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13. ABSTRACT (Maximum 200 words) The aim of the project was to develop an efficient method of helicopter rotor tuning (track and balance) to cope with the potential nonlinearity of the process and to account for the vibration noise. Toward this goal, two methods have been developed. The first method relies on an interval model to represent the range of effect of blade adjustments on helicopter vibration and incorporates learning to adapt the coefficients of the interval model. The coefficients of the model are initially defined according to sensitivity coefficients between the blade adjustments and helicopter vibration, to include the 'a priori' knowledge of the process. These coefficients are subsequently transformed into intervals and updated after each tuning iteration to improve the model's estimation accuracy. The second method of rotor tuning uses a probability model to maximize the likelihood of success of the selected blade adjustments. The underlying model in this method consists of two segments: a linear segment to include the sensitivity coefficients between the blade adjustments and helicopter vibration, and a stochastic segment to represent the probability densities of the vibration components. Based on this model, the blade adjustments with the maximal probability of generating acceptable vibration are selected as recommended adjustments. The effectiveness of the two methods are evaluated in simulation using a series of neural networks trained with actual vibration data. The results indicate that the developed methods improve performance according to several criteria representing various aspects of track and balance.					
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Enclosure 1

# FINAL PROGRESS REPORT

Method for Knowledge-Based Helicopter Track and Balance

Project Number 40144-EG

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## 1 List Of Appendixes

1. Appendix A: Manuscript “An Adaptive Method of Helicopter Track and Balance”
2. Appendix B: Manuscript “A Probability-Based Approach to Helicopter Rotor Tuning”

## 2 Problem Statement

Helicopter track and balance is a tuning procedure for reducing both the chassis vibration and the spread of rotor blades about a mean position. Balance, which is performed for the reduction of vibration, is the more important of the two since it directly affects the performance of the aircraft. Track is performed mainly for aesthetic purposes, as it has been found that well-positioned rotor blades increase pilot’s confidence in the aircraft. Track and balance is performed by making adjustments to the rotor blades, therefore, the tuning process consists of determining the set of blade adjustments that will bring the chassis vibration within specification while simultaneously providing suitable rotor track. Since reduction of vibration is the main goal of the track and balance procedure, adjustments are generally made in such a way that vibration characteristics are not compromised for track.

Track and balance as applied to Sikorsky’s H-60 (Black Hawk) helicopter is performed as follows. For initial measurements, the aircraft is flown through six different regimes during which measurements of rotor track and vibration are recorded. Rotor track is measured by optical sensors which detect the vertical position of the blades. Vibration is measured at the frequency of once per blade revolution (1 per rev) by two accelerometers, ‘A’ and ‘B’, attached to the sides of the cockpit (see Figure 1 in Appendix A). The vibration data is vectorially combined into two components: A+B, representing the vertical vibration of the aircraft, and A-B, representing its roll vibration. A sample of peak vibration levels for the six flight regimes, as well as the angular position of a reference signal corresponding to the peak vibration is given in Table 1 (Appendix A), along with the corresponding track data. The six flight regimes in Table 1 (Appendix A) are: ground (fpm), hover (hov), 80 knots (80), 120 knots (120), 145 knots (145), and maximum horizontal speed (vh). The track data indicate the vertical position of each blade relative to a mean position.

In order to bring track and (1 per rev) vibration within specification, three types of adjustments can be made to the rotor system: pitch control rod adjustments, trim tab adjustments, and balance weight adjustments (see Figure 1 in Appendix A). Pitch control rods can be extended or contracted by a certain number of notches to alter the pitch of the rotor blades. Positive push rod adjustments indicate extension. Trim tabs, which are adjustable surfaces on the trailing edge

of the rotor blades, affect the aerodynamic pitch moment of the air foils and consequently their vibration characteristics. Tab adjustments are measured in thousandths of an inch, with positive and negative changes representing upward and downward tabbing, respectively. Finally, balance weights can be either added to or removed from the rotor hub to tune vibrations through changes in blade mass. Balance weights are measured in ounces with positive adjustments representing the addition of weight. In the case of the Sikorsky H-60 helicopter, which has 4 main rotor blades, a total of twelve adjustments (three adjustments per blade) can be made to tune the aircraft. Ideally, identical adjustments made to any two helicopters should result in identical changes in vibration. In reality, however, this does not occur due to factors such as small differences between individual aircraft and variances in atmospheric flight conditions (i.e., weather).

Virtually all of the current systems of track and balance rely on inverse models that map the track and balance data to blade adjustments. However, these inverse models, that are based on sensitivity coefficients between the blade adjustments and aircraft vibration/rotor track, pose two basic limitations. First, because of one-to-many mapping, the inverse-model solution may not be a comprehensive solution for all of the flight regimes. Second, these inverse models are inherently incapable of coping with the potential nonlinearity between the blade adjustments and aircraft vibration/rotor track, since sensitivity coefficients are often reliable for a limited range of aircraft vibration and rotor track, and do not represent the coupling effects of adjustments. These limitations restrict the number of adjustments made at any one time, therefore, they require more flights than necessary to tune each aircraft. At an approximate cost of \$20,000 per test flight, the cost associated with track and balance is often significant.

### 3 Summary Of Important Results

The aim of this research was to develop a forward-model approach to track and balance to remedy the limitations of the inverse-model solution. Two methods were developed toward this goal. The first method (see Appendix A) uses an interval model and incorporates learning to provide the following advantages:

1. to incorporate the approximate range of sensitivity coefficients, instead of exact values, so that
  - it can cope with the potential nonlinearity of track and balance, due to the piece-wise nature of the interval model, and
  - it can account for the stochastics inherent in vibration measurements.
2. to update its knowledge-base after each flight to account for differences between individual aircraft, and
3. to generate solutions that reduce the vibration of all of the flight regimes, instead of a selected few.

The above method was implemented in a computer program and tested extensively at Sikorsky Aircraft. It was then incorporated with the appropriate interface to be accessed on the web at: <http://mielsvr2.ecs.umass.edu:8080/trackbalance/index.html>

The second method (see Appendix B) uses a probability model to maximize the likelihood of success of the selected blade adjustments. The underlying model in this method consists of two segments: a linear segment to include the sensitivity coefficients between the blade adjustments

and helicopter vibration, and a stochastic segment to represent the probability densities of the vibration components. Based on this model, the blade adjustments with the maximal probability of generating acceptable vibration are selected as recommended adjustments.

Both of the above methods were evaluated in simulation using a series of neural networks trained with actual vibration data. The results indicate that the two methods improve performance according to several criteria representing various aspects of track and balance.

## 4 List Of Publications

### 4.1 Journal Papers

- Wang, S., Danai, K., and Wilson, M., 2004, “An Adaptive Method Of Helicopter Track And Balance,” *ASME J. of Dynamic Systems, Measurement and Control*, in press.
- Wang, S., Danai, K., and Wilson, M., 2004, “A Probability-Based Approach to Helicopter Rotor Tuning,” *J. of the American Helicopter Society*, in press.

### 4.2 Conference Papers

- Yang, D., Wang, S., Danai, K., 2001, “Helicopter Track and Balance by Interval Modeling,” *Proc. of the American Helicopter Society 57th Annual Forum*, Washington DC, May 9-11.
- Wang, S., Danai, K., 2002, “A Forward Approach to Helicopter Track and Balance,” *Proc. of the 2002 ASME Int’l Mechanical Eng. Congress and Exposition*, November 17-22, New Orleans, Louisiana, IMECE2002-33453.
- Wang, S., Danai, K., 2003, “A Probability-Based Approach to Helicopter Track and Balance,” *Proc. of the American Helicopter Society 59th Annual Forum*, Phoenix, Arizona, May 6-8.

### 4.3 Technical Reports Submitted to ARO

- Interim Report 1, March 12, 2001.
- Interim Report 2, March 12, 2002.
- Interim Report 3, March 7, 2003.

### 4.4 Scientific Personnel

1. Dongzhe Yang, Ph.D., 2001, Dissertation Title: Knowledge-Based Interval Modeling Method for Efficient Global Optimization and Process Tuning, Dept. of Mech. and Ind. Eng., University of Massachusetts, Amherst
2. Shengda Wang, Ph.D., 2004, Dissertation Title: Sequential Experimental Design Approaches to Helicopter Track and Balance, Dept. of Mech. and Ind. Eng., University of Massachusetts, Amherst

## APPENDIX A

### AN ADAPTIVE METHOD OF HELICOPTER TRACK AND BALANCE<sup>1</sup>

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#### ABSTRACT

An adaptive method of helicopter track and balance is introduced to improve the search for the required blade adjustments. In this method an interval model is used to represent the range of effect of blade adjustments on helicopter vibration, instead of exact values, to cope with the nonlinear and stochastic nature of aircraft vibration. The coefficients of the model are initially defined according to sensitivity coefficients between the blade adjustments and helicopter vibration, to include the ‘a priori’ knowledge of the process. The model coefficients are subsequently transformed into intervals and updated after each tuning iteration to improve the model’s estimation accuracy. The search for the required blade adjustments is performed according to this model by considering the vibration estimates of all of the flight regimes to provide a comprehensive solution for track and balance. The effectiveness of the proposed method is evaluated in simulation using a series of neural networks trained with actual vibration data. The results indicate that the proposed method improves performance according to several criteria representing various aspects of track and balance.

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<sup>1</sup>To be published in the ASME J. of Dynamic Systems, Measurement and Control

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## Nomenclature

$\Delta \mathbf{x}$	vector of combined blade adjustments
$\Delta x_i$	the $i$ th element of $\Delta \mathbf{x}$
$\Delta \overset{\leftrightarrow}{y}_j$	output of the interval model
$\mathbf{x}_c$	vector of candidate set of blade adjustments
$\mathbf{x}_e$	vector of blade adjustments within the selection region
$\mathbf{x}_s$	vector of previously selected blade adjustments
$k$	flight number, $k = 0$ denotes the initial flight
$V_j(k)$	measured vibration of the $k$ th flight
$\hat{V}_j(k)$	prediction of the $j$ th vibration component for the $k$ th flight
$V_{sj}(k)$	sine of the $j$ th vibration component
$V_{cj}(k)$	cosine of the $j$ th vibration component
$\Delta V_j(k)$	change of the $j$ th vibration component
$\Delta \hat{V}_j(k)$	predicted change of the $j$ th vibration component
$F$	neural network output denoting the vibration change of two consecutive flights
$e_j(k)$	measurement error for the $j$ th vibration element
$\hat{e}_j(k)$	prediction error for the $j$ th vibration element
$C$	coefficient matrix

## 1 INTRODUCTION

Helicopter track and balance is the process of adjusting the rotor blades to reduce the aircraft vibration and the track spread of the rotor blades. Track and balance as applied to Sikorsky's Black Hawk (UH-60) helicopters is performed as follows. For initial measurements, the aircraft is flown through six different regimes during which measurements of rotor track and vibration are recorded. Rotor track is measured by optical sensors which detect the vertical position of the blades. Vibration is measured in the cockpit of the helicopter at the frequency of once per blade revolution (*1 per rev*) by two accelerometers, 'A' and 'B', attached to the sides of the cockpit (see Figure 1). The vibration data is vectorially combined into two components: A+B, representing the vertical vibration of the aircraft, and A-B, representing its roll vibration. A sample of peak vibration levels for the six flight regimes, as well as the vibration phase relative to a reference blade position are given in Table 1, along with a sample of track data. The six flight regimes in Table 1 are: ground (*fpm*), hover (*hov*), 80 knots (*80*), 120 knots (*120*), 145 knots (*145*), and maximum horizontal speed (*vh*). The track data indicate the vertical position of each blade relative to a mean position.

