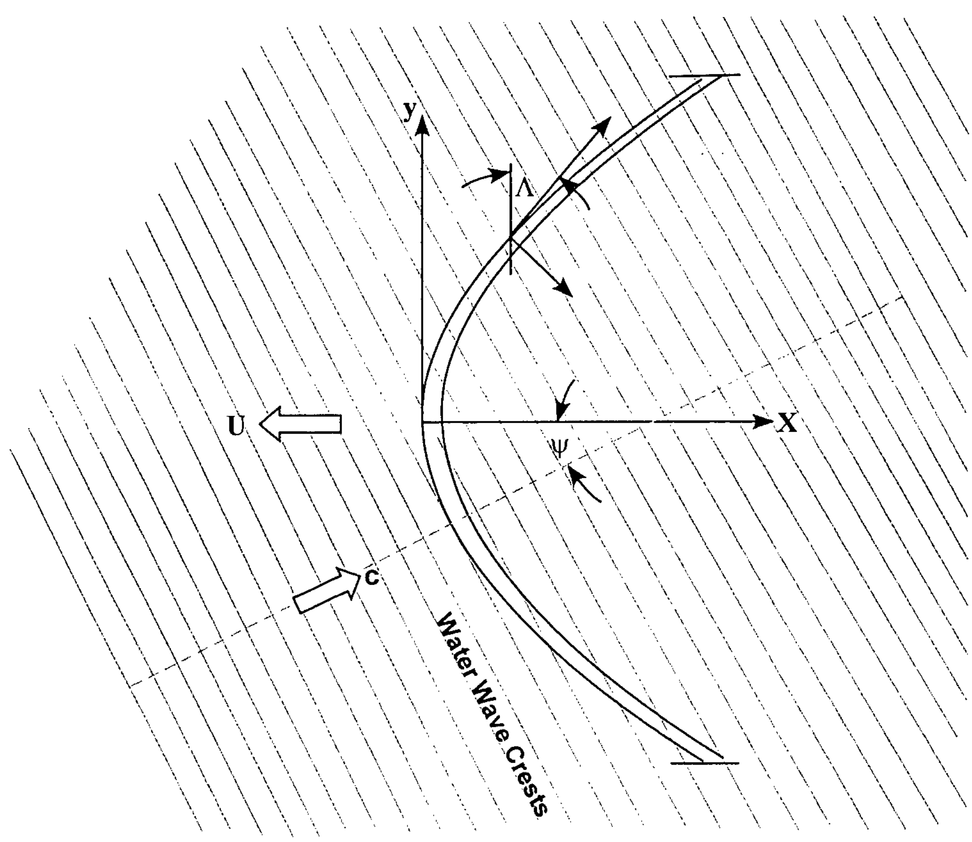

Figure 2. View from above the water.

2.1 Nomenclature

The following nomenclature is used for **dimensional** variables of interest.

$\bar{t}$ = time (sec)
$\bar{z}$ = depth (ft)
$\bar{x}$ = horizontal position, measured positive opposite to flight direction (ft)

$\bar{p}$ = overpressure (psf)
P_{max} = maximum overpressure in sonic boom signature at the water surface (psf)

U = horizontal flight speed of supersonic body (ft/sec)
L = sonic boom signature length at water surface (ft)
a_A = speed of sound in air (ft/sec) [Code assumes a_A =1118.2 ft/sec.]
r = ratio of water sound speed to air sound speed [Code assumes $r = 4.53$]

λ = distance between water wave crests (ft)
$\bar{k}$ = $2\pi / \lambda$ = water wave number (ft^{-1})
A_0 = water wave crest height (ft)
c = water wave phase speed (ft/sec)

Λ = swept angle of impact point (off-track location) (deg)
Ψ = non-alignment angle between water wave propagation and flight direction (deg)

$\bar{\Omega}$ = interaction frequency of sonic boom with crests (rad/sec)

[The lengths, L, λ, and A_0, and the angles, Λ and Ψ, are explained via Figs. 1 and 2.]

Some corresponding **non-dimensional** variables are defined as follows.

t = $\bar{t}\, U / L$ (dimensionless time)
z = $\bar{z} / L$ (dimensionless depth)
x = $\bar{x} / L$ (dimensionless horizontal position)

p = $\bar{p} / P_{max}$ (dimensionless overpressure)

M_A = U / a_A (Mach number on air side of surface)
k = $\bar{k}\, L$ (dimensionless wave number)
δ = A_0 / λ (slope parameter)

Ω = $\bar{\Omega}\, L / U$ (dimensionless interaction frequency)

2.2 Model Parameters

Various parameters which appear in the Cheng & Lee model are defined in the table below. The first column contains an equation number pertaining to the current document. The third and fourth columns contain the cross reference to Ref. 3.

Eq. No.	Definition	Page (Ref. 3)	Equation (Ref. 3)
D.1	$M_A = U / a_A$ (assume a_A =1118.2 ft/sec)	5	line 11
D.2	$M_n = M_A \cos\Lambda$	13	above (5.3)
D.3	$k_1 = k\cos(\Lambda+\Psi)$	13	(5.4b)
D.4	$k_2 = k\sin(\Lambda+\Psi)$	13	(5.4b)
D.5	$B_n = \sqrt{M_n^2 - 1}$	13	below (5.3)
D.6	$P = 2k M_n^2 \cos(\Lambda+\Psi)$	14	(5.7e)
D.7	$Q = k^2\left[M_n^2 - (1+M_n^2)\sin^2(\Lambda+\Psi)\right]$	14	(5.7f)
D.8	$\alpha = \sqrt{P^2 - 4B_n^2 Q} \,/\left(2B_n^2\right)$	14	(5.8b)
D.9	$\mu = P/\left(2B_n^2\alpha\right)$	15	(5.10d)
D.10	$M_W = M_n / r$ (assume $r = 4.53$)	5	line 10
D.11	$B_W = \sqrt{1 - M_W^2}$	11	(4.1c)
D.12	$P_W = 2k M_W^2 \cos(\Lambda+\Psi)$		
D.13	$Q_W = k^2\left[M_W^2 - (1+M_W^2)\sin^2(\Lambda+\Psi)\right]$		
D.14	$S_W = \sqrt{P_W^2 + 4B_W^2 Q_W}$	19	(5.20b)
D.15	$z_S = B_W z$ [Equivalent to Cheng: $\bar{z} = \beta_n z$]	21	above (5.26)

2.3 Interaction Frequency

The interaction frequency is given in terms of wave number by the following relation (see Ref. 3, Eq. 5.2b.)

$$\bar{\Omega} = c\bar{k} + U\bar{k}_1 \tag{2.2}$$

Or, using definition D.3 in the above table (see Ref. 3, Eq 5.4b),

$$\bar{\Omega} = \bar{k}\left[c + U\cos(\Lambda + \Psi)\right] \tag{2.3}$$

In non-dimensional form, this relation is

$$\Omega = k\left[c/U + \cos(\Lambda + \Psi)\right] \tag{2.4}$$

Within the code (and most places within the theoretical development in Ref. 3) the first term in the brackets is neglected, since the water surface wave phase speed is much smaller than the boom speed in air. Thus the frequency relation actually used in the code is

$$\Omega = k\cos(\Lambda + \Psi) = k_1 \tag{2.5}$$

3 Cheng & Lee Theory

With the preliminaries of the first two sections, the theory of Ref. 3 will now be summarized in a manner which places it in correspondence to the code contained in Ref. 1.

The code computes the two overpressure contributions in the following non-dimensional form.

$$p(x,z) = p_1(x,z) + p_2(x,z) \tag{3.1}$$

3.1 Flat-surface Term

The first term is the flat-surface contribution, which is calculated according to the Sawyers theory (see Ref. 6) as

$$p_1(x,z) = -u(x,z) \tag{3.2}$$

where the function $u(x,z)$ is computed within the code as

$$u(x,z) = \mathrm{Re}\left[\frac{i}{\pi}\left\{\int_0^1 \frac{u_0(\xi)-u_0(x)}{\hat{z}_0-\xi}\,d\xi - u_0(x)\left[\mathrm{Log}(\hat{z}_0-1)-\mathrm{Log}(\hat{z}_0)\right]\right\}\right] \tag{3.3}$$

where "Re" means "real part" and $\hat{z}_0$ is the complex position:

$$\hat{z}_0 = x + i\,z_S \qquad (\text{where} \quad z_S = B_W z) \tag{3.4}$$

The function, $u_0(x)$, is the unit waveform of the sonic boom at the water surface. This function is non-zero only for $0 \le x \le 1$. Two "canned" surface waveforms are provided within the code: one for an N-wave (see Fig. 3) and another for a Titan IV (see Fig. 4.) In addition, the user may alter the routine for $u_0(x)$, and thereby specify any arbitrary unit waveform as input.

