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<b>13. ABSTRACT (Maximum 200 words)</b> An approximate analytic theory was developed and implemented numerically for calculating the space charge limited current and electric field of a thin cylindrical beam or current sheet between two wide parallel electrodes. It was found that the average current density scaled as the reciprocal of the beam width when the latter becomes very small. The total cylindrical beam current thus decreases proportionally to its diameter while the total current of a sheet becomes almost independent of the width in this regime.				
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**FINAL REPORT**  
**AFOSR GRANT AF 49620-01-1-0154**  
**BASIC STUDIES IN PLASMA PHYSICS**  
**02/01/01 - 01/31/04**

by

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**1) Space charge limited flow of a thin electron beam confined by a strong magnetic field**

An approximate analytic theory is developed and implemented numerically for calculating the space charge limited current and electric field of a thin cylindrical beam or current sheet between two wide parallel electrodes. The flow is confined by a sufficiently strong magnetic field. Assuming that the potential and current density are almost homogeneous in the direction transversal to the flow we compute the beam current and profile by a variational method. We find that the average current density scales as the reciprocal of the beam width when the latter becomes very small. The total cylindrical beam current thus decreases proportionally to its diameter while the total current of a sheet becomes almost independent of the width in this regime.

**2) Note on a diffraction-amplification problem**

We investigate the solution of the equation  $\partial_t \mathcal{E}(x, t) - iD\partial_x^2 \mathcal{E}(x, t) = \lambda |S(x, t)|^2 \mathcal{E}(x, t)$ , for  $x$  in a circle and  $S(x, t)$  a Gaussian stochastic field with a covariance of a particular

form. It is shown that the coupling  $\lambda_c$  at which  $\langle |\mathcal{E}| \rangle$  diverges for  $t \geq 1$  (in suitable units), is always less or equal for  $\mathcal{D} > 0$  than  $\mathcal{D} = 0$ .

### 3) Behavior of Susceptible Infected Susceptible Epidemics on Heterogeneous Networks with Saturation

We investigate saturation effects in susceptible-infected-susceptible models of the spread of epidemics in heterogeneous populations. The structure of interactions in the population is represented by networks with connectivity distribution  $P(k)$ , including scale-free (SF) networks with power law distributions  $P(k) \sim k^{-\gamma}$ . Considering cases where the transmission of infection between nodes depends on their connectivity, we introduce a saturation function  $C(k)$  which reduces the infection transmission rate  $\lambda$  across an edge going from a node with high connectivity  $k$ . A mean-field approximation with the neglect of degree-degree correlation then leads to a finite threshold  $\lambda_c > 0$  for SF networks with  $2 < \gamma \leq 3$ . We also find, in this approximation, the fraction of infected individuals among those with degree  $k$  for  $\lambda$  close to  $\lambda_c$ . We investigate via computer simulation the contact process on a heterogeneous regular lattice and compare the results with those obtained from mean-field theory with and without neglect of degree-degree correlations.

### 4) Time Asymptotics of the Schrödinger Wave Function in Time-Periodic Potentials

We study the transition to the continuum of an initially bound quantum particle in  $\mathbb{R}^d$ ,  $d = 1, 2, 3$ , subjected, for  $t \geq 0$ , to a time periodic forcing of arbitrary magnitude. The analysis is carried out for compactly supported potentials, satisfying certain auxiliary conditions. It provides complete analytic information on the time Laplace transform of the wave function. From this, comprehensive time asymptotic properties (Borel summable transseries) follow.

We obtain in particular a criterion for whether the wave function gets fully delocalized (complete ionization). This criterion shows that complete ionization is generic and provides

a convenient test for particular cases. When satisfied it implies absence of discrete spectrum and resonances of the associated Floquet operator. As an illustration we show that the parametric harmonic perturbation of a potential chosen to be any nonzero multiple of the characteristic function of a measurable compact set has this property.

