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**MONTEREY, CALIFORNIA**

**A STUDY ON THE TRANSIENT RESPONSE  
OF A STEPPED COLUMN SUBJECTED  
TO AN IMPACT LOAD**

by

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June 2004

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Prepared for: Department of Mechanical and Astronautical Engineering  
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## **ABSTRACT**

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## I. INTRODUCTION

Dynamic buckling of structures has been studied in the field of naval architectural, aerospace, nuclear power plant and civil engineering. Transient or impulsive loads, which may exceed the critical static Euler limit, can also be an important design consideration. Especially, in case of warship or submarine, the design of structures to resist shock loads from hostile weapons or missile launching is important for the protection of crew's safety and environment. On the ground of that, many people have researched dynamic buckling of structures.

Takuo and Yukio [Ref. 1] studied a simply supported column with an initial deflection, which had one end struck by a mass along the axial line at higher impact velocities.

Tanchum and Haim [Ref. 2] calculated and compared the buckling problems of beam and plate subjected to an axial impact. They studied on the initial geometric imperfection of structures and determined a dynamic load amplification factor by analytical studies with ADINA program.

N. G. Pegg [Ref. 3] investigated the effect of a ring stiffener and its size and spacing on the dynamic buckling response of a cylinder.

W. Gu and W. Tang [Ref. 4] studied the dynamic plastic buckling of cylindrical shells subjected to general external impulsive velocity and subjected to asymmetric impulsive loadings.

Murli and Judah [Ref. 5] investigated the dynamic buckling of geometrically imperfect columns with viscous damping under an axial compressive pulse. Viscous damping effects became significant for extremely short pulses and that higher damping caused an increase in the dynamic buckling load and a smoother deflection pattern.

S. Kenny and N. Pegg [Ref. 6] did the paper concerned with dynamic elastic pulse buckling events, characterized by the unacceptable growth of lateral displacements due to an intense transient load that significantly exceeds the critical static Euler limit.

O. Aksogan and A.H. Sofiyev [Ref. 7] studied the dynamic buckling of an elastic cylindrical shell with variable thickness subjected to a uniform external pressure which was a power function of time.

Shijie and Hong [Ref. 8] investigated the dynamic elastic buckling of simply supported columns subjected to intermediate velocity impact. They proposed the dynamic buckling criterion to determine the critical buckling condition and to estimate the dynamic buckling critical load.

Galib H. and Christos C. [Ref. 9] suggested the computational simulation method to evaluate the deterministic and non-deterministic dynamic buckling of adaptive composite shells.

As mentioned above, there are many studies for a continuous beam or a cylinder under axial shock load. But it is not easy to find the paper on the stiffness variation, one of important design factors in the preliminary design stage of the structures such as ship, airplane and missile. Generally, the body section stiffness of those are different each other by access doors, stiffener, skin cutout and so on. Therefore, the cost by a trial and error can be reduced if the structure design process considers the stiffness distribution of a system in preliminary design step. In this report, we investigated the effect of the stiffness variation on the stepped slender column subjected to an axial or a vertical impact load. The theoretical studies, for a simply supported beam under an axial load and a cantilever beam under a vertical load, were carried out. A transient analyses for beams and cylindrical shells were done, and the results were evaluated. Also, it was investigated the effect of the frequency variation of an impact load on the response of structures. A finite element modeling and a numerical analysis were performed by MSC/PATRAN and MSC/NASTRAN program.

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## II. MATHEMATICAL FORMULATION

This chapter is concerned with the equations of motion of a stepped beam subjected to axial or vertical loads. Mathematical formulations are carried out for a simply supported beam and a cantilever beam as follows.

### A. A SIMPLY SUPPORTED BEAM

From H.E. Lindberg and A.L. Florence [Ref. 10], the equations of motion of a simply supported beam under an axial compressive force  $P$  are

$$EI \frac{\partial^4 y}{\partial x^4} + P \frac{\partial^2}{\partial x^2} (y + y_0) + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

Where,  $E$  is an elastic coefficient,  $I$  is an area moment of inertia and  $y_0$  is an initial deflection,  $\rho$  is a mass density and  $A$  is an area of beam.

For static buckling, the inertia term is neglected and Equation (1) becomes

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} = -P \frac{d^2 y_0}{dx^2} \quad (2)$$

Or, substituting  $k^2 = \frac{P}{EI}$

$$\frac{d^4 y}{dx^4} + k^2 \frac{d^2 y}{dx^2} = -k^2 \frac{d^2 y_0}{dx^2} \quad (3)$$

Homogeneous solution is

$$y_h = C_1 \cos kx + C_2 \sin kx + C_3 x + C_4 \quad (4)$$

And, we can get  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$  from the boundary conditions.

In case of the stepped beam as in Figure 1, a homogeneous solution is

$$y_h = \sum_{j=1}^3 [C_{j1} \cos k_j x + C_{j2} \sin k_j x + C_{j3} x + C_{j4}] \quad (5)$$

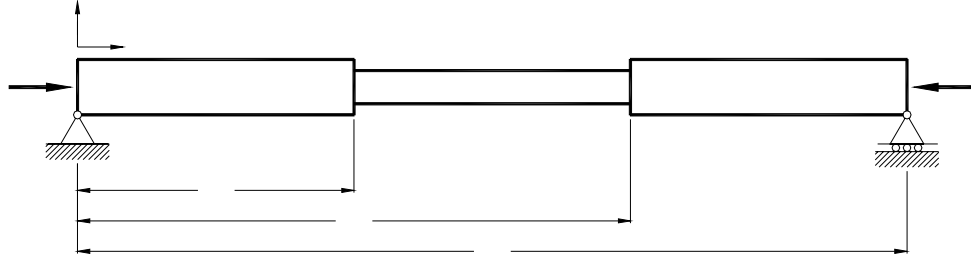


Figure 1. Stepped Simply Supported Beam Subjected to Compressive Force

Applying the boundary and the continuity conditions to get the coefficients of Equation (5)

At  $x=0$

$$y_1(0) = 0 \quad (6)$$

$$\frac{\partial^2}{\partial x^2} y_1(0) = 0 \quad (7)$$

At  $x=x_1, x_2$

$$y_1(x_1) = y_2(x_1) \quad (8)$$

$$y_1'(x_1) = y_2'(x_1) \quad (9)$$

$$y_1''(x_1) = y_2''(x_1) \quad (10)$$

$$y_1'''(x_1) = y_2'''(x_1) \quad (11)$$

$$y_2(x_2) = y_3(x_2) \quad (12)$$

$$y_2'(x_2) = y_3'(x_2) \quad (13)$$

$$y_2''(x_2) = y_3''(x_2) \quad (14)$$

$$y_2'''(x_2) = y_3'''(x_2) \quad (15)$$

At  $x=x_3$

$$y_3(x_3) = 0 \quad (16)$$

$$\frac{\partial^2}{\partial x^2} y_3(x_3) = 0 \quad (17)$$

Rearranging Equations (5 through 17) by the matrix form as follows,

$$[T_{12 \times 12}] \{C_{jk}\} = \{0\} \quad j=1,2,3 \text{ and } k = 1,2,3,4 \quad (18)$$

And  $[T_{12 \times 12}]$  are

$$\begin{aligned} [T_{0101}] &= 1, & [T_{0104}] &= 1, & [T_{0201}] &= -k_1^2, & [T_{0301}] &= \cos k_1 x_1, \\ [T_{0302}] &= \sin k_1 x_1, & [T_{0303}] &= x_1, & [T_{0304}] &= 1, & [T_{0305}] &= -\cos k_2 x_1, \\ [T_{0306}] &= -\sin k_2 x_1, & [T_{0307}] &= -x_1, & [T_{0308}] &= -1, & [T_{0401}] &= -k_1 \sin k_1 x_1, \\ [T_{0402}] &= k_1 \cos k_1 x_1, & [T_{0403}] &= 1, & [T_{0405}] &= k_2 \sin k_2 x_1, & [T_{0406}] &= -k_2 \cos k_2 x_1, \\ [T_{0407}] &= -1, & [T_{0501}] &= -k_1^2 \cos k_1 x_1, & [T_{0502}] &= -k_1^2 \sin k_1 x_1, & [T_{0505}] &= k_2^2 \cos k_2 x_1, \\ [T_{0506}] &= k_2^2 \sin k_2 x_1, & [T_{0601}] &= k_1^3 \sin k_1 x_1, & [T_{0602}] &= -k_1^3 \cos k_1 x_1, & [T_{0605}] &= -k_2^3 \sin k_2 x_1, \\ [T_{0606}] &= k_2^3 \cos k_2 x_1, & [T_{0705}] &= \cos k_2 x_2, & [T_{0706}] &= \sin k_2 x_2, & [T_{0707}] &= x_2, \\ [T_{0708}] &= 1, & [T_{0709}] &= -\cos k_3 x_2, & [T_{0710}] &= -\sin k_3 x_2, & [T_{0711}] &= -x_2, \\ [T_{0712}] &= -1, & [T_{0805}] &= -k_2 \sin k_2 x_2, & [T_{0806}] &= k_2 \cos k_2 x_2, & [T_{0807}] &= 1, \\ [T_{0809}] &= k_3 \sin k_3 x_2, & [T_{0810}] &= -k_3 \cos k_3 x_2, & [T_{0811}] &= -1, & [T_{0905}] &= -k_2^2 \cos k_2 x_2, \\ [T_{0906}] &= -k_2^2 \sin k_2 x_2, & [T_{0909}] &= k_3^2 \cos k_3 x_2, & [T_{0910}] &= k_3^2 \sin k_3 x_2, & [T_{1005}] &= k_2^3 \sin k_2 x_2, \\ [T_{1006}] &= -k_2^3 \cos k_2 x_2, & [T_{1009}] &= -k_3^3 \sin k_3 x_2, & [T_{1010}] &= k_3^3 \cos k_3 x_2, & [T_{1109}] &= \cos k_3 x_3, \\ [T_{1110}] &= \sin k_3 x_3, & [T_{1111}] &= x_3, & [T_{1112}] &= 1, & [T_{1209}] &= -k_3^2 \cos k_3 x_3, \\ [T_{1210}] &= -k_3^2 \sin k_3 x_3 \end{aligned}$$

The others of  $[T_{12 \times 12}]$  are all zero.

For a nontrivial solution of Equation (18), the determinant of the above matrix is set equal to zero,  $|T_{12 \times 12}| = 0$ .

To find a particular solution, we take

$$y_0 = Y_0 e^{ix}, \quad y_p = B e^{ix}, \quad i = \text{imaginary number.} \quad (19)$$

Substitution Equation (19) into Equation (3) gives

$$y_p = -\frac{1}{2} \sum_{j=1}^3 Y_{0j} (\cosh k_j x - \sinh k_j x) \quad (20)$$

Therefore the general solution is

$$y_g = \sum_{j=1}^3 [C_{j1} \cos k_j x + C_{j2} \sin k_j x + C_{j3} x + C_{j4} - \frac{1}{2} Y_{0j} (\cosh k_j x - \sinh k_j x)] \quad (21)$$

The equations of motion for dynamic buckling is the same Equation (1) that was mentioned above. After dividing through by EI, we assume parameters as follows.

$$k^2 = \frac{P}{EI}, \quad r^2 = \frac{I}{A}, \quad c^2 = \frac{E}{\rho} \quad (22)$$

where A is an area and  $\rho$  is a mass density of beam.

Then the Equation (1) becomes

$$\frac{\partial^4 y}{\partial x^4} + k^2 \frac{\partial^2 y}{\partial x^2} + \frac{1}{r^2 c^2} \frac{\partial^2 y}{\partial t^2} = -k^2 \frac{\partial^2 y_0}{\partial x^2} \quad (23)$$

In case of a stepped beam, the equation of motion is

$$\frac{\partial^4 y_j}{\partial x^4} + k_j^2 \frac{\partial^2 y_j}{\partial x^2} + \frac{1}{r_j^2 c_j^2} \frac{\partial^2 y_j}{\partial t^2} = -k_j^2 \frac{\partial^2 y_{0j}}{\partial x^2}, \quad j=1, 2, 3 \quad (24)$$

We assume the solution of Equation (24) as follows.

$$y(x,t) = f(x)g(t), \quad y_0(x,0) = f_0(x) \quad (25)$$

where,

$$f(x) = y(x) = \sum_{j=1}^3 [C_{j1} \cos k_j x + C_{j2} \sin k_j x + C_{j3} x + C_{j4} - \frac{1}{2} Y_{0j} (\cosh k_j x - \sinh k_j x)] \quad (26)$$

$$f_0(x) = y_0(x) = \sum_{j=1}^3 Y_{0j} e^{ir_j x} = \sum_{j=1}^3 Y_{0j} e^{-k_j x} = \sum_{j=1}^3 Y_{0j} (\cosh k_j x - \sinh k_j x) \quad (27)$$

Substituting Equation (25) into Equation (24) and arranging, the following equation is given.

$$H_3 (H_5 - \frac{1}{2} H_2 H_4) \ddot{g}(t) - H_1 H_2 H_4 g(t) = -H_1 H_2 H_4 \quad (28)$$

Dots indicate differentiation with respect to time. Where,

$$H_1 = k_j^4 \quad (29)$$

$$H_2 = Y_{0j} \quad (30)$$

$$H_3 = \frac{1}{r_j^2 c_j^2} \quad (31)$$

$$H_4 = \cosh k_j x - \sinh k_j x \quad (32)$$

$$H_5 = C_{j1} \cos k_j x + C_{j2} \sin k_j x + C_{j3} x + C_{j4} \quad (33)$$

Rewriting Equation (28)

$$\ddot{g}(t) + Sg(t) = S \quad (34)$$

where,

$$S = -\frac{S_b}{S_a} = \frac{-H_1 H_2 H_4}{H_3 (H_5 - \frac{1}{2} H_2 H_4)} \quad (35)$$

The homogeneous solution of dynamic buckling can be expressed as follows. Substituting

$g(t) = e^{\lambda t}$  into Equation (34), a characteristic equation is  $\lambda^2 = -S$ .