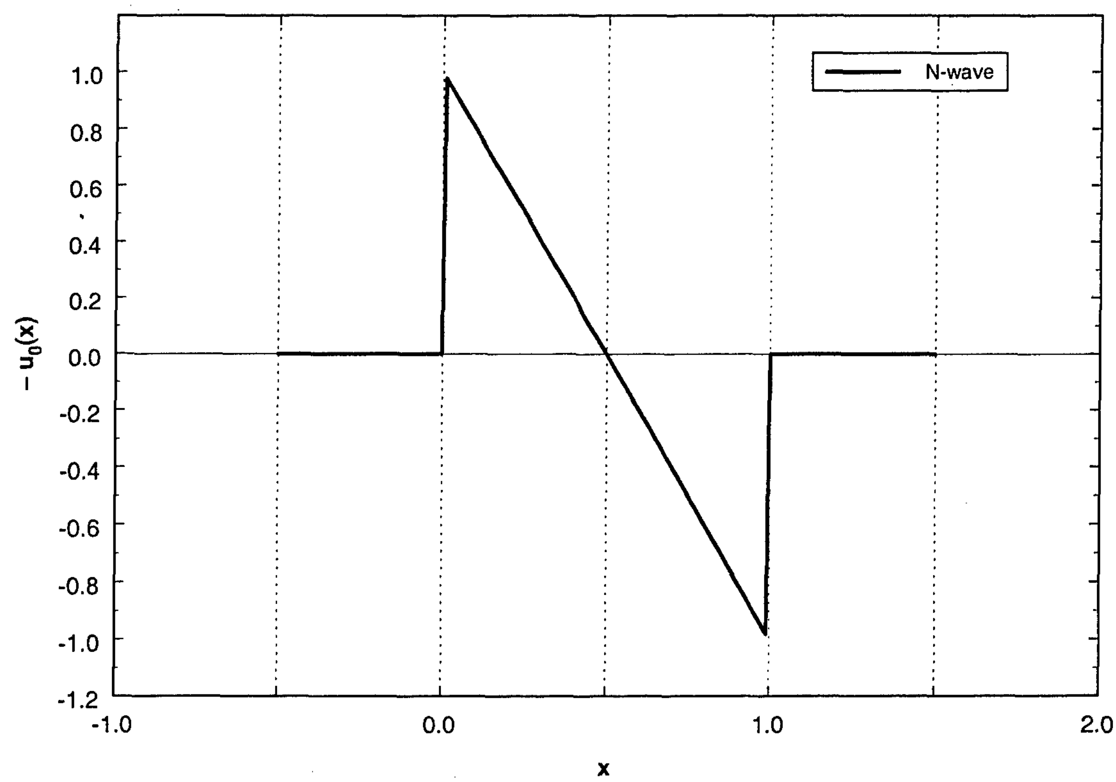

Figure 3. N-wave unit waveform.

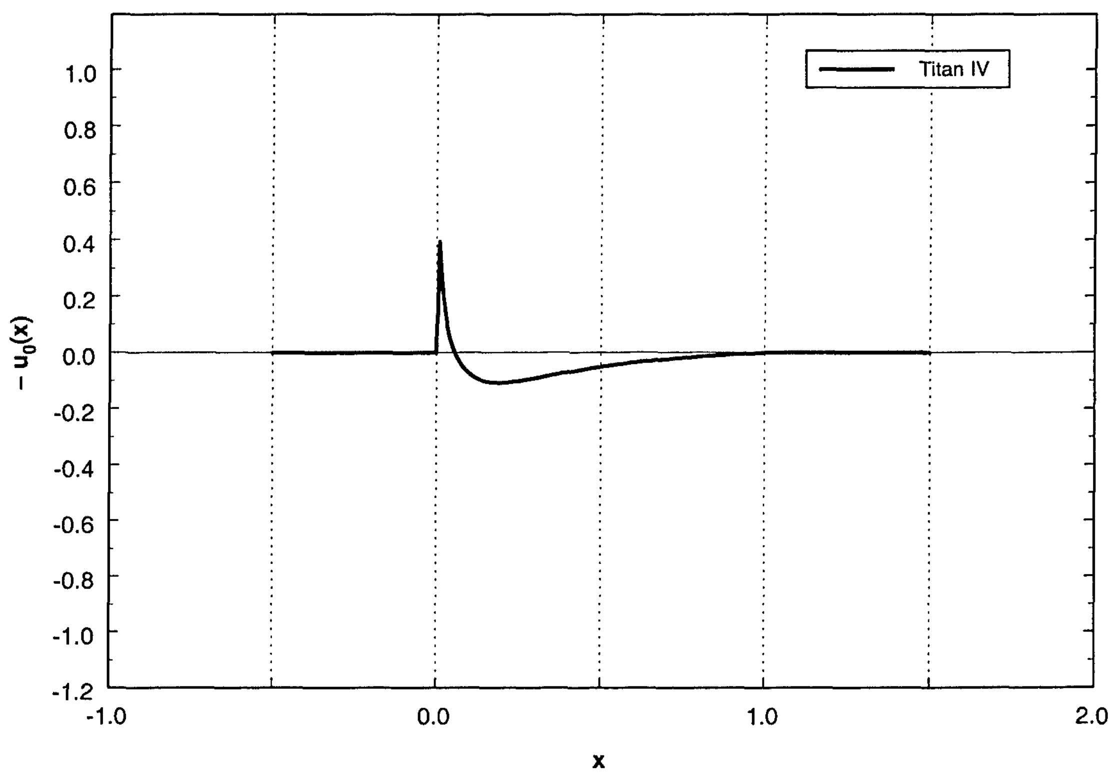

Figure 4. Titan IV unit waveform.

3.2 Wavy Surface Term

The wavy surface term is expressed as follows. [See Ref. 3, Eq. (5.11).]

$$p_2(x,z) = \delta \ \mathrm{Re}\left[\hat{p}_2(x,z)e^{-i\bar{\Omega}\bar{\tau}}\right] \tag{3.5}$$

In the above equation, δ is the slope parameter defined in Section 2.1. In the code, the phase factor is computed using non-dimensional variables as follows.

$$p_2(x,z) = \delta \ \mathrm{Re}\left[\hat{p}_2(x,z)e^{-i\Omega x}\right] \tag{3.6}$$

3.2.1 Complex Potential at Surface, $\hat{\phi}_2(x)$

Calculation of the complex overpressure amplitude, $\hat{p}_2$, at a field location (x, z) involves several steps. One begins by computing the surface ($z = 0$) values for the complex potential $\hat{\phi}_2$ associated with this overpressure. Specifically, the code computes

$$\hat{\phi}_2(x) = 2\frac{e^{ik_1 x}}{B_n}\hat{S}(x) \tag{3.7}$$

where

$$\hat{S}(x) = \int_0^x H(x_1)G(x-x_1)\,dx_1 \quad \text{for} \quad 0 \le x \le 1 \tag{3.8}$$

and

$$\hat{S}(x) = \int_0^1 H(x_1)G(x-x_1)\,dx_1 \quad \text{for} \quad x > 1 \tag{3.9}$$

where the integrand is defined by the product of two functions:

$$H(x_1) = ik_1 u_0(x_1) - B_n^2 u_0'(x_1) \tag{3.10}$$

where u_0' is the derivative of the unit surface waveform, and

$$G(\xi) = e^{-i(k_1-\mu\alpha)\xi}\,J_0(\alpha\xi) \tag{3.11}$$

The latter function (see Ref. 3, Eq. 5.10c) involves J_0, the zero-order Bessel function of the first kind. It can be shown that Eq. (3.7) [along with Eqs. (3.8, 10, and 11)] is exactly equivalent to Eq. (5.8a) on page 14 of Ref. 3, if in the latter equation, one sets $z = 0$ and identifies f' as $-u_0$.