#### **5) Using Kinetic Monte Carlo Simulations to Study Phase Separation in Alloys**

We review recent extensions of the kinetic Ising model used to investigate phase separation in binary alloys. Firstly, vacancies are included to model the diffusion of the atoms on the microscopic scale more realistically. These can change the coarsening rate and the coarsening mechanism. Secondly, the lattice is allowed to deform owing to the different sizes of the atoms and the resulting misfit between precipitates and matrix. The deformability of the lattice induces long-range elastic interactions between the atoms. These change the shape, orientation and arrangement of the precipitates. The growth of the precipitates need not follow the law.

#### **6) The Asymmetric Exclusion Process and Brownian Excursions**

We consider the totally asymmetric exclusion process (TASEP) in one dimension in its maximal current phase. We show, by an exact calculation, that the non-Gaussian part of the fluctuations of density can be described in terms of the statistical properties of a Brownian excursion. Numerical simulations indicate that the description in terms of a Brownian excursion remains valid for more general one dimensional driven systems in their maximal current phase.

#### **7) Stability of Solutions of Hydrodynamic Equations Describing the Scaling Limit of a Massive Piston in an Ideal Gas**

We analyze the stability of stationary solutions of a singular Vlasov type hydrodynamic equation (HE). This equation was derived (under suitable assumptions) as the hydrodynamical scaling limit of the Hamiltonian evolution of a system consisting of a massive

piston immersed in an ideal gas of point particles in a box. We find explicit criteria for global stability as well as a class of solutions that are linearly unstable for a dense set of parameter values. We present some numerical evidence that when the mechanical system (with a large number of particles) has initial conditions 'close' to stationary stable solutions of the HE, then it stays close to these solutions for a long time. On the other hand, if the initial state of the particle system is close to an unstable stationary solution of the HE, then the mechanical system diverges rapidly from that solution and later appears to develop long lasting periodic oscillations. We find similar (approximately periodic) solutions of the HE that are linearly stable.

#### 8) On the (Boltzmann) Entropy of Nonequilibrium Systems

Boltzmann defined the entropy of a macroscopic system in a macrostate  $M$  as the log of the volume of phase space (number of microstates) corresponding to  $M$ . This agrees with the thermodynamic entropy of Clausius when  $M$  specifies the locally conserved quantities of a system in local thermal equilibrium (LTE). Here we discuss Boltzmann's entropy, involving an appropriate choice of macro-variables, for systems not in LTE. We generalize the formulas of Boltzmann for dilute gases and of Resibois for hard sphere fluids and show that for macro-variables satisfying any deterministic autonomous evolution equation arising from the microscopic dynamics the corresponding Boltzmann entropy must satisfy an  $\mathcal{H}$ -theorem.

#### 9) Fourier's Law for a Harmonic Crystal with Self-Consistent Stochastic Reservoirs

We consider a  $d$ -dimensional harmonic crystal in contact with a stochastic Langevin type heat bath at each site. The temperatures of the "exterior" left and right heat baths are at specified values  $T_L$  and  $T_R$ , respectively, while the temperatures of the "interior" baths are chosen self-consistently so that there is no average flux of energy between them and the

system in the steady state. We prove that this requirement uniquely fixes the temperatures and the self consistent system has a unique steady state. For the infinite system this state is one of local thermal equilibrium. The corresponding heat current satisfies Fourier's law with a finite positive thermal conductivity which can also be computed using the Green-Kubo formula. For the harmonic chain ( $d = 1$ ) the conductivity agrees with the expression obtained by Bolsterli, Rich and Visscher in 1970 who first studied this model. In the other limit,  $d \gg 1$ , the stationary infinite volume heat conductivity behaves as  $(\ell_d d)^{-1}$  where  $\ell_d$  is the coupling to the intermediate reservoirs. We also analyze the effect of having a non-uniform distribution of the heat bath couplings. These results are proven rigorously by controlling the behavior of the correlations in the thermodynamic limit.