In case of  $S > 0$ ;  $\lambda = \pm \sqrt{Si} = \pm S_s i$

$$g_h(t) = u_1 \cos S_s t + u_2 \sin S_s t \quad (36)$$

In case of  $S < 0$ ;  $\lambda = \pm S_r$

$$\begin{aligned} g_h(t) &= u_1 e^{S_r t} + u_2 e^{-S_r t} = u_1 \cosh S_r t + u_1 \sinh S_r t + u_2 \cosh S_r t - u_2 \sinh S_r t \\ &= (u_1 + u_2) \cosh S_r t + (u_1 - u_2) \sinh S_r t \end{aligned} \quad (37)$$

In case of  $S = 0$ ;  $\lambda = 0$

$$g_h(t) = u_1 + u_2 t \quad (38)$$

The particular solution of dynamic buckling is as follows. Substituting  $g(t) = \lambda$  into

Equation (34), a characteristic equation is  $\lambda = 1$ .

$$g_p(t) = 1 \quad (39)$$

Therefore the general solution is

$$g_g(t) = g_h(t) + g_p(t) \quad (40)$$

$$y(x,t) = f(x)g_g(t) = f(x)g(t) \quad (41)$$

The beam is assumed to be initially at rest. Also,  $y$  is measured from the initial displacement  $Y_0$ , so the initial conditions are

$$y(x,0) = \dot{y}(x,0) = 0 \quad (42)$$

Applying these to Equation (41) yields  $u_1$  and  $u_2$  for each case. The final solutions are then

$$y(x,t) = \sum_{j=1}^3 [C_{j1} \cos k_j x + C_{j2} \sin k_j x + C_{j3} x + C_{j4} - \frac{1}{2} Y_{0j} (\cosh k_j x - \sinh k_j x)] \times (-\cos S_j t + 1), \quad S > 0 \quad (43)$$

$$y(x,t) = \sum_{j=1}^3 [C_{j1} \cos k_j x + C_{j2} \sin k_j x + C_{j3} x + C_{j4} - \frac{1}{2} Y_{0j} (\cosh k_j x - \sinh k_j x)] \times (-\cosh S_j t + 1), \quad S < 0 \quad (44)$$

$$y(x,t) = 0, \quad S = 0 \quad (45)$$

## B. A CANTILEVER BEAM

If the external force is applied at the free edge of a cantilever beam with stiffness variation as in Figure 2, the equations of motion are given as

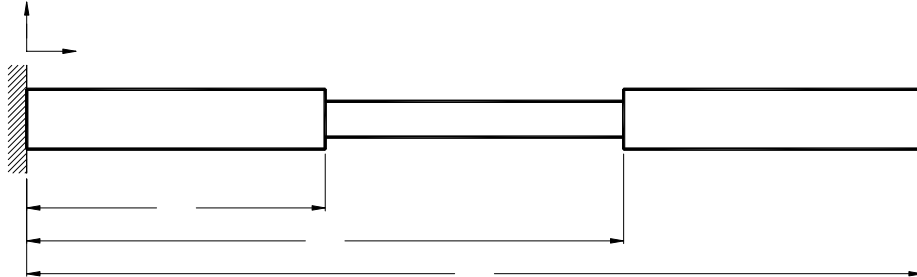


Figure 2. Stepped Cantilever Beam Subjected to a Vertical Load

$$E_j I_j \frac{\partial^4 y_j}{\partial x^4} + \rho_j A_j \frac{\partial^2 y_j}{\partial t^2} = 0, \quad j=1,2,3 \quad (46)$$

We assume that the solution of Equation (46) is

$$y_j = Y_j(x) e^{i\omega t} \quad (47)$$

Substituting Equation. (47) into Equation (46)

$$E_j I_j \frac{\partial^4 Y_j}{\partial x^4} - \rho_j A_j \omega^2 Y_j = 0 \quad (48)$$

$$Y_j(x) = Y_j = C_j e^{\lambda_j x} \quad (49)$$

Substituting Equation (49) into Equation (48)

$$(E_j I_j \lambda_j^4 - \rho_j A_j \omega^2) C_j = 0 \quad (50)$$

For the nontrivial solution of Equation (50)

$$|E_j I_j \lambda_j^4 - \rho_j A_j \omega^2| = 0 \quad (51)$$

We can get twelve roots of  $\lambda$  from Equation (51). Finally, we have

$$Y_j(x) = \sum_{n=1}^4 C_{jn} e^{\lambda_{jn} x}, \quad j=1,2,3 \quad (52)$$

The boundary and the continuity condition to get  $C_{jn}$  of Equation (52) are

At  $x = 0$

$$y_1(0) = 0 \quad (53)$$

$$y_1'(0) = 0 \quad (54)$$

At  $x = x_1, x_2$

$$y_1(x_1) = y_2(x_1) \quad (55)$$

$$y_1'(x_1) = y_2'(x_1) \quad (56)$$

$$y_1''(x_1) = y_2''(x_1) \quad (57)$$

$$y_1'''(x_1) = y_2'''(x_1) \quad (58)$$

$$y_2(x_2) = y_3(x_2) \quad (59)$$

$$y_2'(x_2) = y_3'(x_2) \quad (60)$$

$$y_2''(x_2) = y_3''(x_2) \quad (61)$$

$$y_2'''(x_2) = y_3'''(x_2) \quad (62)$$

At  $x = x_3$

$$y_3''(x_3) = 0 \quad (63)$$

$$y_3'''(x_3) = -\frac{F_0}{E_3 I_3} \quad (64)$$

From Equation (53 through 64), we can get 12  $C_{jn}$  as follows.

$$C_{34} = \frac{F_0}{E_3 I_3 R_3^3 \cos(R_3 x_3)}$$

$$C_{33} = -\frac{\sin(R_3 x_3)}{\cos(R_3 x_3)} C_{34}$$

$$C_{32} = -\frac{\sin(R_3 x_2)}{\cosh(R_3 x_2)} C_{33} + \frac{\cos(R_3 x_2)}{\cosh(R_3 x_2)} C_{34}$$

$$C_{31} = -\frac{\sin(R_3 x_2)}{\cosh(R_3 x_2)} C_{32} + \frac{\cos(R_3 x_2)}{\cosh(R_3 x_2)} C_{33} + \frac{\sin(R_3 x_2)}{\cosh(R_3 x_2)} C_{34}$$

$$C_{24} = \frac{R_3 \sinh(R_3 x_2)}{R_2 \cos(R_2 x_2)} C_{31} + \frac{R_3 \cosh(R_3 x_2)}{R_2 \cos(R_2 x_2)} C_{32} - \frac{R_3 \sin(R_3 x_2)}{R_2 \cos(R_2 x_2)} C_{33} + \frac{R_3 \cos(R_3 x_2)}{R_2 \cos(R_2 x_2)} C_{34}$$

$$C_{23} = -\frac{\sin(R_2 x_2)}{\cos(R_2 x_2)} C_{24} + \frac{\cosh(R_3 x_2)}{\cos(R_2 x_2)} C_{31} + \frac{\sinh(R_3 x_2)}{\cos(R_2 x_2)} C_{32} + \frac{\cos(R_3 x_2)}{\cos(R_2 x_2)} C_{33} + \frac{\sin(R_3 x_2)}{\cos(R_2 x_2)} C_{34}$$

$$C_{22} = -\frac{\sin(R_2 x_1)}{\cosh(R_2 x_1)} C_{23} + \frac{\cos(R_2 x_1)}{\cosh(R_2 x_1)} C_{24}$$

$$C_{21} = -\frac{\sinh(R_2 x_1)}{\cosh(R_2 x_1)} C_{22} + \frac{\cos(R_2 x_1)}{\cosh(R_2 x_1)} C_{23} + \frac{\sin(R_2 x_1)}{\cosh(R_2 x_1)} C_{24}$$

$$C_{14} = \frac{R_2 \sinh(R_2 x_1)}{R_1 \cos(R_1 x_1)} C_{21} + \frac{R_2 \cosh(R_2 x_1)}{R_1 \cos(R_1 x_1)} C_{22} - \frac{R_2 \sin(R_2 x_1)}{R_1 \cos(R_1 x_1)} C_{23} + \frac{R_2 \cos(R_2 x_1)}{R_1 \cos(R_1 x_1)} C_{24}$$

$$C_{13} = -\frac{\sin(R_1 x_1)}{\cos(R_1 x_1)} C_{14} + \frac{\cosh(R_2 x_1)}{\cos(R_1 x_1)} C_{21} + \frac{\sinh(R_2 x_1)}{\cos(R_1 x_1)} C_{22} + \frac{\cos(R_2 x_1)}{\cos(R_1 x_1)} C_{23} + \frac{\sin(R_2 x_1)}{\cos(R_1 x_1)} C_{24}$$

$$C_{12} = -C_{14}$$

$$C_{11} = -C_{13}$$

## II. NUMERICAL ANALYSIS AND CONSIDERATIONS

For investigating what the stiffness variation of structures affect on a system under an impact load, a numerical analysis is carried out for several models. The MSC/PATRAN [Ref. 11] for finite element modeling and the MSC/NASTRAN program [Ref. 12] for transient analysis are used. In this report, a cantilever beam subjected to a vertical load as shown in Figure 2 and a simply supported beam subjected to an axial load as in Figure 3 are analyzed. Then a transient analysis is carried out also for a cylindrical shell of a cantilever condition. The purpose is to confirm whether the analysis result of a beam of one dimensional configuration coincides with that of a cylindrical shell of three dimensional configuration such as Figure 4 or not. In a preliminary design stage, to model the large structures of a cylindrical shell type like ship, missile and airplane as one dimensional beam is very important to save time and effort. The response of structures with the frequency variation of impact loads are also investigated for the beam and the cylinder of a cantilever condition. Table 1 shows the load and the boundary conditions applied in transient analyses. Table 2 shows the dimension and the physical property of material of each model used for a numerical analysis.

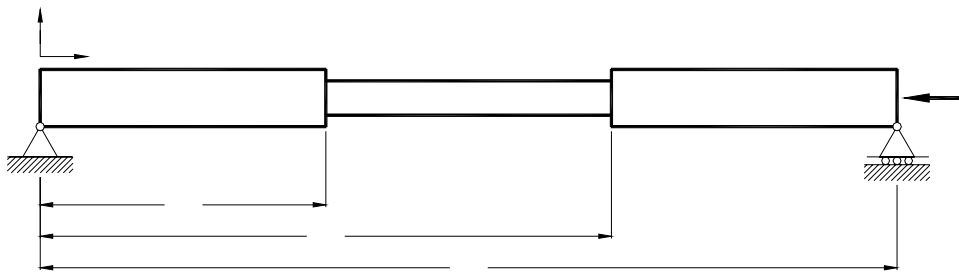


Figure 3. Stepped Simply Supported Beam Subjected to an Axial Load

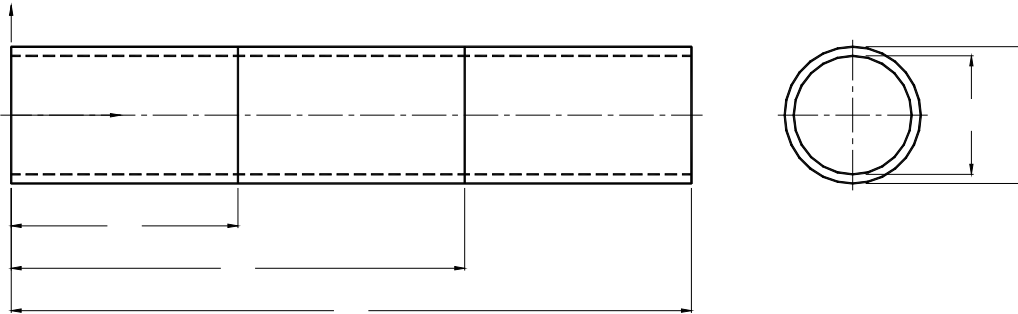


Figure 4. Configuration of a Cylindrical Shell

Table 1. Applied Loads and Boundary Conditions for Transient Analysis

Load and Type	Load : $F(t) = F_0 \sin \omega t$						Boundary Conditions and Models
	Shape	Direction	$F_0$ (kgf)	$\omega$ (hz)	Time (t, sec)		
					$t_1$	$t_2$	
Load 1	Half Sine Wave	Axial	161,167	50	0.0	0.0100	Simply Supported, Beam(ssb)
Load 2	Half Sine Wave	Vertical	1,600	50	0.0	0.0100	Cantilever, Beam and Cylindrical Shell(cb/ccs)
Load 3	Half Sine Wave	Vertical	1,600	100	0.0	0.0050	Cantilever, Beam and Cylindrical Shell(cb/ccs)
Load 4	Half Sine Wave	Vertical	1,600	200	0.0	0.0025	Cantilever, Beam and Cylindrical Shell(cb/ccs)

Table 2. Dimension and Physical Data of Beam and Cylinder Model (Unit: mm)

Dimension and Model Type	Section 1			Section 2			Section 3		
	O.D*	I.D*	Length	O.D*	I.D*	Length	O.D*	I.D*	Length
Model 1	302	298	1000	302	298	1000	302	298	1000
Model 2	302	298	1000	302	299	1000	302	298	1000
Model 3	302	298	1000	302	300	1000	302	298	1000
Model 4	302	298	1000	302	301	1000	302	298	1000
Material	Al 2024-t3								
Elastic Modulus	7,380 (kgf/mm <sup>2</sup> )								
Mass Density	2.853E-10 (kgf.sec <sup>2</sup> /mm <sup>4</sup> )								
Poisson's Ratio	0.33								

\* : O.D and I.D are acronym for Outer and Inner Diameter.

