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**DELOCALIZATION AND VIBRATION CONTROL OF NEARLY PERIODIC  
STRUCTURES USING PIEZOELECTRIC NETWORKS**

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Final Report (January 1, 2001 – June 30, 2004)

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## Objectives

The objective of this research is to achieve effective delocalization and vibration control of nearly periodic structures. The results of this investigation can be applied to vibration control of various Air Force systems, such as space structures, bladed-disk assemblies and satellite antennae. The new idea is to utilize the electromechanical coupling characteristics of piezoelectric materials and shunt circuits (a piezoelectric network configuration) to (a) form an electro-mechanical wave channel to eliminate localization by means of electromechanical coupling and energy redistribution, and (b) simultaneously, provide efficient passive or active-passive hybrid energy dissipation mechanisms for the purpose of vibration suppression.

## Summary of Efforts

- (a) In this investigation, novel piezoelectric networks have been developed to create additional coupling channels among the sub-structures and destroy vibration localization. It was found that with proper selection of circuitry elements, the level of vibration localization of the nearly periodic structure could be reduced.
- (b) Parametric studies were carried out to explore the underlining mechanism for the delocalization approach. The robustness of the proposed delocalization scheme was also examined, where analysis showed that circuitry imperfections have little effect on the system performance.
- (c) Through working with the Wright Patterson Research Lab, the non-dimensional generic study has been extended to investigate the feasibility of utilizing such a technology on turbo-machinery fan structures. It was shown that one can achieve the delocalization effect with very reasonable system parameters.
- (d) Multi-harmonic vibration suppression under external forcing scenario has been investigated. A new algorithm was synthesized to analytically decide the piezoelectric damped absorber parameters. It is shown that, with capacitance networking and electrical coupling, this approach can suppress all the spatial harmonics, which cannot be achieved by traditional piezoelectric shunts.

- (e) At the end of this research program, based on the findings and experience obtained in this research, several challenging issues that need to be addressed in future investigations for realization of the new idea have been identified.

### **Descriptions of Accomplishments/New Findings**

A basic piezoelectric network configuration utilized in this investigation is shown in Figure 1. In general, with piezoelectric patches mounted on or embedded in the substructures, an inductive circuit (LC shunt) can absorb significant amount of vibration energy from the substructure to which it is attached and store that portion of energy in the electrical form. While in most cases directly increasing the mechanical coupling between substructures is difficult to achieve due to various design limitations, one can easily introduce strong electrical coupling, such as connecting the LC shunts with capacitors as shown in Figures 1b and 1c, to create a new wave channel. With this coupled piezoelectric circuits design, the otherwise localized vibration energy in a nearly periodic structure can now be transferred into electrical energy, and this energy can freely propagate in the integrated system through the newly created electro-mechanical wave channel with strong electrical coupling. This idea stems from previous findings on multi-coupled nearly periodic structures, and will “destroy” the intrinsic mechanism for vibration localization.

For the purpose of illustration and without loss of generality, the periodic structure that we consider in this research is assumed to consist of  $N$  identical cantilever beams coupled by  $N$  springs (Figure 1a). Here we use coupling spring to emulate the coupling effects in generic periodic structures, such as the stiffness coupling due to disk in turbine bladed-disks. Let  $\phi$  be the first local beam mode without the piezoelectric shunt circuit. The transversal displacement of the  $j$ -th beam is approximated as

$$w_j(x,t) \approx \phi(x)q_j(t) \quad (1)$$

The equations of motion for the periodic structure integrated with piezoelectric patches can be derived using Hamilton's principle, which are

$$\begin{aligned} m\ddot{q}_j + g\dot{q}_j + kq_j + k_1Q_j + k_c(q_j - q_{j-1}) + k_c(q_j - q_{j+1}) &= f_j \\ k_2Q_j + k_1q_j &= V_{oj} \end{aligned} \quad (2a,b)$$

where  $m$ ,  $g$ ,  $k$ ,  $k_c$ ,  $f_j$ ,  $k_1$  and  $k_2$  are, respectively, mass, damping, substructure stiffness, coupling stiffness, external force, electro-mechanical coupling coefficient, and inverse of the capacitance of piezoelectric patch, which are given by,

$$\begin{aligned} m &= \int_0^l \rho_b A_b \phi^2 dx + \int_0^l \rho_p A_p \phi^2 \Delta H dx, & g &= \int_0^l c_b \phi^2 dx \\ k &= \int_0^l E_b I_b \phi''^2 dx + \int_0^l E_p I_p \phi''^2 \Delta H dx, & k_c &= k_s \phi^2(x_s) \\ k_1 &= \frac{1}{w_p l_p} \int_0^l F_p h_{31} \phi'' \Delta H dx, & k_2 &= \frac{A_p \beta_{33}}{w_p^2 l_p} \end{aligned}$$