#### 10) A Random Matrix Model of Relaxation

We consider a two level system,  $\mathcal{S}_\epsilon$ , coupled to a general  $n$  level system,  $\mathcal{S}_\setminus$ , via a random matrix. We derive an integral representation for the mean reduced density matrix  $\rho(t)$  of  $\mathcal{S}_\epsilon$  in the limit  $n \rightarrow \infty$ , and we identify a model of  $\mathcal{S}_\setminus$  which possesses some of the properties expected for macroscopic thermal reservoirs. In particular, it yields the Gibbs form for  $\rho(\infty)$ . We consider also an analog of the van Hove limit and obtain a master equation (Markov dynamics) for the evolution of  $\rho(t)$  on an appropriate time scale

#### 11) Absolute Continuity of Projected SRB Measures of Coupled Arnold Cat Map Lattices

We study a  $d$ -dimensional coupled map lattice consisting of hyperbolic toral automorphisms (Arnold cat maps) that are weakly coupled by an analytic map. We construct the Sinai-Ruelle-Bowen measure for this system and study its marginals on the tori. We prove that they are absolutely continuous with respect to Lebesgue measure if and only if the coupling satisfies a nondegeneracy condition.

#### 12) The Boltzmann Entropy for Dense Fluids Not in Local Equilibrium

Using computer simulations we investigate the time evolution of the (Boltzmann) entropy of a dense fluid not in local equilibrium. The macrovariables  $M$  describing the system are the (empirical) particle density  $f = \{f(\mathbf{x}, \mathbf{v})\}$  and the total energy  $E$ . We find that  $S(f_t, E)$  is monotone increasing in time even when its kinetic part is decreasing. We argue that for isolated Hamiltonian systems monotonicity of  $S(M_t) = S(M_{X_t})$  should hold generally for “typical” (the overwhelming majority of) initial microstates (phase-points)  $X_0$  belonging to the initial macrostate  $M_0$ , satisfying  $M_{X_0} = M_0$ . This is a consequence of Liouville’s theorem when  $M_t$  evolves according to an autonomous deterministic law.

### 13) Space-Charge-Limited 2 – d Electron Flow Between Two Flat Electrodes in a Strong Magnetic Field

An approximate analytic solution is constructed for the 2-d space charge limited emission by a cathode surrounded by non emitting conducting ledges of width  $\Lambda$ . An essentially exact solution (via conformal mapping) of the electrostatic problem in vacuum is matched to the solution of a linearized problem in the space charge region whose boundaries are sharp due to the presence of a strong magnetic field. The current density growth in a narrow interval near the edges of the cathode depends strongly on  $\Lambda$ . We obtain an empirical formula for the total current as a function of  $\Lambda$  which extends to more general cathode geometries.

### 14) Exact Large Deviation Functional of a Stationary Open Driven Diffusive System: The Asymmetric Exclusion Process

We consider the asymmetric exclusion process (ASEP) in one dimension on sites  $i = 1, \dots, N$ , in contact at sites  $i = 1$  and  $i = N$  with infinite particle reservoirs at densities  $\rho_a$  and  $\rho_b$ . As  $\rho_a$  and  $\rho_b$  are varied, the typical macroscopic steady state density profile  $\bar{\rho}(x)$ ,  $x \in [a, b]$ , obtained in the limit  $N = L(b - a) \rightarrow \infty$ , exhibits shocks and phase transitions. Here we derive an exact asymptotic expression for the probability of observing

an arbitrary macroscopic profile  $\rho(x)$ :  $P_N(\{\rho(x)\}) \sim \exp[-L\mathcal{F}_{[a,b]}(\{\rho(x)\}); \rho_a, \rho_b]$ , so that  $\mathcal{F}$  is the large deviation functional, a quantity similar to the free energy of equilibrium systems. We find, as in the symmetric, purely diffusive case  $q = 1$  (treated in an earlier work), that  $\mathcal{F}$  is in general a non-local functional of  $\rho(x)$ . Unlike the symmetric case, however, the asymmetric case exhibits ranges of the parameters for which  $\mathcal{F}(\{\rho(x)\})$  is not convex and others for which  $\mathcal{F}(\{\rho(x)\})$  has discontinuities in its second derivatives at  $\rho(x) = \bar{\rho}(x)$ ; the fluctuations near  $\bar{\rho}(x)$  are then non-Gaussian and cannot be calculated from the large deviation function.