## A. A SIMPLY SUPPORTED BEAM UNDER AN AXIAL IMPACT LOAD

### 1. The Effect of a Stiffness Variation

Figure 5 through 14 show the displacements and the stresses of a simply supported beam subjected to an axial impact load, the load 1 in Table 1. The purpose is to understand the phenomenon varying the displacements and the stresses by the thickness reduction of section 2 in Table 2, namely, the stiffness change of structures.

Figure 5, which is in case of model 1 without the thickness variation of each section, shows axial displacements on a time domain. From the graphs, we know that the axial displacements are the largest at the sta.3000 applied an axial impact and the others are decreasing in order of sta. 2000, 1500, 1000 and 500. The displacements have the characteristics appearing mainly for 0.01sec applying an axial impact, then reducing abruptly and disappearing finally.

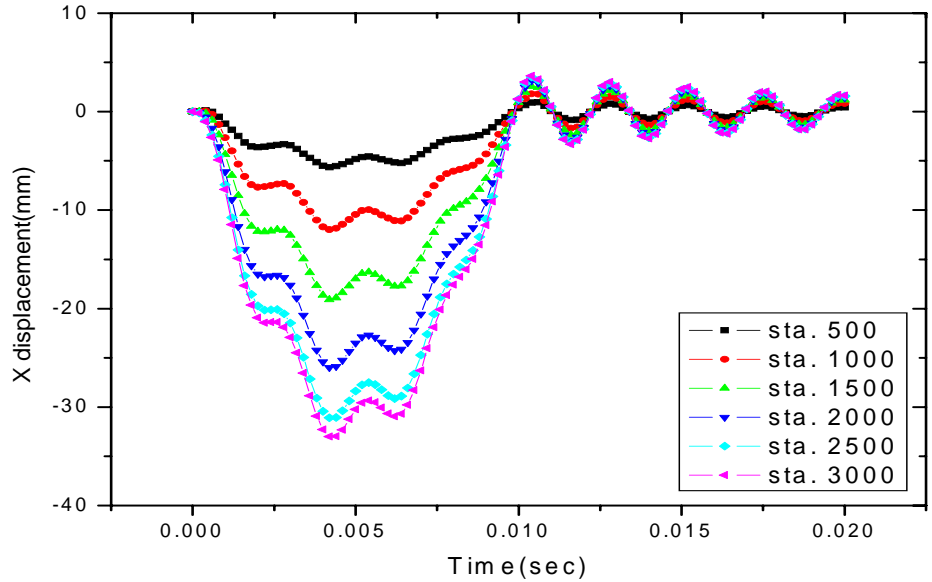


Figure 5. Displacements of Model 1(ssb, 2mm) under Load 1(axial, 50hz)

Figure 6 is the stress curve of model 1. The stresses are the largest at the sta. 1500, the mid-position of the section 2 among three sections. It is natural that the largest compressive stress of a simply supported beam subjected to an axial impact is to come out at the mid position of the beam. The stress like the displacement happened mainly for applying an axial impact, and it disappeared. The magnitude of the axial impact is 161,167 kg, a critical buckling load of the model 1. Therefore, from the figure, we know that the stresses of all models exceed the ultimate tensile strength of Al2024-t3, 45 kg/mm<sup>2</sup>, already.

Figure 7 is the displacement curve of model 2 that the thickness of section 2 is 1.5mm. A general trend is similar to Figure 5, but the magnitudes of the displacements are increasing large.

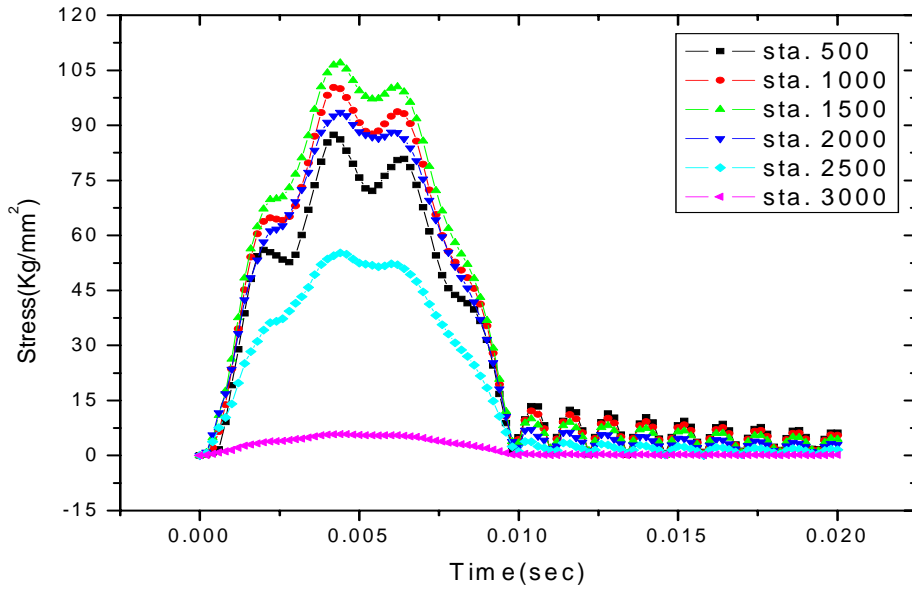


Figure 6. Stresses of Model 1(ssb, 2mm) under Load 1(axial, 50hz)

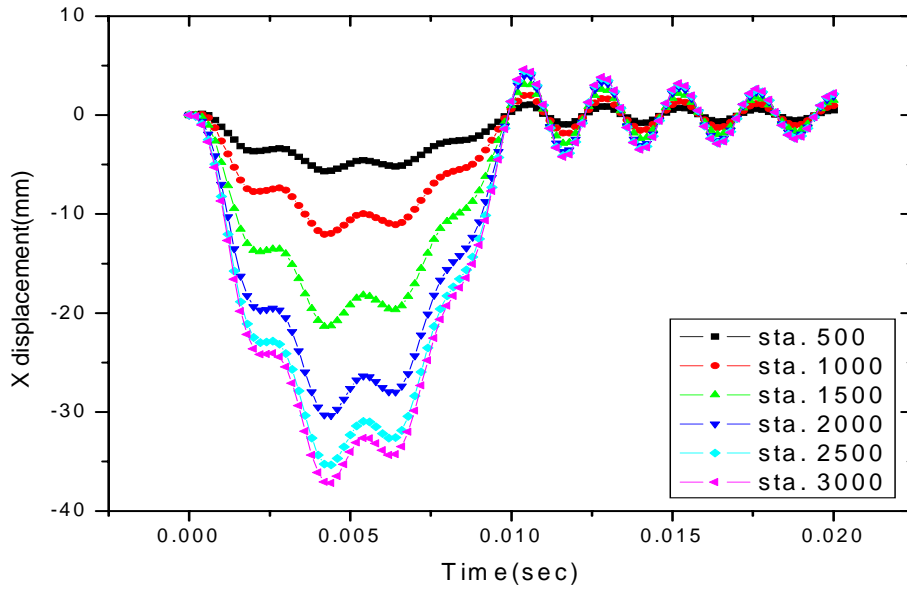


Figure 7. Displacements of Model 2(ssb, 1.5mm) under Load 1(axial, 50hz)

Figure 8 is the stress curve of model 2. The compressive stress is the maximum at the mid position of the beam and increased than the Figure 6. It is because of a stiffness weakening by the thickness reduction of section 2.

Figure 9 and 10 are the displacement and the stress graphs of model 3. The thickness of section 2 is 1.0mm which is the half of model 1. The trend of the displacement and the stress is similar to Figure 7 and 8, but the magnitude increases.

Figure 11 and 12 are of model 4 that the thickness of section 2 is 0.5mm. It shows that the displacement is about double and the stress is about 4 times larger than Figure 5 and 6 of model 1.

From Figure 5 through 12, we know that the displacement and the stress, at the sta. 3000 and at the sta. 1500, are increased due to the stiffness weakening of structures, namely, the thickness reduction of section 2.

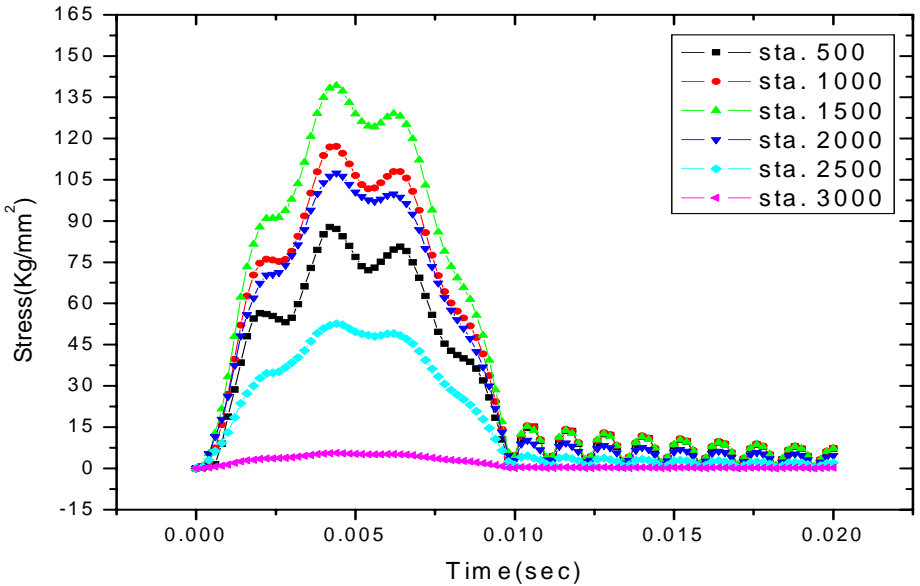


Figure 8. Stresses of Model 2(ssb, 1.5mm) under Load 1(axial, 50hz)

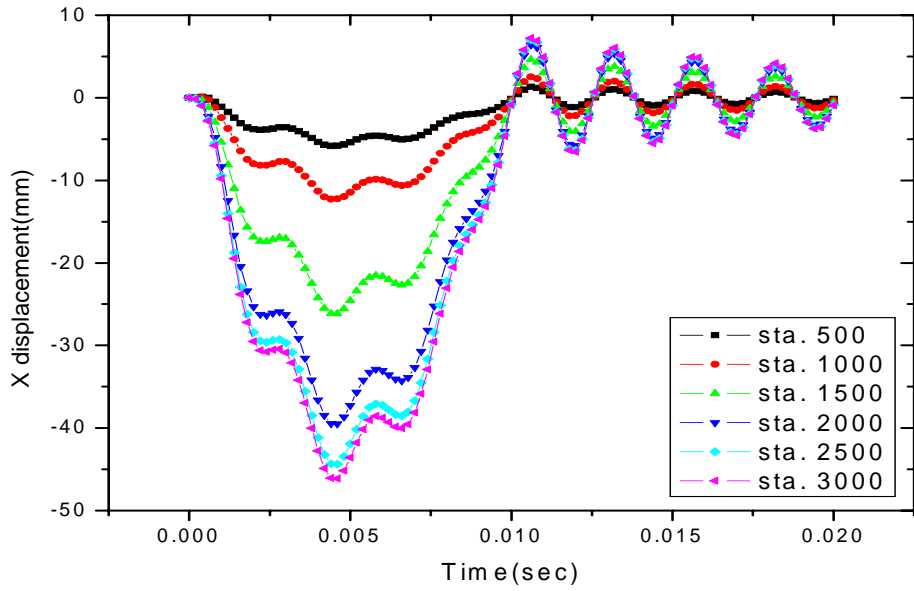


Figure 9. Displacements of model 3(ssb, 1mm) under load 1(axial, 50hz)

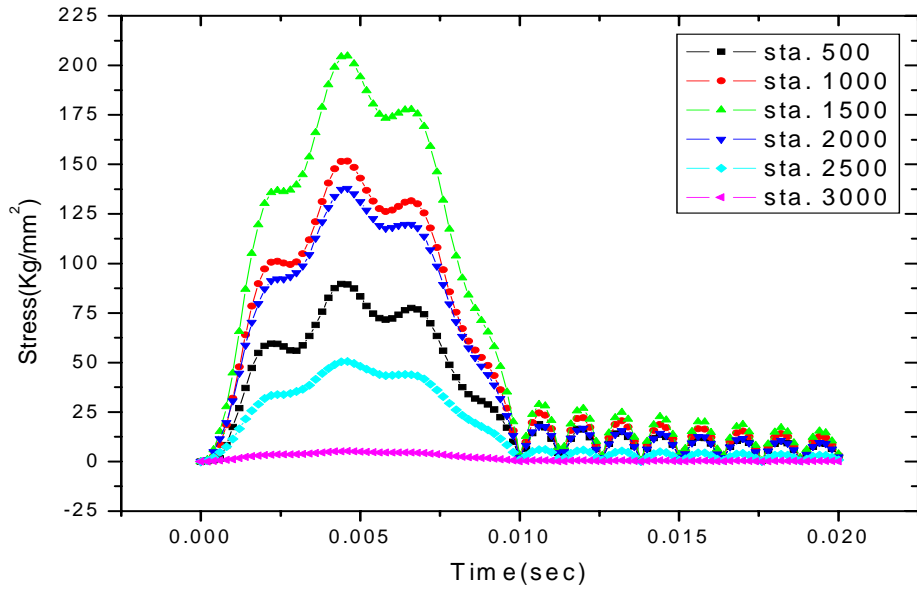


Figure 10. Stresses of Model 3(ssb, 1mm) under Load 1(axial, 50hz)

