

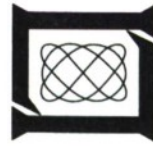
# Correlation of Nonlinear Distortion in Digital Phased Arrays: Measurement and Mitigation

D.J. Rabideau  
L.C. Howard

Presented at the 2002 IEEE MTT-S International Microwave Symposium, 4 June 2002

Issued 1 November 2004

**Lincoln Laboratory**  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LEXINGTON, MASSACHUSETTS



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# Correlation of Nonlinear Distortion in Digital Phased Arrays: Measurement and Mitigation

Daniel J. Rabideau  
L. Cole Howard

Lincoln Laboratory, Massachusetts Institute of Technology

Presented at the 2002 IEEE MTT-S International Microwave Symposium, 4 June 2002

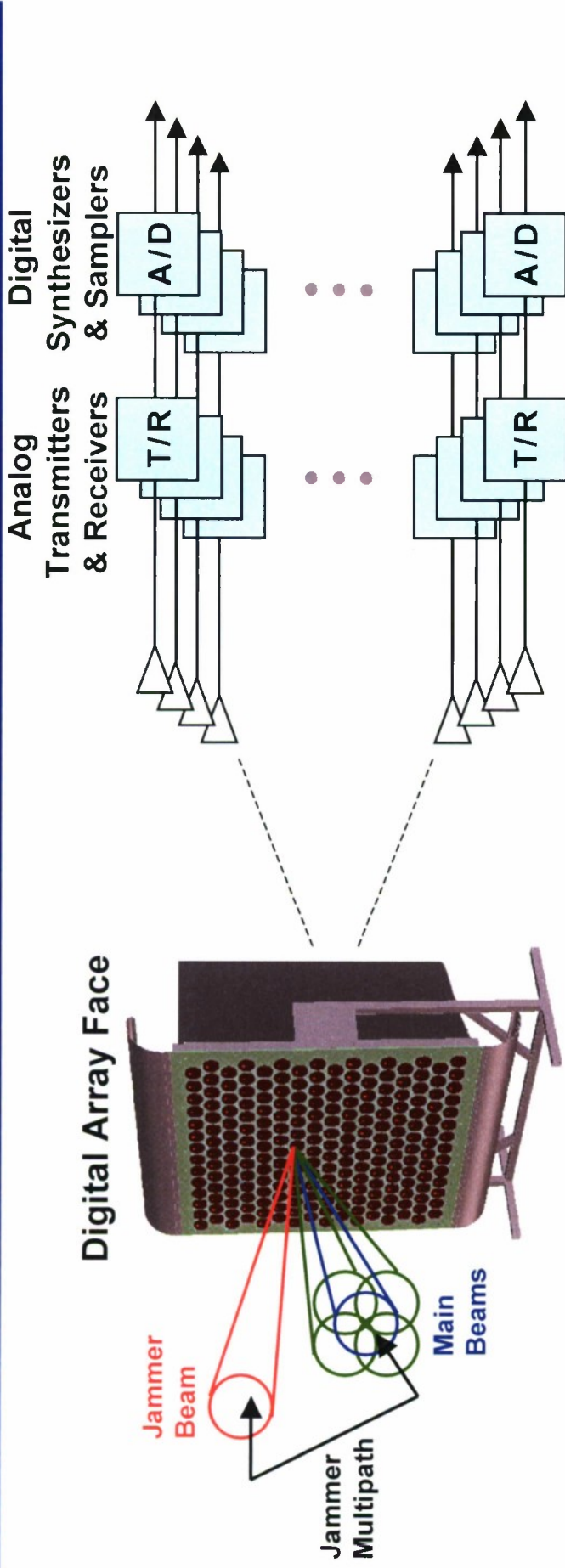
This work was sponsored under Air Force Contract F19628-00-C-0002.  
Opinions, interpretations, conclusions and recommendations are those of the author  
and are not necessarily endorsed by the Department of Defense.

50 years





# Digital Arrays & Their Benefits




- Digital sampling & filtering behind each traditional analog receiver
    - Digital Synthesis (DDS) in front of each traditional analog transmitter
  - Enables:
    - Multiple simultaneous beams; Adaptive beams
    - Flexible scan patterns; Faster area search
    - **Improved dynamic range & linearity**
    - Potential for modular, open-systems design
- The subject of this presentation*



# Outline

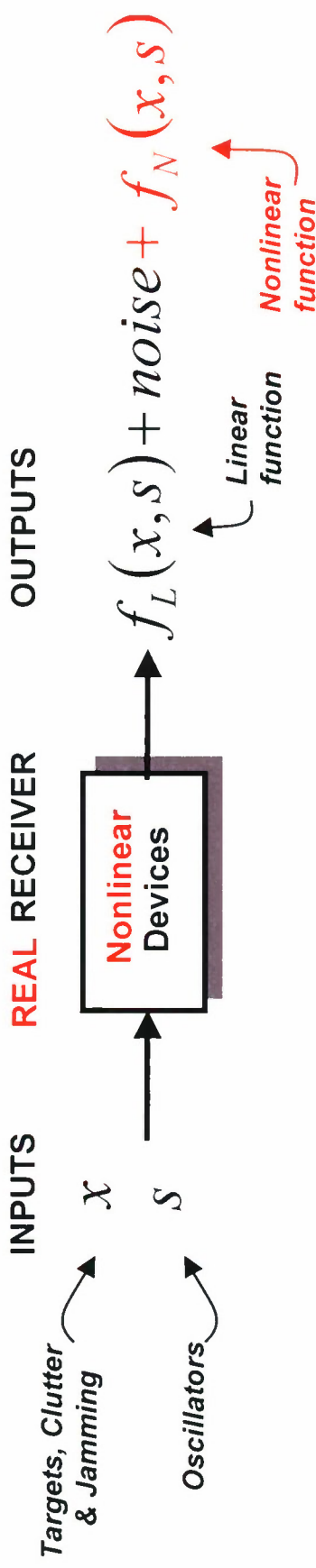
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- Introduction
-  Importance of linearity and dynamic range
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- Summary and future work



# Linearity and Dynamic Range

- An idealized receiver performs linear operations



- Real receivers attempt to approximate this ...
- If nonlinear term exceeds noise, it prevents noise limited detection, i.e., it decreases radar sensitivity

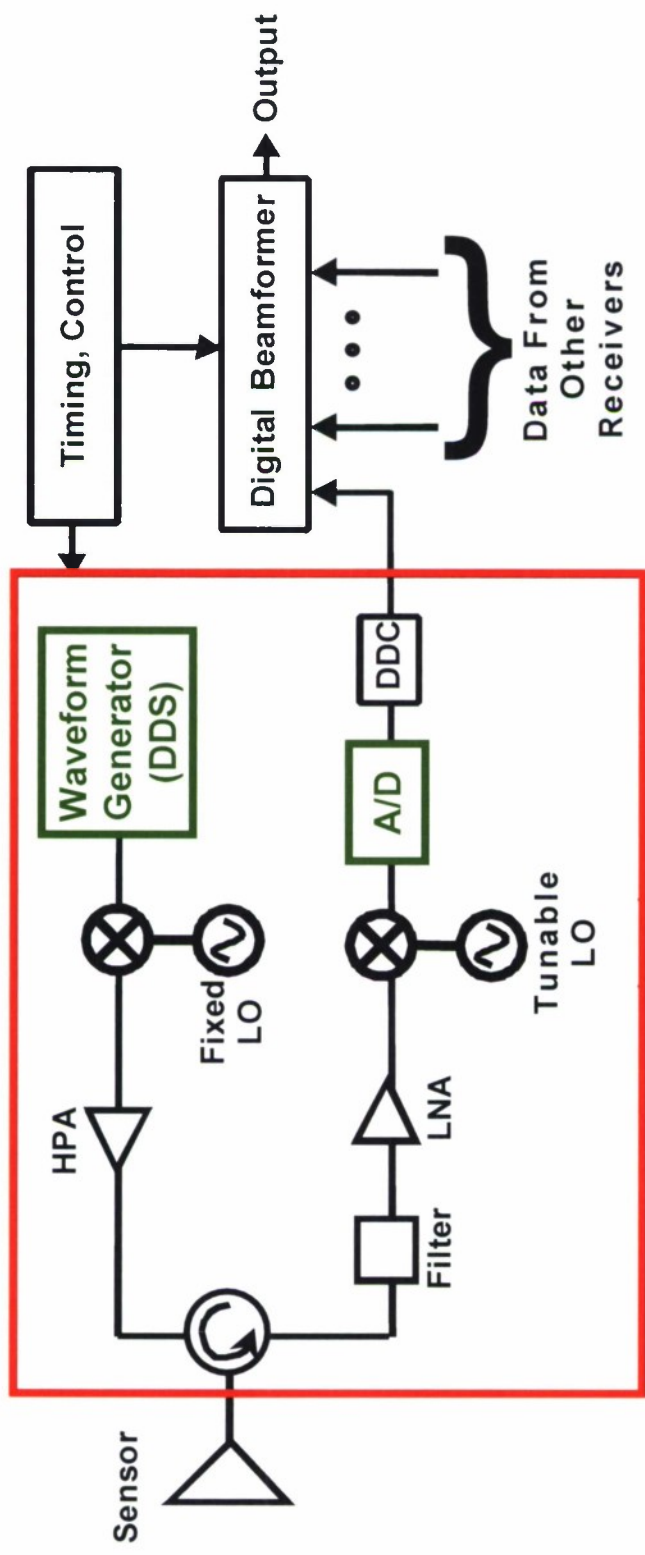
$$\text{Dynamic Range} = \frac{P_{max}}{P_{min}}$$