### 15) Information Loss in Coarse Graining of Polymer Configurations via Contact Matrices

Contact matrices provide a coarse grained description of the configuration  $\omega$  of a linear chain (polymer or random walk) on  $\mathbb{Z}^n$ :  $C_{ij}(\omega) = 1$  when the distance between the positions of the  $i$ th and  $j$ th steps are less than or equal to some distance  $a$  and  $C_{ij}(\omega) = 0$  otherwise. We consider models in which polymers of length  $N$  have weights corresponding to simple and self-avoiding random walks, SRW and SAW, with  $a$  the minimal permissible distance. We prove that to leading order in  $N$ , the number of matrices equals the number of walks for SRW, but not for SAW. The coarse grained Shannon entropies for SRW agree with the fine grained ones for  $n \leq 2$ , but differs for  $n \geq 3$ .

### 16) Transition to the Continuum of a Particle in Time-Periodic Potentials

We present new results for the transition to the continuum of an initially bound quantum particle subject to a harmonic forcing. Using rigorous exponential asymptotics methods we obtain explicit expressions, as generalized Borel summable transseries, for the probability of localization in a specified spatial region at time  $t$ . The transition to the continuum occurs for general compactly supported potentials in one dimension and our results extend easily to higher dimensional systems with spherical symmetry. This of course implies the absence of discrete spectrum of the corresponding Floquet operator.

### 17) Hydrodynamics of Binary Fluid Phase Segregation

Starting with the Vlasov-Boltzmann equation for a binary fluid mixture, we derive an equation for the velocity field  $\mathbf{u}$  when the system is segregated into two phases (at low temperatures) with a sharp interface between them.  $\mathbf{u}$  satisfies the incompressible Navier-Stokes equations together with a jump boundary condition for the pressure across the interface which, in turn, moves with a velocity given by the normal component of  $\mathbf{u}$ . Numerical simulations of the Vlasov-Boltzmann equations for shear flows parallel and perpendicular to the interface in a phase segregated mixture support this analysis. We expect similar behavior in real fluid mixtures.

### 18) Free Energy Minimizers for a Two-Species Model with Segregation and Liquid-Vapor Transition

We study the coexistence of phases in a two-species model whose free energy is given by the scaling limit of a system with long range interactions (Kac potentials) that are attractive between particles of the same species and repulsive between different species.

### 19) Generation of Primordial Cosmological Perturbations from Statistical Mechanics Models

The initial conditions describing seed fluctuations for the formation of structure in standard cosmological models, i.e. the Harrison-Zeldovich distribution, have very characteristic "super-homogeneous" properties: they are statistically translation invariant and isotropic, and the variance of the mass fluctuations in a region of volume  $V$  grows more slowly than  $V$ . We discuss the geometrical construction of distributions of points in  $bfR^3$  with similar properties encountered in tiling and in statistical physics, e.g. the Gibbs distribution of a one-component system of charged particles in a uniform background [one-component plasma (OCP)]. Modifications of the OCP can produce equilibrium correlations of the kind assumed in the cosmological context. We then describe how such systems can

be used for the generation of initial conditions in gravitational N-body simulations

## 20) Exact Free Energy Functional for a Driven Diffusive Open Stationary Nonequilibrium System

We obtain the exact probability  $\exp[-L\mathcal{F}(\{\rho(x)\})]$  of finding a macroscopic density profile  $\rho(x)$  in the stationary nonequilibrium state of an open driven diffusive system, when the size of the system  $L \rightarrow \infty$ .  $\mathcal{F}$ , which plays the role of a nonequilibrium free energy, has a very different structure from that found in the purely diffusive case. As there,  $\mathcal{F}$  is nonlocal, but the shocks and dynamic phase transitions of the driven system are reflected in non-convexity of  $\mathcal{F}$ , in discontinuities in its second derivatives, and in non-Gaussian fluctuations in the steady state.