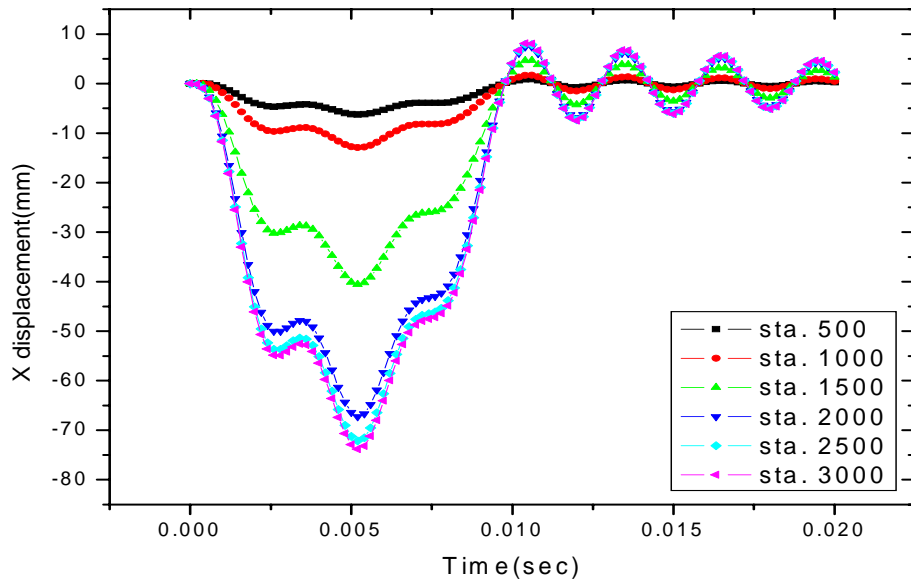


Figure 11. Displacements of Model 4(ssb, 0.5mm) under Load 1(axial, 50hz)

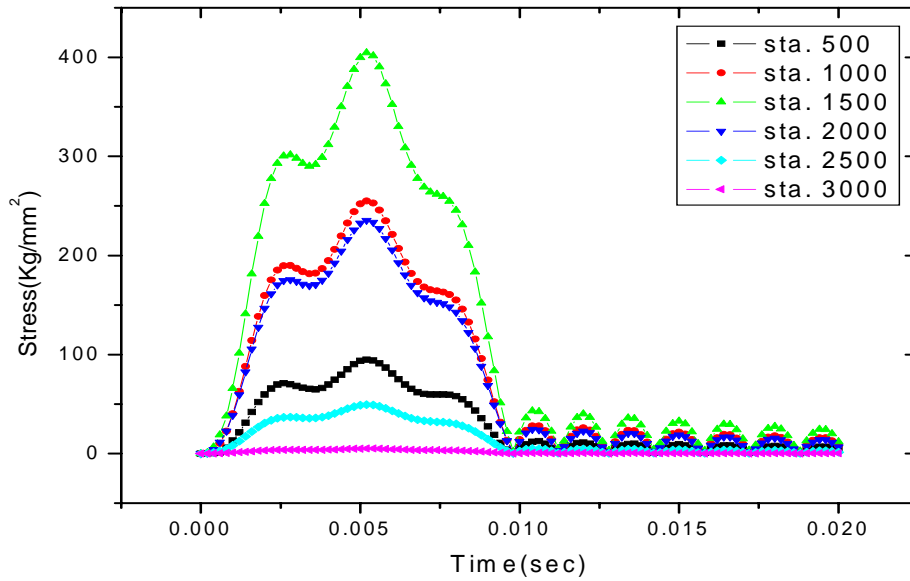


Figure 12. Stresses of Model 4(ssb, 0.5mm) under Load 1(axial, 50hz)

Figure 13 is the graph that done a comparative study on the displacements of models at the sta. 3000 occurred the largest displacement. The increasing rate of the displacement is larger than the decreasing rate of stiffness, and the order is model 4, 3, 2 and 1. The time occurring the largest displacement is delaying by the thickness reduction of the section 2.

Figure 14 is to compare the stresses of models at the sta.1500 occurred the largest stress. The stress has also a similar trend like the displacement, and the stress of model 4 is the largest.

Table 3 is the results gotten by a normal mode analysis and a transient analysis to investigate what the thickness reduction of section 2 affects on the dynamic characteristics of structures. The area moments of inertia and the first natural frequency are reduced by the thickness reduction of models, but the displacements and the stresses are increased about double and four times each.

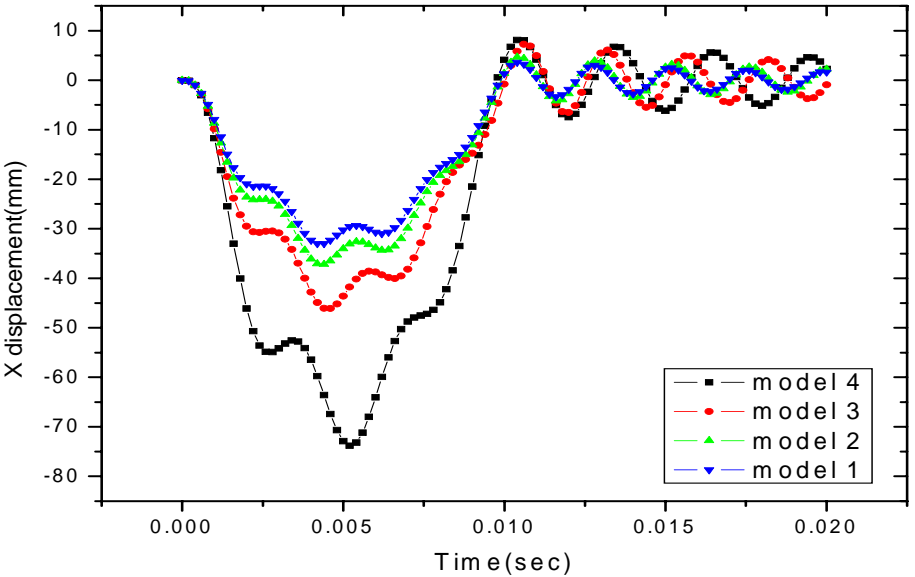


Figure 13. Displacements of Models(ssb) at sta. 3000 under Load 1(axial, 50hz)

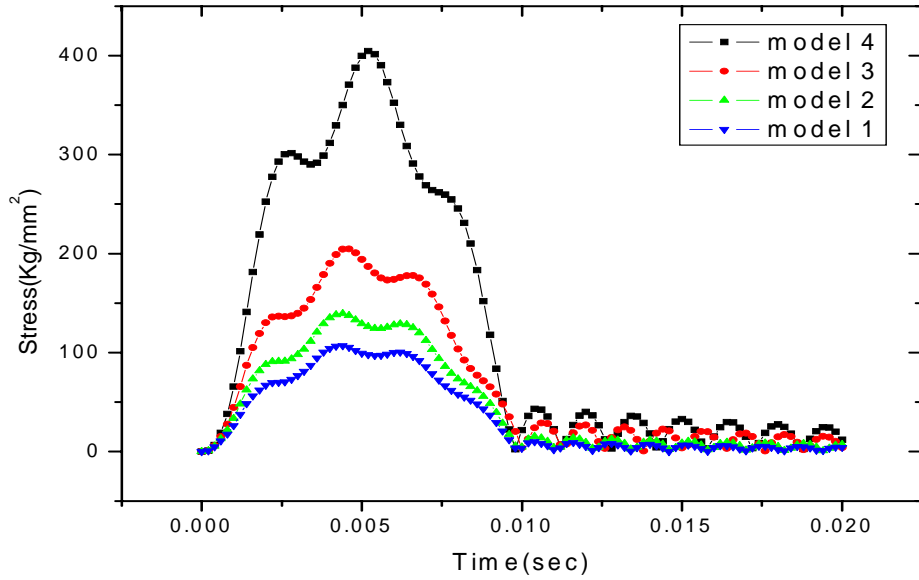


Figure 14. Stress of Models(ssb) at sta. 1500 under Load 1(axial, 50hz)

Table 3. Analysis Results of Simply Supported Beams

Results and Model Type	Area Moment of Inertia		First Natural Frequencies		Transient Analysis			
	Mag.* <sup>1</sup> (mm <sup>4</sup> )	Rate (%)	Mag.* <sup>1</sup> (hz)	Rate (%)	Maximum Displ.* <sup>2</sup> (mm)	Rate (%)	Maximum Stress (kg <sub>f</sub> /mm <sup>2</sup> )	Rate (%)
Model 1	2.12E07	100	91.4	100	33.0	100	107	100
Model 2	1.60E07	75	91.0	99.6	37.2	113	139	130
Model 3	1.07E07	50	88.2	96.5	46.1	140	205	192
Model 4	5.38E06	25	77.6	84.9	73.8	224	405	379

\*<sup>1</sup>: Mag. is an abbreviation of magnitude.

\*<sup>2</sup>: Displ. is an abbreviation of displacement.

## **B. A CANTILEVER BEAM UNDER A VERTICAL IMPACT LOAD**

### **1. The Effect of a Stiffness Variation**

From Figure 5 through 14, we knew that the stiffness variation of a simply supported beam subjected to axial impact load affects large on the displacement and the stress of structures. Therefore, we are going to investigate that how a vertical impact affects on a cantilever beam, one among the boundary conditions used often in a structural design. A vertical load is selected because the bending stress often induces the maximum stress in structure. The magnitude of an impact load, 1600kg, was determined by a static analysis. That is the load just before the working stress is over the allowable stress of material.

Figure 15 is the displacement graph of model 1. The displacement is increasing in proportion to the arm length of a bending moment because the stiffness of model 1 is not changed. The difference from a simply supported beam subjected to an axial impact is that the displacement is decreasing a little and occurring alternately after 0.01sec done an impact. It means that a cantilever beam vibrates above and below because the vertical impact load is applied at the free edge of that.

Figure 16 is the stress curve of model 1. Also, the stress is occurring at all position of a cantilever beam after 0.01sec done an impact. This is also a different point from a simply supported beam under axial load. The magnitude order of stress is opposite from the displacement. It is because a bending stress is the largest at the longest arm of a bending moment as the static analysis.

Figure 17 is the displacement curve of model 2 that the thickness of section 2 is 1.5mm. The displacement is increasing than model 1, but the trend is similar to model 1.

Figure 18 is the stress curve of model 2. The stress is increasing and the trend is similar to model 1.

Figure 19 is to show the displacements of model 3 that the thickness of section 2 is 1.0mm. The general trend is similar to model 2, but the magnitudes of displacements are increasing.

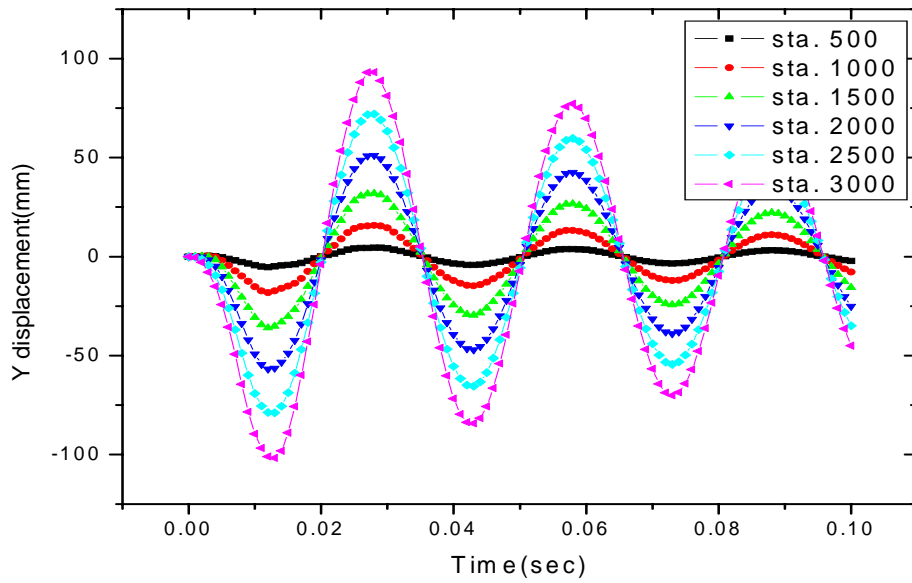


Figure 15. Displacements of Model 1(cb, 2mm) under Load 2(vertical, 50hz)

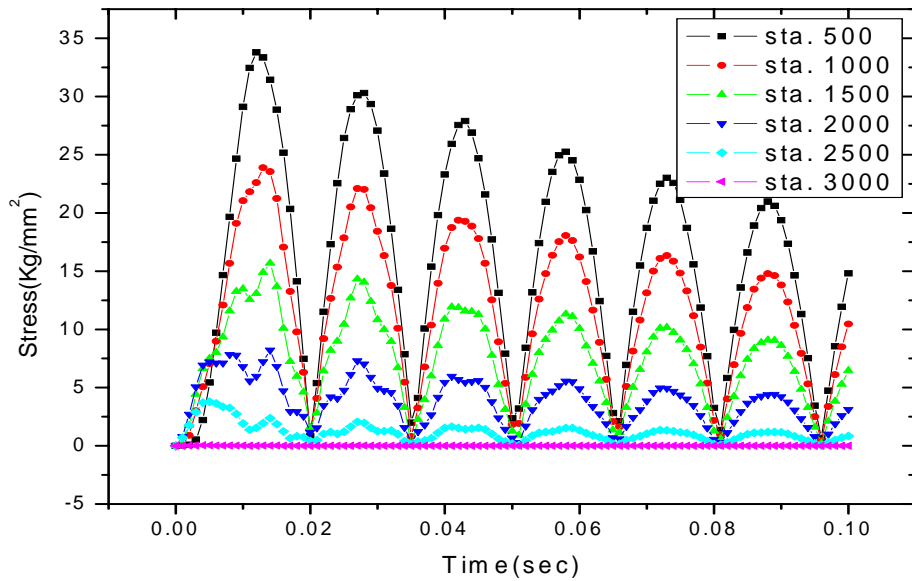


Figure 16. Stresses of Model 1(cb, 2mm) under Load 2(vertical, 50hz)