Power of strongest signal receivable w/o generating a detectable nonlinearity

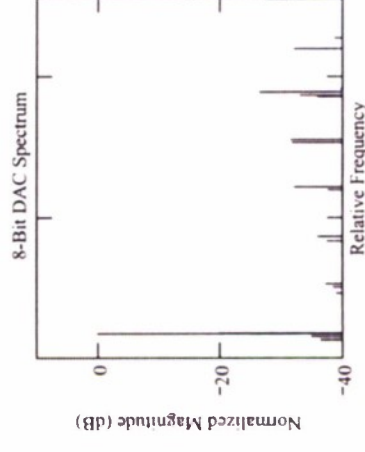
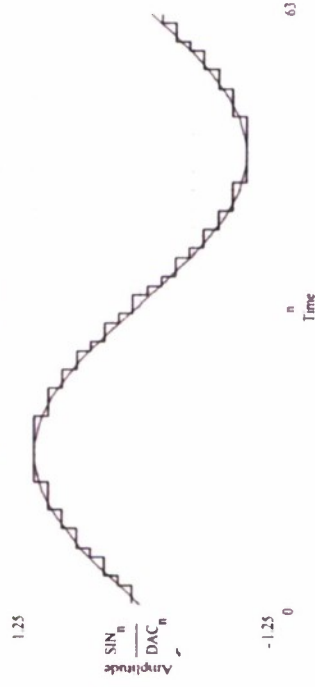
Weakest detectable signal



# Some Types of Nonlinearity in a Typical Digital T/R Module

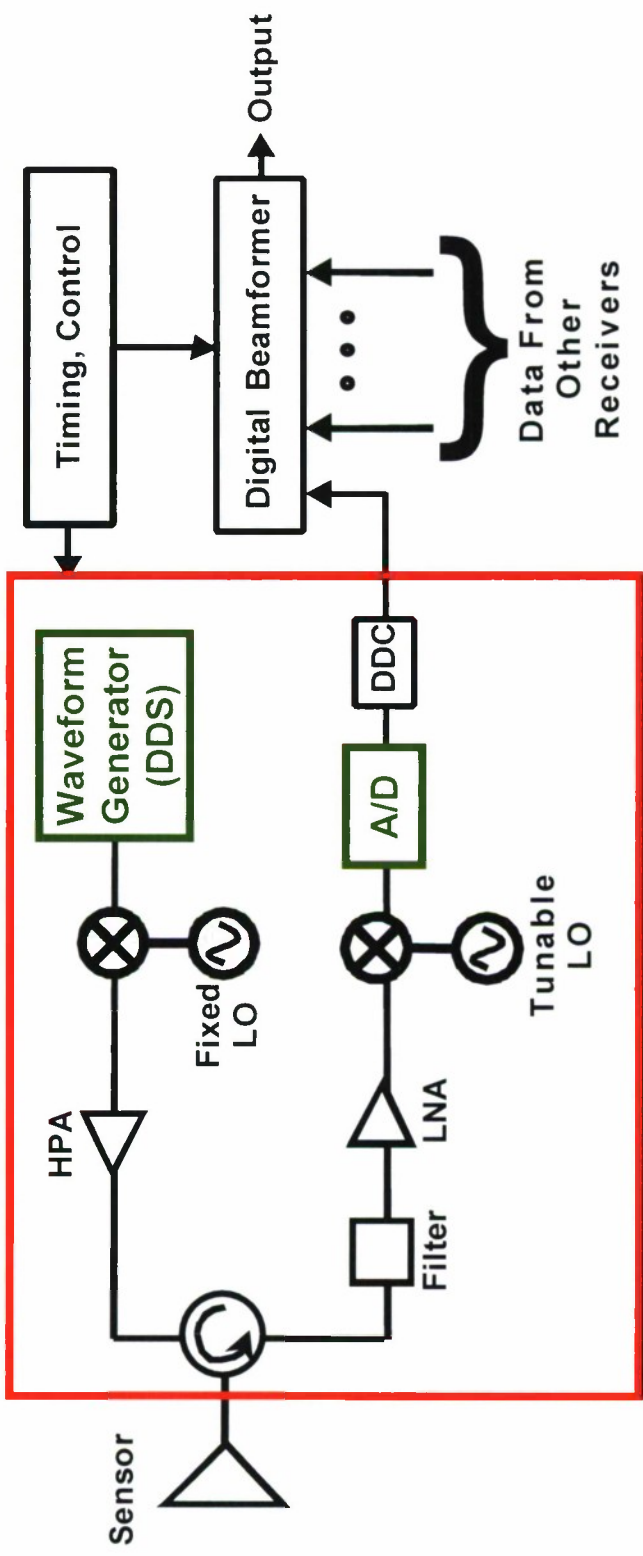


- Quantization (A/Ds, DACs)

