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13. ABSTRACT (Maximum 200 words)

We study the effects of static interqubit interactions on the accuracy of various quantum algorithms. Extensive numerical simulations show that their effect is stronger compared to external decoherence. Analytical approach based on Random Matrix Theory is developed. It gives universal law for fidelity decay induced by interqubit static interactions. This determines the time scale for reliable quantum computation in presence of realistic static imperfections and external decoherence. New polynomial algorithms are developed for simulation of complex dynamics in the regime of classical and quantum chaos, and Anderson metal-insulator transition. A generic quantum error correction method is developed; it allows to eliminate coherent effect of static errors. The theoretical results are confirmed by numerical computations with up to 28 qubits.

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FINAL PROGRESS REPORT

The project was directed towards the investigation of the effects of imperfections in a realistic quantum computer with many qubits. Such imperfections can be generated by decoherence induced by coupling with the external world and noisy gates, but also by static residual interqubit couplings in a completely isolated system. An isolated quantum computer is characterized by exponentially small energy level spacings between multi-qubit states. Unavoidably a residual interaction between qubits is present, and is much larger than this spacing even for computers of a few tens of qubits. The results obtained in Toulouse before the beginning of the grant showed that two-body interaction leads to emergence of quantum chaos, characterized by ergodic eigenstates of the quantum computer hardware if the strength of interaction excedes the quantum chaos border. In this regime each ideal quantum register state disintegrates quickly into exponentially many states. This process destroys the operability of the quantum computer after a characteristic time scale, without any coupling to the external world. It brings an additional source of errors besides imperfections during gate operations in time and external decoherence. The aim of the proposal was to study the effects of such imperfections on different quantum algorithms and to determine the accuracy of quantum computation in their presence. In parallel the project aimed to develop new quantum algorithms for simulation of complex dynamics and Anderson metal-insulator transition and test on them the effects of static imperfections with the help of extensive numerical simulations, with up to 24 qubits. Error-correcting codes adapted to static imperfections induced by residual couplings between qubits were supposed to be developed and tested during the project.

The main results of the project can be classified in four main areas.

I The conditions for emergence of quantum chaos, ergodicity and dynamical thermalization were determined for an isolated quantum computer composed of a two-dimensional lattice of qubits with static random energy shifts of individual qubit energies and residual short-range inter-qubit couplings (Ref [1]). It was shown that quantum chaos sets in for the strength of inter-qubit couplings being larger than the average static one-qubit energy shift divided by the number of qubits in the quantum computer. Above this border the eigenstates are described by thermodynamical Fermi-Dirac distribution. It was shown that the relaxation rate to this equilibrium distribution is proportional to the square of the coupling strength multiplied by number of qubits and divided by the average energy shift. The theory

was tested by numerical simulations with up to 24 qubits. Together with the results obtained by the group before the beginning of the project, this determines the global conditions for onset of quantum chaos in quantum computer hardware in presence of static imperfections.

II It is important to investigate how the effects described in part I influence the accuracy of quantum computation while performing a specific quantum algorithm. To this aim, we developed several new algorithms simulating complex classical and quantum evolution. These quantum algorithms simulate classical symplectic dynamics (e.g. generalized Arnold cat map) (Refs [3][4]), classical strange attractors (Ref [6]), quantum sawtooth map (Ref [2]), double-well map with chaos assisted tunneling (Ref [8]), dynamical localization (Ref [9]), kicked rotator model (Ref [10]), a dynamical model based on the wavelet transform (Ref [12]), Anderson metal-insulator transition (Ref [14]), quantum tent map (Ref [17]), electrons in magnetic field and kicked Harper model (Ref [20]). In all these algorithms, it was shown that the evolution of exponentially large state vectors can be simulated in a number of elementary gates quadratic or cubic in the number of qubits. However, if the extraction of information via quantum measurements is taken into account, then only quadratic speed up can be reached for the determination of critical point of the Anderson transition [14] and dynamical localization length [9]. At the same time, for classical chaotic dynamics, we argue that exponential gain can be obtained for certain characteristics of the Liouville density function [3,4,6]. The developed algorithms simulate rich nontrivial dynamics and show interesting behaviours already with 5-7 qubits. Thus they may serve as the most optimal test ground for the first generation of quantum computers with up to 10 qubits. We think that rigorous mathematical analysis of the efficiency of these algorithms is highly desirable since it may provide mathematical proofs that quantum computers exponentially overcome the classical ones.