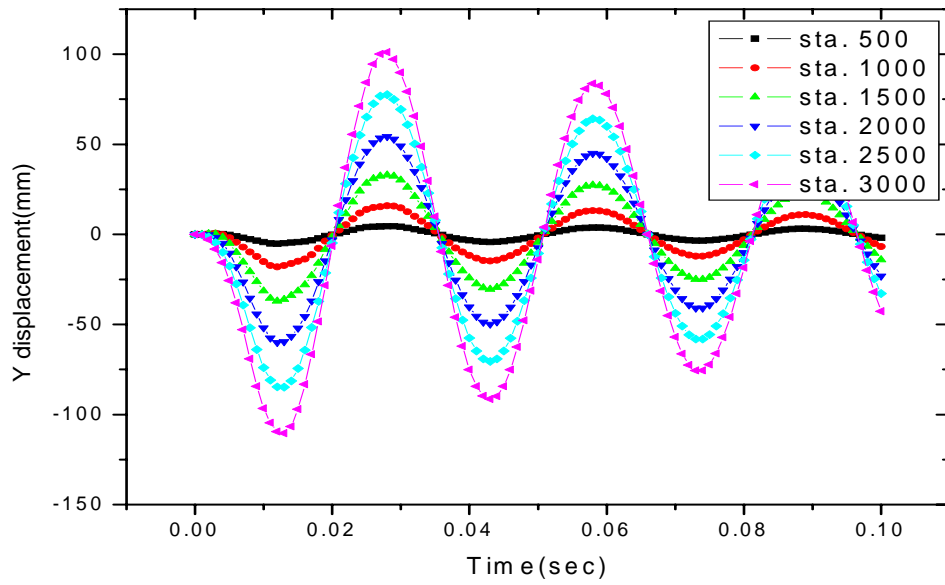


Figure 17. Displacements of Model 2(cb, 1.5mm) under Load 2(vertical, 50hz)

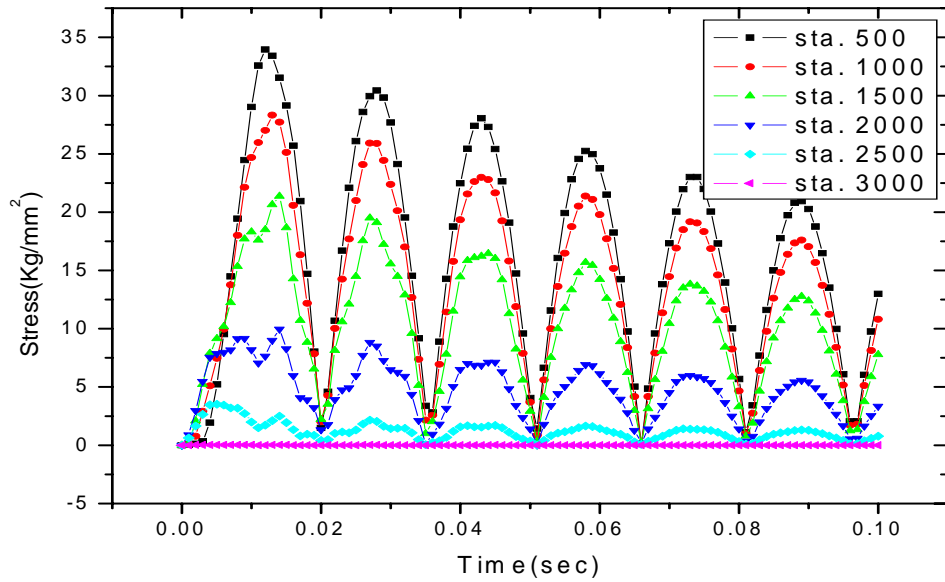


Figure 18. Stresses of Model 2(cb, 1.5mm) under Load 2(vertical, 50hz)

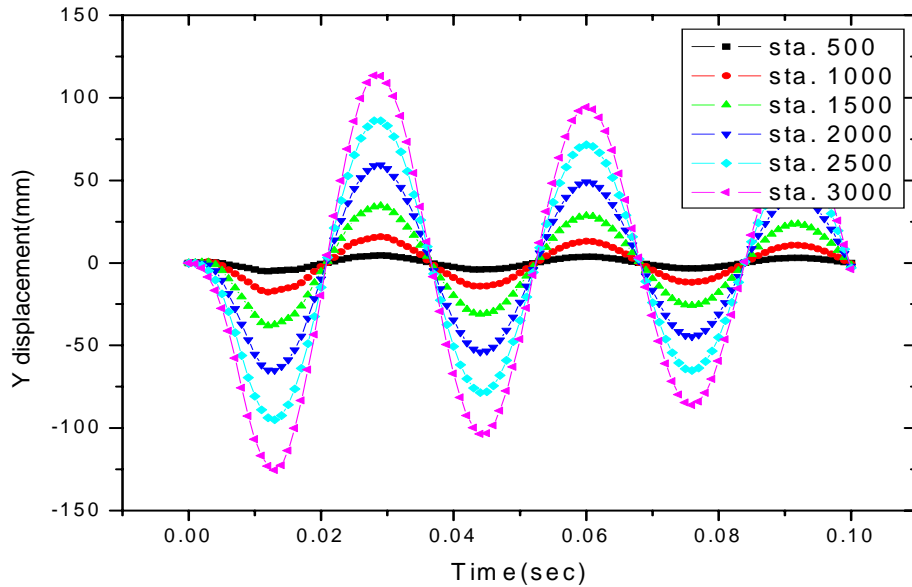


Figure 19. Displacements of Model 3(cb, 1mm) under Load 2(vertical, 50hz)

Figure 20 shows the stress of model 3. The different from the stress of model 2 is that the stress is increasing abruptly at the sta. 1000 and 1500. The stress distribution is changing by that the stiffness of section 2 is reduced to 50%.

Figure 21 is the displacement curve of model 4 that the thickness of section 2 is 0.5mm. The displacement is the largest among models. But the trend is also similar to others.

Figure 22 is the stress curve of model 4. It shows that the stress is the largest at the sta. 1500 of the mid position of a cantilever beam. It is because the stiffness reduced to 75% than that of model 1, and the stress values are exceeding the ultimate tensile strength of an Al2024-t3,  $45\text{kg/mm}^2$ .

Table 4 is the analysis results of cantilever beams subjected to a vertical impact of load 2. The stress and the displacement are increased by thickness reduction of section 2.

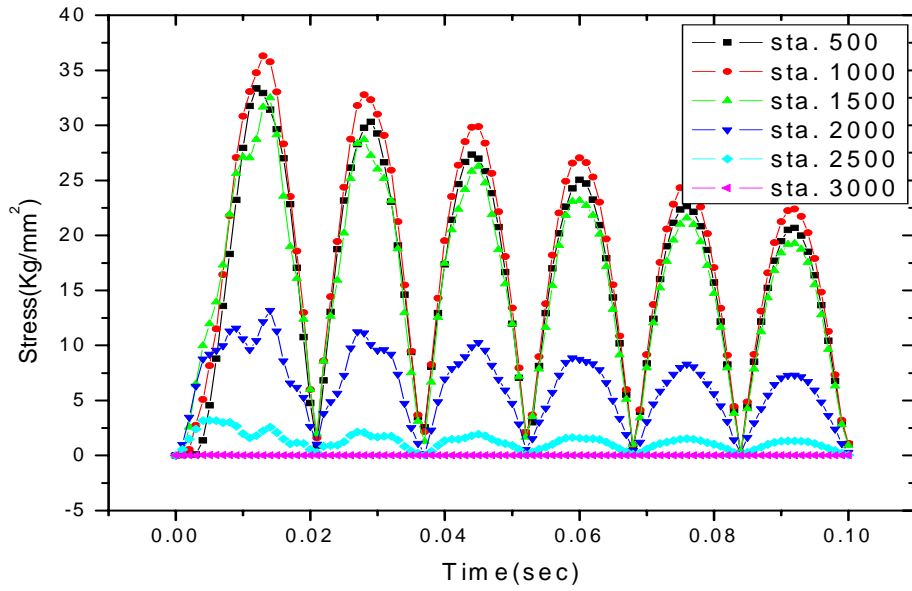


Figure 20. Stresses of Model 3(cb, 1mm) under Load 2(vertical, 50hz)

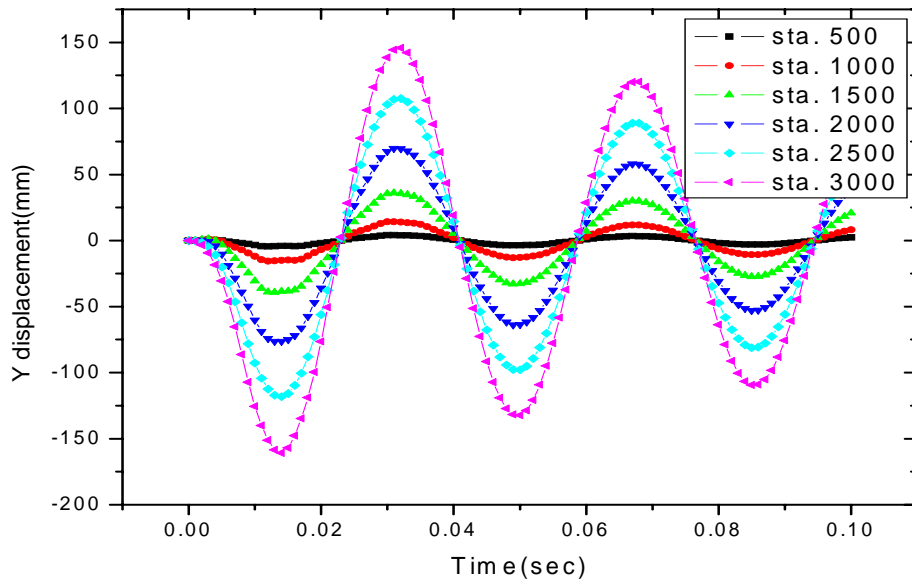


Figure 21. Displacement of Model 4(cb, 0.5mm) under Load 2(vertical, 50hz)

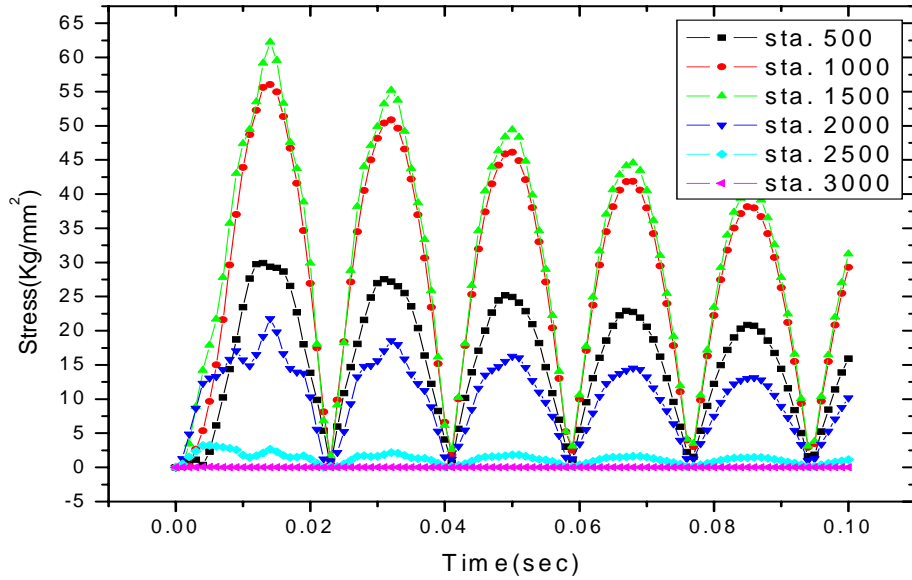


Figure 22. Stresses of Model 4(cb, 0.5mm) under Load 2(vertical, 50hz)

Table 4. Analysis Results of Cantilever Beams

Model Type and Results	Area Moment of Inertia		First Natural Frequencies		Transient Analysis			
	Mag.* <sup>1</sup> (mm <sup>4</sup> )	Rate (%)	Mag.* <sup>1</sup> (hz)	Rate (%)	Maximum Disp.* <sup>2</sup> (mm)	Rate (%)	Maximum stress (kg <sub>f</sub> /mm <sup>2</sup> )	Rate (%)
Model 1	2.12E07	100	33.0	100	101.8	100	33.8	100
Model 2	1.60E07	75	32.8	99.4	110.5	108.5	34.0	100.6
Model 3	1.07E07	50	31.7	96.1	125.4	123.2	36.3	107.4
Model 4	5.38E06	25	28.0	84.8	160.9	158.1	62.3	184.3

\*<sup>1</sup>: Mag. is an abbreviation of magnitude.

\*<sup>2</sup>: Displ. is an abbreviation of displacement.

It is investigated the displacements and the stresses in all cantilever models with a stiffness variation subjected to a vertical impact load. The displacement is the largest at the free edge of a cantilever beam and the farther from that, it is lessened. In case of stress, it is the largest at the fixed position of a cantilever beam but it shows the phenomenon that the stress is moving to the place that the stiffness reduction is large.

## **2. The Effect of a Frequency Variation**

Figure 23 through 28 are to investigate what the frequency change of an impact load affects on the models

Figure 23 shows the displacements of models at the sta. 3000 when the impact of 50 hz applies a cantilever beam. The displacement of model 4 is the largest, and the response is delaying than model 1, 2 and 3. The reason is that the stiffness reduction rate of section 2 is larger than other models.

Figure 24 is the stress curve at the sta.1500 which is the position occurring the maximum stress when the impact of 50 hz is applied. The stress of model 4 is larger than those of model 1, 2 and 3. This is one of important factors which should be considered when design structures. Model 1, 2 and 3 are safe because the stresses of those are smaller than the ultimate tensile strength of material, but model 4 is not safe for it is over the ultimate tensile strength.

Figure 25 is the displacement curves of models subjected to the impact load of 100hz. The displacements are decreasing than the models of 50hz. The reason is that the applied energy is reduced and it makes shorten the response of the structures because the magnitude of an impact does not change but time only shortens.