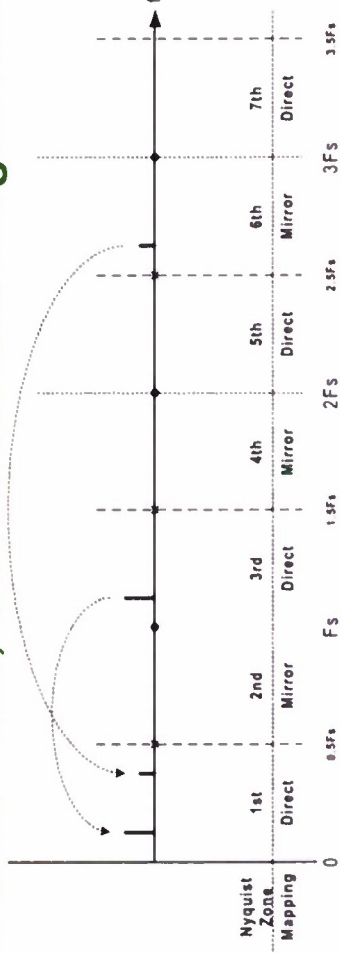




# Some Types of Nonlinearity in a Typical Digital T/R Module

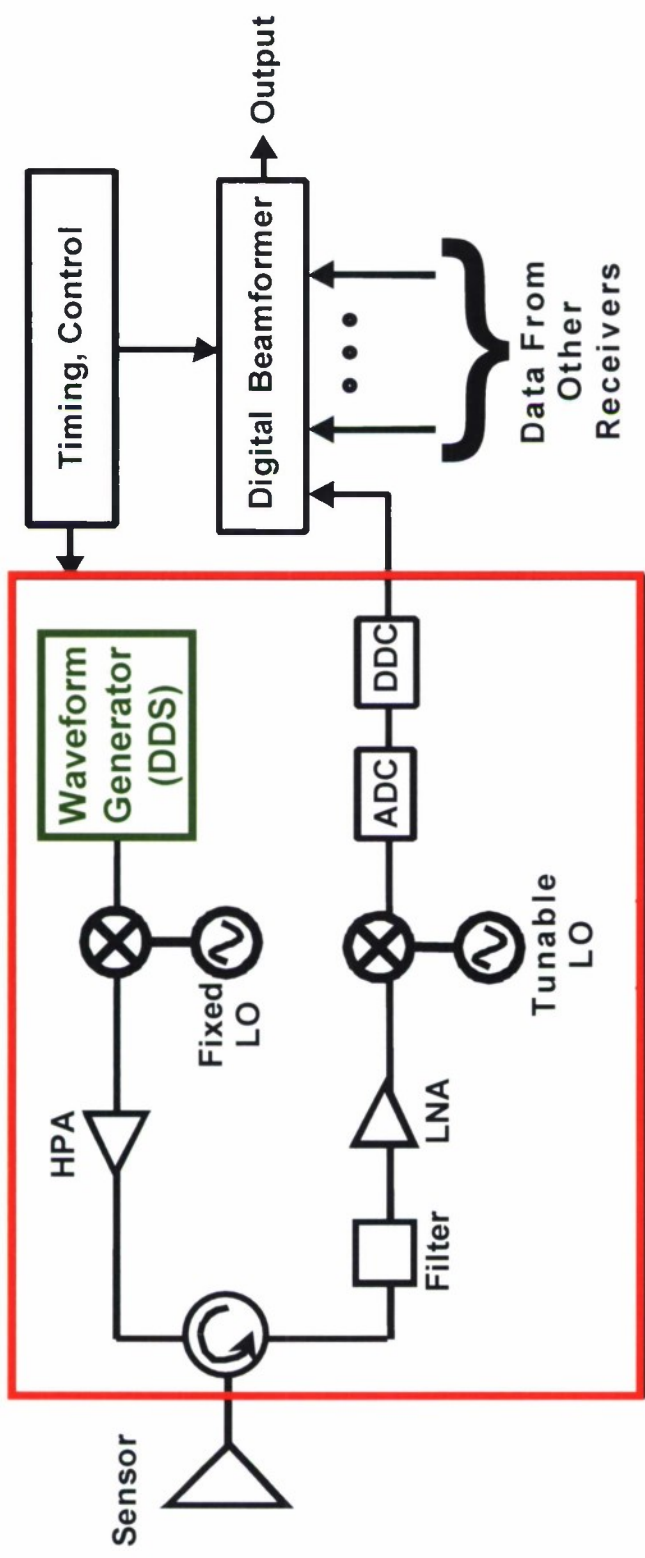


- Harmonic distortions, differential and integral nonlinearities

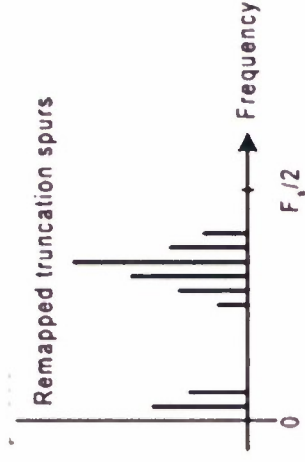




# Some Types of Nonlinearity in a Typical Digital T/R Module

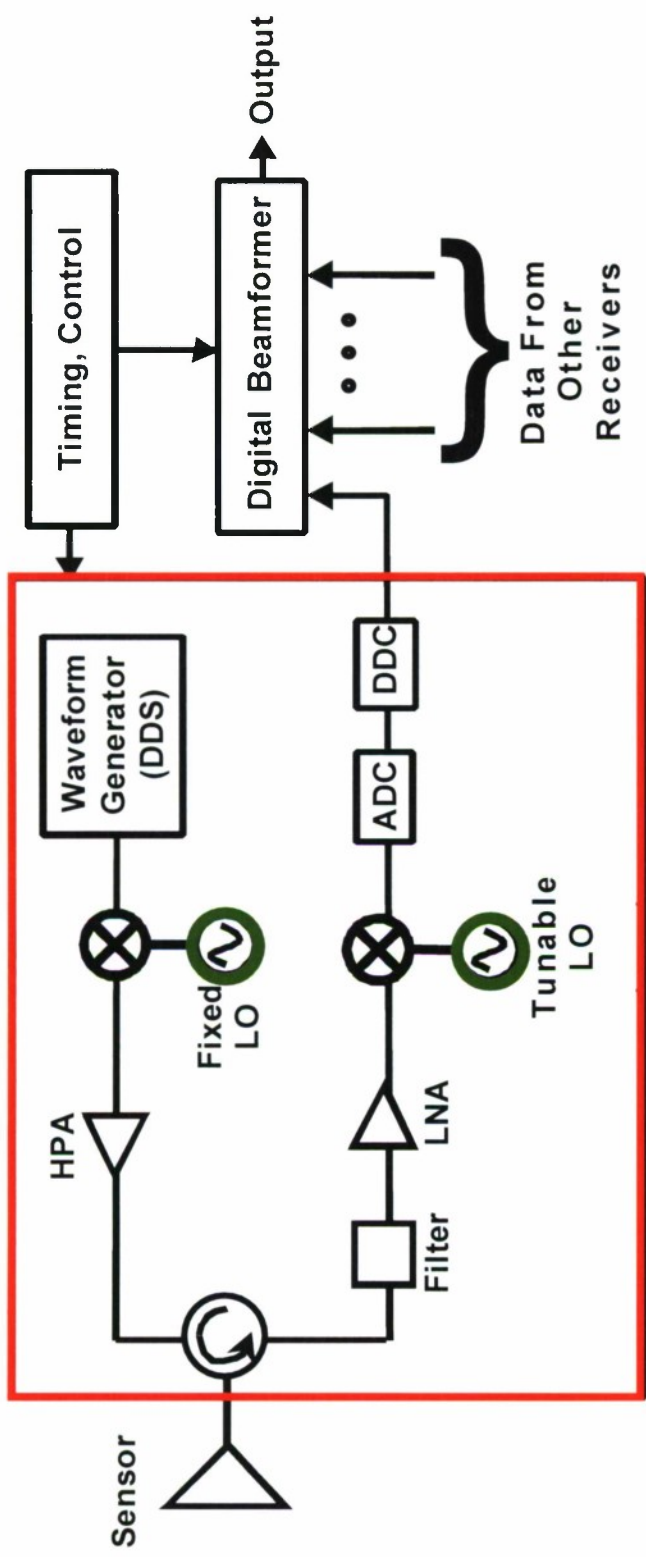


- **Phase Truncation (DDSs)**

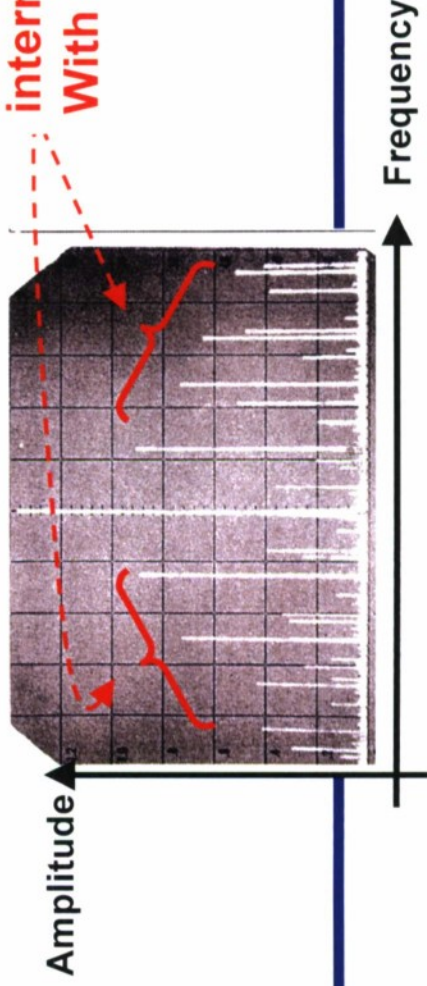