III The effects of static imperfections were tested on the algorithms described in part II by analytical methods and numerical simulations with up to 28 qubits (see e.g. Ref [6]). We compared these effects with the effects of random errors in quantum gates and showed that static errors lead to much faster decay of fidelity (Refs [2], [12], [17], [21]). Indeed, we demonstrated that for random errors in quantum gates, the fidelity of quantum computation for a wide class of algorithms decays exponentially with the number of performed gates and the decay rate is given by the square of the noise strength (the numerical constant was also determined) (Refs [3],[6],[8],[10],[12],[15],[17]). The effects of static imperfections were investigated for different algorithms in Refs [2],[5],[7],[9],[12],[17],[20],[21]. In this case, the fidelity decay is given by a combination of exponential and gaussian decays with two time scales, one of them is given by the Fermi Golden Rule and is similar to the Thouless time in mesoscopic systems while the other one is the Heisenberg time scale proportional to the size of the Hilbert space of the quantum computer [17]. This decay law is obtained on the basis of the Random Matrix Theory and gives a *universal*

decrease of fidelity induced by static imperfections in a quantum computer simulating complex quantum dynamics. The theory has been confirmed in extensive numerical simulations where the scaled fidelity varied by 10 orders of magnitude. However, if the dynamics is integrable (e.g. inside the Kolmogorov-Arnold-Moser islands of stability in symplectic maps), then the Random Matrix Theory cannot be applied and the decay of fidelity is described by a more complicated law, which deserves further investigation. Also the stability of the Grover algorithm in presence of static imperfections should be analyzed separately. This was done in Ref [19] where it was shown the existence of regular and chaotic phases depending on the imperfection strength. The critical border between two phases drops polynomially with the number of qubits, being exponentially larger than the frequency of Grover oscillations. In the regular phase, the algorithm remains robust against imperfections, still showing quadratic speed up over classical computation. In the chaotic phase, the algorithm is completely destroyed. The ensemble of these investigations give a complete physical picture of the accuracy bounds for the effects of static imperfections on a large class of algorithms, including wavelet transform, arithmetical functions and quantum Fourier transform which are essential to the Shor algorithm. These studies also demonstrated that fidelity remains close to 1 on a time scale which depends polynomially on the imperfection strength and drops not faster than polynomially with the number of qubits. At the same time, we showed that the eigenstates of repetitive quantum algorithms (like quantum sawtooth, tent map, etc...) are exponentially sensitive to the static perturbation (Refs[5],[7],[20]).

IV To suppress effects of static imperfections, we developed two quantum error-correcting methods specifically adapted to them. One of them uses random SWAP operations between different qubits performed after certain number of gates in a quantum algorithm (e.g. Grover algorithm [21]). This allowed to increase the probability of the searched state by a factor close to 10. A more efficient method was developed and numerically tested in Ref.[22]. This generic error-correction method is capable of correcting coherent errors originated from static residual inter-qubit couplings in a quantum computer. It is based on a randomization of static imperfections in a many-qubit system by the repeated application of Pauli operators which change the computational basis. This Pauli random error-correction method eliminates coherent errors produced by static imperfections and increases parametrically the maximum time over which realistic quantum computations can be performed reliably. This method doesn't require redundancy so that all physical qubits involved can be used for logical purposes. The numerical tests were done for the quantum tent map algorithm with up to 10 qubits, and it was shown that the method gives improvement of the fidelity by two orders of magnitude.

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