3.2.2 Interaction Overpressure at Surface, $\hat{p}_{2,\text{surf}}(x)$

At the surface, the overpressure due to interaction is computed by a surface routine in the code as follows.

$$\hat{p}_{2,\text{surf}}(x) = -\left(\frac{2\pi}{k_1}\right)\left(\frac{d\hat{\phi}_2}{dx} - ik_1\hat{\phi}_2\right) \tag{3.12}$$

The above expression corresponds to Eqs. (5.8e,f) in Ref. 3.

NOTE: The surface routine provided on the CD contains two minor errors. It reverses the sign in (3.12) and incorrectly uses k rather than k_1 in the computation. The output for the sample problem given in Section 5 incorporates the sign correction.

3.2.3 Complex Transformed Amplitude, $\hat{A}(\xi)$

Having computed $\hat{\phi}_2(x)$, a complex function is now computed which is part of the Fourier transform of the desired function, $\hat{p}_2(x, z)$. Within the code, this function is computed as follows.

$$\hat{A}(\xi) = \hat{A}_\phi(\xi) + \hat{A}_B(\xi) \tag{3.13}$$

where

$$\hat{A}_\phi(\xi) = i\left(\frac{2\pi}{k_1}\right)(\xi + k_1)\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{i\xi x}\hat{\phi}_2(x)\,dx \tag{3.14}$$

and

$$\hat{A}_B(\xi) = 2\left(\frac{2\pi}{k_1}\right)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{i\xi x}\left[e^{ik_1 x}\,q(x)\right]dx \tag{3.15}$$

where, in (3.15), the derivative of the Sawyers solution with depth, evaluated at the surface, has been denoted by

$$q(x) \equiv \left.\frac{\partial u}{\partial z}\right|_{z=0} \tag{3.16}$$

The computation of $\hat{A}(\xi)$ via Eqs. (3.13) – (3.16) is equivalent to Eqs. (5.18a,b) on page 18 of Ref. 3.

3.2.4 Underwater Interaction Overpressure, $\hat{p}_2(x,z)$

With the transformed amplitude $\hat{A}(\xi)$ obtained in section 3.2.3, the overpressure due to wavy surface interaction is computed as the following inverse Fourier transform. [See Ref. 3, Eq. (5.17).]

$$\hat{p}_2(x,z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{A}(\xi)\,\sigma(\xi,z)\,d\xi \tag{3.17}$$

where

$$\sigma(\xi,z) = \begin{cases} \exp\left[i\sqrt{|K(\xi)|}\,z\right] & \text{for } K(\xi) < 0 \\ \exp\left[-\sqrt{|K(\xi)|}\,z\right] & \text{for } K(\xi) \ge 0 \end{cases} \tag{3.18}$$

and

$$K(\xi) = B_W^2 \xi^2 - P_W \xi - Q_W \tag{3.19}$$

and B_W, P_W, and Q_W are defined in Eqs. D.11 – D.13 in Section 2.2. Note that Eqs. (3.18) and (3.19) above correspond, respectively, to Eqs. (5.15b) and (5.13b) on page 17 of Ref. 3.

3.2.5 Far-Field (Deep Water) Solution, $\hat{p}_{2,\text{far}}(x,z)$

In addition to the exact solution computed as the inverse Fourier transform in Eq. (3.17), the code also computes an asymptotic approximation, which is valid for large z. This is computed as

$$\hat{p}_{2,\text{far}}(x,z) = \frac{\sqrt{S_W}\,\hat{A}(\xi_*)}{\sqrt{2}\,B_W\sqrt{z_S}\left(1+\eta^2\right)^{3/4}} \exp(iE) \tag{3.20}$$

where

$$E = \left(\frac{S_W}{2B_W^2}\right)\left(\sqrt{1+\eta^2} - \frac{P_W}{S_W}\eta\right) z_S - \frac{\pi}{4} \tag{3.21}$$

Eqs. (3.20) and (3.21) are equivalent to Eq. (5.26) on page 21 of Ref. 3. The parameters S_W and z_S in the above equations are defined in Eqs D.14 and D.15 in Section 2.2. In Eqs. (3.20) and (3.21), two additional definitions are required. These are given below, and correspond to Eqs. (5.23c) and (5.24) on page 21 of Ref. 3.

$$\eta = x/(B_W z) = x/z_S \tag{3.22}$$

$$\xi_* = \frac{P_W - S_W \eta / \sqrt{1+\eta^2}}{2B_W^2} \tag{3.23}$$

4 Cheng & Lee Code

4.1 Contents of the Distribution CD

The code which performs the calculations described in the previous section was received on a CD (Ref. 1.) The relevant files on that CD are discussed below.

4.1.1 FORTRAN Source Code

There are six files on the CD which contain the FORTRAN source code required to perform the calculations. (See the following table.)

Step	File	Purpose	See Section:
1	2d_gen26.for	Computes the Sawyers solution, $u(x,z)$	3.1
2	Sw_st16.for	Computes surface potential, $\hat{\phi}_2(x)$	3.2.1
3	SUR_P26.FOR	Computes interaction overpressure at surface, $\hat{p}_{2,\text{surf}}(x)$	3.2.2
4	Sw_st27.for	Computes amplitude, $\hat{A}(\xi)$	3.2.3
5	Sw_st34.for	Computes $\hat{p}_2(x,z)$ and $\hat{p}_{2,\text{far}}(x,z)$	3.2.4 & 3.2.5
	df2.for	Function which defines unit surface waveform, $u_0(x)$	3.1

The above six files were copied to a folder, and the latter was renamed to "df.for" in order to be recognized in the "include" statement in the other modules. The first five programs in the above table were then compiled (under Lahey 95.) These five codes were then run sequentially, using inputs to the screen for the sample problem described in the User's Guide (Ref. 2.) The text files output by these five codes were examined. [NOTE: For the sample problem, the selected unit surface waveform, $u_0(x)$, is an N-wave.]

4.1.2 Supplied Executables

The CD also contains ten executable files. However, only five of them are relevant. The others apparently represent an earlier version of the code. The relevant ones are given in the second column of the following table.

Step	Supplied on CD	Corresponds to Source File:	Test Results Agree?
1	2d_gen26.exe	2d_gen26.for	Yes
2	Sw_st15.exe	Sw_st16.for	Yes
3	SUR_P26.exe	SUR_P26.FOR	Yes
4	Sw_st25.exe	Sw_st27.for	Yes
5	Sw_st35.exe	Sw_st34.for	Yes

In the above table, the last column means that the executable supplied on the CD was run with the same inputs as were used for the executable generated by compiling the source file.

A "Yes" entry means the two executables produced the same results. It is clear from these tests, that the six FORTRAN files contain the source code which was used to generate the five *relevant* executables (second column of above table) supplied on the CD.

4.1.3 EXCEL Spreadsheets

There are three relevant EXCEL files on the CD. These are used to process and plot output from the five execution steps given above. The following table gives the utility of the spreadsheets.

EXCEL Spreadsheet	Requires Pasting Output From:
Plot_Phi_Fta.xls	phi_st.txt (Step 2) and fta_st.txt (Step 4)
P_Zeq_0.xls	p2_zeq0.txt (Step 3) and p0_zeq0.txt (Step 3)
P_Zeq_1.xls	p0_z1.txt (Step 1) , p2_st.txt (Step 5), and p2_far.txt (Step 5)

4.1.4 User's Guide

The final file of interest on the distribution CD is "User_Guide_IV.ppt" which contains the User's Guide (Ref. 2.)

4.2 Running a Sample Problem

The steps required to obtain the solution to a sample problem are now described. Screen inputs are given to the right of the equal signs (courier font). [All numerical results which follow were generated using compiled source code. However, the resulting executables have been shown in Section 4.1.2 to be equivalent to the executables supplied on the CD.]

4.2.1 Step 1: Generate the Sawyers Solution

Run the code for the Sawyers solution, "2d_gen26.exe."