Figure 26 is the stress curves of models subjected to an impact load of 100hz. The stresses like the displacement are decreasing than those of the 50hz models. And another difference from the results of 50hz is that the stress value is the maximum at the second peak than at the first one. Because it is added before the first peak does not finish completely.

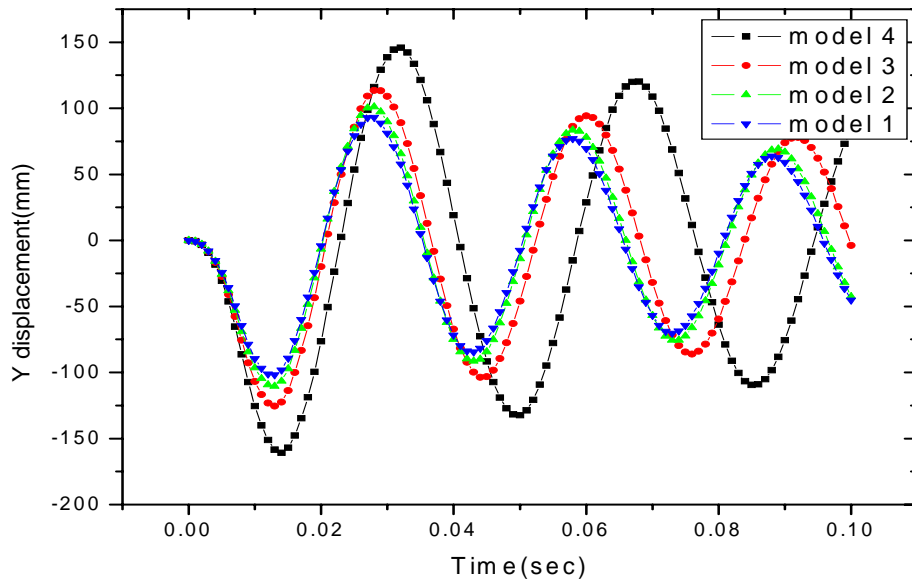


Figure 23. Displacements of Models(cb) at sta. 3000 under Load 2(vertical, 50hz)

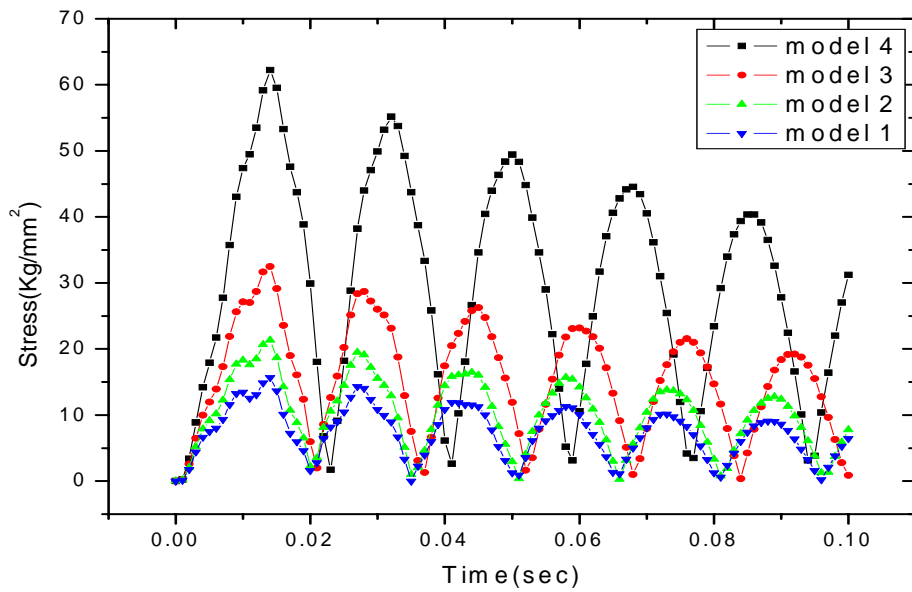


Figure 24. Stresses of Models(cb) at sta. 1500 under Load 2(vertical, 50hz)

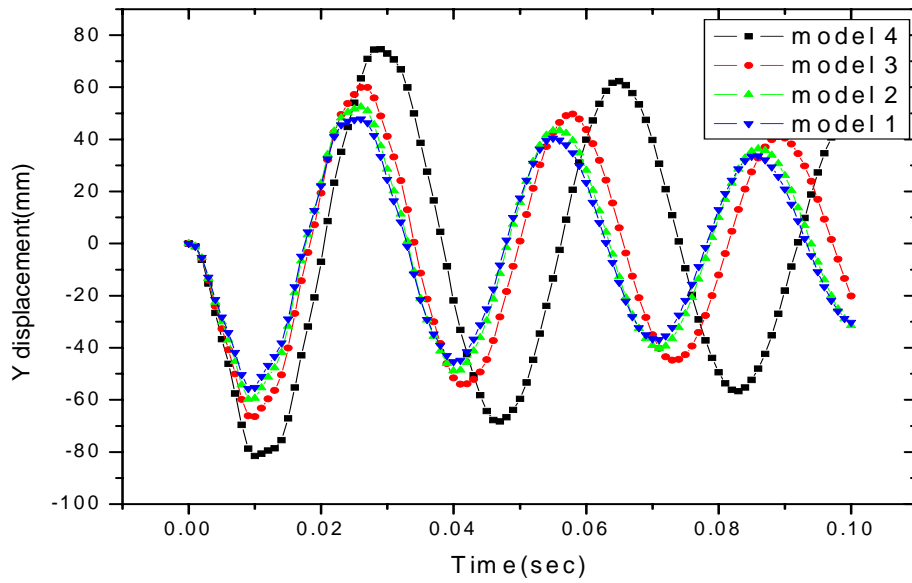


Figure 25. Displacements of Models(cb) at sta. 3000 under Load 3(vertical, 100hz)

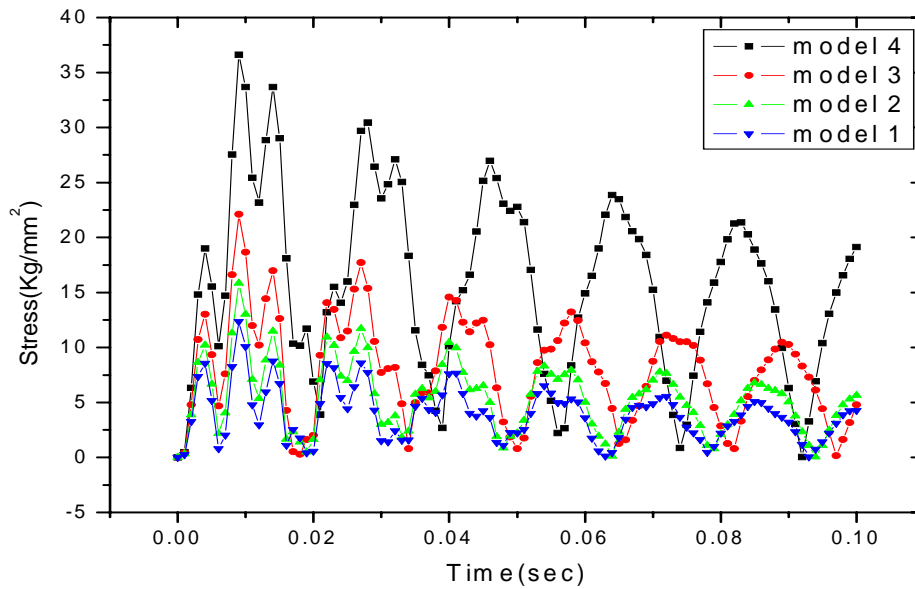


Figure 26. Stresses of Models(cb) at sta. 1500 under Load 3(vertical, 100hz)

Figure 27 is the displacement curves of models under an impact load of 200hz. The displacements are decreased large because the shock energy is reduced by the reduction of an applying time.

Figure 28 is the stress curves of models under an impact load of 200hz. The stresses are decreasing than the models of 100hz, but the maximum stress value is the second peak as that of 100hz.

It has been investigated what the frequency variation of impact load affects on the stresses of models. The increment of a frequency only of an applied impact load would rather reduce the stress of each model because of the reduction of an applied energy.

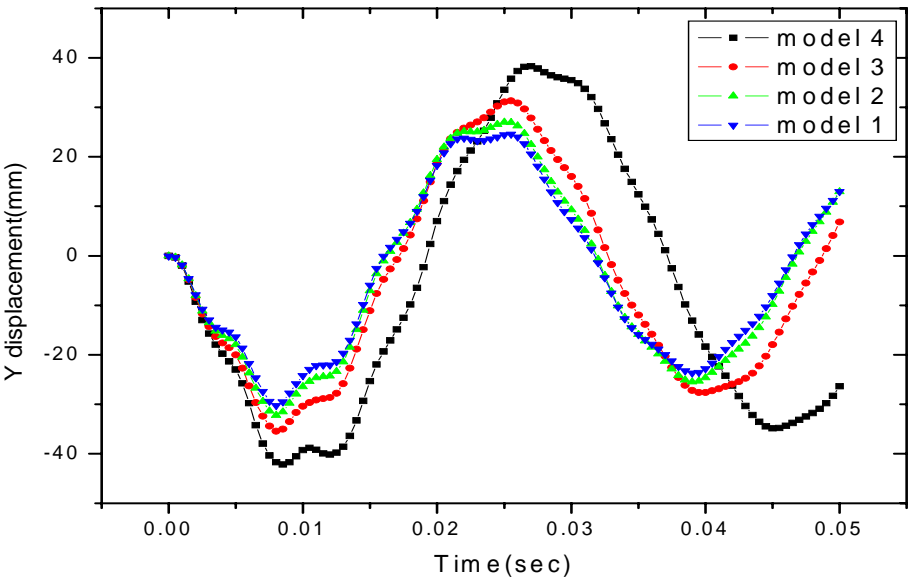


Figure 27. Displacement of Models(cb) at sta. 3000 under Load 4(vertical, 200hz)

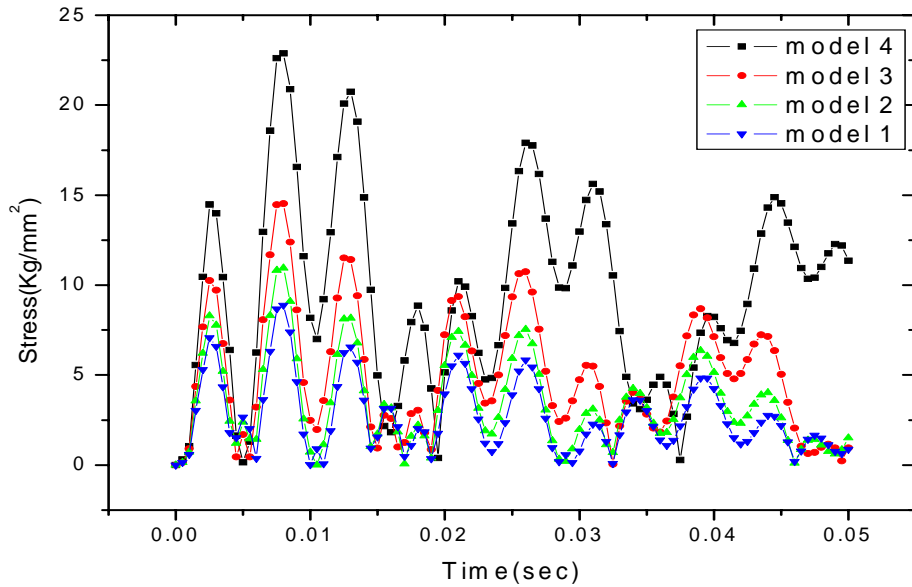


Figure 28. Stresses of Models(cb) at sta. 1500 under Load 4(vertical, 200hz)

### C. A CYLINDRICAL SHELL UNDER A VERTICAL IMPACT LOAD

Till now, studies were conducted on the simply supported beam and the cantilever beam subjected to an axial and a vertical impact load. We are going to apply the same load that applied a cantilever beam to a cylindrical shell as in Figure 4, and to compare the results of a beam and a cylindrical shell. Physical properties of material, force and boundary condition used on this occasion are the same. In the preliminary design stage, modeling and analyzing three dimensional cylinder to one dimensional beam are important in the point of view saving time and effort.

Figure 29 and 30 are the displacement and the stress curves of the cylindrical shell with a cantilever boundary condition subjected to a vertical impact

From Figure 29, the displacements of a cylindrical shell are the maximum at the sta. 3000, and the order is sta. 2500, 2000, 1500 and 500. It shows the same trends as a cantilever beam.