Here  $\Delta H = H(x - x_j) - H(x - x_r)$  and  $H(x)$  is the Heaviside step function.  $Q_j$  and  $V_{oj}$  are the charge flowing to and the voltage across the piezoelectric patch attached to the  $j$ -th beam, respectively.

Observing Figure 1c, from Kirchhoff's current law, we have

$$\begin{aligned} Q_a &= Q_j - Q_{j+1} \\ Q_b &= Q_{j-1} - Q_j \end{aligned}$$

which, in virtue of the voltage law, lead to

$$V_{oj} = -L\ddot{Q}_j - k_a(Q_j - Q_{j-1}) + k_a(Q_{j-1} - Q_j) \quad (3)$$

Hereafter we let  $k_a = 1/C_a$  be the inverse of the coupling capacitance. Substituting (3) into (2b), we have the equations of motion of the electro-mechanically integrated system as

$$\begin{aligned} m\ddot{q}_j + g\dot{q}_j + kq_j + k_1Q_j + k_c(q_j - q_{j-1}) + k_c(q_j - q_{j+1}) &= f_j \\ L\ddot{Q}_j + k_2Q_j + k_1q_j + k_a(Q_j - Q_{j-1}) + k_a(Q_j - Q_{j+1}) &= 0 \end{aligned} \quad (4a,b)$$

For the original periodic structure without the piezoelectric circuits, the equation of motion is

$$m\ddot{q}_j + g\dot{q}_j + kq_j + k_c(q_j - q_{j-1}) + k_c(q_j - q_{j+1}) = f_j \quad (5)$$

In the above derivation, we assume that the system is perfectly periodic. In reality, however, there will be imperfections. Without loss of generality, we assume that the structural imperfection (or called mistuning) only exists in the substructure mechanical stiffness, which is the common practice in localization study. The stiffness of the  $j$ -th beam with mistuning then is

$$\tilde{k}_j = k + \Delta k_j \quad (6)$$

where  $k$  is the nominal stiffness of the perfectly periodic structure, and  $\Delta k_j$  is the zero-mean random mistuning. In the following, we consider free vibration and neglect damping, and seek harmonic solutions. After nondimensionalization, we have the equations of the system with and without the piezoelectric circuits as, respectively,

$$\begin{aligned} -\Omega^2 x_j + (1 + \Delta s_j)x_j + R_c^2(x_j - x_{j-1}) + R_c^2(x_j - x_{j+1}) + \delta \xi y_j &= 0 \\ -\Omega^2 y_j + \delta^2 y_j + R_a^2(y_j - y_{j-1}) + R_a^2(y_j - y_{j+1}) + \delta \xi x_j &= 0 \end{aligned} \quad (7a,b)$$

and

$$-\Omega^2 x_j + (1 + \Delta s_j) x_j + R_c^2 (x_j - x_{j-1}) + R_c^2 (x_j - x_{j+1}) = 0 \quad (8)$$

where,

$$\begin{aligned} \omega_m &= \sqrt{k/m}, \quad \omega_e = \sqrt{k_2/L}, \quad \delta = \omega_e / \omega_m \\ \Omega &= \omega / \omega_m, \quad x_j = \sqrt{m} q_j, \quad y_j = \sqrt{L} Q_j \\ \xi &= k_1 / \sqrt{kk_2}, \quad R_c = \sqrt{k_c/k}, \quad R_a = \sqrt{k_a/k_2}, \quad \Delta s_j = \Delta k_j / k \end{aligned}$$

Here  $\omega$  and  $\Omega$  are the dimensional and nondimensionalized harmonic frequencies,  $\omega_m$  and  $\omega_e$  are the natural frequencies of uncoupled substructure and circuit, respectively;  $\delta$  is the frequency tuning ratio which characterizes the circuitry inductance;  $\xi$  is the generalized electro-mechanical coupling coefficient which reflects the energy transfer capability of the piezoelectric patch;  $R_c$  is the mechanical coupling ratio between the substructures,  $R_a$  is the electrical coupling ratio which is related to the coupling capacitance, and  $\Delta s_j$  is the mistuning ratio which is a zero-mean random number with standard deviation  $\sigma$ . All the analyses in this research are performed in nondimensionalized manner.