In order to bring track and *1 per rev* vibration within specification (usually below 0.2 inches per second (ips)), three types of adjustments can be made to the rotor system: pitch control rod, trim tab, and balance weight (see Fig. 1). Pitch control rods can be extended or contracted by a certain number of notches to alter the pitch of the rotor blades; positive push rod adjustments indicate extension. Trim tabs, which are adjustable surfaces on the trailing edge of the rotor blades, affect the aerodynamic pitch moment of the blade and consequently the overall *1 per rev* vibration characteristics of the rotor. Tab adjustments are measured in thousandths of an inch, with positive and negative changes representing upward and downward tabbing, respectively. Finally, balance weights can be either added to or removed from the rotor hub to tune vibrations through changes in the center of gravity of the rotor. Balance weights are measured in ounces

Table 1: Typical track and balance data recorded during a flight.

Flight Regime	Vibration			
	A+B		A-B	
	Mag.	Phase	Mag.	Phase
	(ips)	(deg.)	(ips)	(deg.)
<i>fpm</i>	0.19	332	0.38	272
<i>hov</i>	0.07	247	0.10	217
<i>80</i>	0.02	86	0.04	236
<i>120</i>	0.04	28	0.04	333
<i>145</i>	0.02	104	0.07	162
<i>vh</i>	0.10	312	0.12	211

Flight Regime	Track (mm)			
	Blade #			
	1	2	3	4
<i>fpm</i>	-2	3	1	-2
<i>hov</i>	-1	3	0	-2
<i>80</i>	1	11	1	-13
<i>120</i>	2	13	-1	-14
<i>145</i>	5	18	-3	-20
<i>vh</i>	2	13	-1	-14

with positive adjustment representing the addition of weight. In the case of the Sikorsky UH-60 helicopter, which has 4 main rotor blades, a total of twelve adjustments can be made to tune the rotors (i.e., three adjustments per blade). Among them, balance weights mostly affect the ground vibration of the UH-60 helicopter, so they are not commonly used for in-flight tuning. Furthermore, since the symmetry of rotor blades in four-bladed aircraft produces identical effects for equal adjustment of opposite blades, the combined form of blade adjustment to opposite blade-pairs can be used as inputs. Accordingly, the input vector can be defined as:

$$\Delta \mathbf{x} = [\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T \quad (1)$$

where  $\Delta x_1$  and  $\Delta x_3$  denote the combined trim tab adjustments ( $\Delta TAB$ ) to blade combinations 1-3 and 2-4, respectively, and  $\Delta x_2$  and  $\Delta x_4$  represent the combined pitch control rod adjustments ( $\Delta PCR$ ) to blade combinations 1-3 and 2-4, respectively. The relationships between the combined and individual adjustments are in the form:

$$\Delta x_1 = \Delta TAB_3 - \Delta TAB_1 \quad (2)$$

$$\Delta x_2 = \Delta PCR_3 - \Delta PCR_1 \quad (3)$$

$$\Delta x_3 = \Delta TAB_4 - \Delta TAB_2 \quad (4)$$

$$\Delta x_4 = \Delta PCR_4 - \Delta PCR_2 \quad (5)$$

Generally the *1 per rev* vibration is not sufficient for rotor tuning, and additional information in the form of either blade track or vibration at higher rotation orders is required [1]. In practice, track and balance is performed by first specifying a combined set of adjustments to reduce *1 per rev* vibration. These adjustments are then expanded into a detailed set that best minimizes track spread (without compromising the vibration reductions).

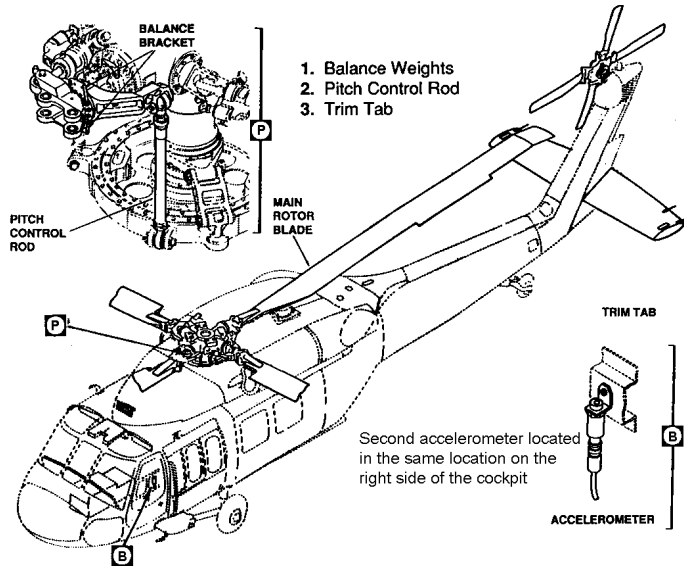


Figure 1: Illustration of the position of accelerometers A and B on the aircraft, and the rotor blade adjustments (pitch control rod, trim tab and hub weights).

Ideally, identical adjustments made to two different helicopters of the same model type should result in identical changes in vibration. In reality, however, significant inconsistency in vibration changes may be present for identical adjustments to different helicopters of the same model type. This inconsistency is attributed to several factors [2]: (1) noise in vibration measurements (sensor); (2) nonuniformity of flight conditions, such as weather; (3) error in implementing blade adjustments; and (4) dissimilarities between aircraft and rotor blades. Since it is impossible to identify the source, the vibration inconsistency is treated as stochastics in this research.

Virtually all of the current systems of rotor track and balance rely on the strategy shown in Fig. 2, whereby the measurements of the flight just completed are used as the basis to search for blade adjustments. The search for blade adjustments is guided by the “Process Model” (see Fig. 2) which represents the relationship between vibration changes and blade adjustments. A difficulty of track and balance is the excess equations to degrees of freedom (4 inputs to control 24 outputs, i.e. of Table 1). Another difficulty is caused by the high level of noise present in the vibration measurements.

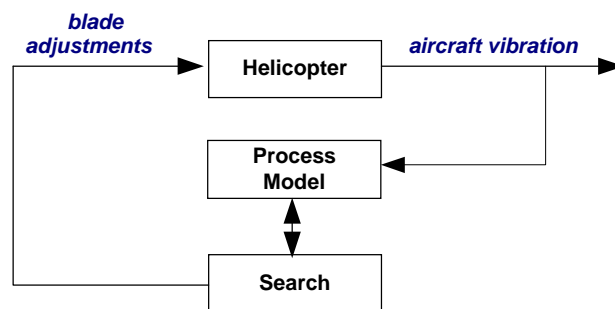


Figure 2: Tuning strategy of the current methods.

The traditional approach to track and balance uses linear relationships to define the Process Model [3, 4] and uses model inversion to streamline the search. The drawback of the traditional approach, therefore, is its neglect of the potential nonlinearity of track and balance and the vibration noise. In an attempt to include the potential nonlinearity of the process, Taitel et al. [2] trained a set of neural networks with actual track and balance data to map vibration measurements to blade adjustments and to evaluate the efficacy of the solution. In effect, they trained an inverse-model to produce the solutions available in the historical track and balance data, and trained a forward-model to evaluate the solution. The potential advantage of this method is that it can interpolate among the historical solutions to address potential nonlinearity and vibration noise. It is disadvantaged, however, in that it is only applicable to helicopters with extensive track and balance history and that its solutions are constrained by those contained in the historical data.

Another deviation from the traditional approach is introduced by Ventres and Hayden [1] who define the relationships between adjustments and vibration in the frequency domain, and provide an extension of these relationships to higher order vibrations. They use an optimization method to search for the adjustments to reduce *1 per rev* vibration as well as higher order vibrations. Accordingly, this approach has the capacity to provide a comprehensive solution, but it too neglects the potential nonlinearity between the blade adjustments and aircraft vibration.

In this paper an adaptive method of track and balance is introduced to remedy the shortcomings of the existing methods. This method uses an interval model to cope with the potential nonlinearity of the process and to account for the vibration noise. It also incorporates learning to provide adaptation to the track and balance process. With this method, the coefficients of the interval model are initialized according to the sensitivity coefficients between the blade adjustments and helicopter vibration to take advantage of the *a priori* knowledge of track and balance, but these coefficients are modified after the first iteration to better represent the vibration measurements. This method takes into account vibration data from all of the flight regimes during the search for the appropriate blade adjustments, therefore, a comprehensive solution is provided. The combination of a versatile modeling framework, learning, and comprehensive search has been shown to lead to better blade adjustment choices.

## 2 THE PROPOSED METHOD

The schematic of the proposed method is shown in Fig. 3. As in the other methods, it uses a process model as the basis of search for the appropriate blade adjustments, but instead of a linear model it uses an interval model to accommodate process nonlinearity and measurement noise. According to this model, the feasible region of the process is estimated to consist of the adjustments that will result in acceptable vibration estimates. This feasible region is then used as the basis of search for the blade adjustments that will minimize the modeled vibration. If the application of these adjustments does not result in satisfactory vibration, the interval model will be updated to better estimate the feasible region and improve the choice of blade adjustments for the next flight.

### 2.1 The Interval Model

In order to account for the stochastics and nonlinearity of vibration, an interval model [5] is defined to represent the range of aircraft vibration caused by blade adjustments, instead of single

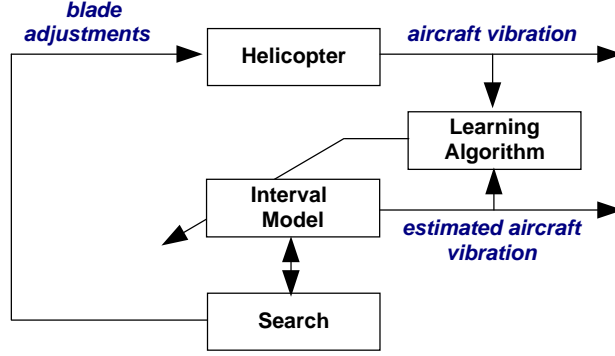


Figure 3: The strategy of the proposed tuning method.

values defined by pointwise models. The interval model used here has the form:

$$\Delta \overleftrightarrow{y}_j = \sum_{i=1}^n \overleftrightarrow{C}_{ji} \Delta x_i \quad j = 1, \dots, m \quad (6)$$

where each coefficient is defined as an interval,

$$\overleftrightarrow{C}_{ji} = [C_{Lji}, C_{Uji}]$$

In the above model, the variables with the two-sided arrow ‘ $\leftrightarrow$ ’ denote intervalled variables,  $C_{Lji}$  and  $C_{Uji}$  represent, respectively, the current values of the lower and upper bounds of the sensitivity coefficients between each input  $\Delta x_i$  and output  $\Delta \overleftrightarrow{y}_j$ . The interval  $\Delta \overleftrightarrow{y}_j$  denotes the estimated range of change of the  $j$ th output caused by the change to the current inputs  $\Delta x_1, \dots, \Delta x_n$ .

The fit provided by the interval model for a mildly nonlinear input/output relationship is illustrated in Fig. 4 when the output range is estimated relative to one explored input<sup>3</sup>. According to Eq. (6), the estimated range of the output becomes larger and, therefore, less accurate as the potential input is selected farther from the current input (resulting in a large  $\Delta x_i$ ). This potential drawback of the interval model is considerably reduced when multiple inputs have been explored so that the interval model can take advantage of several inputs for estimating the output range.

The estimated output  $\overleftrightarrow{y}_j$  at a potential input  $x_i$  may be computed relative to any set of previously explored inputs, yielding different estimates of  $\overleftrightarrow{y}_j$  (due to different values of  $\Delta x_i$ ). In order to cope with the multiplicity of estimates,  $\overleftrightarrow{y}_j$  is defined as the common range among all of the  $\overleftrightarrow{y}_j$  estimates [6]. The estimation of  $\overleftrightarrow{y}_j$  using this commonality rule is illustrated in Fig. 5, which illustrates that using this estimation approach for  $\overleftrightarrow{y}_j$  enables representation of the system non-linearities in a piece-wise fashion. It can be shown that the lack of commonality between the estimated ranges of output will cause a part of the input-output relationship to not be represented by the interval model. In such cases, however, the lack of compliance between the interval model and the input-output relationship can be corrected by adaptation of the coefficient intervals through learning.