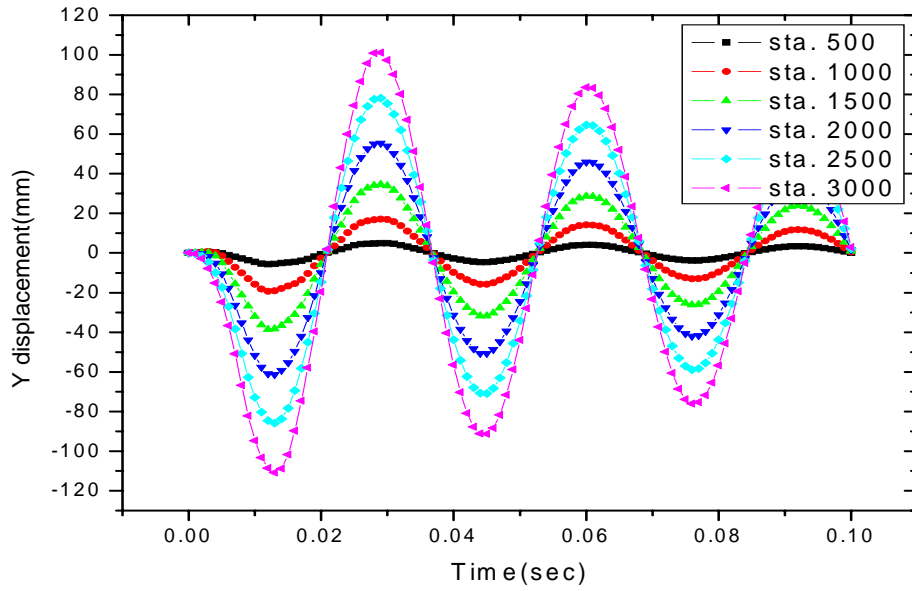


Figure 29. Displacements of Model 1(ccs, 2mm) under Load 2(vertical, 50hz)

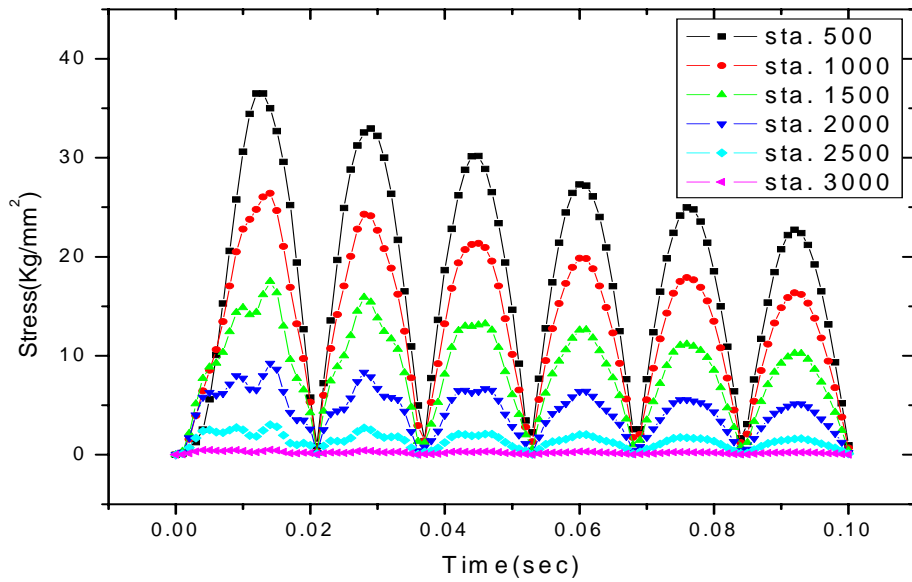


Figure 30. Stresses of Model 1(ccs, 2mm) under Load 2(vertical, 50hz)

In case of stress, the order of the magnitude is sta. 500, 1000, 1500, 2000, 2500 and 3000. This coincides the magnitude order of a bending moment and the trend is the same as beam.

Figure 31 through 42 is the graphs to compare the analysis results of cantilever beams and cylindrical shells with frequency variation at the position of maximum stress.

Figure 31 is the stress curves of model 1 on the frequency 50hz at the sta. 1500. It shows a similar trend, but the stress of a cylindrical shell is larger than that of beam. It is because of the model characteristics of a beam and a cylindrical shell. It is the result to compare the mean stress of the element for a beam with that at the farthest of the element for a cylindrical shell. The stiffness of cylindrical shell is reduced than that of beam because the finite element of a cylindrical shell is not fine mesh. Whether the element of a cylindrical shell make a fine mesh or not, that is the best way to decide when required more detail analysis after an analysis of a beam model. This is to reduce a computing time and an effort. The time occurring the peak value of a cylindrical shell is delaying than that of a beam after the third peak value. It takes time to respond a structure after an external load is applied because a cylindrical shell is three dimensional structure and is more complex than a beam.

Figure 32, 33 and 34 are the result applied the load of 50hz for the model 2, 3 and 4. It shows a similar trend with model 1. But the stresses are increasing little by little. It is because of the stiffness reduction of section 2.

Figure 35, 36, 37 and 38 are the stress curves of the case applied the load of 100 hz for each model. The analysis results for a beam and a cylindrical shell are similar, and the less the stiffness, the more the stress. But the stresses are decreasing than that of 50hz. It is because the application time is shortened and the energy is lessened. And the second peak is the largest because it is added before the first peak does not finish.

Figure 39 through 42 are the case applied the load of 200hz for each model. It shows a similar trend with that of 100hz. But the stresses are decreasing than that of 100hz.

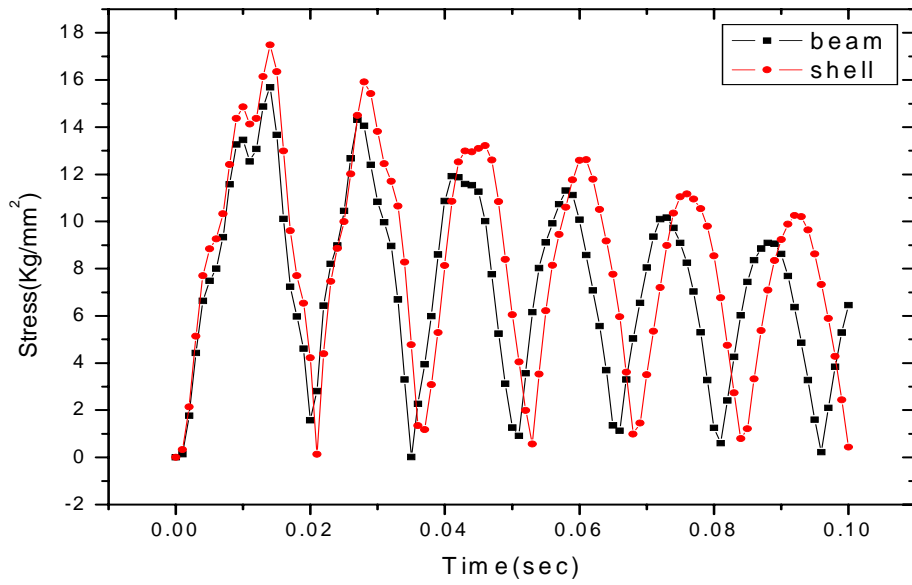


Figure 31. Stresses of Model 1(cb/ccs, 2mm) at sta. 1500 under Load 2(vertical, 50hz)

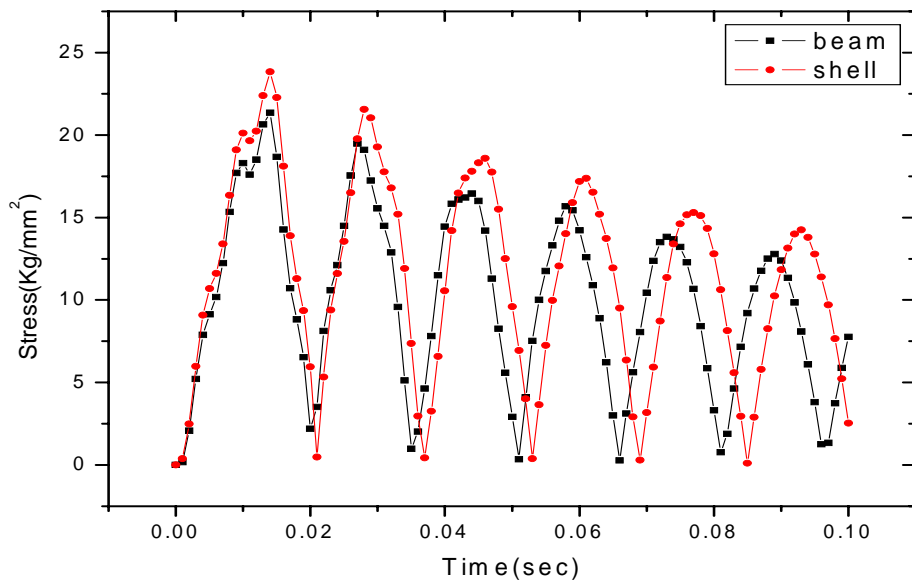


Figure 32. Stresses of Model 2(cb/ccs, 1.5mm) at sta. 1500 under Load 2(vertical, 50hz)

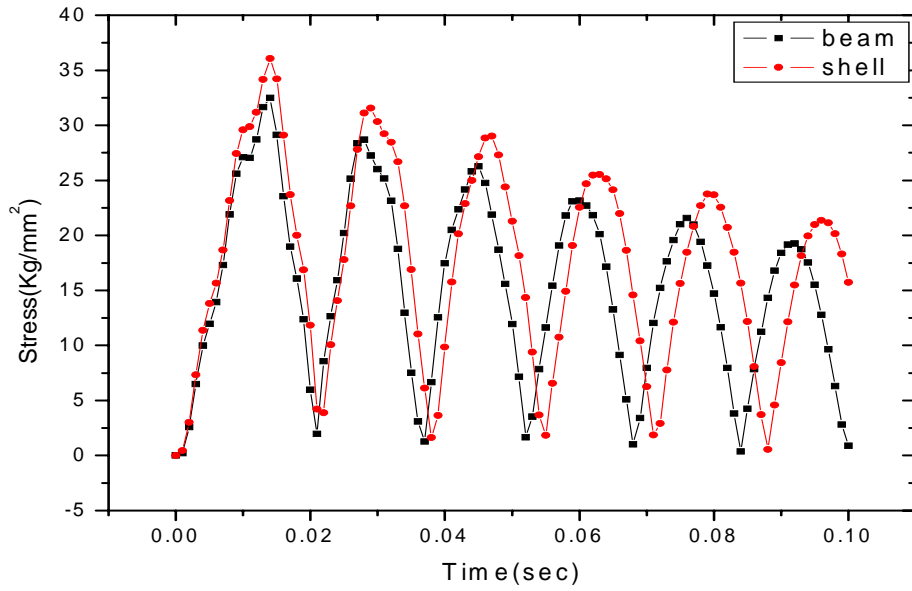


Figure 33. Stresses of Model 3(cb/ccs, 1mm) at sta. 1500 under Load 2(vertical, 50hz)

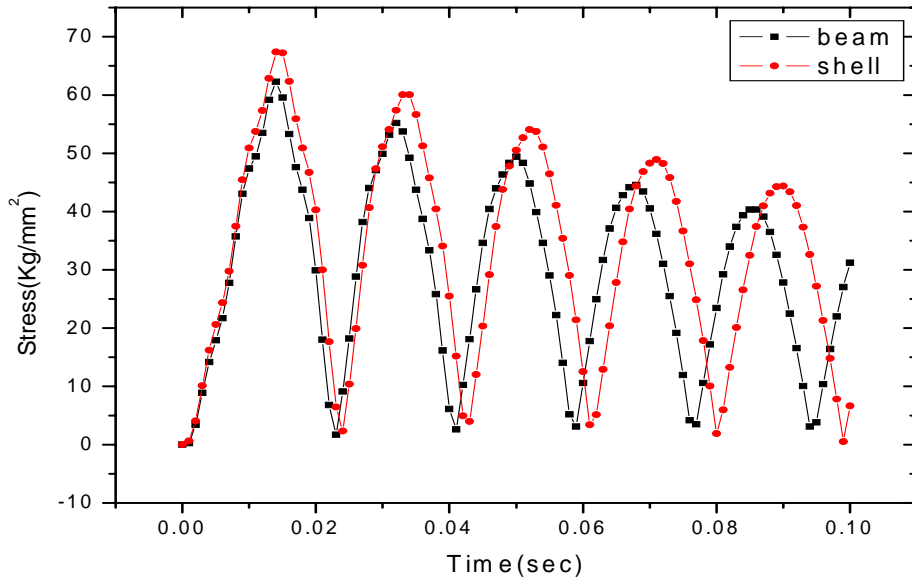


Figure 34. Stresses of Model 4(cb/ccs, 0.5mm) at sta. 1500 under Load 2(vertical, 50hz)

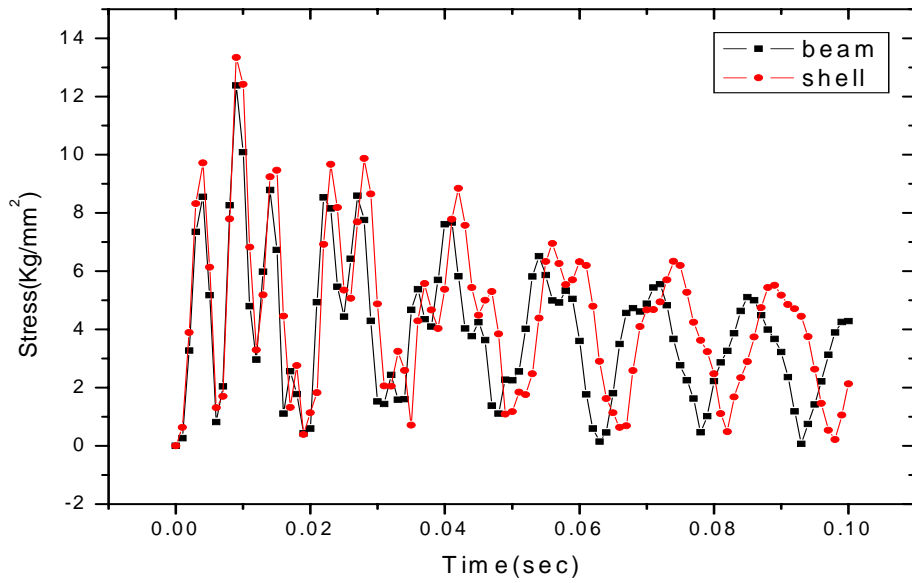


Figure 35. Stresses of Model 1(cb/ccs, 2mm) at sta. 1500 under Load 3(vertical, 100hz)