# Some Types of Nonlinearity in a Typical Digital T/R Module

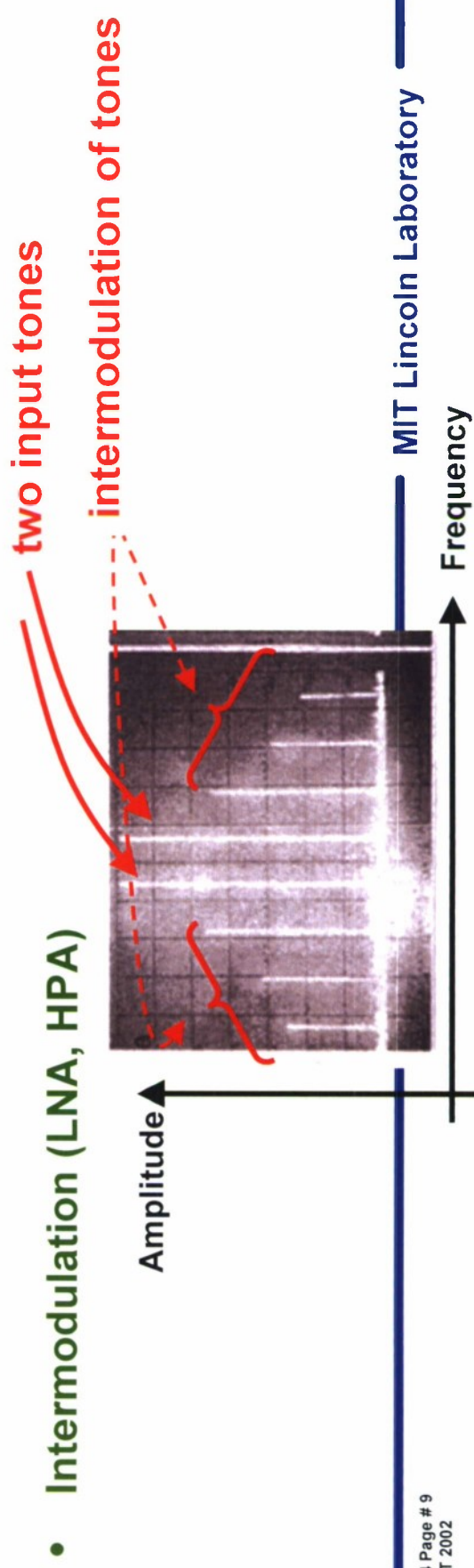
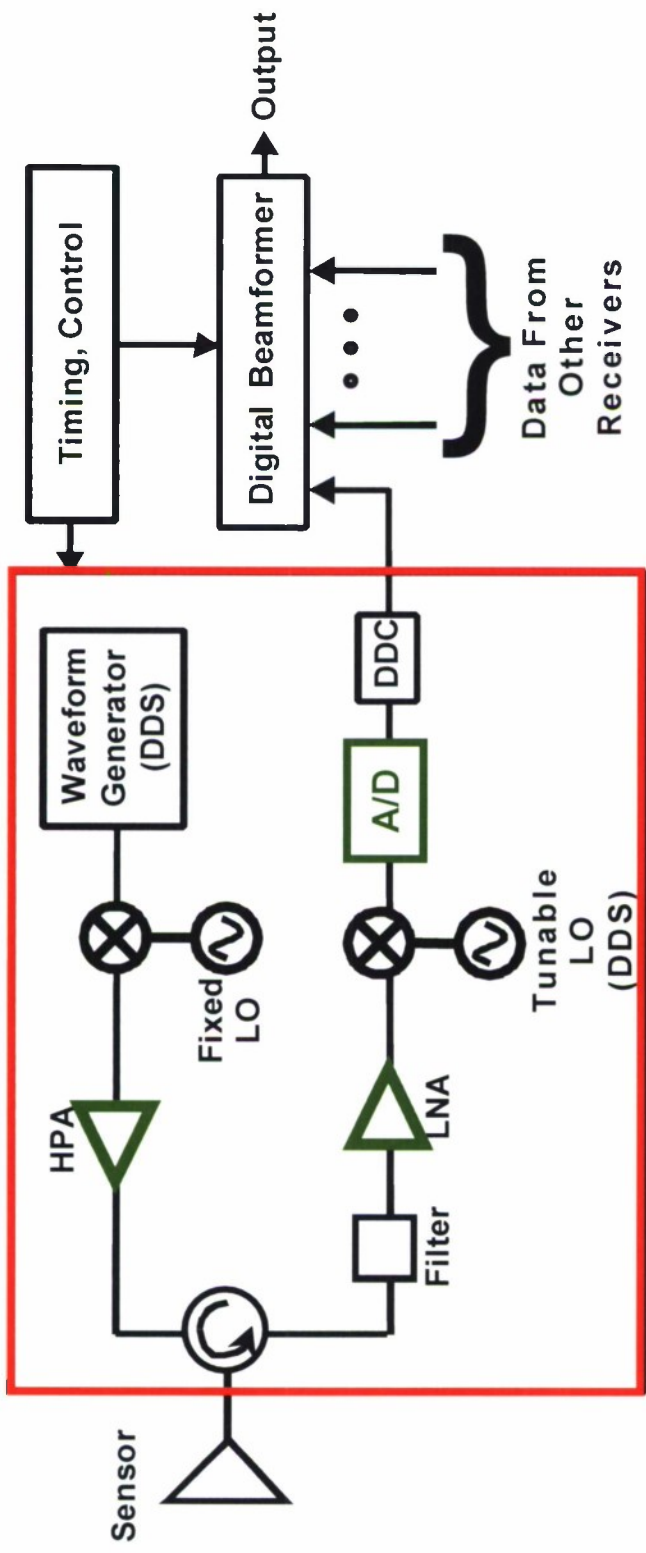


- **M x N Products (Mixers)**
- **intermodulation of tone With LO**





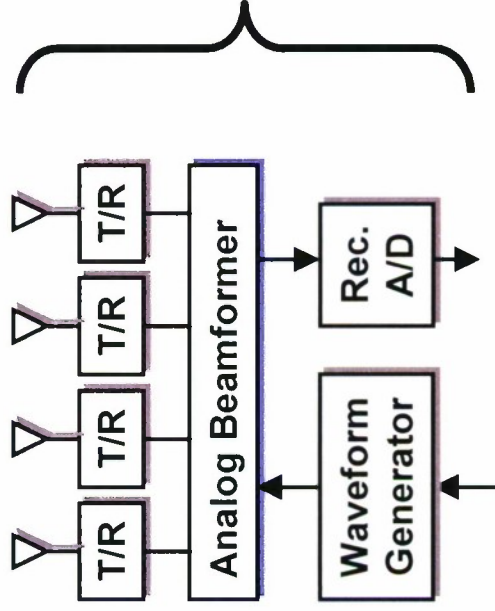
# Some Types of Nonlinearity in a Typical Digital T/R Module





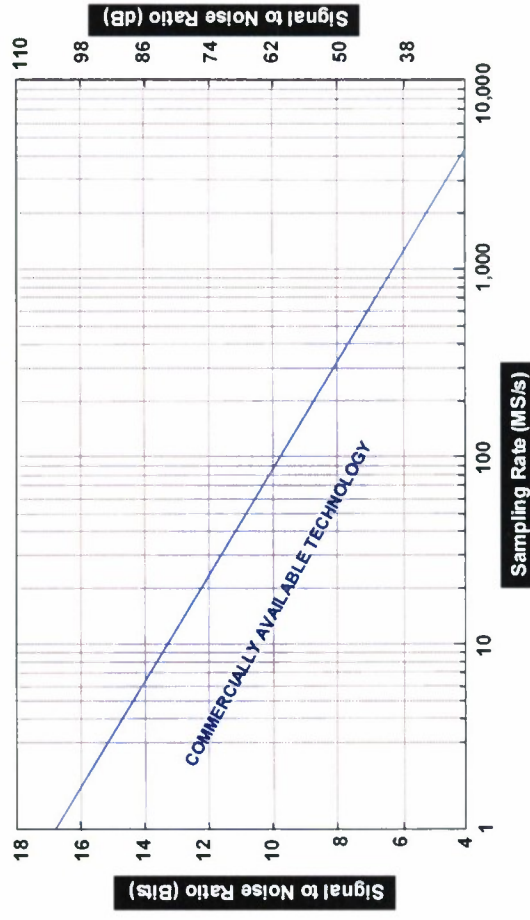
# Linearity Requirements in Analog Arrays