## 21) Decay Versus Survival of a Localized State Subjected to Harmonic Forcing: Exact Results

We investigate the survival probability of a localized 1D quantum particle subjected to a time-dependent potential of the form  $rU(x)\sin\omega t$  with  $U(x) = 2\delta(x-a)$  or  $U(x) = 2\delta(x-a) - 2\delta(x+a)$ . The particle is initially in a bound state produced by the binding potential  $-2\delta(x)$ : We prove that this probability goes to zero as  $t \rightarrow \infty$  for almost all values of  $r$ ,  $\omega$  and  $a$ . The decay is initially exponential followed by a  $t^{-3}$  law if  $\omega$  is not close to resonances and  $r$  is small; otherwise the exponential disappears and Fermi's golden rule fails. For exceptional sets of parameters  $r$ ,  $\omega$  and  $a$  the survival probability never decays to zero, corresponding to the Floquet operator having a bound state. We show similar behavior even in the absence of a binding potential: permitting a free particle to be trapped by harmonically oscillating delta function potential.

## 22) Dynamics of a Massive Piston in an Ideal Gas: Oscillatory Motion and Approach to Equilibrium

We study numerically and theoretically (on a heuristic level) the time evolution of a gas confined to a cube of size  $L^3$  divided into two parts by a piston with mass  $M_L \sim L^2$  which can only move in the  $x$ -direction. Starting with a uniform “double-peaked” (non Maxwellian) distribution of the gas and a stationary piston, we find that (a) after an initial quiescent period the system becomes unstable and the piston performs a damped oscillatory motion, and (b) there is a thermalization of the system leading to a Maxwellian distribution of the gas velocities. The time of the onset of the instability appears to grow like  $L \log L$  while the relaxation time to the Maxwellian grows like  $L^{7/2}$ .

### 23) Properties of Stationary Nonequilibrium States in the Thermostatted Periodic Lorentz Gas II: The Many Point Particles System

We study the stationary nonequilibrium states of  $N$ -point particles moving under the influence of an electric field  $\mathbf{E}$  among fixed obstacles (disk) in a two-dimensional torus. The total kinetic energy of the system is kept constant through a Gaussian thermostat that produces a velocity dependent mean field interaction between the particles. The current and the particle distribution functions are obtained numerically and compared for small  $|\mathbf{E}|$  with analytic solutions of a Boltzmann-type equation obtained by treating the collisions with the obstacles as random independent scatterings. The agreement is surprisingly good for both small and large  $N$ . The latter system in turn agrees with a self-consistent one-particle evolution expected to hold in the  $N \rightarrow \infty$  limit.

### 24) Large Deviations in Rarefied Quantum Gases

The probability of observing a large deviation (LD) in the number of particles in a region  $\Lambda$  in a dilute quantum gas contained in a much larger region  $V$  is shown to decay as  $\exp[-|\Lambda|\Delta F]$ , where  $|\Lambda|$  is the volume of  $\Lambda$  and  $\Delta F$  is the change in the appropriate free energy density, the same as in classical systems. However, in contrast with the classical case, where this formula holds at all temperatures and chemical potentials our proof is

restricted to rarefied gases, both for the typical and observed density, at least for Bose or Fermi systems. The case of Boltzmann statistics with a bounded repulsive potential can be treated at all temperatures and densities. Fermions on a lattice in any dimension, or in the continuum in one dimension, can be treated at all densities and temperatures if the interaction is small enough (depending on density and temperature), provided one assumes periodic boundary conditions.

### **25) Scaling Dynamics of a Massive Piston in a Cube Filled with Ideal Gas: Exact Results**

We continue the study of the time evolution of a system consisting of a piston in a cubical container of large size  $L$  filled with an ideal gas. The piston has mass  $M \sim L^2$  and undergoes elastic collisions with  $N \sim L^3$  gas particles of mass  $m$ . In a previous paper, Lebowitz, Piasecki and Sinai considered a scaling regime, with time and space scaled by  $L$ , in which they argued heuristically that the motion of the piston and the one particle distribution of the gas satisfy autonomous coupled differential equations. Here we state exact results and sketch proofs for this behavior.