<sup>3</sup>An explored input represents an input for which the exact value of the output is available. In track and balance, an explored input would denote a blade adjustment that has been applied to the helicopter.

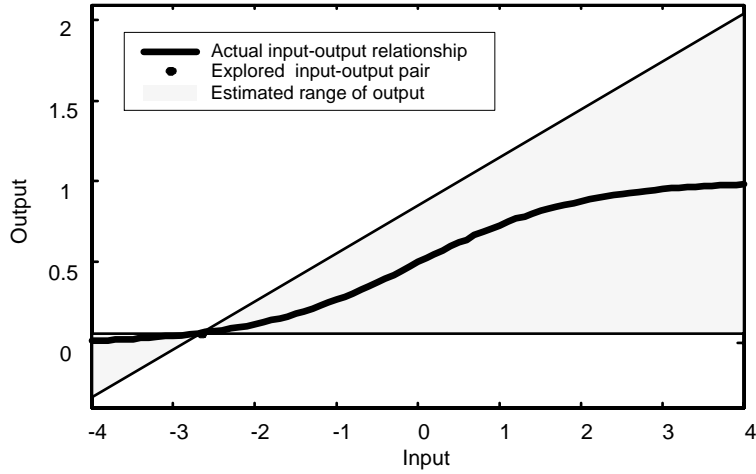


Figure 4: Estimated range of output by the interval model using one reference input.

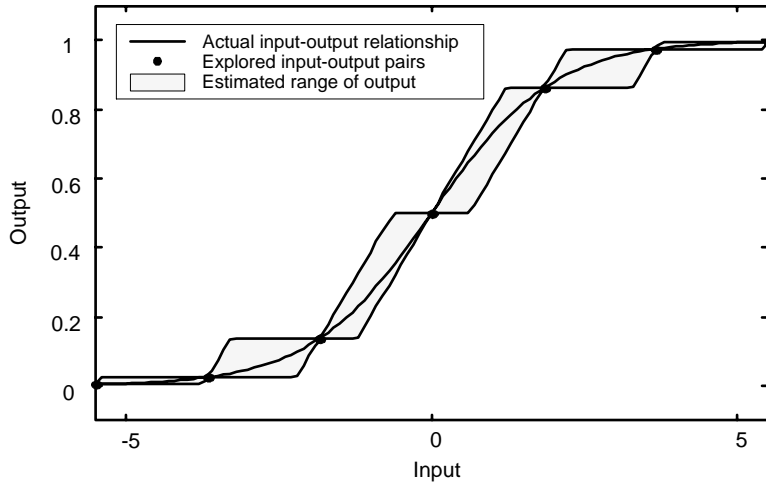


Figure 5: Estimated range of output by the interval model using seven reference inputs.

## 2.2 Estimation of Feasible Region

The feasible region comprises all sets of blade adjustments that will reduce the aircraft vibration within specifications (usually 0.2 inches per second (ips) peak). The feasible region is estimated here by comparing the individually estimated  $\vec{y}_j$  with their corresponding constraints, to decide whether the corresponding blade adjustments belong to the feasible region. In this study, even when the interval  $\vec{y}_j$  partly overlaps the vibration constraint, the corresponding blade adjustments are included in the estimated feasible region. The above procedure of estimating the feasible region based on individual outputs is then extended to multiple outputs by forming the conjunction of the estimated feasible regions from each output.

### 2.3 Selection of Blade Adjustments

The blade adjustments provide the coordinates of the feasible region, therefore, they need to provide a balanced coverage of the input space when used for feasible region estimation. As such, blade adjustment selection becomes synonymous with maximizing the distance of the selected blade adjustments from the previous blade adjustments, as well as bringing them closer to the center of the feasible region. This objective can be pursued by minimizing the following objective function:

$$S = \frac{\sum_{e=1}^{N_e} \text{Distance}(\mathbf{x}_c, \mathbf{x}_e)}{(\prod_{s=1}^{N_s} \text{Distance}(\mathbf{x}_c, \mathbf{x}_s))^{\frac{1}{N_s}}}, \quad (7)$$

where  $\mathbf{x}_c$  represents a candidate set of blade adjustments within the feasible region,  $\mathbf{x}_e$  represents any candidate set of blade adjustments at the core of the feasible region,  $\mathbf{x}_s$  denotes each of the previously selected blade adjustments, and  $N_e$  and  $N_s$  represent the number of the estimated feasible blade adjustments and the previously selected blade adjustments, respectively. Note that  $\mathbf{x}_c$ ,  $\mathbf{x}_e$  and  $\mathbf{x}_s$  represent the total blade adjustments from the initial flight. In Eq. (7), when the candidate set  $\mathbf{x}_c$  is close to the previously selected blade adjustments,  $(\prod_{s=1}^{N_s} \text{Distance}(\mathbf{x}_c, \mathbf{x}_s))^{\frac{1}{N_s}}$  becomes small, and when it is far from the center of the feasible region, the value of  $\sum_{e=1}^{N_e} \text{Distance}(\mathbf{x}_c, \mathbf{x}_e)$  becomes large. By minimizing  $S$ , the candidate blade adjustments are selected such that the above extremes are avoided.

### 2.4 Learning

Although the interval model based on the sensitivity coefficients may provide a suitable initial basis for tuning, it may not be the most representative of the current track and balance data. As such, it may not be able to carry the search process to the end. A noted feature of the proposed method is its learning capability which enables it to refine its knowledge-base. To this end, the coefficients of the model are updated by considering new values for each of the upper and lower limits of individual coefficients. The objective is to make the range of the coefficients as small as possible while the interval model encompasses the acquired input-output data. The learning problem can be defined as:

$$\text{Minimize } E = \sum_{m=1}^{K-1} \sum_{k>m}^K \{[y_L(m, k) - y(k)]^2 + [y_U(m, k) - y(k)]^2\} \quad (8)$$

Subject to:

$$y_U(m, k) \geq y(k) \quad (9)$$

$$y_L(m, k) \leq y(k) \quad (10)$$

$$C_{U_i} - \gamma \geq C_{L_i} \quad (11)$$

where  $K$  represents the total number of sample points collected so far,  $y_L(m, k)$  and  $y_U(m, k)$  represent, respectively, the lower and upper limits of the estimated output range at the  $k$ th sample point relative to the  $m$ th sample point,  $y(k)$  denotes the actual output value at the  $k$ th sample point, and  $C_{U_i}$  and  $C_{L_i}$  represent the upper and lower limits of the  $i$ th coefficient interval, respectively. The parameter  $\gamma$  is a small positive number to control the range of the coefficients.

Most of the approaches that can be potentially used for adapting the coefficient intervals, such as gradient descent [7, 8, 9] or nonlinear programming [10], cannot be applied to track and

balance due to their demand for rich training data and their impartiality to the initial value of the coefficients containing the *a priori* knowledge of the process. As an alternative, a learning algorithm is devised here to cope with the scarcity of track and balance data while staying true to the initial values of the coefficients. In this algorithm, the coefficients of the interval model are initially set at the sensitivity coefficients and subsequently adapted after each flight in two steps: enlargement and shrinkage. First, the vibration measurements from all of the flights completed for the present helicopter are matched against the estimated output ranges from the current interval model. If any of the measurements do not fit the upper or lower limits of the estimates, the coefficient intervals are enlarged in small steps iteratively and the output ranges are re-estimated at each iteration using the updated interval model. Enlargement of the coefficient intervals stops when the estimated output ranges include all of the measurements. At this point, even though the updated interval model provides a fit for the input-output data it may be over-compensated. In order to rectify this situation, the coefficient intervals are shrunk individually by selecting new candidates for their upper and lower limits.

The shrinkage-enlargement learning algorithm, has the form

$$\Delta C_{Li} = -\eta\delta_L\Delta x_i(m, k) \quad (12)$$

$$\Delta C_{Ui} = -\eta\delta_U\Delta x_i(m, k) \quad (13)$$

where during the enlargement phase,  $\delta_L$  and  $\delta_U$  are defined as:

$$\delta_L = \begin{cases} \Delta y_L & \text{if } \Delta x_i(m, k) > 0 \text{ and } \Delta y_L > 0 \\ \Delta y_U & \text{if } \Delta x_i(m, k) < 0 \text{ and } \Delta y_U < 0 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

$$\delta_U = \begin{cases} \Delta y_U & \text{if } \Delta x_i(m, k) > 0 \text{ and } \Delta y_U < 0 \\ \Delta y_L & \text{if } \Delta x_i(m, k) < 0 \text{ and } \Delta y_L > 0 \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

and during the shrinkage phase, they are defined as:

$$\delta_L = \begin{cases} \Delta y_L & \text{if } \Delta x_i(m, k) > 0 \text{ and } \Delta y_L < 0 \\ \Delta y_U & \text{if } \Delta x_i(m, k) < 0 \text{ and } \Delta y_U > 0 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

$$\delta_U = \begin{cases} \Delta y_U & \text{if } \Delta x_i(m, k) > 0 \text{ and } \Delta y_U > 0 \\ \Delta y_L & \text{if } \Delta x_i(m, k) < 0 \text{ and } \Delta y_L < 0 \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

with

$$\Delta x_i(m, k) = x_i(k) - x_i(m) \quad (18)$$

$$\Delta y_L = y_L(m, k) - y(k) \quad (19)$$

$$\Delta y_U = y_U(m, k) - y(k) \quad (20)$$

This procedure is repeated for each coefficient interval in an iterative fashion until the objective function  $E$  (Eq. (8)) is minimized. The minimization of  $E$  ensures limited adaptation of the coefficient intervals within the smallest possible range.

At the beginning of tuning, the limited number of input-output data available for learning will not provide a comprehensive representation of the process. Therefore, the coefficient intervals

should not be shrunk drastically until enough input-output data have become available. For this, the length of each coefficient interval  $[C_{Li}, C_{Ui}]$  is constrained by the minimal interval length for each tuning iteration as,

$$\min L = \{C_{Ui}(0) - C_{Li}(0)\}(1 - \beta)^n \quad (21)$$

where  $\beta \in [0, 1]$  controls the shrinkage rate of the coefficient interval, and  $n$  denotes the number of tuning iteration. The coefficient interval cannot be shrunk when  $\beta = 0$  and can be shrunk without limit when  $\beta = 1$ . Usually  $\beta$  is selected closer to 0.

In order to evaluate the effectiveness of the learning algorithm, it was tested in mirroring a linear model (i.e., a linear model was used to represent the ‘‘Helicopter’’ in Fig. 3). The upper and lower limits of the interval model coefficients were initially set arbitrarily close to those of the linear model to avoid a long learning exercise. Note that in this case, the interval model was initially set as a pointwise model, although its coefficients were subsequently adapted into intervals. The results indicate that the two limits of the coefficients converge into the actual coefficient values of the linear model, verifying the effectiveness of the learning algorithm. The performance of the learning algorithm was also tested in presence of noise by adding random numbers to the output of the linear model. The last adaptation iterations of two of the coefficient intervals are shown in Fig. 6. The results are equally reassuring, in that although the coefficients remain as intervals, they both include the actual linear model coefficient value within their limits. This shows the effectiveness of the interval model in accommodating the stochastics of the process.

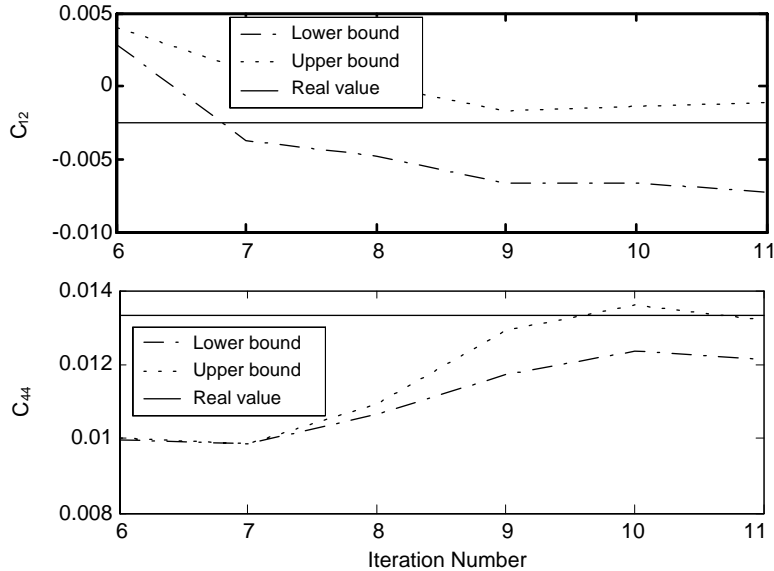


Figure 6: Typical adaptation of the coefficient intervals to the noise-contaminated outputs of a linear model.