## ANALOG ARRAY



$$\text{Dynamic Range} = \frac{P_{max}}{P_{min}}$$

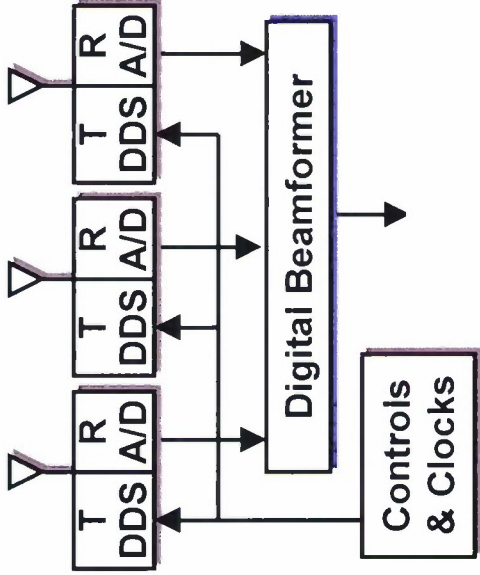
Due to array gain, the dynamic range requirement is usually greatest at the output of beamformer (i.e. the A/D requires the greatest dyn. range)



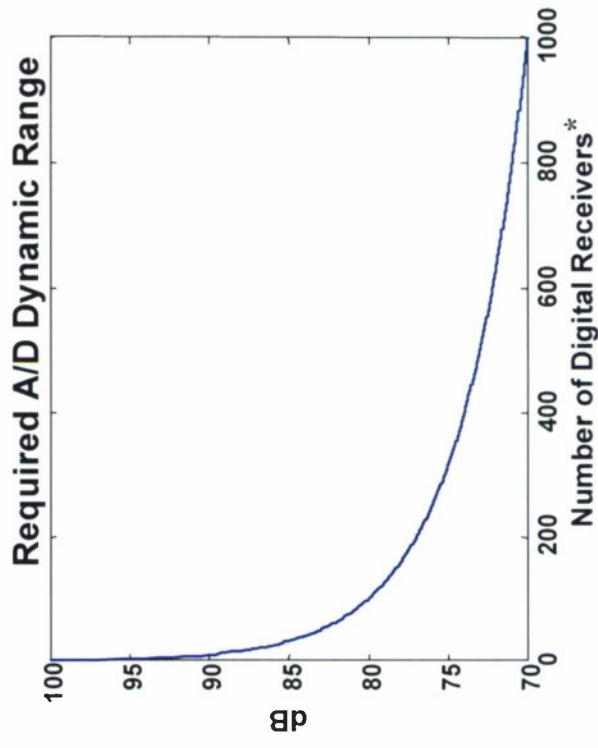


# How Digital Arrays “Theoretically” Improve Dynamic Range

## DIGITAL ARRAY



- Digital arrays move the waveform generator and A/D closer to the elements
  - Sampled signals are smaller
  - Requirement on A/D, DDS dynamic range is lessened





## ... but there's one BIG problem

---

- Even if each receiver's nonlinearities lie below the noise level, the nonlinearities may be correlated spatially (or even coherent).\*


If so, the digital beamformer will integrate them (spatially) above the noise, limiting the system's overall dynamic range

*\* [ We had originally conjectured (and proven via mathematic models) that many of these nonlinearities would be correlated. Thanks to the ADC, we've also collected data to show this experimentally ... as you shall see. ]*



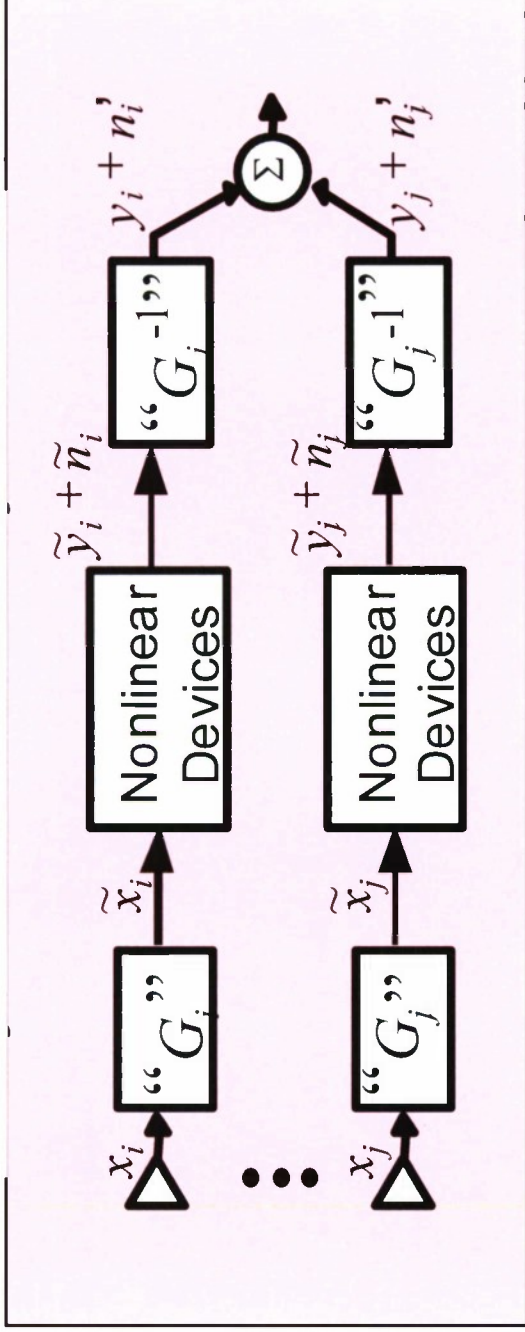
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# Novel Approach: -- Decorrelation of Array Nonlinearities --



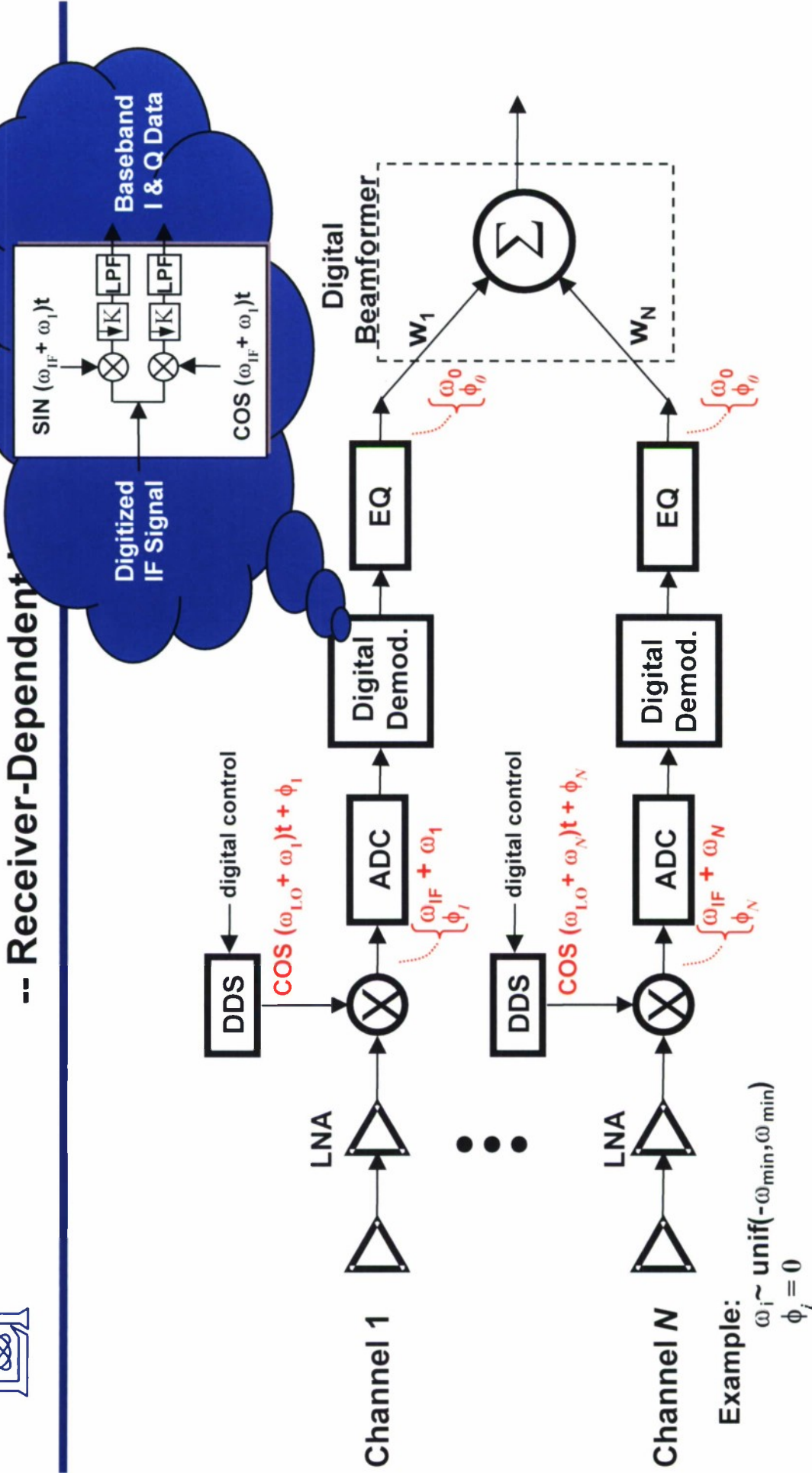
- Input signals are transformed differently at each channel
- “Inverse” transformation later restores the linear component
  - But nonlinear terms still vary from channel to channel, decorrelating them and/or making them sum incoherently.