### **26) Dynamics of a Massive Piston in an Ideal Gas**

We study a dynamical system consisting of a massive piston in a cubical container of large size  $L$  filled with an ideal gas. The piston has mass  $M \sim L^2$  and undergoes elastic collisions with  $N \sim L^3$  non-interacting gas particles of mass  $m = 1$ . We find that, under suitable initial conditions, there is, in the limit  $L \rightarrow \infty$ , a scaling regime with time and space scaled by  $L$ , in which the motion of the piston and the one particle distribution of the gas satisfy autonomous coupled equations (hydrodynamical equations), so that the mechanical trajectory of the piston converges, in probability, to the solution of the hydrodynamical equations for a certain period of time. We also discuss heuristically the dynamics of the system on longer intervals of time.

## 27) Large Deviation of the Density Profile in the Steady State of the Open Symmetric Simple Exclusion Process

We consider an open one dimensional lattice gas on sites  $i = 1, \dots, N$ , with particles jumping independently with rate 1 to neighboring interior empty sites, the *simple symmetric exclusion process*. The particle fluxes at the left and right boundaries, corresponding to exchanges with reservoirs at different chemical potentials, create a stationary nonequilibrium state (SNS) with a steady flux of particles through the system. The mean density profile in this state, which is linear, describes the typical behavior of a macroscopic system, i.e., this profile occurs with probability 1 when  $N \rightarrow \infty$ . The probability of microscopic configurations corresponding to some other profile  $\rho(x)$ ,  $x = i/N$ , has the asymptotic form  $\exp[-N\mathcal{F}(\{\rho\})]$ ;  $\mathcal{F}$  is the *large deviation functional*. In contrast to equilibrium systems, for which  $\mathcal{F}_{eq}(\{\rho\})$  is just the integral of the appropriately normalized local free energy density, the  $\mathcal{F}$  we find here for the nonequilibrium system is a nonlocal function of  $\rho$ . This gives rise to the long range correlations in the SNS predicted by fluctuating hydrodynamics and suggests similar non-local behavior of  $\mathcal{F}$  in general SNS, where the long range correlations have been observed experimentally.

## 28) Nonperturbative Analysis of a Model Quantum System under Time Periodic Forcing

We analyze the time evolution of a one-dimensional quantum system with an attractive delta function potential whose strength is subjected to a time periodic (zero mean) parametric variation  $\eta(t)$ . We show that for generic  $\eta(t)$ , which includes the sum of any finite number of harmonics, the system, started in a bound state, will get fully ionized as  $t \rightarrow \infty$  irrespective of the magnitude or frequency of  $\eta(t)$ . For the case  $\eta(t) = r \sin(\omega t)$  we find an explicit representation of the probability of ionization. There are, however, exceptional, very non-generic  $\eta(t)$ , that do not lead to full ionization. These include rather simple explicit periodic  $\eta(t)$  for which the system evolves to a nontrivial localized stationary

state related to eigenfunctions of the Floquet operator.

### 29) Microscopic Origins of Irreversible Macroscopic Behavior: An Overview

Time-asymmetric behavior as embodied in the second law of thermodynamics is observed in *individual macroscopic* systems. It can be understood as arising naturally from time-symmetric microscopic laws when account is taken of a) the great disparity between microscopic and macroscopic scales, b) initial conditions, and c) the fact that what we observe is “typical” behavior of real systems—not all imaginable ones. This is in accord with the ideas of Maxwell, Thomson, and Boltzmann and their natural quantum extensions. Common alternate explanations, such as those based on equating irreversible macroscopic behavior with the ergodic or mixing properties of probability distributions (ensembles) already present for chaotic dynamical systems having only a few degrees of freedom or on the impossibility of having a truly isolated system, are either unnecessary, misguided or misleading. Specific features of macroscopic evolution, such as the diffusion equation, do however depend on the dynamical instability (deterministic chaos) of trajectories of isolated macroscopic systems.