An important characteristic of an improved model is its enhanced representation of the feasible region. This point is illustrated for a hypothetical case in Fig. 7 in terms of the overlap between the feasible regions of the trained model and the process. An increased overlap would mean a higher possibility that the selected adjustments, within the estimated feasible region, will be positioned within the feasible region of the process.

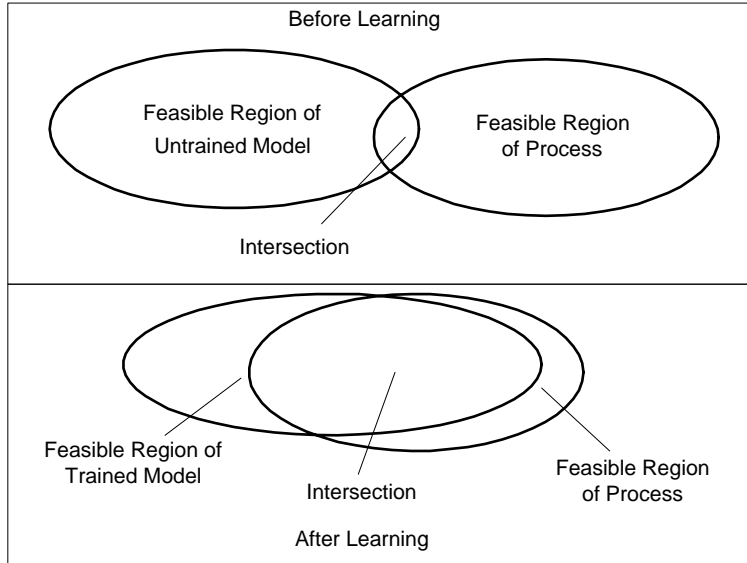


Figure 7: Variation of the feasible region due to learning.

### 3 IMPLEMENTATION

Ideally, the performance of the proposed method should be evaluated ‘side by side’ against that of the traditional method. However, such an evaluation would require tuning the aircraft with one method, undoing changes, and tuning the aircraft with another. Since such testing is prohibitively costly, a compromise approach of evaluating the method in simulation is utilized. A process simulation model is therefore used to represent the “Helicopter” in Fig. 3, with the “Forward Model” represented by an Interval Model.

#### 3.1 Simulation Model

Considering the potential nonlinearity of the effect of blade adjustments on the helicopter vibration, and the high level of noise present in vibration measurements, multi-layer neural networks offer the most suitable framework for modeling these relationships [11]. As such, a series of neural networks were trained with historical balance data to represent the relationships between vibration changes and blade adjustments, with the stochastics of vibration represented by the addition of random numbers to the outputs of the networks.

A total of 102 sets of vibration data were used to train and test the neural networks. These data, which were obtained from actual flight tests at Sikorsky Aircraft in the course of rotor tuning of 39 new production UH-60 helicopters, represent vibration changes between consecutive flights caused by blade adjustments. The inputs to these networks were the combined blade adjustments of push rods and trim tabs, and their outputs were the resulting vibration changes between two consecutive flights. Since the vibration data are vector quantities (see Table 1) that are represented by both magnitude and phase components, the vibration data were transformed into Cartesian coordinates, so that each vector element would denote the change in the cosine or sine component of the A+B or A-B vibration of each of the six flight regimes (see Table 1). In this study, each neural network model consisted of four inputs and one output, so a total of 24

networks were trained to represent all of the vibration components. To avoid overtraining, the 102 sets of data were divided into two equal subsets, one set to train the network and the other for cross validation. Alternatively, all of the vibration measurements can be represented by one neural network, but such a network is more difficult to train. Our experience indicates that the single neural network model was not as accurate as the 24-network model in representing the test data. Formally, the outputs of the neural networks, which represent the cosine or sine component of the vibration at different regimes,  $v_{cj}(k)$  and  $v_{sj}(k)$ , respectively, are defined as:

$$\hat{V}_{sj}(k) = V_{sj}(k-1) + \Delta V_{sj}(k) + R_{sj}(k) \quad (22)$$

$$\hat{V}_{cj}(k) = V_{cj}(k-1) + \Delta V_{cj}(k) + R_{cj}(k) \quad (23)$$

$$\Delta V_{sj}(k) = F_{sj}(\Delta \mathbf{x}) \quad (24)$$

$$\Delta V_{cj}(k) = F_{cj}(\Delta \mathbf{x}) \quad (25)$$

$$\hat{V}_j(k) = \sqrt{\hat{V}_{sj}(k)^2 + \hat{V}_{cj}(k)^2} \quad (26)$$

where the input vector  $\Delta \mathbf{x} = \{\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4\}$  denotes the set of combined blade adjustments, each of the functionals  $F_{sj}$  or  $F_{cj}$  represent the change in vibration between two consecutive flights by a neural network, and  $R_{sj}(k)$  and  $R_{cj}(k)$  denote random numbers added to the outputs of the networks to account for measurement noise. Each of the networks consisted of two hidden layers, with 4 and 8 processing elements in the first and second layer, respectively. The random numbers  $R_{cj}$  and  $R_{sj}$  were generated according to the Gaussian distribution  $N(\mu, \sigma^2)$ , with the mean  $\mu$  and variance  $\sigma^2$  defined as

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N e_i \quad (27)$$

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^N (e_i - \hat{\mu})^2 \quad (28)$$

In the above formulation,  $e_j(k)$  denotes the difference between the measured and expected value of vibration, defined as

$$e_j(k) = V_j(k) - V_j(k-1) - \Delta V_j(k) \quad (29)$$

A sample of estimated vibration changes generated by the neural network model is compared side-by-side with the actual vibration changes in Fig. 8. Considering the randomness associate with the vibration measurements, the results indicate close agreement between the predicted and actual vibration changes. The suitability of using a neural network as the framework of simulation was further verified by comparing the estimation error of the trained neural network with those of two linear models trained with the same training data. One linear model was trained with all of the available 102 data sets, whereas the other was trained with the same data sets as those used in training the neural network model. The neural network produced the smallest mean square error, providing further evidence that it is the best form for representing the vibration estimates.

### 3.2 Interval Modeling

A total of 24 interval models were constructed to approximate the changes in the cosine and sine components of the A+B and A-B vibrations at each of the six flight regimes. The interval models had the form

$$\overleftrightarrow{V}_{cj}(k) = V_{cj}(k-1) + \sum_{i=1}^4 \overleftrightarrow{C}_{cji}(k-1) \Delta x_i(k) \quad (30)$$

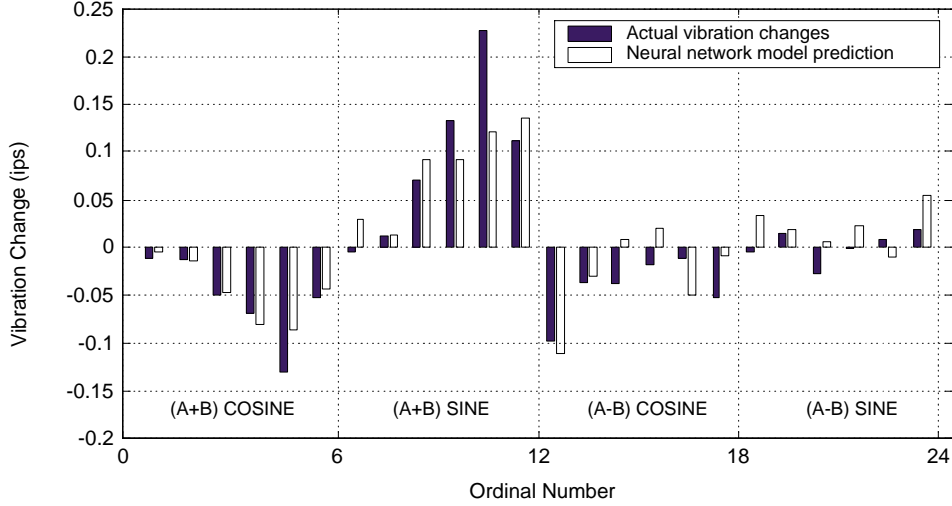


Figure 8: A sample set of simulated vibration changes shown side-by-side the actual vibration changes.

$$\vec{V}_{sj}(k) = V_{sj}(k-1) + \sum_{i=1}^4 \vec{C}_{sji}(k-1) \Delta x_i(k) \quad (31)$$

$$\vec{V}_j(k) = \sqrt{\vec{V}_{sj}^{\leftrightarrow 2}(k) + \vec{V}_{cj}^{\leftrightarrow 2}(k)} \quad (32)$$

$$i = 1, \dots, 4; j = 1, \dots, 12 \quad (33)$$

where the  $\vec{V}_{cj}(k)$  and  $\vec{V}_{sj}(k)$  represent, respectively, the estimated cosine and sine components of A+B or A-B vibration at each of the six flight regimes,  $\vec{V}_j(k)$  denotes the magnitude of the vibration, and  $\Delta x_i$  are the same as those in Eqs. (2-5). For this study, the feasible region was defined to include all of the blade adjustments associated with vibration estimates that satisfied the following specification

$$\max\{\min(\vec{V}_1), \dots, \min(\vec{V}_{12})\} \leq 0.2 \quad (34)$$

The above specification ensures that the lower limit of the estimated vibration range of the largest vibration component will be less than 0.2 ips (an industry standard). The selection of the lower limit here is to ensure that the feasible region is as large as possible, so as not to eliminate any potentially good candidate blade adjustments. The computation of the feasible region was based on an input space ranging  $[-15, 15]$  notches for push rods and  $[-0.035, 0.035]$  inches for trim tabs, within which vibration ranges for 20,000 random sets of blade adjustments were estimated. The blade adjustments associated with vibration ranges satisfying Eq. (34) were included in the feasible region.

As noted earlier, the proposed method uses the feasible region as the basis of search for the blade adjustments. For this study, the blade adjustments that produced the smallest value for the objective function  $S$  (Eq. (7)) were selected as those to be applied to the helicopter. It should be noted that given the stringent constraints on the vibration components, there were cases where the search algorithm could not find any feasible blade adjustments that would satisfy all of the

constraints. In such cases, the set of blade adjustments that produced the smallest lower limit of the maximum estimated vibration was used as a compromise solution.

The interval model was updated after each tuning iteration. For shrinkage-enlargement learning, the parameter  $\beta$  in Eq. (21) was set to 1 and  $\gamma$  to 0, so that the coefficient intervals could be shrunk without limits. Learning was performed separately for each helicopter dataset to customize the interval model to the individual helicopters, i.e., the interval model was set to the sensitivity coefficients for each helicopter and was adapted after the first tuning iteration. As such, the interval model was actually a point-wise model for the first iteration and had the form of an interval model only thereafter.

## 4 PERFORMANCE EVALUATION

The proposed method was tested with actual data from 39 UH-60 helicopters. These datasets were generated from test flights performed by Sikorsky during the production acceptance process. For each helicopter, the method was applied iteratively until the simulated vibrations were within their specifications, or an upper limit of 5 process iterations had been reached.