One can solve for the vibration modes of the original structure from Equation (8). Here, as an example, we consider a periodic structure where the number of substructures  $N = 80$ , and the coupling coefficient  $R_c^2 = 0.0025$ . With  $\sigma = 0.0025$ , the vibration localization phenomenon in the three modes (the second, 12<sup>th</sup> and 32<sup>nd</sup> modes) shown in Figure 2(a) is clearly significant. However, as we apply the tuned network treatment, the three equivalent modes in the integrated electro-mechanical system (solve Equations 7a and 7b) are no longer localized (Figure 2(b)).

To gain more insight of the proposed concept, a series of analyses and syntheses were performed. Parametric studies were carried out to explore the underlining mechanism

for the delocalization approach. Throughout the study, the Lyapunov exponents were used to quantify the localization level. Design guidelines were also established through utilizing an approximate solution of the Lyapunov exponent. The localization factors of the original structure and the electro-mechanically integrated system were compared. It was found that with proper selection of circuitry elements, the localization factor of the nearly periodic structure can be significantly reduced. For example, for the cases where  $\sigma = 0.0025$  and  $\sigma = 0.01$ , the maximum values of the localization factors in the pass-band frequency ranges are 0.0804 and 0.1522, respectively, under the tuning of  $\delta = 0.5$  and  $R_a = 1.5$ . Compared these numbers to the results without the tuned circuit where the minimum values of the localization factors in the pass-band frequency ranges are 0.3291 and 0.9373, respectively, it is clear that vibration localization can be significantly reduced with the network treatment.

To examine the robustness of the proposed scheme, the positive Lyapunov exponents of the integrated system with mistuning and circuitry imperfections were derived (see Figure 3). In these simulations, we assumed that the standard deviations of the  $\delta$  and  $R_a$  variations were both 0.01, which is significant compared to the assumed standard deviation of the structural stiffness  $\sigma = 0.0025$  and  $\sigma = 0.01$ . As illustrated in Figure 3, the circuitry imperfections have very little effect on the localization factors. This is not a surprise, because compared to the electrical coupling  $R_a^2$  the circuitry imperfections are several orders of magnitude smaller and therefore they will cause very little reflection in the newly-created electro-mechanical wave channel. For the cases where  $\sigma = 0.0025$  and  $\sigma = 0.01$ , the maximum values of the localization factors in the pass-band frequency ranges with circuitry imperfections are 0.0830 and 0.1552, respectively, which are only slightly larger than the localization factors of the system without circuitry imperfections. Therefore, we can conclude that the proposed scheme can tolerate significant amount of implementation imperfections and achieve satisfactory result.

Through working with the Wright Patterson Air Force Lab researchers, the nondimensional study results have been extended to evaluate the feasibility of utilizing such a treatment on turbomachinery fan structures. A wide beam model, matching the first bending natural frequency of a CRFER compressor fan blade (Figure 4), has been developed. Through dimensionalizing the optimal non-dimensional piezoelectric network parameters, one can derive the physical values of the required design variables. It was concluded that the optimal design can be achieved with piezoelectric patches that are 4% (thickness) and 10% (length) of the fan blade, which makes the application very feasible, in terms of size and space.

In addition to delocalization, the vibration suppression capability of the piezoelectric network for forced excitation scenarios has been examined. For such cases, resistors, in series with the inductors, are added to the individual branches of the network. It has been well known that piezoelectric shunts with properly tuned inductance and resistance can achieve the damped dynamic absorber effect. However, due to mechanical coupling among the substructures, the classical configuration and approach with identical passive piezoelectric shunts can only optimally control one spatial harmonic, and it is ineffective for all other spatial harmonic excitations. The proposed approach used the network capacitance to compensate for the mechanical coupling effect. That is, it can "retune" the piezoelectric absorbers/dampers to be optimal for *all* spatial harmonics, thereby significantly outperforms the classical approach (see Figure 5). The piezoelectric networks on all substructures have the same parameters, which will lead to convenience in practical implementations.