Screen Inputs:

```
output file name,                    = p0_z1.txt
Mach number at the air side, M_A     =   1.5
Begin calculation at Xbegin          = -20.0
End calculation at Xend              =  20.0
Number of divisions, Nx              =   200
Scaled Depth, z                      =   0.5
```

Output:

🗀 Two-column text file "p0_z1.txt" which contains:

x $u(x,z=0.5)$

from x = Xbegin to x = Xend at intervals of Δx = (Xend - Xbegin)/Nx .

4.2.2 Step 2: Compute Potential at Surface

Run the code for the surface potential, "Sw_st16.exe."

Screen Inputs:

```
Wave number,                          k  =  14.0
Mach number at the air side,         M_A =   1.5
Swept angle (deg),                    Λ  =   0.0
Non-alignment angle (deg),            Ψ  =   0.0
End of Phi calculation,          Xstop  =  20.0
Integration step,                Xstep  =   0.01
```

Output:

🗀 Three-column text file "phi_st.txt" which contains real and imaginary parts of $\hat{\phi}_2(x)$:

$x \qquad \mathrm{Re}\left[\hat{\phi}_2(x)\right] \qquad \mathrm{Im}\left[\hat{\phi}_2(x)\right]$

from $x = 0$ to $x = \mathrm{Xstop}$ at intervals of $\Delta x = \mathrm{Xstep}$.

🗀 Unformatted (binary) file "phi_dat" needed for steps 3 and 4.

4.2.3 Step 3: Compute Interaction Overpressure at Surface

Run the code for the surface overpressure, "SUR_P26.exe."

Screen Inputs:

```
input file name,                   = phi_dat
```

Output:

🗀 Two-column text file "p0_zeq0.txt" which contains:

$x \qquad u(x, z = 0) = u_0(x)$

from $x = 0$ to $x = \mathrm{Xstop}$ at intervals of $\Delta x =$ 5*Xstep.

🗀 Three-column text file "p2_zeq0.txt" which contains real and imaginary parts of $\hat{p}_{2,\mathrm{surf}}(x)$:

$x \qquad \mathrm{Re}\left[\hat{p}_{2,\mathrm{surf}}(x)\right] \qquad \mathrm{Im}\left[\hat{p}_{2,\mathrm{surf}}(x)\right]$

from $x = 0$ to $x = \mathrm{Xstop}$ at intervals of $\Delta x =$ 5*Xstep.

4.2.4 Step 4: Compute Transformed Amplitude

Run the code for the amplitude, "Sw_st27.exe."

Screen Inputs:

```
input Phi file name,                    = phi_dat
Begin calculation at Xibegin            = -20.0
End calculation at Xiend                =  20.0
Integration step,  dxi                  =  0.01
End of Phi Calculation, Xstop           =  20.0
```

Output:

- Three-column text file "fta_st.txt" which contains real and imaginary parts of $\hat{A}(\xi)$:

$$\xi \qquad \mathrm{Re}\left[\hat{A}(\xi)\right] \qquad \mathrm{Im}\left[\hat{A}(\xi)\right]$$

from ξ = Xibegin to ξ = Xiend at intervals of $\Delta\xi$ = dxi .

- Unformatted (binary) file "fta_dat" needed for step 5.

4.2.5 Step 5: Compute Interaction Overpressure at Depth

Run the code to compute the wavy surface overpressure at depth, "Sw_st34.exe."

Screen Inputs:

```
Input A(xi) file name,                  = fta_dat
Scaled Depth, z                         =   0.5
Begin calculation at Xbegin             = -20.0
End calculation at Xend                 =  20.0
Number of divisions, Nx                 =   200
```

Output:

- Three-column text file "p2_st.txt" which contains real and imaginary parts of $\hat{p}_2(x,z)$:

$$x \qquad \mathrm{Re}\left[\hat{p}_2(x,z)\right] \qquad \mathrm{Im}\left[\hat{p}_2(x,z)\right]$$

from x = Xbegin to x = Xend at intervals of Δx = (Xend - Xbegin)/Nx .

- Three-column text file "p2_far.txt" which contains real and imaginary parts of $\hat{p}_{2,\mathrm{far}}(x,z)$:

$$x \qquad \mathrm{Re}\left[\hat{p}_{2,\mathrm{far}}(x,z)\right] \qquad \mathrm{Im}\left[\hat{p}_{2,\mathrm{far}}(x,z)\right]$$

from x = Xbegin to x = Xend at intervals of Δx = (Xend - Xbegin)/Nx .

4.2.6 Step 6: Examine Output Using Spreadsheets

Examine the output of Steps 1-5 using the spreadsheets provided.

Spreadsheet 1: "Plot_Phi_Fta.xls"
Paste the output in the files "phi_st.txt" (Step 2) and "fta_st.txt" (Step 4) into this spreadsheet to look at plots of the real and imaginary parts of $\hat{\phi}_2(x)$ and $\hat{A}(\xi)$. **This spreadsheet performs no calculations.**

Spreadsheet 2: "P_Zeq_1.xls"
This is the most important spreadsheet. It allows display of the final result, $\overline{p}(\overline{t},\overline{z})$, given in Eq. (2.1). (The horizontal field location is always taken as $\overline{x}=0$.) To use this spreadsheet, paste the three-column output in "p2_st.txt" (Step 5) into columns B, C, and D. Paste the three-column output in "p2_far.txt" (Step 5) into columns F, G, and H. Finally, paste the two-column output in "p0_z1.txt" (Step 1) into columns J and K. At the top of the spreadsheet, it is also necessary to enter the values of the parameters: $M_A, k, \Psi, \Lambda, P_{\max}, \delta,$ and L.

In cell K3, the non-dimensional interaction frequency is computed according to Eq. (2.5).

$$\Omega = k\cos(\Lambda + \Psi) = k_1$$

In cell K5, the velocity of the supersonic body is computed:

$$U = a_A M_A \quad \text{(assumption: } a_A = 1118.2 \text{ ft/sec)}$$

In column M, the spreadsheet transforms the non-dimensional horizontal position, x, in column B to dimensional time as follows.

$$\overline{t} = x L / U$$

In column N, the flat surface overpressure is computed from the Sawyers result in column K. [See Eq. (3.2).]

$$\overline{p}_1(\overline{t},\overline{z}) = -P_{\max}\, u(x,z)$$

In column O, the wavy surface overpressure is computed from the position, x, in column B and the real and imaginary parts of $\hat{p}_2(x,z)$ in columns C and D. [See Eq. (3.6).]

$$\overline{p}_2(\overline{t},\overline{z}) = P_{\max}\,\delta\,\mathrm{Re}\left[\hat{p}_2(x,z)\,e^{-i\Omega x}\right]$$

In column P, the final result, i.e., the total overpressure at depth, is tabulated as the sum of column N and O. [See Eq. (2.1).]

$$\bar{p}(\bar{t},\bar{z}) = \bar{p}_1(\bar{t},\bar{z}) + \bar{p}_2(\bar{t},\bar{z})$$

Spreadsheet 3: "P_Zeq_0.xls"
This spreadsheet is very similar to "P_Zeq_1.xls" discussed above. It allows the display of the *surface* value of overpressure, $\bar{p}(\bar{t},\bar{z}=0)$. To use this spreadsheet, paste the three-column output in "p2_zeq0.txt" (Step 3) into columns B, C, and D. Paste the two-column output in "p0_zeq0.txt" (Step 3) into columns F and G. At the top of the spreadsheet, it is also necessary to enter the values of the parameters: $M_A, k, \Psi, \Lambda, P_{max}, \delta$, and L.

Calculations proceed exactly as for the above spreadsheet, with the final result of total surface overpressure, $\bar{p}(\bar{t},\bar{z}=0)$, being tabulated in column L.

4.3 Code Inputs: Summary and Utilization

The following table summarizes the inputs needed to specify a particular case.

Input No.	Variable	Units	Input in Step(s):	Input Value for Sample Problem
V.1	Nx	---	1 and 5	200
V.2	Xbegin	---	1 and 5	-20.0
V.3	Xend	---	1 and 5	20.0
V.4	M_A	---	1 and 2	1.5
V.5	z	---	1 and 5	0.5
V.6	k	---	2	14.0
V.7	Λ	deg	2	0.0
V.8	Ψ	deg	2	0.0
V.9	Xstop	---	2 and 4	20.0
V.10	Xstep	---	2	0.01
V.11	Xibegin	---	4	-20.0
V.12	Xiend	---	4	20.0
V.13	dxi	---	4	0.01
V.14	P_{max}	psf	Spreadsheets 2 and 3	1.0
V.15	L	ft	Spreadsheets 2 and 3	668.
V.16	δ	---	Spreadsheets 2 and 3	0.0267
V.17	a_A	ft/sec	Assumed = 1118.2 Spreadsheets 2 and 3	1118.2
V.18	$u_0(x)$	---	1, 2, 3, and 4	N-wave

Inputs V.4 – V.8 and V.14 – V.17 are defined in Section 2.1.