Due to the stochastic nature of the vibration measurements, the track and balance process contains some degree of randomness. As such, it cannot be evaluated by deterministic measures. This calls for the creation of performance measures that would account for uncertainty. One such measure that assesses tuning efficiency is the *Average Tuning Iteration Number* (ATIN) which represents the average number of adjustment iterations taken for each helicopter to bring vibration within the acceptable level of 0.2 ips. The number of flights indicated by the method for the 39 helicopters is included in Table 2 along with those actually performed during the production acceptance process. The results indicate that the proposed method indicates a smaller ATIN relative to that actually performed. This result, although not conclusive due to the nonuniformity of the test-beds (simulation vs. real aircraft), lends credence to the efficiency of the proposed method.

Table 2: The number of tuning iterations indicated by the Interval Model Method and those actually applied during production acceptance.

Helicopter # (39)	Number of Tuning Iterations	
	Actual	IM Method
176	1	1
178	1	1
179	3	2
180	1	1
184	2	1
⋮	⋮	⋮
260	4	1
⋮	⋮	⋮
861	1	1
Total	71	48
ATIN	1.82	1.23

Another potentially significant aspect of the proposed method is its adaptation capability, that enables it to transform a point-wise model into an interval model and to subsequently update it after the first iteration. Adaptation capability, however, may not be as significant in track and balance when limited data is available for training. In order to evaluate the significance of learning in the performance of the proposed method, the results in Table 2 were reproduced in Table 3 with the learning feature turned off. The ATINs indicate that with learning the proposed method requires fewer iterations for tuning each helicopter, and that, in turn, indicates that the interval model enhances the performance of the proposed method, since without learning the model remains point-wise at the sensitivity coefficients. But perhaps an equally interesting observation from Table 3 is that even without learning the proposed method requires fewer iterations than those actually performed during the production acceptance process. Given that the adjustments associated with both sets of results were selected from the same model (i.e., sensitivity coefficients), the better performance of the proposed method can only be attributed to its more effective search strategy that leads to more comprehensive solutions.

Table 3: The number of tuning iterations indicated by the proposed method (with and without learning) along with those actually applied during production acceptance.

Helicopter # (39)	Tuning Iteration Number		
	Actual	IM Method	
		With Learning	Without Learning
185	3	2	3
186	3	2	3
208	2	2	3
245	3	2	3
260	4	2	3
802	3	2	3
822	3	2	3
⋮	⋮	⋮	⋮
Total	71	48	62
ATIN	1.82	1.23	1.59

A preferred aspect of a system of track and balance is its ability to tune the aircraft in one iteration. This aspect of the method was evaluated by checking the number of helicopters tuned in one iteration. For these results, in order to eliminate the difference between the simulation model and the helicopter, only the vibration estimates from simulation were used to evaluate the suitability of the adjustments. The results of this evaluation are shown in Table 4 where the helicopters tuned in one iteration are shown by a  $\checkmark$  and those requiring more than one iteration are denoted by  $\times$ . The results indicate that the proposed method satisfies this more stringent criterion better than the actual adjustments. This further validates the claim that the proposed method benefits from a more effective search engine.

Due to the randomness of the vibration measurements, repeated applications of an adjustment set may lead to slightly different vibration measurements. This, in turn, may cause a variance in the number of iterations produced by adjustments when the resulting vibration is close to the specified threshold. It would be beneficial, therefore, to devise a measure for the probability of

Table 4: Tally of the helicopters tuned in one iteration.

Helicopter # (39)	Tuned in one iteration	
	Actual	IM Method
176	✓	✓
178	×	✓
179	✓	✓
⋮	⋮	⋮
822	×	✓
858	×	✓
859	✓	✓
861	✓	✓
Total	19	30

success of adjustments. The empirical measure, Acceptability Index ( $AI$ ), defined here to denote the percentage of times an adjustment set will result in the vibration satisfying the specification, has the form:

$$AI = \frac{1}{N} \sum_{l=1}^N s_l \quad (35)$$

where  $N$  represents the total number of flights simulated to represent the repeated application of the same adjustment set, and

$$s_l = \begin{cases} 1 & \text{if vibration of the } l\text{th simulation flight is acceptable} \\ 0 & \text{if vibration of the } l\text{th simulation flight is unacceptable} \end{cases}$$

The Acceptability Index ( $AI$ ) computed for both the actual and selected adjustments at the first iteration are included in Table 5. The results indicate that the IM method provides adjustments with a higher probability of success as judged by the acceptability of vibration estimates from the simulation model. These results, which indicate that the selected adjustments from the proposed method can more consistently tune the helicopter in one iteration, imply the better positioning of the adjustments within the feasible region.

Another criterion for the adjustments can be established by comparing them to the actual cumulative adjustments performed during helicopter production acceptance. The cumulative adjustment set,  $\Sigma x$ , can be defined as

$$\Sigma x = \sum_{k=1}^N \Delta x_k \quad (36)$$

where  $N$  represents the total number of tuning iterations performed during production acceptance for each helicopter, with  $\Delta x_k$  denoting the adjustments applied at the  $k$ th iteration. A sample of actual first iteration adjustments, actual cumulative adjustments, and first iteration adjustments from the IM method is shown in Table 6. The results indicate that the adjustments from the IM method are closer to the actual cumulative adjustments than are the actual first iteration adjustments. Although the cumulative adjustments may not be the most desirable ones for the aircraft, they represent an acceptable set that have been proven during the production acceptance

process. The closeness of the IM Method’s solutions to the actual cumulative adjustments provides further evidence about the effectiveness of the search strategy.

## 5 CONCLUSIONS

A method is introduced for helicopter track and balance that can incorporate the *a priori* knowledge of the system in the form of sensitivity coefficients between the blade adjustments and aircraft vibration. It uses an interval model to cope with the potential nonlinearity of track and balance as well as to account for the high level of noise commonly present in the vibration measurements. This method also benefits from learning, which enables it to update its knowledge base to accommodate small differences between aircraft and blades. As such, this method offers the versatility to be readily customized for new helicopters for which a record of track and balance does not exist to yield accurate sensitivity coefficients. The results obtained with actual track and balance data indicate superior performance according to several performance measures. Specifically, the results indicate that the method (1) requires fewer iterations than those actually performed during the production acceptance process, (2) provides adjustments with a higher probability of success, and (3) provides adjustments at the first iteration that are close to actual cumulative adjustments of several iterations performed during the production acceptance process. All these provide assurance about the comprehensiveness of the method’s solutions.

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Table 5: The value of Acceptability Index ( $AI$ ) Computed for both the actual and selected adjustments at the first flight.

Helicopter # (39)	Acceptability Index	
	Actual	IM Method
176	0.92	0.87
178	0	0.54
179	0.61	0.52
⋮	⋮	⋮
260	0.40	0.89
261	0.09	0.95
263	0.18	0.12
⋮	⋮	⋮
822	0.00	0.64
857	0.62	0.55
858	0.64	0.93
859	0.94	0.67
861	0.96	0.74
Average	0.581	0.724

Table 6: Comparison of the first iteration solutions of IM method and actual solutions from Sikorsky's production line with the cumulative acceptable adjustments.

Helicopter #	Adjustments											
	Actual First Iteration Solution				Actual Cumulative Adjustment Set				IM Method First Iteration Solution			
	$PCR_{13}$	$PCR_{24}$	$TAB_{13}$	$TAB_{24}$	$PCR_{13}$	$PCR_{24}$	$TAB_{13}$	$TAB_{24}$	$PCR_{13}$	$PCR_{24}$	$TAB_{13}$	$TAB_{24}$
801	6	-4	-10	11	2	-4	-4	14	3	-5	-6	12
802	5	2	0	0	9	0	-10	10	8	-2	-5	10
822	6	0	-20	0	10	-4	-23	13	8	-4	-22	6
858	7	0	-14	0	9	-2	-10	3	9	-2	-15	4

## APPENDIX B

### A Probability-Based Approach to Helicopter Rotor Tuning<sup>1</sup>

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#### ABSTRACT

A method of helicopter rotor balance is introduced that uses a probability model to maximize the likelihood of success of the selected blade adjustments. The underlying model in this method consists of two segments: a linear segment to include the sensitivity coefficients between the blade adjustments and helicopter vibration, and a stochastic segment to represent the probability densities of the vibration components. Based on this model, the blade adjustments with the maximal probability of generating acceptable vibration are selected as recommended adjustments. The effectiveness of the proposed method is evaluated in simulation using a series of neural networks trained with actual vibration data. The results indicate that the proposed method improves performance according to several criteria representing various aspects of track and balance.

#### Nomenclature

$\Delta \mathbf{x}$	vector of combined blade adjustments
$\Delta x_i$	the $i$ th element of $\Delta \mathbf{x}$
$k$	flight number, $k = 0$ denotes the initial flight
$V_j(k)$	measured vibration of the $k$ th flight
$\hat{V}_j(k)$	prediction of the $j$ th vibration component for the $k$ th flight
$V_{sj}(k)$	sine of the $j$ th vibration component
$V_{cj}(k)$	cosine of the $j$ th vibration component
$\Delta V_j(k)$	change of the $j$ th vibration component
$e_j(k)$	measurement error of the $j$ th vibration element
$\hat{e}_j(k)$	prediction error of the $j$ th vibration element
$p(\mathbf{x})$	joint probability density function
$S, \Gamma$	integration region
$\sigma^2$	variance of random number
$\mu$	mean value of random number
$C$	coefficient matrix

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## Introduction

Helicopter track and balance is the process of adjusting the rotor blades to reduce the aircraft vibration and deviation in blade track. Track and balance as applied to Sikorsky's Black Hawk (UH-60) helicopters is performed as follows. For initial measurements, the aircraft is flown through six different regimes during which measurements of rotor track and vibration are recorded. Rotor track is measured by optical sensors which detect the vertical position of the blades. Vibration is measured in the cockpit of the helicopter at the frequency of once per blade revolution (*1 per rev*) by two accelerometers, 'A' and 'B', attached to the sides of the cockpit (see Figure 1). The vibration data is vectorially combined into two components: A+B, representing the vertical vibration of the aircraft, and A-B, representing its roll vibration. A sample of peak vibration levels for the six flight regimes, as well as the vibration phase relative to a reference blade position are given in Table 1, along with a sample of track data. The six flight regimes in Table 1 are: ground (*fpm*), hover (*hov*), 80 knots (*80*), 120 knots (*120*), 145 knots (*145*), and maximum horizontal speed (*vh*). The track data indicate the vertical position of each blade relative to a mean position.

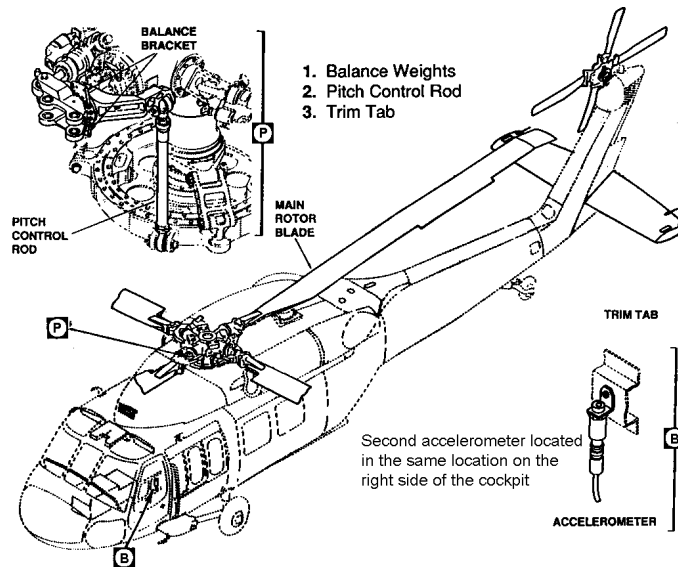


Figure 1: Illustration of the position of accelerometers A and B on the aircraft, and the rotor blade adjustments (push control rod, trim tab and hub weights).