### **Summary of Technical Challenges and Future Directions**

While the recent investigations have illustrated promising delocalization and vibration control results through utilizing piezoelectric networks, they also illustrated the technical barriers that need to be overcome before the technology can be fully realized. Based on our experience, we have identified the new research challenges during the last phase of this program, which are summarized as follows:

(a) The recent investigations [Zhang and Wang, 2002; Tang and Wang, 2003] have shown that the effectiveness of the treatment is limited by the level of electromechanical coupling of the piezoelectric patches. From observing Figure 1, one can see that since the electrical coupling established through the external capacitors virtually has no limits, the bottleneck of the piezoelectric network is the electromechanical coupling, i.e. how much mechanical energy can be transferred into electrical energy. For most of the cases investigated in previous studies, the electromechanical coupling coefficient is not high enough to ensure that all the modes be significantly delocalized. In other words, while the degrees of localization of the modes were reduced with the treatment, the improvements in some modes were marginal. To examine the effect of the electromechanical coupling coefficient on the level of vibration localization of the system, an illustration is shown in Figure 6. From this figure, it is obvious that with the increase of the electromechanical coupling coefficient ( $\xi$ ), the localization index becomes smaller, indicating that the modes are less localized. The localization index is reduced from 0.9 (corresponding to  $\xi=0.1$ ) to about 0.068 (corresponding to  $\xi=0.8$ ). Physically, it means that much more mechanical energy can be transferred into electrical energy and distributed throughout the system when electromechanical coupling is increased [Zhang and Wang, 2003]. However, this coupling coefficient is in general difficult to change with only passive designs. An innovative method is needed to increase the coupling coefficient so that the mechanical energy can be better redistributed and delocalized. A possible scheme is to utilize the negative capacitance concept [Tang and Wang, 2001], but more detailed study is definitely required to synthesize such an approach.

(b) The study to date has mainly focused on the delocalization effects using piezoelectric networking. Although by adding damping actions [Zhang and Wang, 2002], we have also used the network to suppress vibration, a comprehensive study and good insight of *simultaneous* delocalization/ vibration suppression effects using piezoelectric networking has yet to be performed. In fact, it is clear that the circuit

and active controller design for vibration suppression of an idea structure will not always be effective when the system is mistuned [Zhang and Wang, 2002]. That is, the two issues (*vibration delocalization* and *vibration suppression*) could have contradicting requirements. More and better understanding of the tradeoffs between the two issues and a concurrent methodology to balance the two are needed to achieve a truly effective network.

(c) Most of the previous investigations and the feasibility study on periodic structure control have focused on *simple* structures, mainly with single mode, single circuit, and single piezoelectric patch per substructure. To extend the findings to realistic systems, one has to consider the fact that for complex structures, each substructure could have multiple dominant modes, multiple actuator patches and multiple circuits. Such configurations will create new wave channels that could be even more beneficial for energy redistribution and propagation (better delocalization). On the other hand, they will also create new research challenges to be addressed. One issue is that with such an arrangement, comprehensive but yet efficient analysis effort needs to be carried out for the understanding of the network. The other challenge will be that with multiple patches and circuits, the network topology needs to be reconfigured. The interconnection among the local circuits should be synthesized in a way such that we can utilize more wave channels for energy redistribution.

(d) The other major thrust that should be seriously addressed is to pursue experimental investigations. Through utilizing a recent DURIP grant from AFOSR, a comprehensive experimental test stand is being set up at Penn State. Through working with the Wright Patterson Air Force Lab, an eighteen blade, one-foot diameter test bladed-disk specimen has been selected, fabricated, and treated with piezoelectric actuators. In addition to the test specimen, the overall experimental system consists of an electro-magnetic shaker mounted to the hub of the blisk, a laser vibrometer for vibration measurement, a spectrum analyzer for data acquisition and analysis, and a controller for sensor positioning and mapping. This

comprehensive system will be fully utilized in future investigations that will provide validations to the analytical predictions and also generate new insight and ideas.