Utilization of V.1 – V.3 in Steps 1 and 5
The first three inputs (Nx, Xbegin, and Xend) allow the user to choose the x-interval over which solution output is desired, and also the step size for that output. For example, using the Sample Problem values of (200, -20.0, and 20.0) gives output from x = -20 to x = 20 at intervals of $\Delta x = 40 / 200 = 0.2$. Note that these three inputs are **not** used for any numerical evaluations of integrals or derivatives.

Evaluation of the Integral in Eq. (3.3) in Step 1
The integral in this equation is evaluated in subroutine **s2d_gen1**, which is contained in file "2d_gen26.for." [This is the code for Step 1, which computes the Sawyers (flat-surface) term.] The integral is evaluated numerically (see next section.) The code employs no user inputs to establish the first-order method here. The interval (0,1) is always divided evenly into 2000 intervals.

$$N = 2000 \quad \text{and} \quad \Delta\xi = 1/2000$$

Numerical Evaluations in Step 2
The inputs, Xstop (V.9) and Xstep (V.10), are used in the routine (Step 2) which computes $\hat{\phi}_2(x)$. Values of $\hat{\phi}_2$ are computed starting at x = 0 and stopping at x = Xstop, at intervals of Δx = Xstep. Recommended input values (see Ref. 2) are Xstop = 20 and Xstep = 0.01.

The integral $\hat{S}(x)$ defined in Eqs. (3.8) and (3.9) is evaluated numerically (see next section) with an integration step size of $\Delta x_1 = 0.0001$. The derivative of the unit surface waveform which appears in Eq. (3.10) is computed using a central finite difference with a "grain" of $\Delta x_1 = 0.0001$.

Numerical Evaluation in Step 3
Values of $\hat{p}_{2,\mathrm{surf}}$ [see Eq. (3.12)] are evaluated in Step 3 by finite differencing consecutive values of $\hat{\phi}_2$ which were computed in Step 2. Thus the "grain" of the finite difference derivative is Δx = Xstep. The code writes only every fifth value to output. Thus, the usual situation, after execution of Step 3, is to have output values of $\hat{p}_{2,\mathrm{surf}}$ at intervals of Δx = 0.05. (NOTE: The calculation of the surface values of overpressure due to wavy interaction is a side calculation. The results are not used in subsequent steps.)

Numerical Evaluations in Step 4
The variables V.11 – V.13 are used in the routine (Step 4) which computes $\hat{A}(\xi)$. Recommended values (see Ref. 2) are Xibegin = -20, Xiend = 20, and dxi = 0.01. Values of $\hat{A}$ are computed starting at ξ = Xibegin and stopping at ξ = Xiend, at intervals of $\Delta\xi$ = dxi.

The integral denoted by $\hat{A}_\phi$ in Eq. (3.14) is computed numerically (see next section) with Δx = Xstep. For numerical evaluation, the upper limit on the integral ($x = \infty$) is taken as x = Xstop.

The integral denoted by $\hat{A}_B$ in Eq. (3.15) is computed numerically (see next section) with $\Delta x = 0.01$. For numerical evaluation, the lower and upper infinite limits on the integral are taken as x = -Xstop and x = Xstop.

The function $q(x)$ defined in Eq. (3.16) is evaluated using a forward finite difference with $\Delta z = 0.0001$.

Numerical Evaluation in Step 5

The inverse Fourier transform in Eq. (3.17) is evaluated numerically (see next section.) For numerical purposes, the integration step size and the infinite limits on the integral are defined by the inputs as follows.

$$\Delta\xi = \text{dxi}, \quad \xi_{\text{min}} = \text{Xibegin}, \text{ and } \quad \xi_{\text{max}} = \text{Xiend}$$

4.4 Numerical Evaluation of Integrals

There are five integrals evaluated by the code. These are given in Eqs. (3.3), (3.8), (3.9), (3.14), (3.15), and (3.17). In each case, the following method is used. Suppose the integral of interest is:

$$I(x_{LO}, x_{HI}) = \int_{x_{LO}}^{x_{HI}} f(x)dx \qquad (4.1)$$

If the integral is computed using N evaluations of the integrand, then define

$$\Delta x = (x_{HI} - x_{LO}) / N \qquad (4.2)$$

$$x_j = x_{LO} + \Delta x/2 + (j-1)\Delta x \quad j = 1,2,...,N \qquad (4.3)$$

$$f_j = f(x_j) \quad j = 1,2,...,N \qquad (4.4)$$

The numerical approximation of the integral is then given by:

$$I(x_{LO}, x_{HI}) = \Delta x \sum_{j=1}^{N} f_j \qquad (4.5)$$

For the five integrals of interest, the values of the integrand, f_j, are complex numbers. However, the variable of integration (denoted by x, x_1 or ξ) is always real.

5 Code Results for a Sample Problem

A sample problem is now presented to serve as a test case for the code. [To reiterate the statement made at the beginning of Section 4.2, all numerical results were generated using compiled source code.] Suppose a sonic boom sweeps over an area of deep ocean where the wave height (trough-to-crest) is 16 feet and the wavelength is 300 feet. That is,

$$A_0 = 8 \ \text{ft} \tag{5.1}$$

$$\lambda = 300 \ \text{ft} \tag{5.2}$$

Suppose also that the boom moves along the water surface with a velocity of 1677 ft/sec in a direction exactly opposite to that of the propagation of the water waves. That is,

$$U = 1677 \ \text{ft/sec} \tag{5.3}$$

$$\Psi = 0^\circ \tag{5.4}$$

Assume further that the boom signature at the water surface is an N-wave, 668 feet in length, with a maximum overpressure of 1 psf, and that we are interested in the transmitted sound directly below the flight path of the supersonic body.

$$u_0(x) = \text{N-wave} \tag{5.5}$$

$$L = 668 \ \text{ft} \tag{5.6}$$

$$P_{\max} = 1 \ \text{psf} \tag{5.7}$$

$$\Lambda = 0^\circ \tag{5.8}$$

From the above parameters, one calculates the following inputs needed for running the code.

$$M_A = U / a_A = 1677 / 1118.2 = 1.5 \tag{5.9}$$

$$k = (2\pi / \lambda) L = 2\pi \cdot 668 / 300 = 14 \tag{5.10}$$

$$\delta = A_0 / \lambda = 8 / 300 = 0.0267 \tag{5.11}$$

Equations (5.4) through (5.11) specify 8 of the 18 inputs used for the sample problem. Values used for the remaining 10 inputs are shown in the table in Section 4.3. (For example: Nx = 200, Xbegin = -20, Xend = 20, Xstop = 20, Xstep = 0.01, dxi = 0.01.) The problem was run for four different scaled depths: $z = 0.01, \ 0.15, \ 0.5, \text{ and } 1.0$.

Output results for the flat surface (Sawyers) term, p_1, are shown in Fig. 5. Results for the wavy surface term, p_2, are shown in Fig. 6. Results for the total disturbance, $p_1 + p_2$, are shown in Fig. 7.

The results for the two separate overpressures and the total are shown at each of the depths in Figs. 8 – 12.