To bring track and *1 per rev* vibration within specification (usually below 0.2 inches per second (ips)), three types of adjustments can be made to the rotor system: pitch control rod, trim tab, and balance weight (see Fig. 1). Pitch control rods can be extended or contracted by a certain number of notches to alter the pitch of the rotor blades; positive push rod adjustments indicate extension. Trim tabs, which are adjustable surfaces on the trailing edge of the rotor blades, affect the aerodynamic pitch moment of the blade and consequently the overall *1 per rev* vibration characteristics of the rotor. Tab adjustments are measured in thousandths of an inch, with positive and negative changes representing upward and downward tabbing, respectively. Finally, balance weights can be either added to or removed from the rotor hub to tune vibrations through changes in the center of gravity of the rotor. Balance weights are measured in ounces with positive adjustments representing the addition of weight. In the case of the Sikorsky UH-60

Table 1: Typical track and balance data recorded during a flight.

Flight Regime	Vibration			
	A+B		A-B	
	Mag. (ips)	Phase (deg.)	Mag. (ips)	Phase (deg.)
<i>fpm</i>	0.19	332	0.38	272
<i>hov</i>	0.07	247	0.10	217
<i>80</i>	0.02	86	0.04	236
<i>120</i>	0.04	28	0.04	333
<i>145</i>	0.02	104	0.07	162
<i>vh</i>	0.10	312	0.12	211

Flight Regime	Track (mm)			
	Blade #			
	1	2	3	4
<i>fpm</i>	-2	3	1	-2
<i>hov</i>	-1	3	0	-2
<i>80</i>	1	11	1	-13
<i>120</i>	2	13	-1	-14
<i>145</i>	5	18	-3	-20
<i>vh</i>	2	13	-1	-14

helicopter, which has 4 main rotor blades, a total of twelve adjustments can be made to tune the rotors (i.e., three adjustments per blade and 24 outputs). Among them, balance weights mostly affect the ground vibration of the UH-60 helicopter, so they are not commonly used for in-flight tuning. Furthermore, since the symmetry of rotor blades in four-bladed aircraft produces identical effects for equal adjustment of opposite blades, the combined form of blade adjustments to each pair of opposing blades can be used as inputs. Accordingly, the input vector can be defined as:

$$\Delta \mathbf{x} = [\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T \quad (1)$$

where  $\Delta x_1$  and  $\Delta x_3$  denote the combined trim tab adjustments ( $\Delta TAB$ ) to blade combinations 1-3 and 2-4, respectively, and  $\Delta x_2$  and  $\Delta x_4$  represent the combined pitch control rod adjustments ( $\Delta PCR$ ) to blade combinations 1-3 and 2-4, respectively. The relationships between the combined and individual adjustments are in the form:

$$\Delta x_1 = \Delta TAB_3 - \Delta TAB_1 \quad (2)$$

$$\Delta x_2 = \Delta PCR_3 - \Delta PCR_1 \quad (3)$$

$$\Delta x_3 = \Delta TAB_4 - \Delta TAB_2 \quad (4)$$

$$\Delta x_4 = \Delta PCR_4 - \Delta PCR_2 \quad (5)$$

Generally the *1 per rev* vibration is not sufficient for rotor tuning, and additional information in the form of either blade track or vibration at higher rotation orders is required (Ref. [?]). In practice, track and balance is performed by first specifying a combined set of adjustments to reduce *1 per rev* vibration. These adjustments are then expanded into a detailed set that best minimizes track spread (without compromising the vibration reductions). In this research, only the combined blade adjustments are determined to reduce the helicopter vibration.

Ideally, identical adjustments made to different helicopters of the same model type should result in identical changes in vibration. In reality, however, significant inconsistency in vibration changes may be present for identical adjustments to different helicopters of the same model type. This inconsistency is attributed to several factors (Ref. [?]): (1) noise in vibration measurements (sensor); (2) nonuniformity of flight conditions, such as weather; (3) error in implementing blade adjustments ; and (4) dissimilarities between aircraft and rotor blades. Since it is impossible to identify the source, the vibration inconsistency is treated as stochastics in this research.

Virtually all of the current systems of rotor track and balance rely on the strategy shown in Fig. 2, whereby the measurements of the flight just completed are used as the basis to search for blade adjustments. The search for blade adjustments is guided by the “Process Model” (see Fig. 2) which represents the relationship between vibration changes and blade adjustments. A difficulty of track and balance is the excess equations to degrees of freedom (4 inputs to control 24 outputs, i.e. of Table 1). Another difficulty is caused by the high level of noise present in the vibration measurements.

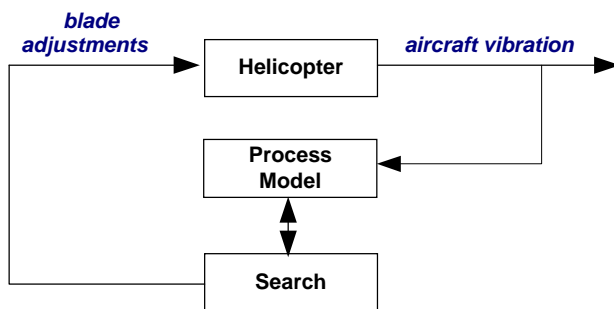


Figure 2: Tuning strategy of the current methods.

The traditional approach to track and balance uses linear relationships to define the Process Model and uses model inversion to streamline the search. The drawback of the traditional approach, therefore, is its neglect of the stochastics of track and balance. In an attempt to cope with the stochastics of the process, Taitel et al. (Ref. [?]) trained a set of neural networks with actual track and balance data to map vibration measurements to blade adjustments and to evaluate the efficacy of the solution. In effect, they developed an inverse-model to produce the solutions available in the historical track and balance data, and provided a forward-model to evaluate the solution. The potential advantage of this method is that it can interpolate among the historical solutions to account for stochastics. It is disadvantaged, however, in that it is only applicable to helicopters with extensive track and balance history and that its solutions are limited to the solutions contained in the historical data.

Another deviation from the traditional approach is by Ventres and Hayden (Ref. [?]) who define the relationships between adjustments and vibration in the frequency domain, and provide an extension of these relationships to higher order vibrations. They use an optimization method to search for the adjustments to reduce *1 per rev* vibration as well as higher order vibrations. This approach has the capacity to provide a comprehensive solution, but it too neglects the stochastic relationship between the blade adjustments and aircraft vibration.

The purpose of this paper is to introduce a method of track and balance that accounts for the stochastics of track and balance. The underlying model in this method comprises two components: a deterministic component as well as a probabilistic one. It relies on the probability model to

estimate the likelihood of the measured vibration satisfying the specifications as the basis of a search for blade adjustments. The likelihood measures used for blade adjustment selections are computed according to the probability distribution of vibration derived from historical track and balance data. Although the method introduced here is demonstrated with four-bladed helicopters, it can be easily extended to cope with helicopters with more than four blades.

### The Proposed Method

The noted feature of the proposed method is its introduction of likelihood of success as a criterion in the search for the optimal blade adjustments. It estimates this likelihood according to the probability density of prediction error between the estimation model and vibration measurements. For selection of the blade adjustments, it quantifies the effectiveness of various adjustments sets in reducing the vibration and selects the set with the maximum probability of producing acceptable vibration.

The proposed method is best explained in the context of a simple example. If the measured vibration from the current flight is denoted by  $V_j(k-1)$  and the estimated vibration change from the model is represented by  $\Delta\hat{V}_j(k) = f(\Delta x)$  as a function of the blade adjustments  $\Delta x$ , then the predicted vibration of the next flight  $\hat{V}_j(k)$  can be defined as

$$\hat{V}_j(k) = V_j(k-1) + \Delta\hat{V}_j(k) \quad (6)$$

$$V_j(k) = \hat{V}_j(k) + \hat{e}_j(k) \quad (7)$$

where  $V_j(k)$  denotes the measured vibration for the next flight. In track and balance, the adjustments are selected according to the predicted vibration  $\hat{V}_j(k)$ , whereas the objective is defined in terms of the measured vibration. The inclusion of the probability model here is to account for the inevitable uncertainty in the actual measured vibration. In this research, the selection of adjustments is performed to maximize the probability that the future measured vibration will be within the specifications. According to Eq. (7), the mean value of the measured vibration is equal to the value of the predicted vibration plus the mean value of the prediction error. But since the predicted vibration is a deterministic entity, the probability distribution of the measured vibration is the same as that of the prediction error. Accordingly, whereas the nominal value of the measured vibration can be controlled by the blade adjustment, its optimum position within the specification region should be determined according to its probability distribution, which is the same as that of the prediction error. For a case where the prediction error  $\hat{e}_j(k)$  is zero-mean normally distributed, placing the predicted vibration at the center of the specification range will be synonymous with maximizing the probability that the measured vibration will be within the range, as illustrated in Fig. 3. The likelihood of success of blade adjustments can therefore be measured by the area under the probability density function of prediction error located within the specification region. The blade adjustment set that produces the highest likelihood will be the preferred adjustment set.

However, the main difficulty with track and balance is the limited number of degrees of freedom, which precludes the ability to perfectly position the predicted vibration. This point is illustrated in Fig. 4 for a simple case where two vibration components are to be positioned at the center of the specification region with only one adjustment. If one assumes that the effect of adjustment  $\Delta x$  on the two vibration components  $\Delta\hat{V}_j(k)$  can be represented by a linear model such as

$$\Delta\hat{V}_j(k) = a_{ij}\Delta x$$

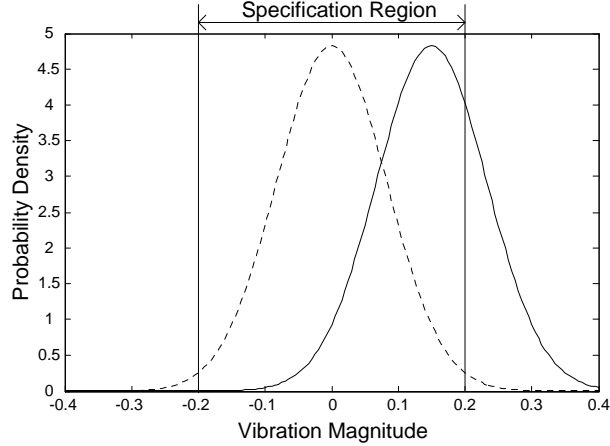


Figure 3: Illustration of improved placement of the predicted vibration within the specification range.

then the movement of the two predicted vibration components will be constrained to the line L in Fig. 4. As illustrated in this figure, since it will be impossible to place the predicted vibration components at the center due to lack of degrees of freedom, a compromised position needs to be selected. In this research, the best compromised position for the predicted vibration is that which renders the largest probability for the measured vibration satisfying the specifications. This position, for the two-component vibration example is the one that maximizes

$$P_r[(V_1, V_2) \in S] = \int_{(V_1, V_2) \in S} p(V_1, V_2) dV_1 dV_2 \quad (8)$$

The above formulation indicates that the placement of the predicted vibration requires knowledge of the joint probability density function  $p(V_1, V_2)$  of the vibration components. In the hypothetical case of independent vibration components with equal probability distributions, the loci of the points with equal probabilities  $P_r[(V_1, \dots, V_n) \in S]$  are the surfaces of hyper-spheres. For the two-component vibration example, the loci of equal probabilities are circles centered at the origin (see Fig. 4). Therefore, the best compromised position for the constrained case is point P, which is the closest point on line L to the center of the specification circle.

Point P, however, does not represent the best position if the two vibration components are dependent or have unequal distributions. The locus of equal probabilities for this more general case is elliptical, as shown in Fig. 5, which locates point Q as the best position on line L (and on the ellipse) for placing the predicted vibration. The inadequacy of degrees of freedom is exacerbated in track and balance where 24 correlated vibration components need to be positioned within the specification region using only 4 blade adjustments.

### Error analysis

As discussed earlier, the placement of the predicted vibration within the specification region will be based upon the probability density of the prediction error. Here, the probability distribution of the prediction error is determined in relation to the measurement error (see Fig. 6). The prediction error, defined as

$$\hat{e}_j(k) = V_j(k) - \hat{V}_j(k) \quad (9)$$

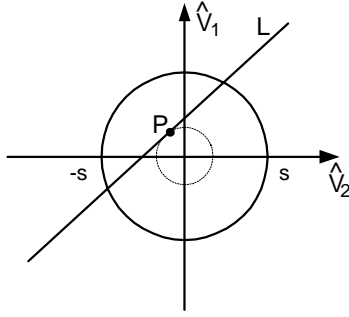


Figure 4: Restricted placement of vibration components within the specification region for a two-dimensional case.