Based on the current study and the above arguments, we recognized that there is an excellent opportunity to build upon the successful research experience in this area at Penn State and advance the state of the art of piezoelectric networking for vibration control of mistuned periodic structures. In other words, in future research, we will address the aforementioned issues, and pursue new and important efforts that are related to our experience but fall outside the scope of the past programs.

#### **Personnel Supported**

Other than the PI, Dr. K. W. Wang, the project has involved two Ph.D. students (Jiong Tang and H. Yu) and one Post-Doc Fellow (Dr. Jianhua Zhang).

#### **Publications**

J. Tang and K. W. Wang, "Vibration Delocalization of Nearly Periodic Structures using Coupled Piezoelectric Networks," *ASME Journal of Vibration and Acoustics*, 125(1), pp. 95-108, 2003.

J. Zhang and K. W. Wang, "Electromechanical Tailoring of Piezoelectric Networks for Vibration Delocalization and Suppression of Nearly-Periodic Structures," *Proc. of International Conference on Adaptive Structure Technology*, Potsdam-Berlin, Germany, 2002.

J. Zhang, K. W. Wang, H. Yu, "Active Coupling Enhancement of Piezoelectric Networks for Vibration Delocalization of Nearly Periodic Structures," *Proc. SPIE Conf. on Smart Structures and Materials*, 5056-22, San Diego, CA, 2003.

#### **Interactions/Transitions**

This research is very relevant to AFOSR's mission, since the results can be applied to vibration control of various Air Force systems, such as space structures, satellite

antennae, and bladed-disk assemblies (e.g., fans) in gas turbine engines. Dr. K. W. Wang has visited Wright Patterson AFRL in June 2001, presented the progress on this project, and carried out technical discussions with Dr. Charles Cross at the Wright Patterson AFRL. Follow up discussions with Dr. Cross have also been pursued on issues regarding fan structure implements and experimental set ups. Dr. Wang has also visited Kirtland AFRL in August 2002, presented the progress on this project and carried out technical discussions with Dr. Lawrence Robertson and his team.

#### **Honors/Awards**

Dr. K. W. Wang is a Fellow of the ASME and the holder of the Diefenderfer Chair in Mechanical Engineering at Penn State. He is also the recipient of the 2004 Penn State Engineering Society Premier Research Award.