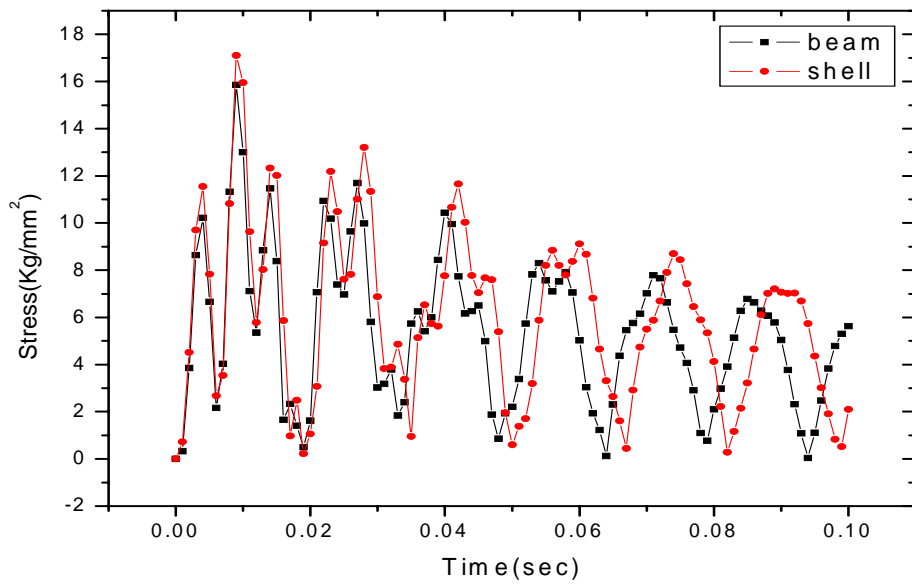


Figure 36. Stresses of Model 2(cb/ccs, 1.5mm) at sta. 1500 under Load 3(vertical, 100hz)

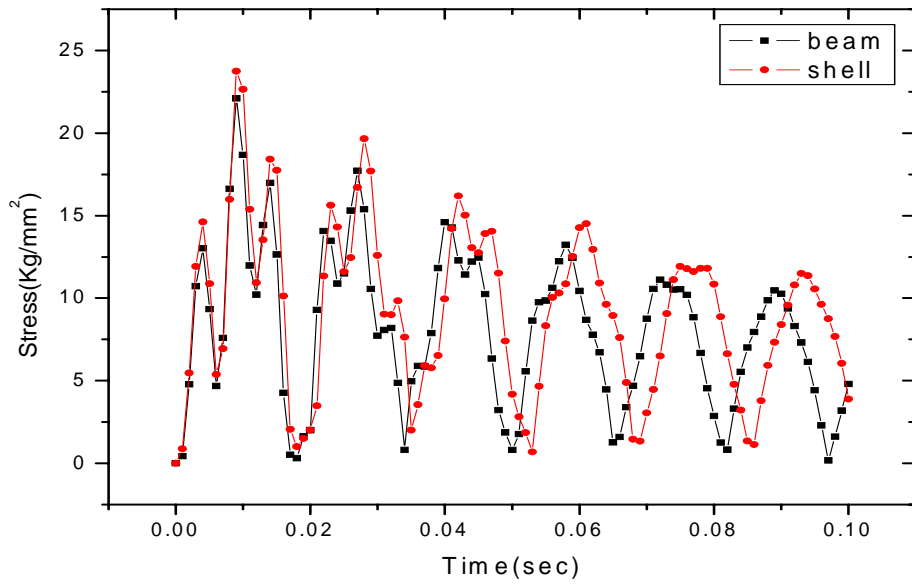


Figure 37. Stresses of Model 3(cb/ccs, 1mm) at sta. 1500 under Load 3(vertical, 100hz)

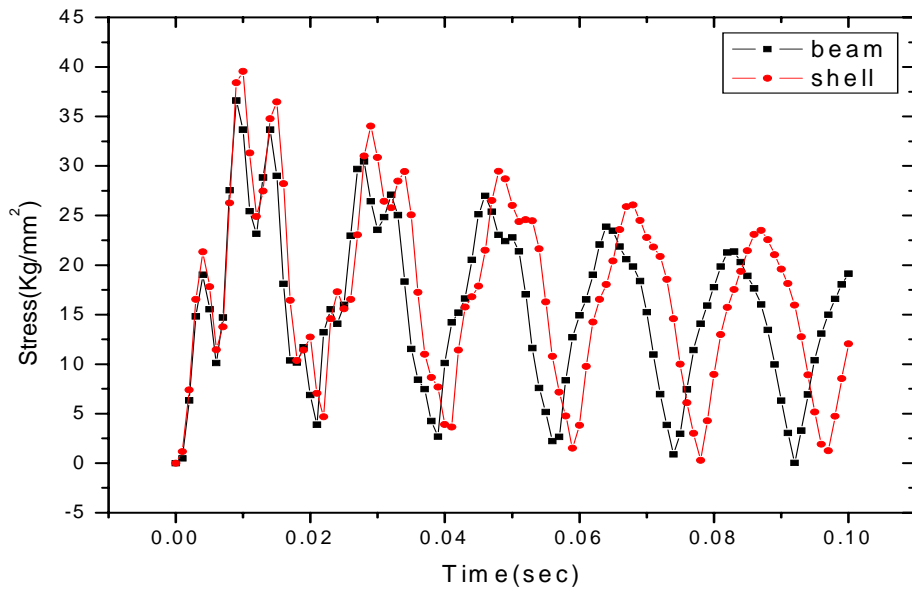


Figure 38. Stresses of Model 4(cb/ccs, 0.5mm) at sta. 1500 under Load 3(vertical, 100hz)

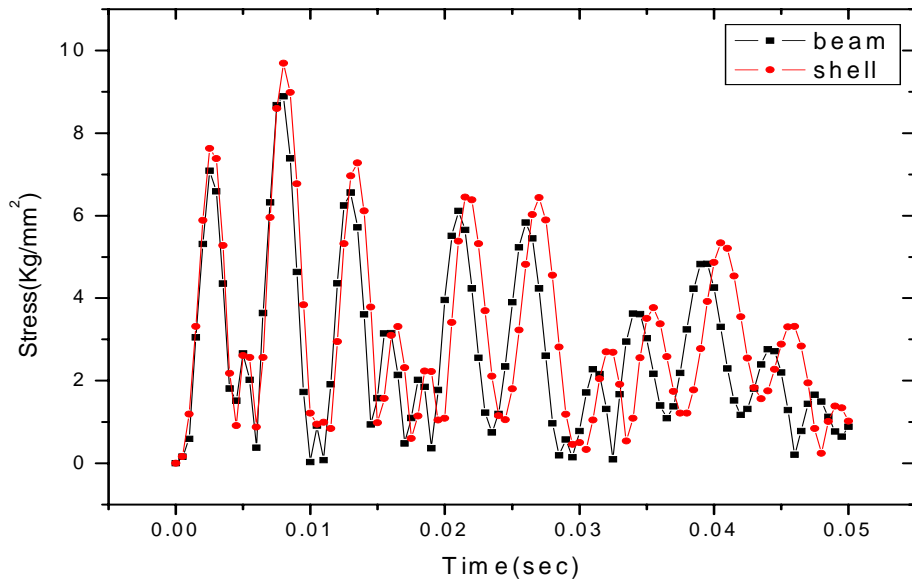


Figure 39. Stresses of model 1(cb/ccs, 2mm) at sta. 1500 under load 4(vertical, 200hz)

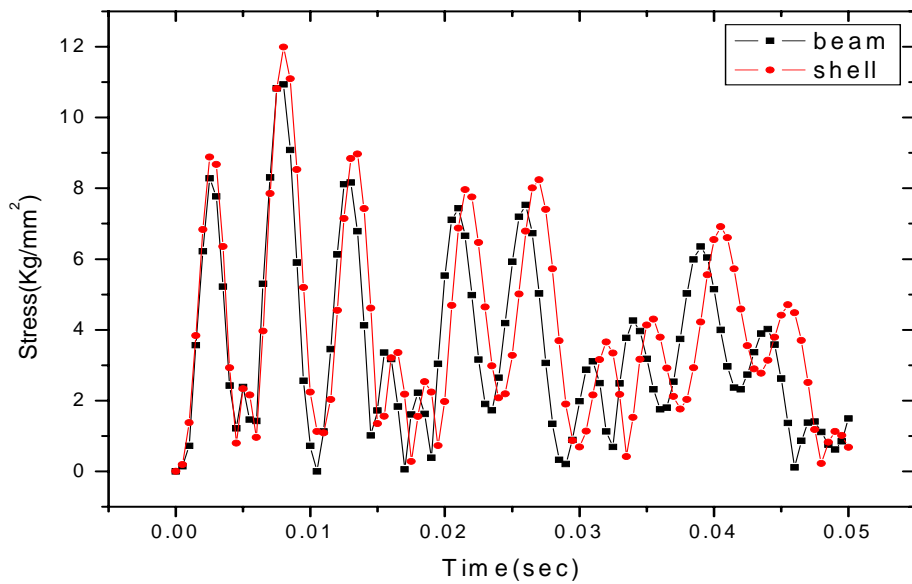


Figure 40. Stresses of model 2(cb/ccs, 1.5mm) at sta. 1500 under load 4(vertical, 200hz)

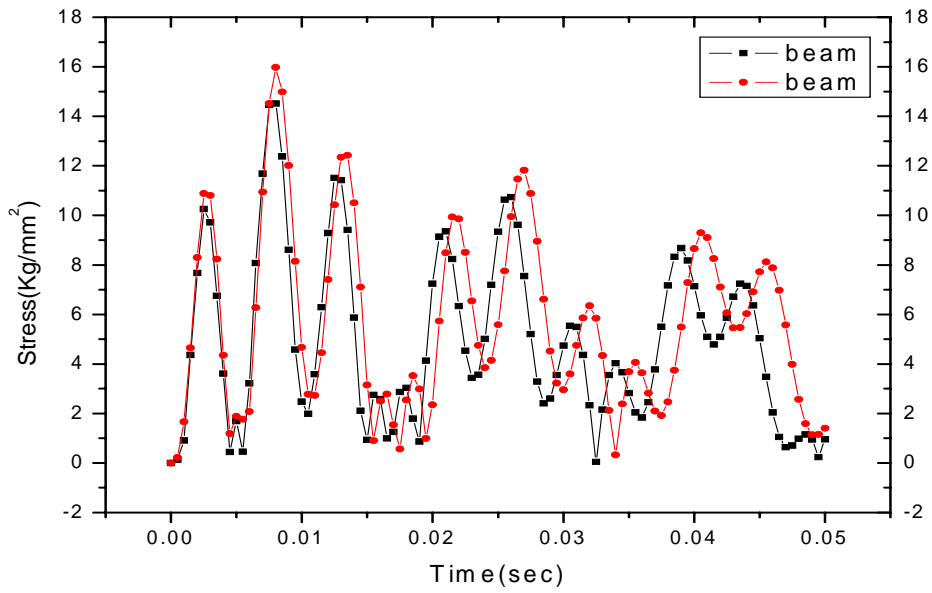


Figure 41. Stresses of model 3(cb/ccs, 1mm) at sta. 1500 under load 4(vertical, 200hz)

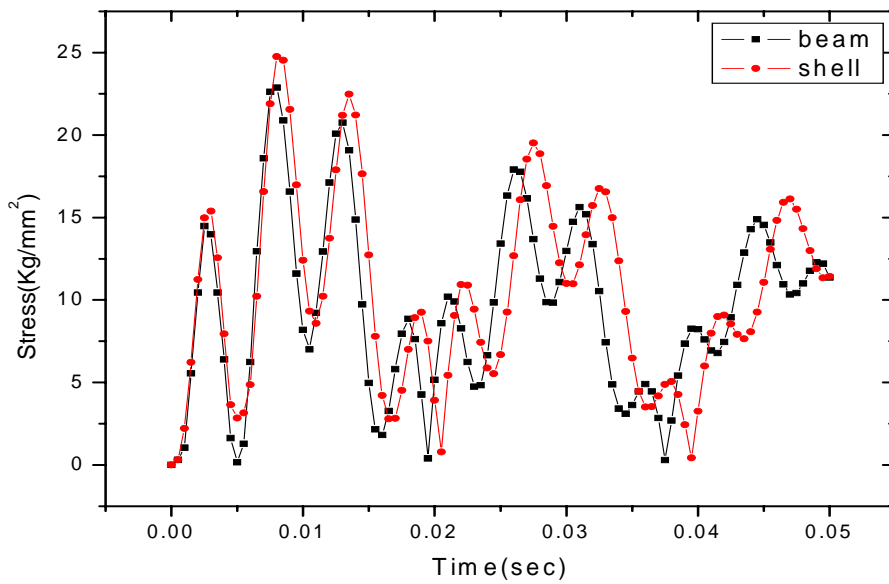


Figure 42. Stresses of model 4(cb/ccs, 0.5mm) at sta. 1500 under load 4(vertical, 200hz)

It has been investigated on the difference between a beam model and a cylindrical shell model. The results were similar if the condition was the same.

## IV. CONCLUSIONS

The research results on the dynamic response of stepped beams and stepped cylindrical shells subjected to an impact load are as follows.

1. The theoretical studies on the analytic solutions of a simply supported beam and a cantilever beam were carried out.

2. It was analyzed that a stiffness reduction affects on the dynamic characteristics of a simply supported beam subjected to an axial impact. By the thickness reduction of models, the displacement and the stress were increased about double and four times each.

3. In case of a cantilever beam subjected to a vertical impact load, the structural response was increased by the stiffness reduction as the simply supported beam. The stress was the largest at the fixed position of beam, but it moved and increased toward which the stiffness reduction was large.

4. The analysis results of a beam and a cylindrical shell were similar if the physical property of a material, boundary and load conditions were same.

5. The stiffness reduction was a factor increasing a stress and a displacement of structures regardless of frequency variation. Because the magnitude of an impact load does not change but time only shortens.

It is still needed parametric study as an initial deflection and an eccentric load. But, we hope this is to be a little help and is to be the first step to reduce an effort and time in the preliminary design stage of complex structures like ship, airplane and missile.

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