**NOTE: We do not remove nonlinearities ...  
but we do prevent them from being correlated (i.e., no array gain)**



# Example of "New Approach"

-- Receiver-Dependent

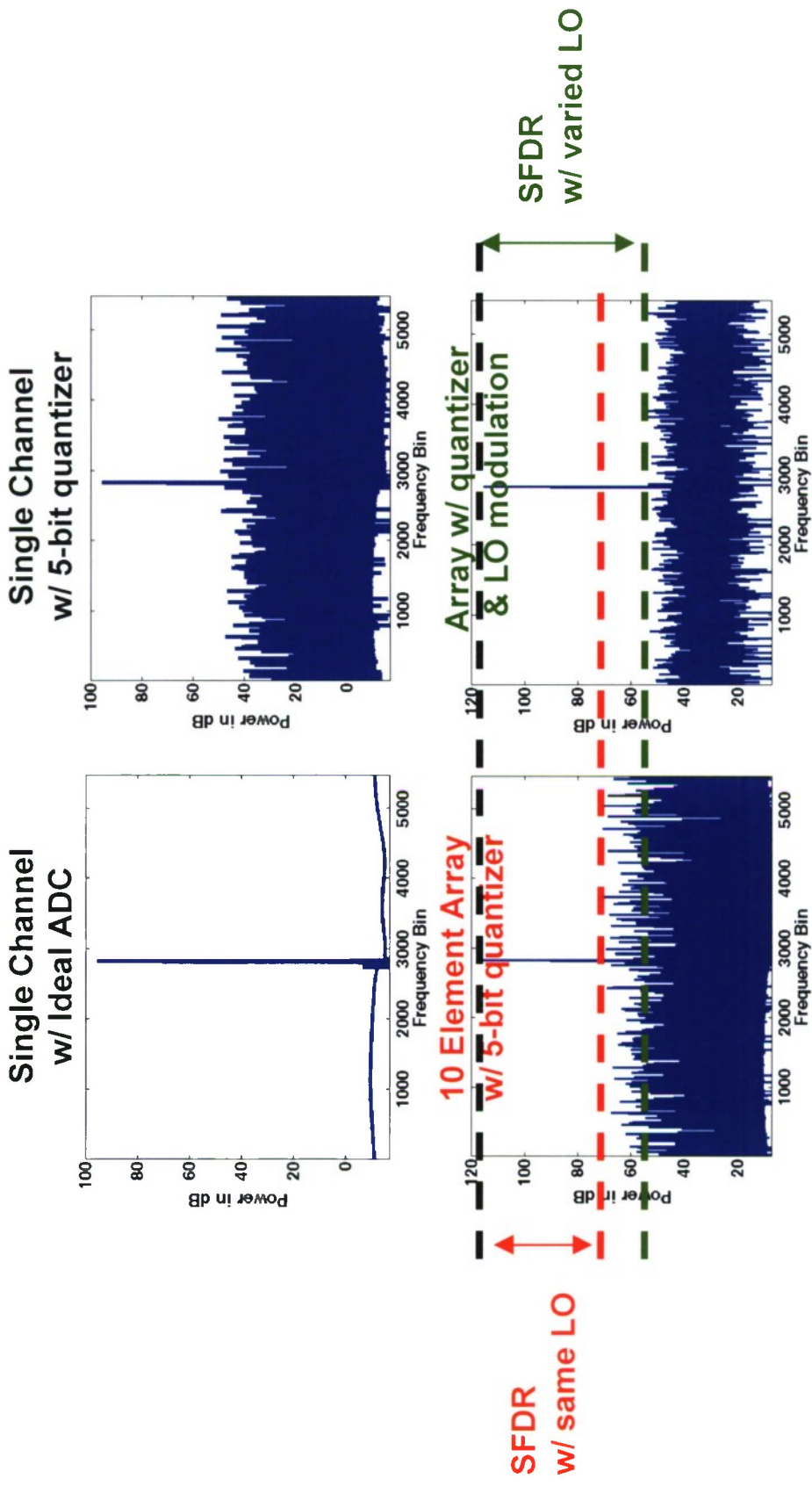


Example:  
 $\omega_i \sim \text{unif}(-\omega_{\min}, \omega_{\min})$   
 $\phi_i = 0$

- Addresses: Harmonically related spurs generated by ADC, DDS and DAC,  $M \times N$  products in mixer, and LO and ADC phase noise
  - Varied LO causes different sets of frequencies to be excited



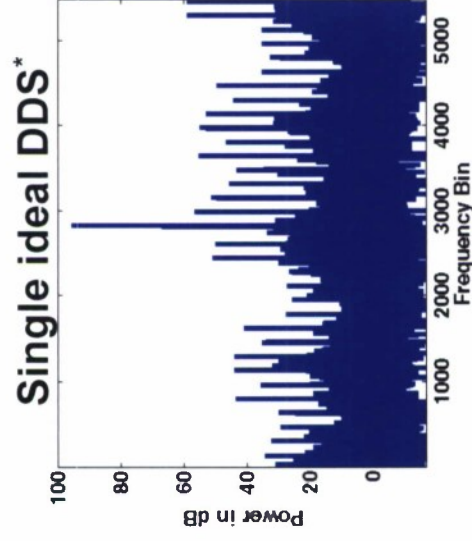
# Predicted A/D Nonlinearities with Receiver-Dependent LO



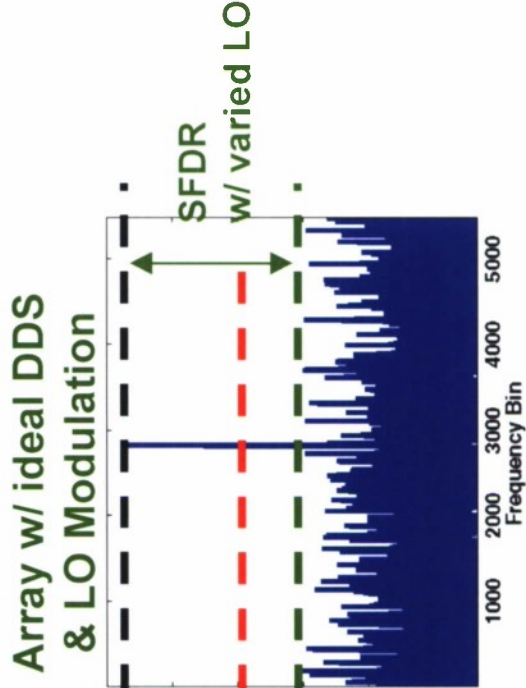
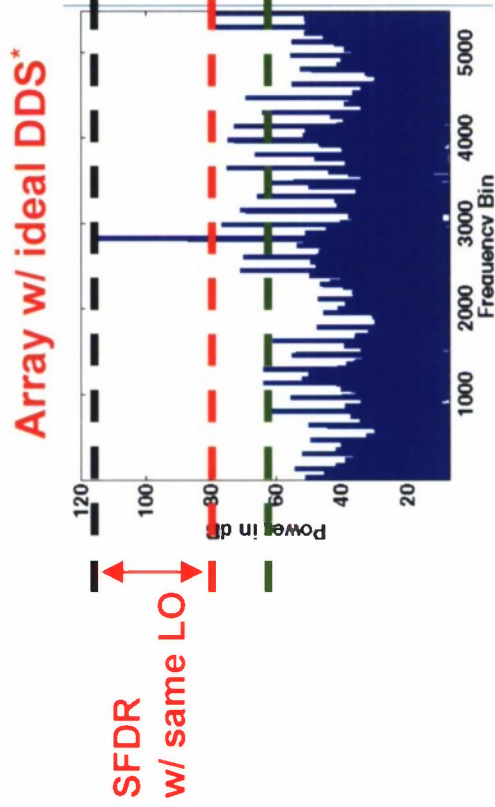
- Randomized LO modulation makes spurs appear uncorrelated, thus avoiding array gain