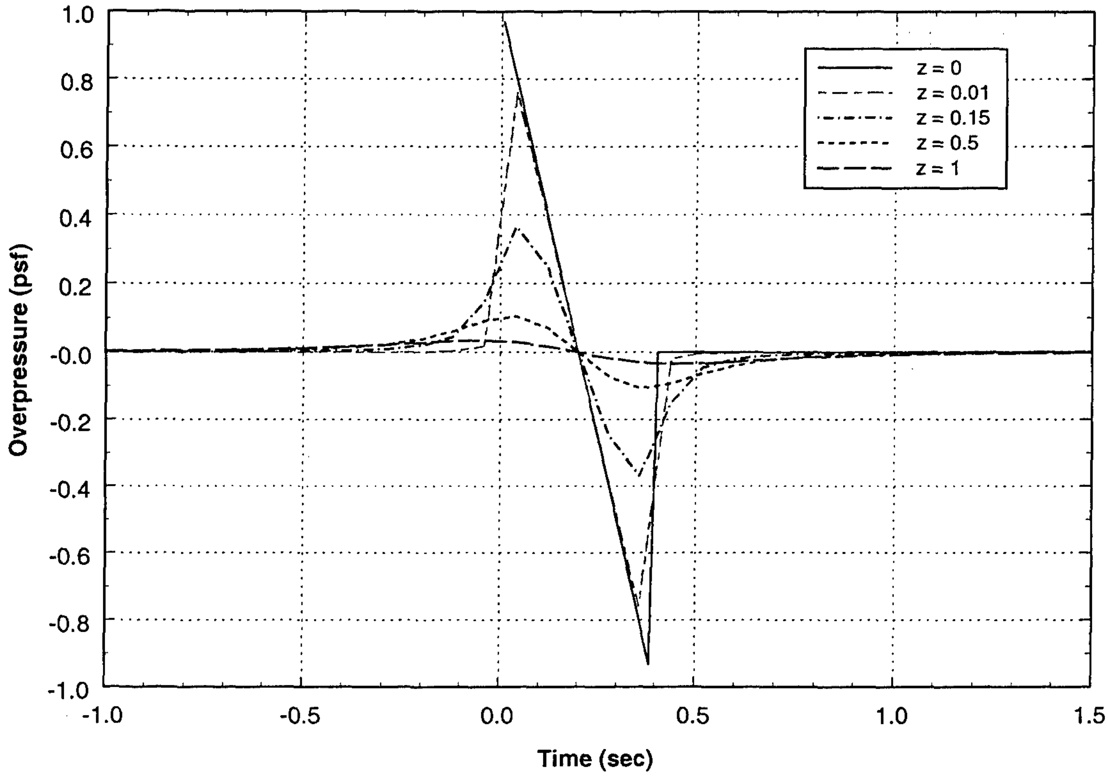

Figure 5. Flat surface term, p_1, at various depths.

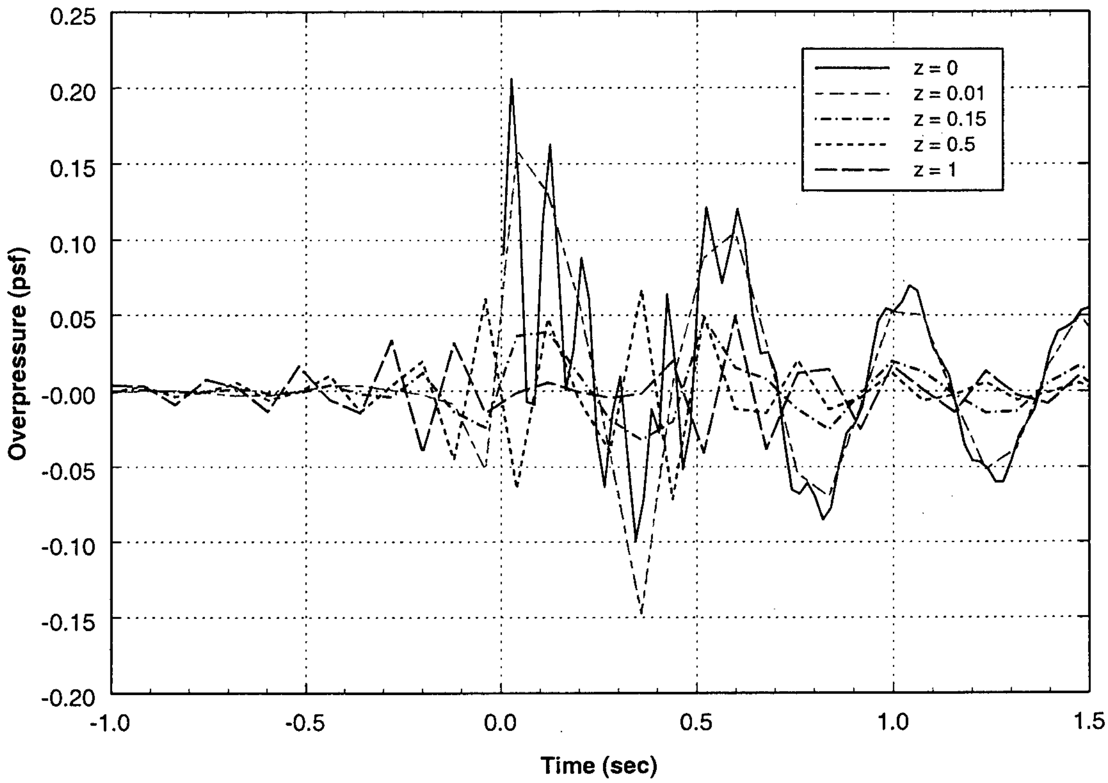

Figure 6. Wavy surface term, p_2 , at various depths

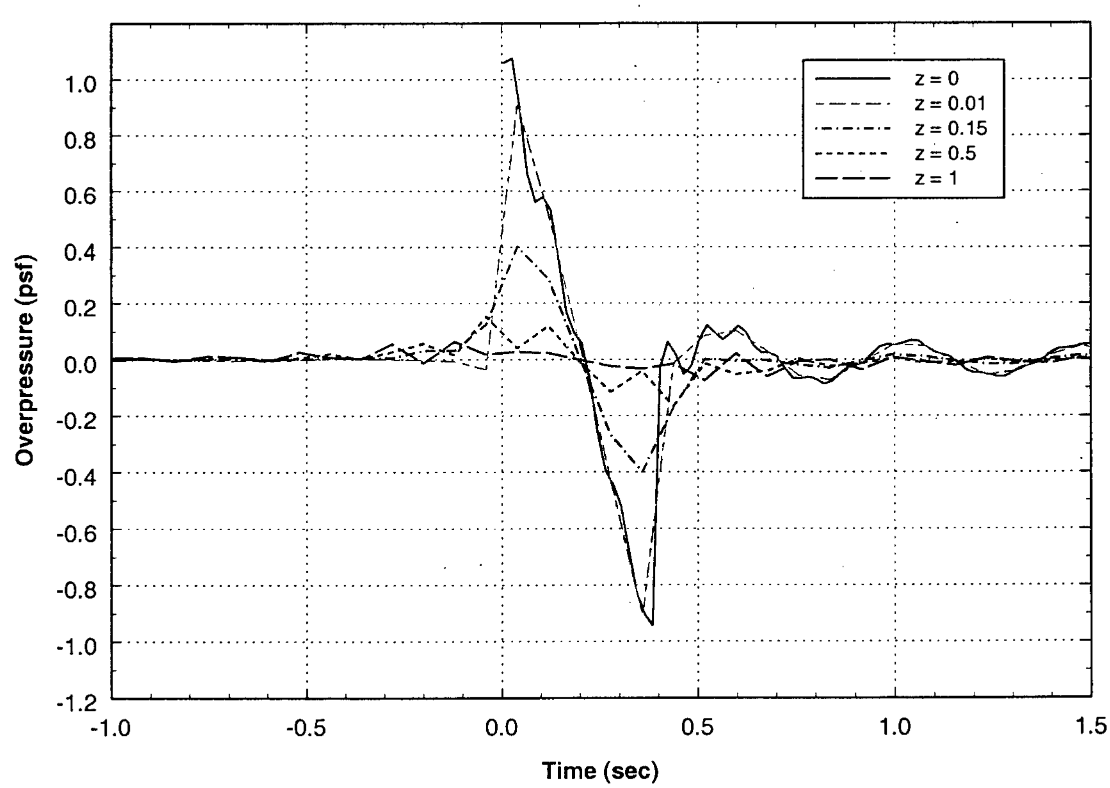

Figure 7. Total disturbance, $p_1 + p_2$, at various depths.