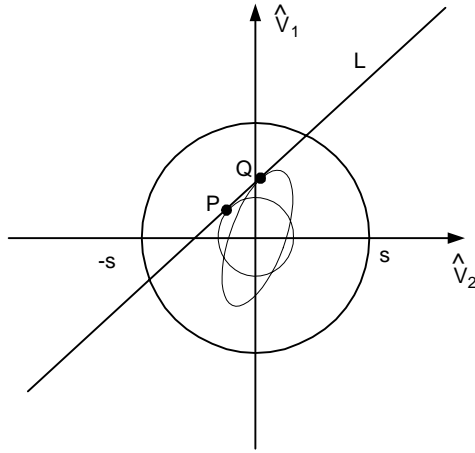


Figure 5: The change in location of the maximal probability point when the two vibration components are dependent or have unequal distributions.

consists of three components: (1) noise (sensors, weather, etc.), (2) modeling error, and (3) adjustment error. Similarly, the measurement error  $e_j(k)$  representing the difference between the measured vibration  $V_j(k)$  and its expected value  $E[V_j(k)]$

$$e_j(k) = V_j(k) - E[V_j(k)] \quad (10)$$

comprised of measurement noise. In the absence of modeling error, one can write

$$\Delta \hat{V}_j(k) = E[\Delta V_j(k)] \quad (11)$$

and

$$E[V_j(k)] = E[V_j(k-1)] + E[\Delta V_j(k)] \quad (12)$$

Now, combining the relationship for the predicted vibration  $\hat{V}_j(k)$

$$\hat{V}_j(k) = V_j(k-1) + \Delta \hat{V}_j(k) \quad (13)$$



vibration at different regimes,  $V_{cj}(k)$  and  $V_{sj}(k)$ , respectively, are defined as:

$$\hat{V}_{sj}(k) = V_{sj}(k-1) + \Delta V_{sj}(k) \quad (16)$$

$$\hat{V}_{cj}(k) = V_{cj}(k-1) + \Delta V_{cj}(k) \quad (17)$$

$$\Delta V_{sj}(k) = F_{sj}(\Delta \mathbf{x}) \quad (18)$$

$$\Delta V_{cj}(k) = F_{cj}(\Delta \mathbf{x}) \quad (19)$$

$$\hat{V}_j(k) = \sqrt{\hat{V}_{sj}(k)^2 + \hat{V}_{cj}(k)^2} \quad (20)$$

where the input vector  $\Delta \mathbf{x} = \{\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4\}$  denotes the set of combined blade adjustments, and each of the functionals  $F_{sj}$  or  $F_{cj}$  represent the change in vibration between two consecutive flights. Two standard model validation practices were used to check the suitability of the linear model (Ref. [?]). The data were separated into two equal size sets (each including 51 input-output data pairs). For the first validation practice, two sets of linear regression models (each consisting of 24 linear regression models) were trained. For the first set of models, the first data set was used for training and the second set for testing. The second model set was trained with interchanged data sets for training and testing. The models were trained using the Least Square Error (LSE) Method (Ref. [?]). The training and testing Mean Square Errors (MSEs) for the two model sets were fairly close, indicating the linear model's effectiveness in representing the effect of blade adjustments on the aircraft vibrations.

For the second model validation practice the coefficients of the two linear regression models were compared for their similarity. The results indicated that the two sets of coefficients were quite consistent for the 24 outputs, further validating the suitability of linear regression models in estimating the changes in aircraft vibrations due to blade adjustments.

A set of neural networks were trained next to provide a basis for evaluating the suitability of the linear model. In this study, each neural network model consisted of four inputs and one output, so a total of 24 networks were trained to represent all of the vibration components. Alternatively, all of the vibration measurements can be represented by one neural network, but such a network is more difficult to train. Each of the networks consisted of two hidden layers, with 4 and 8 processing elements in the first and second layers, respectively. A logarithmic sigmoid transfer function was used for the two hidden layers and a linear transfer function was used for the output layer. Mean square error (MSE) was used to measure the network performance during training. As before, the data were divided into two equal subsets, one set to train the network and the other to test its performance. To avoid overtraining, each training iteration included two steps: in the first step the network was trained, and in the second step it was tested with the test data. The training process was stopped when the performance of the network began to deteriorate.

Samples of predicted vibration changes from the neural network and linear model are compared with the actual vibration changes in Fig. 7. The results indicate close agreement between the predicted and actual vibration changes. A comparison of the mean square error values for the two models indicates that while the neural network produces better prediction of the vibration change, its performance is not drastically better. An analysis of the prediction errors from the linear model and neural networks indicates that both errors are normally distributed with very similar mean and variance values. The cumulative probability density functions of the errors from the two models are compared in Fig. 8. The results indicate that the two density functions are very similar, and that the linear model provides a good approximation, despite the potential nonlinearity in the relation between vibration changes and blade adjustments. This provides assurance of the adequacy of the linear model in blade adjustment selection.

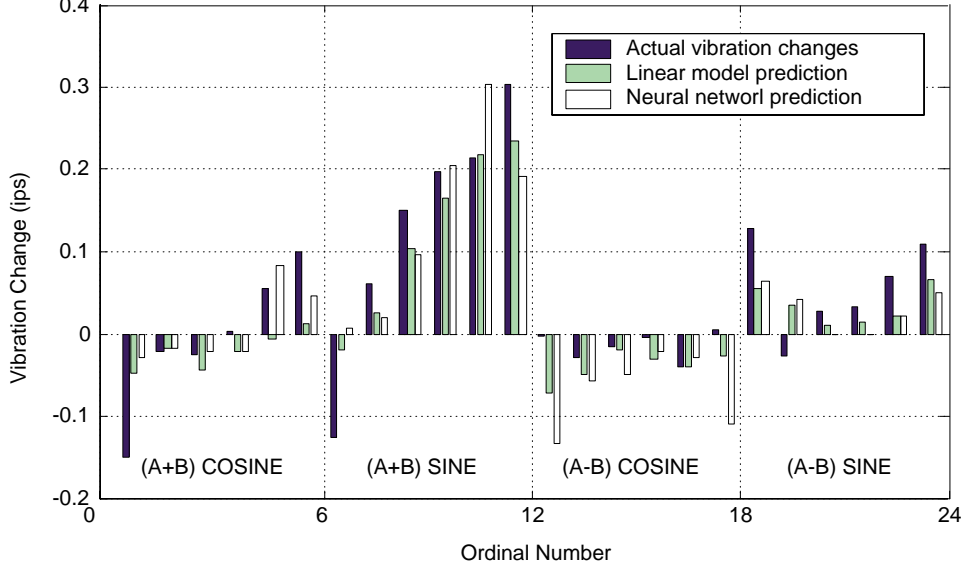


Figure 7: The predicted vibration changes by the linear and neural network models compared with the actual vibration changes.

### Probability analysis

Having identified the probability distribution of the prediction error as Gaussian, significantly facilitates its estimation by narrowing the distribution parameters to the mean ( $\mu$ ) and variance ( $\sigma^2$ ) values, which can be empirically estimated as

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N \hat{e}_i \quad (21)$$

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^N (\hat{e}_i - \hat{\mu})^2 \quad (22)$$

The estimation results indicate that the mean values of all of the vibration components are approximately zero, as anticipated from our previous assessment. The variance values are included in Table 2.

In addition to the mean and variance values, the covariance matrix between the 24 components of the vibration is necessary for defining the probability distribution of the prediction error. The computed covariance matrix indicates that the off-diagonal elements are mostly non-zero and that the elements closer to the diagonal are larger than others, indicating a stronger correlation between the vibration of consecutive flight regimes. As an example, the cross-correlation between the prediction errors of the (A+B)cosine components at 80 knots and 120 knots is shown in Fig. 9, which indicates the close correlation between the two errors. While the results indicate similarly strong correlation between the sine components, there is little correlation between the prediction errors of cosine and sine components. Other observations are that the prediction errors of (A+B)cosine and (A-B)cosine are uncorrelated and that those of ground vibration are uncorrelated with the other five flight regimes.

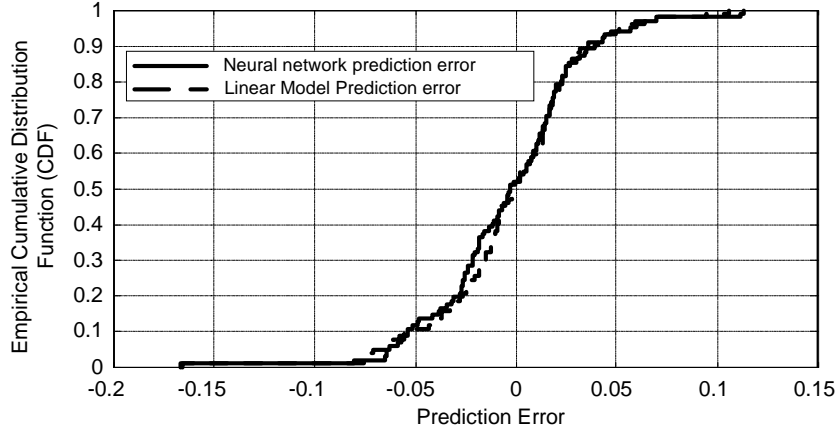


Figure 8: Illustration of the closeness to the normal distribution of the prediction errors of the linear and neural network models.

Table 2: Estimated variance values of the 24 vibration components.

Flight Regime	$\hat{\sigma}$			
	A+B		A-B	
	cos	sin	cos	sin
<i>fpm</i>	0.0826	0.0943	0.0989	0.0873
<i>hov</i>	0.0380	0.0333	0.0506	0.0740
<i>80</i>	0.0558	0.0657	0.0388	0.0527
<i>120</i>	0.0761	0.0831	0.0489	0.0630
<i>145</i>	0.0993	0.1019	0.0521	0.0699
<i>vh</i>	0.1105	0.1086	0.0686	0.0916

As discussed earlier, the probability distribution of the prediction error is identical to that of the measured vibration. So, for the 24-component vector of measured vibration

$$\mathbf{V}(k) = [V_{c1}(k), V_{s1}(k), \dots, V_{c12}(k), V_{s12}(k)]^T \quad (23)$$

the joint probability density function of the measured vibration for the  $k$ th flight,  $\mathbf{V}(k)$ , can be characterized as an N-dimensional Gaussian function

$$p(\mathbf{V}(k)) = \frac{1}{(2\pi)^{\frac{N}{2}} |\mathbf{\Phi}|^{\frac{1}{2}}} \exp\left[-\frac{1}{2} \hat{\mathbf{e}}(k)^T \mathbf{\Phi}^{-1} \hat{\mathbf{e}}(k)\right] \quad (24)$$

$$\hat{\mathbf{e}}(k) = \mathbf{V}(k) - \mathbf{V}(k-1) - C\Delta\mathbf{x}(k) \quad (25)$$

where  $\mathbf{\Phi}$  represents the covariance matrix of the prediction error. Now, if  $\Gamma = \{|\mathbf{V}_j| = \sqrt{V_{c_j}^2 + V_{s_j}^2} \leq \alpha, j = 1, \dots, 12\}$  denotes the specification region in 24-dimensional Euclidean space, the blade adjustments  $\Delta\mathbf{x}$  need to be selected such that the probability that the measured vibration is within the acceptable range is maximized. Formally,

$$\arg_{\Delta\mathbf{x}} \max \left[ Pr(\mathbf{V}(k) \in \Gamma) = \int_{\Gamma} p(\mathbf{V}(k)) d\mathbf{V}(k) \right] \quad (26)$$

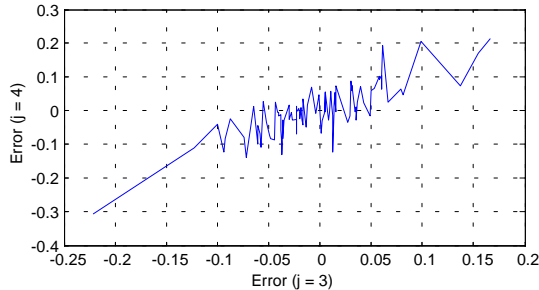


Figure 9: Correlation between the prediction errors of (A+B)cosine components at 80 and 120 knots.