### 30) Precipitate Size Distribution in Alloys with and without Lattice Misfit

We investigate, via three-dimensional atomistic computer simulations, the influence of a lattice misfit between precipitated and matrix on the precipitate size distribution during phase separation in a binary alloy. The elastic interactions are modeled by springs connecting nearest neighbor atoms. The difference in size between the two species of atoms, specified by the misfit parameter  $\delta$ , leads to effective long range interactions. Performing simulations with five different values of  $\delta$  we find a broadening of the scaled precipitate size distribution for  $\delta > 0.5\%$  in comparison with the case of no misfit. Additionally, the size of the largest precipitate in the system increases with increasing misfit. We also observe a percolated network-like precipitate structure for a concentration of solute atoms as low

as  $c_A = 0.2$  at large misfit. We interpret these effects as due to an increased tendency of the precipitates to coalesce, which is caused by their non-spherical shape, their alignment along the  $\langle 100 \rangle$ -directions and their regular arrangement.

### **31) Microscopic Computer Simulations of Directional Coarsening in Face-centered Cubic Alloys**

We carried out Monte Carlo simulations of phase separation in a three-dimensional binary alloy with misfitting phases subjected to uniaxial stress. A lattice of cylindrical or plate-like precipitates is formed at the mesoscale, as observed in real alloys. The rate of precipitate growth is much slower than the conventional  $R(t) \sim t^{\frac{1}{3}}$  behavior in systems with no elastic misfit. Once a well-defined precipitate microstructure is formed, the reversal of external applied load has only a small effect.

### **32) On the Complete Ionization of a Periodically Perturbed Quantum System**

We analyze the time evolution of a one-dimensional quantum system with zero range potential under time periodic parametric perturbation of arbitrary strength and frequency. We show that the projection of the wave function on the bound state vanishes, i.e. the system gets fully ionized, as time grows indefinitely.

### **33) Bounded Fluctuations and Translation Symmetry Breaking: A Solvable Model**

The variance of the particle number (equivalently the total charge) in a domain of length  $\mathcal{L}$  of a one-component plasma (OCP) on a cylinder of circumference  $W$  at the reciprocal temperature  $\beta = 2$ , is shown to remain bounded as  $\mathcal{L} \rightarrow \infty$ . This exactly solvable system with average density  $\rho$  has a density profile which is periodic with period  $(\rho W)^{-1}$  along the axis of the infinitely long cylinder. This illustrates the connection between bounded variance and periodicity in (quasi) one-dimensional systems. When  $W \rightarrow \infty$  the system approaches the two-dimensional OCP and the variance in a domain

$\Lambda$  grows like its perimeter  $|\partial\Lambda|$ . In this limit, the system is translation invariant with rapid decay of correlations.

### 34) Bounded Fluctuations and Translation Symmetry Breaking in One-Dimensional Particle Systems

We present general results for one-dimensional systems of point charges (signed point measures) on the line with a translation invariant distribution  $\mu$  for which the variance of the total charge in an interval is uniformly bounded (instead of increasing with the interval length). When the charges are restricted to multiples of a common unit, and their average charge density does not vanish, then the boundedness of the variance implies translation-symmetry breaking — in the sense that there exists a function of the charge configuration that is nontrivially periodic under translations — and hence that  $\mu$  is not “mixing.” Analogous results are formulated also for one dimensional lattice systems under some constraints on the values of the charges at the lattice sites and their averages. The general results apply to one-dimensional Coulomb systems, and to certain spin chains, putting on common grounds different instances of symmetry breaking encountered there.