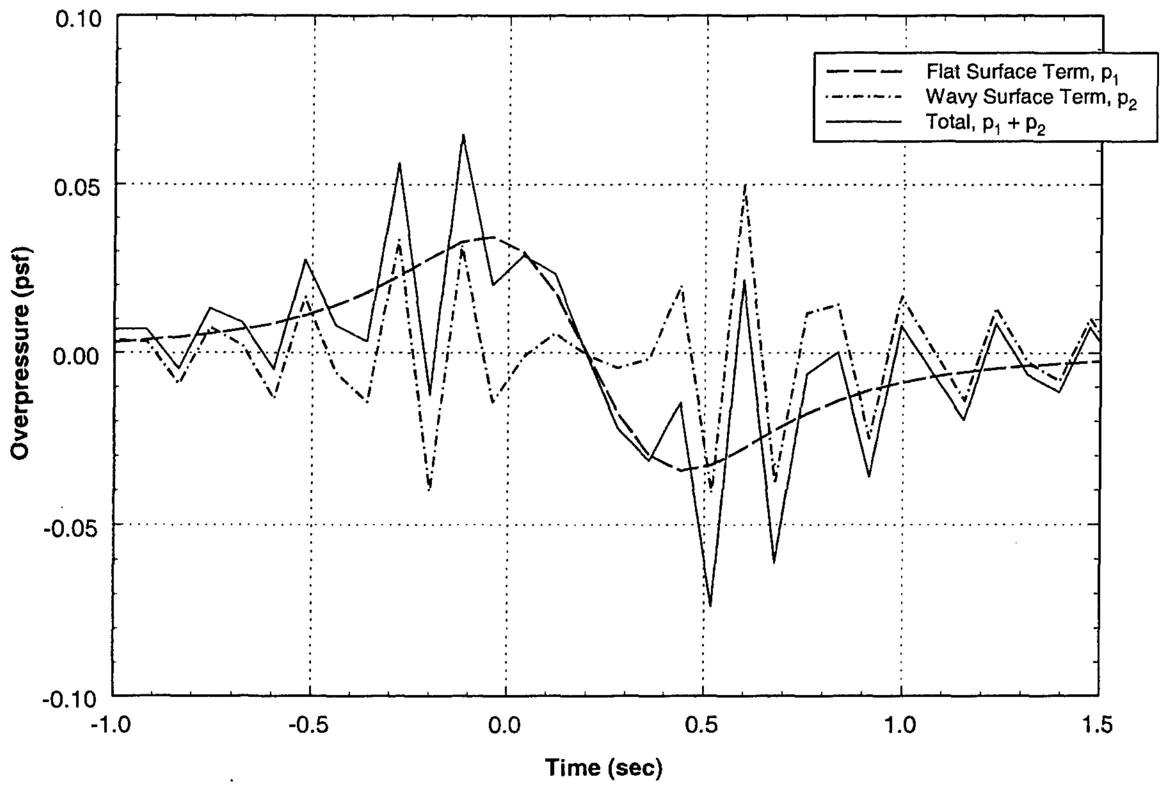

Figure 8. Overpressures at $z = 1$.

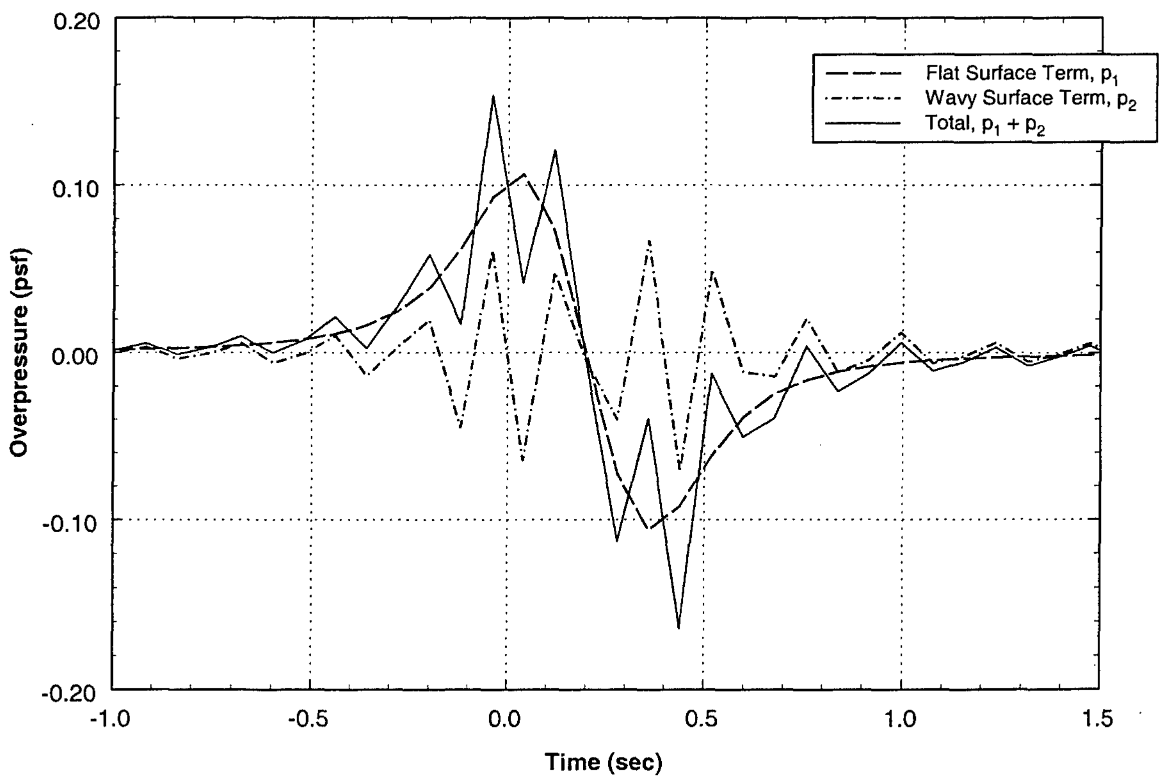

Figure 9. Overpressures at $z = 0.5$.

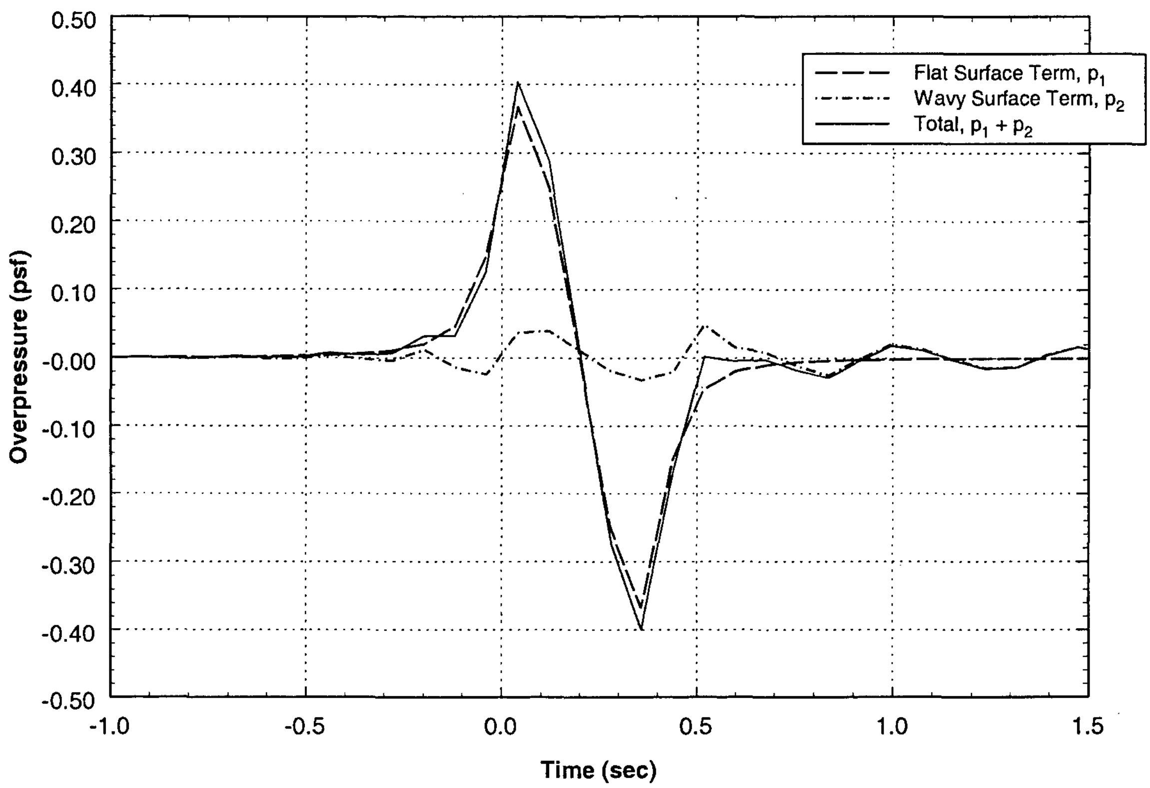

Figure 10. Overpressures at $z = 0.15$.