# Predicted DDS Nonlinearities with Receiver-Dependent LO



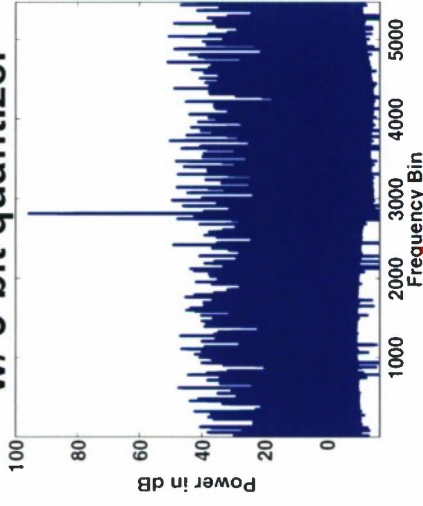
\* 16 bit phase accumulator  
6 bit phase word  
5 bit DAC



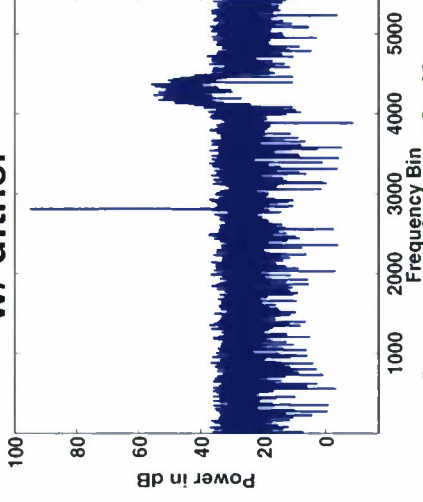


# Example 2: Receiver-Dependent Dither

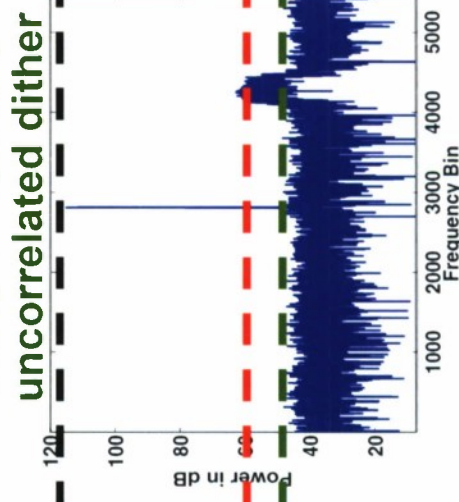
Single Channel  
w/ 5-bit quantizer



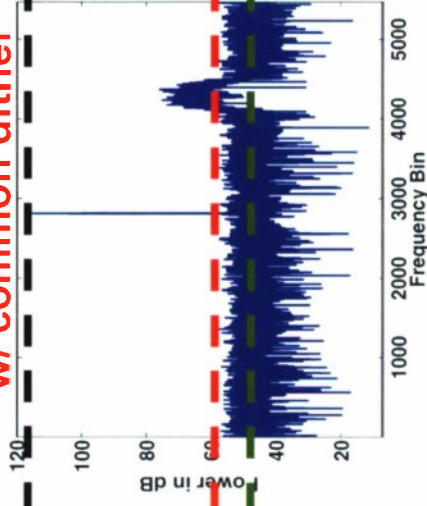
Single Channel  
w/ dither



Array w/ spatially uncorrelated dither



Array w/ common dither



- Randomized dither makes spurs appear uncorrelated, thus avoiding array gain (ADCs and DACs only)



# Other Types of Distortion


---

- $M \times N$  products and harmonics can often be addressed by phase transformations
- Intermods from well-separated sources can be addressed by nonlinear-phase all-pass filtering and subsequent equalization
- Closer-in intermods are not addressed directly
  - However, by first mitigating other complex nonlinearities as described (so that they're below the noise), a relatively simple equalizer can be used to remove the remaining close-in intermods (example given later).
- Method above can be applied on transmit in analogous fashion



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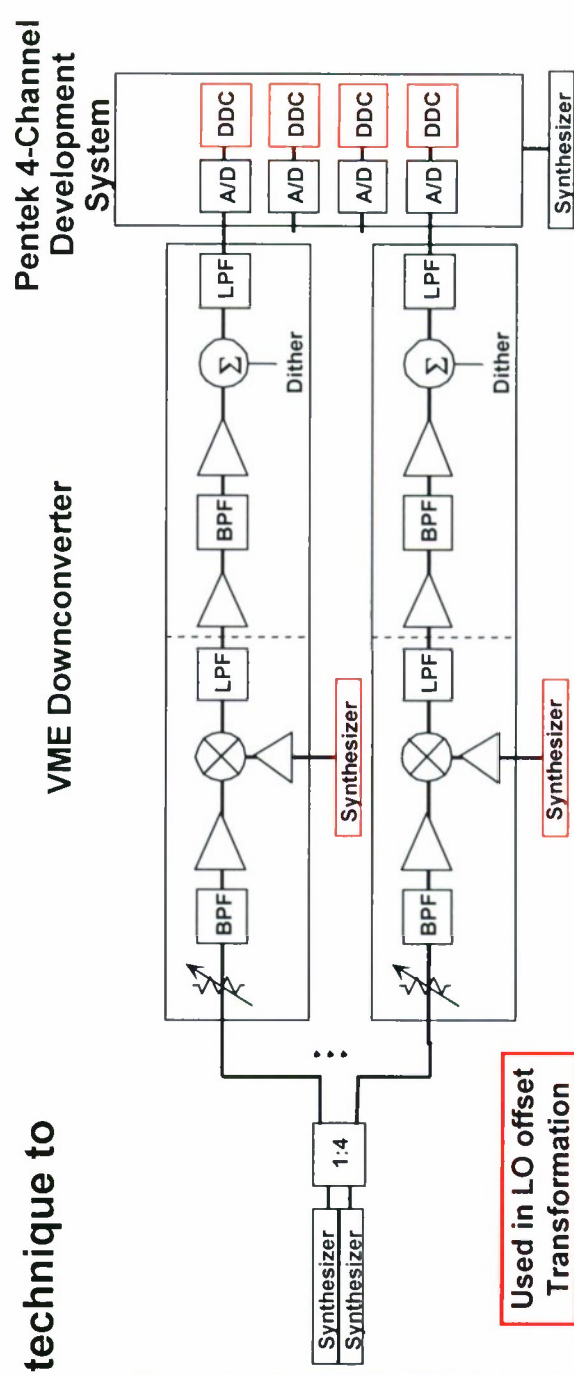
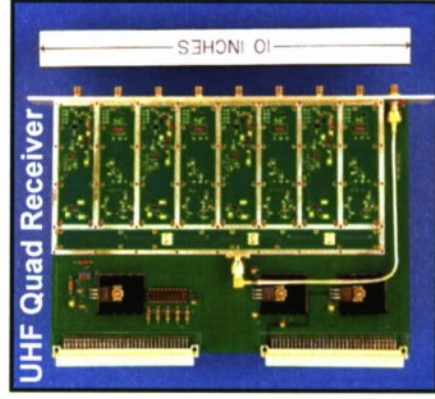
# ADC Testbed Objectives

## Goals

- Experiment fit within time and hardware constraints
  - Only four channels available
  - Linear Array Upgrade program had 2-week window of availability for hardware
- Test experimentally whether or not distortions add in beamformer
- Apply mitigation technique to these cases

## Hardware

- Group 101 Quad UHF receiver
- Pentek 4290 DSP with pair of 6216 dual digital receiver
  - NCO in DDC has < 1 Hz resolution
  - 12 Bit 65 MSPS ADC
- Graychip Digital Down-Converter (DDC) GC1016