### 35) Exact Results for Ionization of a Model Atom

We investigate the ionization of a 1-d quantum system with a zero-range attractive potential at  $x = 0$  when it is subjected to a time dependent harmonic zero range potential forcing of frequency  $\omega$ , strength  $r$ , at position  $x = a$ . We show that the ionization probability goes to 1 as time  $t \rightarrow \infty$  for almost all values of  $r$ ,  $\omega$  and  $a$ . The initially exponential decay of the probability to stay in the bound state is followed at later time by a  $t^{-3}$  law if  $r$  is small and  $\omega$  is not close to resonances. When this is not true the separation between the exponential and power law decay regimes disappears and Fermi’s golden rule fails. There exist exceptional sets of parameters where the survival probability never decays to zero and complete ionization is possible.

### 36) Spatial Structure in Low Dimensions for Diffusion Limited Two-Particle Reactions

Consider the system of particles on  $\mathbb{Z}^d$  where particles are of two types,  $A$  and  $B$ , and execute simple random walks in continuous time. Particles do not interact with their own type, but when a type  $A$  particle meets a type  $B$  particle, both disappear. Initially, particles are assumed to be distributed according to homogeneous Poisson random fields, with equal intensities for the two types. This system serves as a model for the chemical reaction  $A + B \rightarrow inert$ . In our earlier work the densities of the two types of particles were shown to decay asymptotically like  $1/t^{d/4}$  for  $d < 4$  and  $1/t$  for  $d \geq 4$ , as  $t \rightarrow \infty$ . This change in behavior from low to high dimensions corresponds to a change in spatial structure. In  $d < 4$ , particle types segregate, with only one type present locally. After suitable rescaling, the process converges to a limit, with density given by a Gaussian process. In  $d > 4$ , both particle types are, at large times, present locally in concentrations not depending on the type, location or realization. In  $d = 4$ , both particle types are present locally, but with varying concentrations. Here, we analyze this behavior in  $d < 4$ ; the behavior for  $d \geq 4$  will be handled in a future work.

### 37) Thermodynamic Entropy Production Fluctuation in a Two Dimensional Shear Flow Model

We investigate fluctuations in the momentum flux across a surface perpendicular to the velocity gradient in a stationary shear flow maintained by either thermostated deterministic or by stochastic boundary conditions. In the deterministic system the Gallavotti-Cohen (GC) relation for the probability of large deviations, which holds for the phase space volume contraction giving the Gibbs ensemble entropy production, never seems to hold for the flux which gives the hydrodynamic entropy production. In the stochastic case the GC relation is found to hold for the total flux, as predicted by extensions of the GC theorem but not for the flux across part of the surface. The latter appear to satisfy a modified

GC relation. Similar results are obtained for the heat flux in a steady state produced by stochastic boundaries at different temperatures.

### **38) Evolution of a Model Quantum System under Time Periodic Forcing: Conditions for Complete Ionization**

We analyze the time evolution of a one-dimensional quantum system with an attractive delta function potential whose strength is subjected to a time periodic (zero mean) parametric variation  $\eta(t)$ . We show that for generic  $\eta(t)$ , which includes the sum of any finite number of harmonics, the system, started in a bound state will get fully ionized as  $t \rightarrow \infty$ . This is irrespective of the magnitude or frequency (resonant or not) of  $\eta(t)$ . There are however exceptional, very non-generic  $\eta(t)$ , that do not lead to full ionization, which include rather simple explicit periodic functions. For these  $\eta(t)$  the system evolves to a nontrivial localized stationary state which is related to eigenfunctions of the Floquet operator.

### **39) Free Energy Functional for Nonequilibrium Systems : An Exactly Solvable Case**

We consider the steady state of an open system in which there is a flux of matter between two reservoirs at different chemical potentials. For a large system of size  $N$ , the probability of any macroscopic density profile  $\rho(x)$  is  $\exp[-N\mathcal{F}(\{\rho\})]$ ;  $\mathcal{F}$  thus generalizes to nonequilibrium systems the notion of free energy density for equilibrium systems. Our exact expression for  $\mathcal{F}$  is a nonlocal functional of  $\rho$ , which yields the macroscopically long range correlations in the nonequilibrium steady state previously predicted by fluctuating hydrodynamics and observed experimentally.

## List of Publications

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