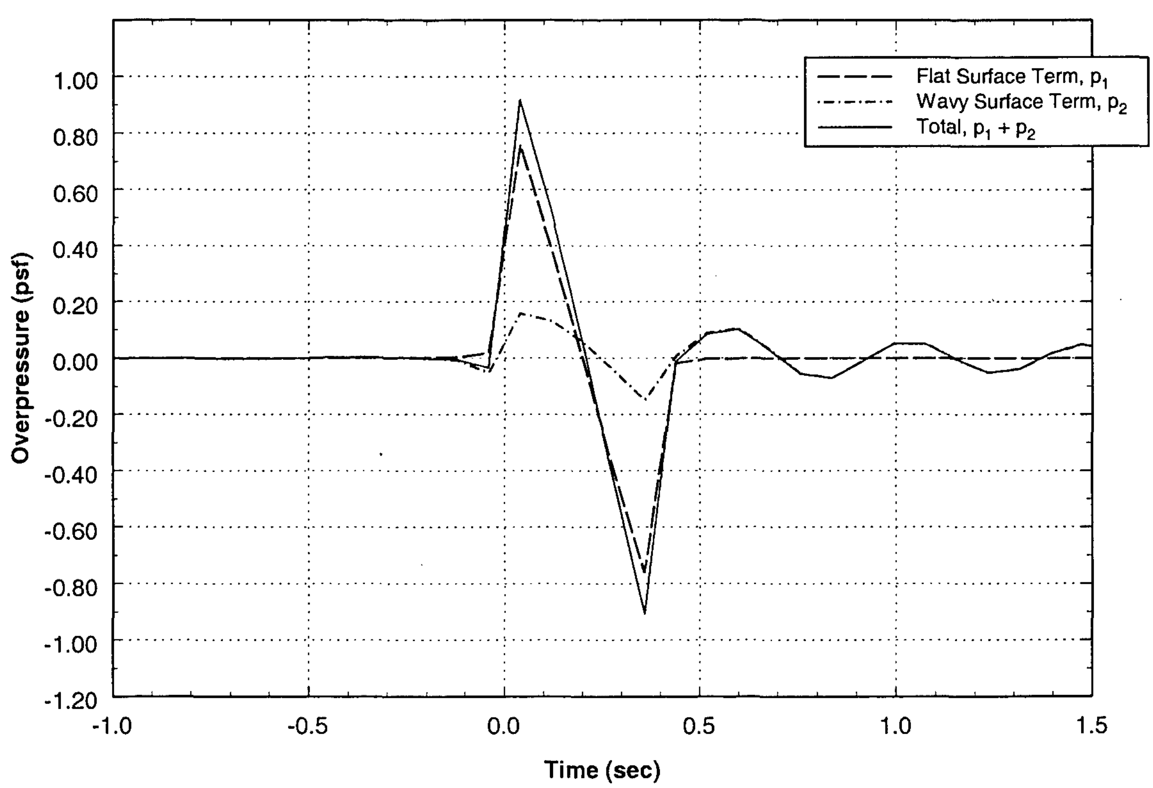

Figure 11. Overpressures at $z = 0.01$.

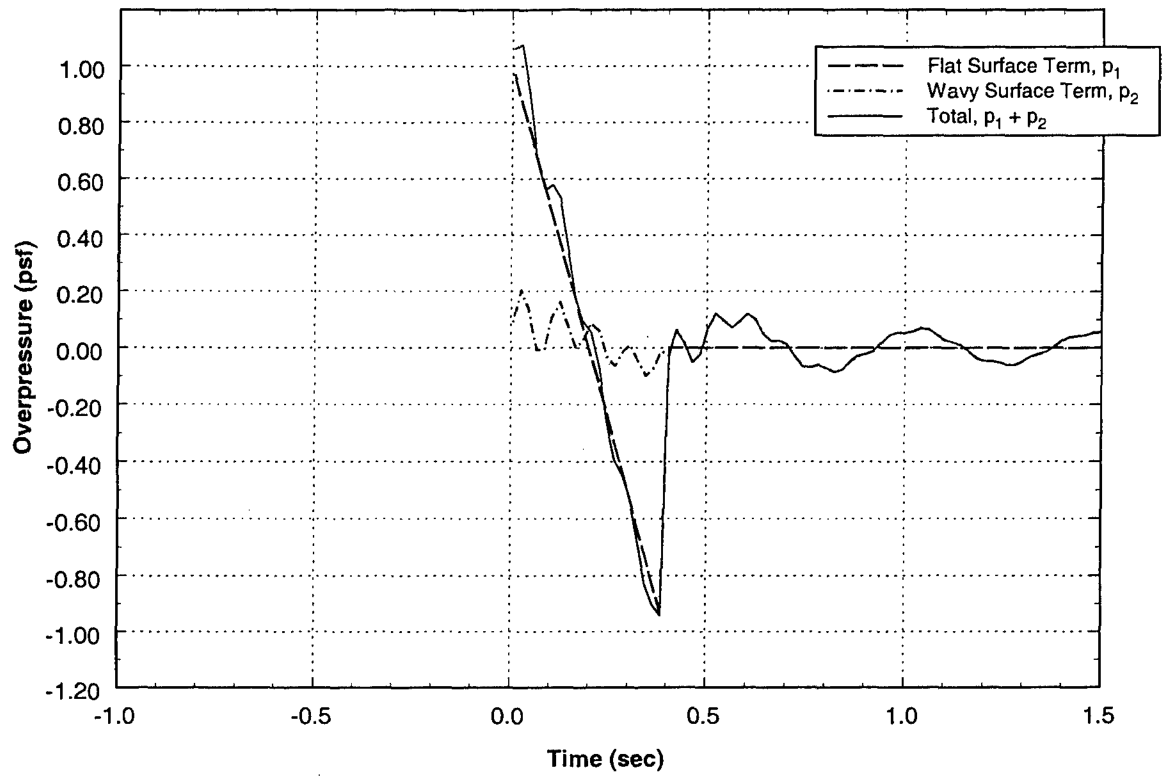

Figure 12. Overpressures at $z = 0$.

In addition to this particular sample problem, the reader may consult Ref. 5 for additional sample solutions.

6 Summary

The sonic boom wavy surface code (Ref. 1) has been documented. Explanations have been given of the function and execution of the various code modules. Results from a sample problem have been plotted in order to provide a "benchmark" for future users and against which to compare possible future releases of this code by Cheng and Lee.

References

1. H. K. Cheng & C. J. Lee, "Sonic Boom Noise Penetration Under a Wavy Surface," CD from HKC Research, (Contains code and User's Guide.), April 2003.

2. C. J. Lee & H. K. Cheng, "Sonic Boom Noise Penetration Under a Wavy Surface," User's Guide, (Contained on CD in Ref. 1.), February 18, 2003.

3. H. K. Cheng & C. J. Lee, "Sonic Boom Noise Penetration Under a Wavy Ocean: Part I, Theory" USC Report AME 11-11-2000, (http://www-bcf.usc.edu/~hkcheng), November 2000.

4. H. K. Cheng, C. J. Lee, & J. R. Edwards, "Sonic Boom Noise Penetration Under a Wavy Ocean: Part II, Examples and Extensions" USC Report AME 4-4-2001, April 2001.

5. H. K. Cheng, "Final Report: Post Ocean Sonic Boom Investigations in Support of Los Angeles Air Force Base," HKC Research, submitted to Air Force Space and Missile System Center, SMC/AXF, Los Angeles Air Force Base, through PARSONS, Pasadena, February 2003.

6. K. N. Sawyers, "Underwater Sound Pressure from Sonic Booms," J. Acoust. Soc. Amer. 44, 523-524, 1968.