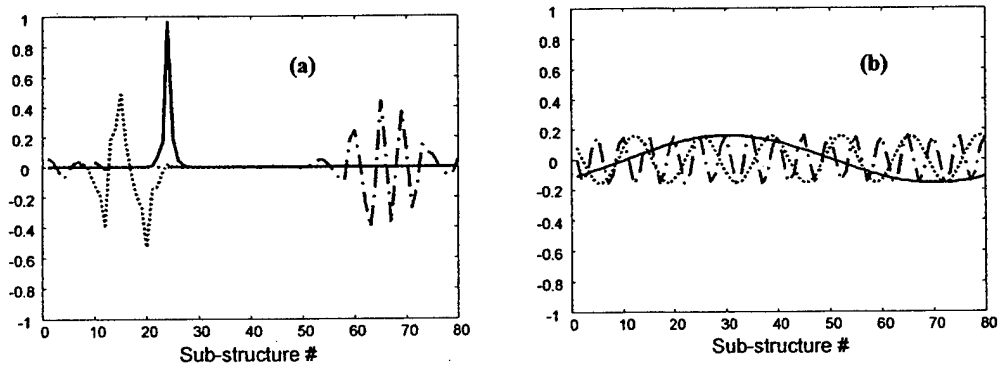
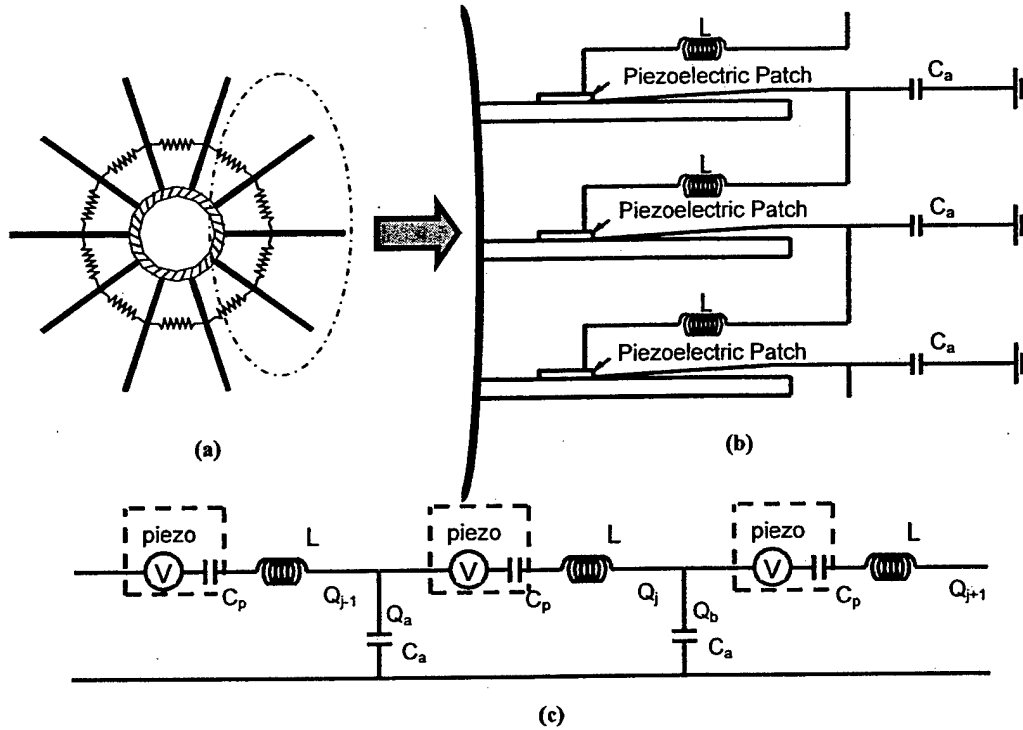


Figure 2. Vibration modes of nearly periodic structure, with miss tuning stiffness ratio  $\sigma = 0.0025$ ; (a) without circuit; (b) with circuit

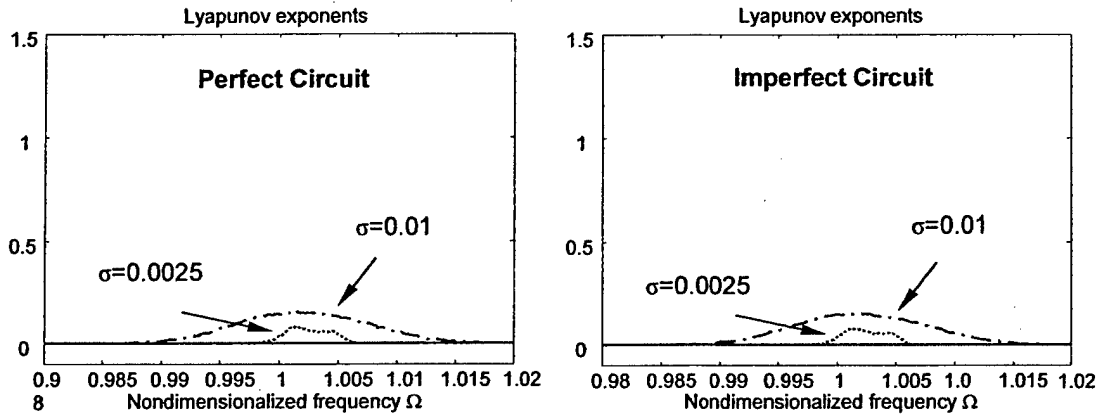


Figure 3. Localization factors of structure augmented with coupled piezoelectric circuits, with and without imperfections in circuitry elements