### Implementation

Ideally, the performance of the proposed method should be evaluated ‘side by side’ against that of the traditional method. However, such an evaluation would require tuning the aircraft with one method, undoing changes, and tuning the aircraft with another. Since such testing is prohibitively costly, a compromise approach of evaluating the method in simulation is utilized. A process simulation model is therefore used to represent the “Helicopter” in Fig. 2, with the probability-based model, consisting of the deterministic and probability models, representing the “Process Model” in this figure.

#### Simulation model

The neural network models were used for simulation to take advantage of their better representation capacity. In addition, random numbers were added to the outputs of the networks to represent the noise in the vibration measurements. These random numbers were generated according to the Gaussian distribution  $\mathcal{N}(\mu_m, \sigma_m^2)$ , where  $\mu_m$  and  $\sigma_m^2$  were estimated from the mean and variance of the prediction error as  $\hat{\mu}_m = 0$  and  $\hat{\sigma}_m^2 = \hat{\sigma}^2/2$ .

#### Selection of blade adjustments

The blade adjustments entail the solution to Eq. (26), which is a nonlinear optimization problem. Solving this problem requires that the integral  $\int_{\Gamma} p(\mathbf{V}(k))d\mathbf{V}(k)$  be computed for the candidate adjustments. Among the different numerical methods to compute N-dimensional integrals (Ref. [?, ?, ?, ?, ?, ?]), Monte Carlo or adaptive sub-region approaches can be used to cope with the ‘curse of dimensionality’. But even these methods require considerable computation time for integrals with more than ten variables. To reduce the computation time, the search space for the blade adjustments was narrowed to

$$\arg_{\Delta \mathbf{x}} \max \left[ -\hat{\mathbf{V}}(k)^T \Phi \hat{\mathbf{V}}(k) \right] \quad (27)$$

as an approximation to Eq. (26). The  $\hat{\mathbf{V}}(k) = \mathbf{V}(k-1) + C\Delta \mathbf{x}(k)$  in the above equation represents the predicted vibration of the  $k$ th flight using the linearly approximated change in vibration. The closeness of Eqs. (26) and (27) can be examined by comparing the contours of equal values of  $-\hat{\mathbf{V}}(k)^T \Phi \hat{\mathbf{V}}(k)$  and  $Pr(\mathbf{V}(k) \in \Gamma)$ . Such contours are shown in Fig. 10 for a two-dimensional case. The results indicate that the contours are very similar, implying the closeness of the approximate solution of Eq. (27) to that of Eq. (26). The candidate blade adjustments were, therefore sought

around the solution to Eq. (27) and their likelihood values were estimated by approximating the integral  $\int_{\Gamma} p(\mathbf{V}(k))d\mathbf{V}(k)$  with a summation. The vibration components resulting from each set of blade adjustments were simulated 100 to 200 times and the probability of success was approximated by the average number of times the simulated vibration satisfied the specifications. The set of blade adjustments rendering the maximum probability of success was then selected as the recommended adjustment set.

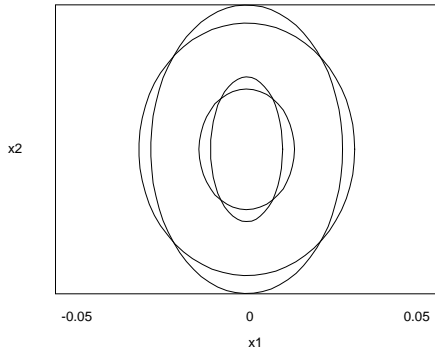


Figure 10: The equal value contours of approximation to  $Pr(\mathbf{V}(k) \in \Gamma)$  for a two dimensional case.

### Performance Evaluation

The proposed method was tested with actual data from 39 UH-60 helicopters. These datasets were generated from test flights performed by Sikorsky during the production acceptance process. For each helicopter, the method was applied iteratively until the simulated vibrations were within their specifications, or an upper limit of 5 process iterations had been reached.

Due to the stochastic nature of the vibration measurements, the track and balance process cannot be evaluated by deterministic measures. Several performance measures have therefore been devised to account for the uncertainty of the process. One such measure that assesses tuning efficiency is the *Average Tuning Iteration Number* (ATIN) which represents the average number of adjustment iterations taken for each helicopter to bring vibration within the acceptable level of 0.2 ips. The number of flights indicated by the method for the 39 helicopters is included in Table 3 along with those actually performed during the production acceptance process. The results reveal that the proposed method indicates a smaller ATIN relative to that actually performed. This result, although not conclusive due to the nonuniformity of the evaluation formats (simulation vs. real aircraft), lends credence to the efficiency of the proposed method.

A fundamental issue to be studied is the impact of the probability component of the model on tuning efficiency, as represented by ATIN. The significance of the probability component in the performance of the method can be analyzed by removing it from the model and using only the deterministic component for blade selection. For this, the blade adjustments can be found by minimizing the distance of the predicted vibration to the center of the specification region (see Fig 4). The results from this deterministic component, obtained by minimizing the mean square value of the predicted vibration, are listed in Table 4 along with the results from the probability-based model in Table 3. The results indicate that the deterministic component alone is not as

Table 3: The number of tuning iterations indicated by the proposed method and those actually applied during production acceptance (Actual versus Probability-based).

Helicopter # (39)	Tuning iteration Number	
	Actual	Probability-based
176	1	1
179	3	2
180	1	1
184	2	1
⋮	⋮	⋮
260	4	2
⋮	⋮	⋮
861	1	1
Total	71	46
ATIN	1.82	1.18

effective in tuning and that accounting for the non-uniformity of the probability distributions of the vibration components is important in blade selection.

Table 4: The number of tuning iterations indicated by the proposed method and those actually applied during production acceptance (Probability-based versus Deterministic).

Helicopter # (39)	Tuning iteration Number	
	Probability-based	Deterministic
176	1	1
179	2	2
180	1	2
184	1	1
⋮	⋮	⋮
260	2	1
⋮	⋮	⋮
861	1	1
Total	46	57
ATIN	1.18	1.46

A preferred aspect of a system of track and balance is its ability to tune the aircraft in one adjustment iteration. This aspect of the method was evaluated by checking the number of helicopters tuned in one iteration. For these results, in order to eliminate the difference between the simulation model and the helicopter, only the vibration estimates from simulation were used to evaluate the suitability of the adjustments. The results of this evaluation are shown in Table 5 where the helicopters tuned in one iteration are shown by a  $\checkmark$  and those requiring more than one iteration are denoted by  $\times$ . The results indicate that the proposed method satisfies this more stringent criterion better than the actual adjustments. This further validates the claim that the proposed method benefits from a more effective search engine.

Table 5: Tally of helicopters tuned in one iteration.

Helicopter # (39)	Vibration of the second flight	
	Actual	Proposed Method
176	✓	✓
178	×	✓
179	✓	✓
⋮	⋮	⋮
822	×	✓
858	×	✓
859	✓	✓
861	✓	✓
Total	19	33

Due to the randomness of the vibration measurements, repeated application of an adjustment set may lead to slightly different vibration measurements. This, in turn, may cause a variance in the number of iterations produced by adjustments when the resulting vibration is close to the specified threshold. It would be beneficial, therefore, to devise a measure for the probability of success of the adjustments. The empirical measure, Acceptability Index ( $AI$ ), is defined as

$$AI = \frac{1}{M} \sum_{l=1}^M s_l \quad (28)$$

to denote the percentage of times an adjustment set will result in the vibration satisfying the specification. In the above equation,  $M$  represents the total number of flights simulated to represent the repeated applications of the same adjustment set, and

$$s_l = \begin{cases} 1 & \text{if vibration of the } l\text{th simulation flight} \\ & \text{is acceptable} \\ 0 & \text{if vibration of the } l\text{th simulation flight} \\ & \text{is unacceptable} \end{cases}$$

The Acceptability Index ( $AI$ ) computed for both the actual and selected adjustments at the first iteration are included in Table 6. The results indicate that the proposed method provides adjustments with a higher probability of success as judged by the acceptability of vibration estimates from the simulation model. These results, which indicate that the selected adjustments from the proposed method can more consistently tune the helicopter in one iteration, imply the better positioning of the adjustments within the feasible region.

Another evaluation basis for the adjustments can be established by comparing them to the actual cumulative adjustments performed during production acceptance. The cumulative adjustment set,  $\Sigma x$ , can be defined as

$$\Sigma x = \sum_{k=1}^N \Delta x_k \quad (29)$$

where  $N$  represents the total number of tuning iterations performed during production acceptance for each helicopter and  $\Delta x_k$  denotes the adjustments applied at the  $k$ th iteration. A sample of

Table 6: The value of Acceptability Index ( $AI$ ) computed for both the actual and proposed adjustments at the first flight.

Helicopter # (39)	Acceptability Index	
	Actual	Proposed Method
176	0.92	0.94
179	0.61	0.80
⋮	⋮	⋮
260	0.40	0.74
261	0.09	0.86
263	0.18	0.13
⋮	⋮	⋮
822	0.0	0.28
857	0.62	0.62
858	0.64	0.94
859	0.94	0.96
861	0.96	0.98
Average	0.581	0.754

actual first iteration adjustments, actual cumulative adjustments, and first iteration adjustments from the proposed method is shown in Table 7. The results indicate that the adjustments from the proposed method are closer to the actual cumulative adjustments than are the actual first iteration adjustments. Although the cumulative adjustments may not be the most desirable ones for the aircraft, they represent an acceptable set that have been proven during the production acceptance process. The closeness of the proposed method’s solutions to the actual cumulative adjustments provides further evidence of the effectiveness of the search strategy.

### Conclusions

A method is introduced for helicopter track and balance that takes advantage of the probability distribution of vibration measurements to cope with their stochastics. In the proposed method, the underlying model in this method comprises two components: a deterministic component as well as a probabilistic one. The method relies on the probability model to estimate the likelihood of the measured vibration satisfying the specifications, and to select the set of blade adjustments with the maximum probability of producing acceptable vibration. The likelihood measures used for blade selection are computed according to the probability distribution of vibration derived from historical track and balance data. Several kinds of performance measures were also proposed to account for process uncertainty. The proposed method has been shown to improve the number of iterations used for track and balance based on these performance measures.

### Acknowledgements

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Table 7: Comparison of the first iteration solutions of the proposed method and actual solutions from Sikorsky’s production line with the cumulative acceptable adjustments.

Helicopter #	Adjustments											
	Proposed Method				Actual Cumulative				Actual First			
	First Iteration Solution				Adjustment Set ( $\sum x$ )				Iteration Solution			
	<i>PCR</i> <sub>13</sub>	<i>PCR</i> <sub>24</sub>	<i>TAB</i> <sub>13</sub>	<i>TAB</i> <sub>24</sub>	<i>PCR</i> <sub>13</sub>	<i>PCR</i> <sub>24</sub>	<i>TAB</i> <sub>13</sub>	<i>TAB</i> <sub>24</sub>	<i>PCR</i> <sub>13</sub>	<i>PCR</i> <sub>24</sub>	<i>TAB</i> <sub>13</sub>	<i>TAB</i> <sub>24</sub>
801	5	-4	-8	11	2	-4	-4	14	6	-4	-10	11
802	7	-1	-3	5	9	0	-10	10	5	2	0	0
822	6	-3	-21	5	10	-4	-23	13	6	0	-20	0
858	8	0	-15	3	9	-2	-10	3	7	0	-14	0