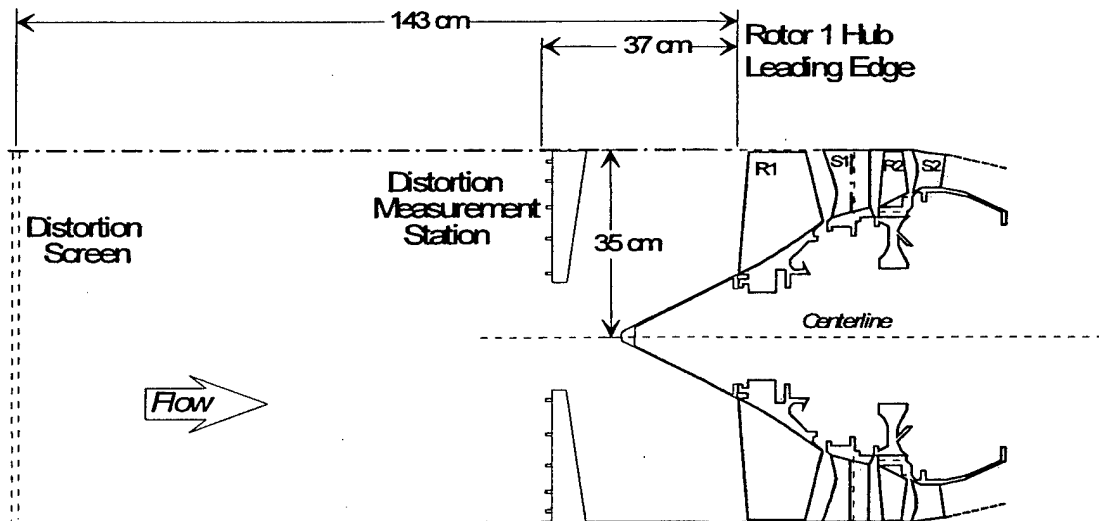


Figure 4. Schematic Cross Section of CRFER Compressor Configuration

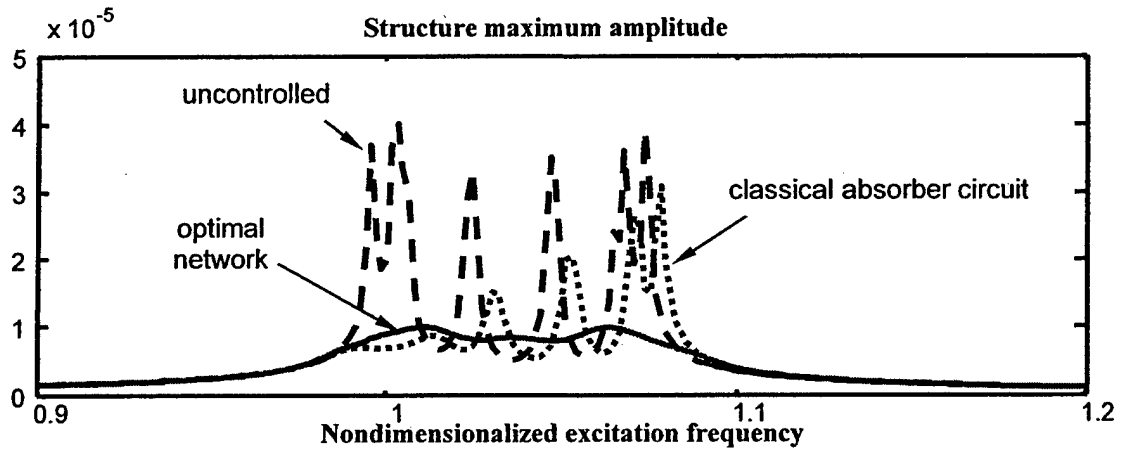


Figure 5. Forced vibration amplitude.

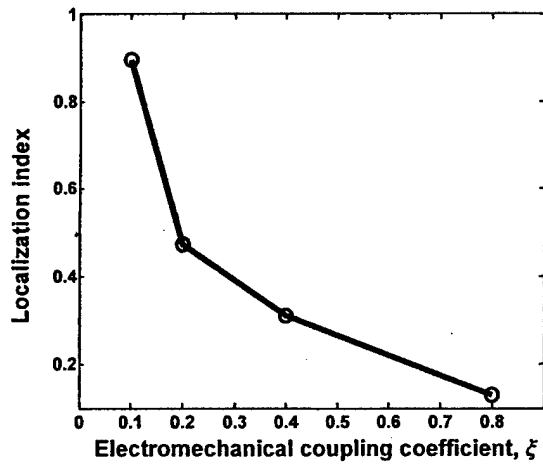


Figure 6. Localization index as a function of the electromechanical coupling coefficient