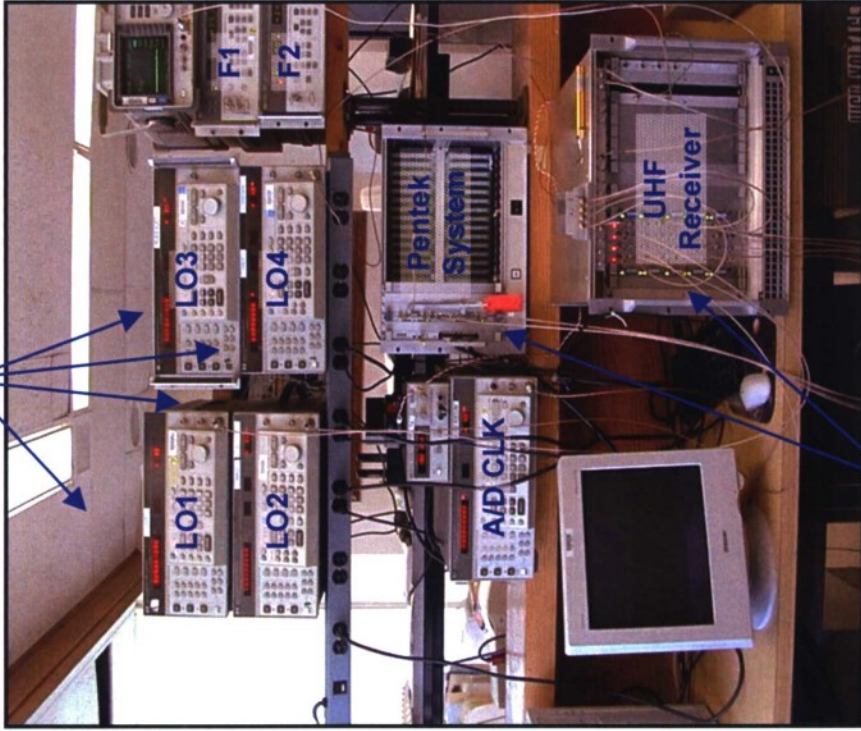


Used in LO offset Transformation



# Experiments

Distributed LO System



Digital Receivers

Case #	Spur investigated	F1 (MHz)	F2 (MHz)	LO1 (MHz)	LO2 (MHz)	LO3 (MHz)	LO4 (MHz)
1	a	-2 x 2 product	422	458	493	-	-
	b	-2 x 2 product	422	458	493	$493+2*\delta 1$	$493+3*\delta 1$
	c	-2 x 2 product	422	458	493	$493+2*\delta 2$	$493+3*\delta 2$
2	a	-3 x 3 product	420	468	491	-	-
	b	-3 x 3 product	420	468	491	$491+2*\delta 1$	$491+3*\delta 1$
	c	-3 x 3 product	420	468	491	$491+2*\delta 2$	$491+3*\delta 2$
3	a	A/D Harmonic without Dither	75				
	a	Receiver and A/D Harmonics	435	505	-	-	-
4	b	Receiver and A/D Harmonics	435	505	$505+\delta 1$	$505+2*\delta 1$	$505+3*\delta 1$
	c	Receiver and A/D Harmonics	435	505	$505+\delta 2$	$505+2*\delta 2$	$505+3*\delta 2$
	a	Intermodulation	435	435	505	-	-
5	b	Intermodulation	435	435	505	$505+\delta 1$	$505+2*\delta 1$
	c	Intermodulation	435	435	505	$505+\delta 2$	$505+2*\delta 2$
	d	Intermodulation	435	435	505	-	-
	e	Intermodulation	435	435	505	-	-
6	a	Receiver noise			505	-	-
	b	Receiver noise			505	$505+\delta 1$	$505+2*\delta 1$
	c	Receiver noise			505	$505+\delta 2$	$505+2*\delta 2$
7	a	A/D noise					



# Outline

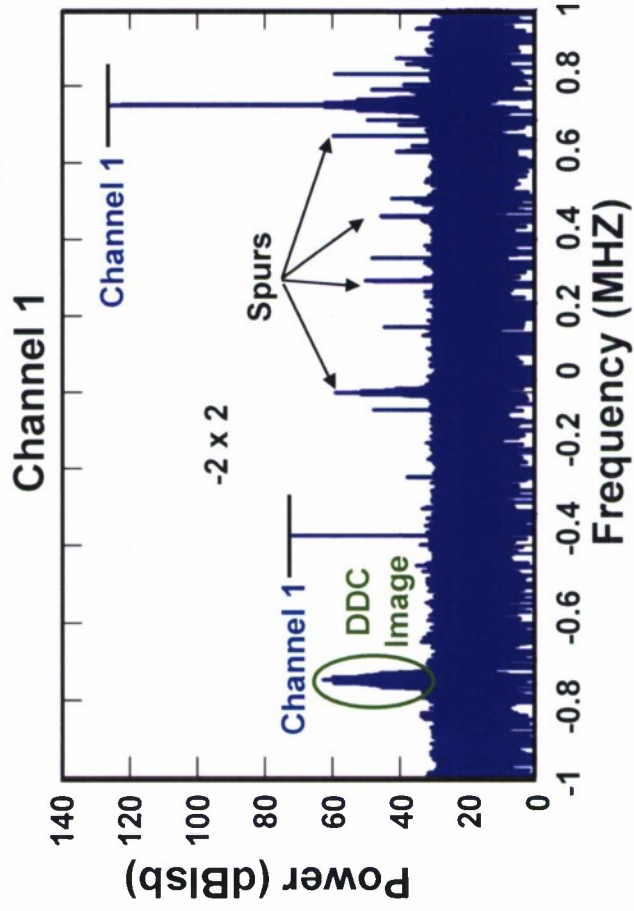
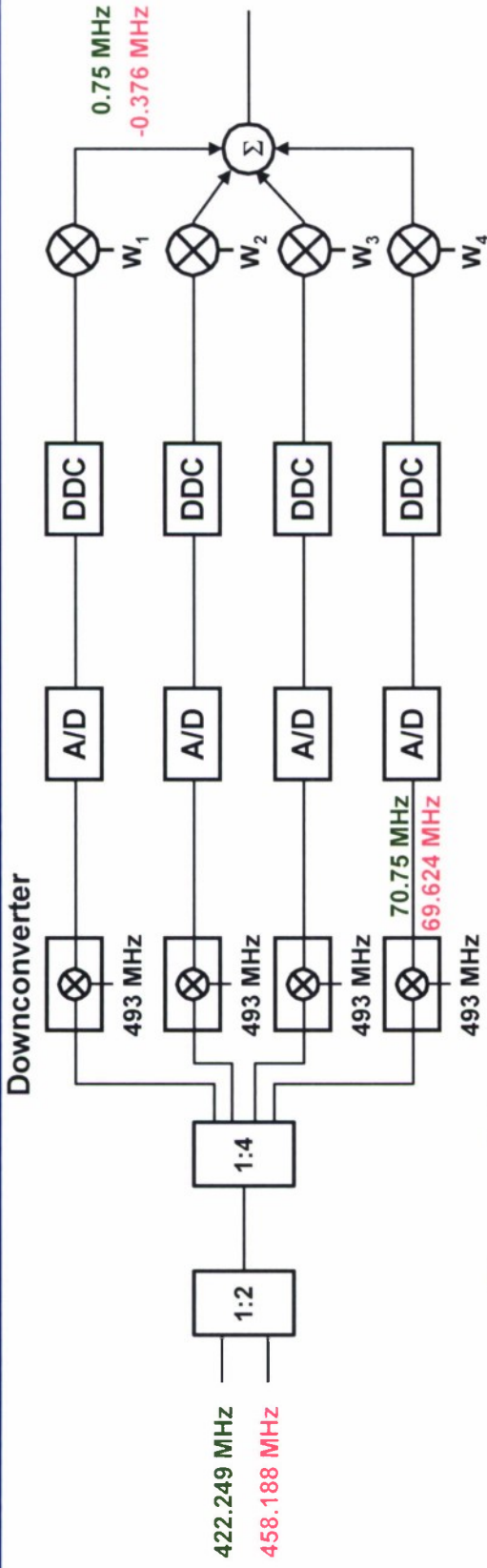
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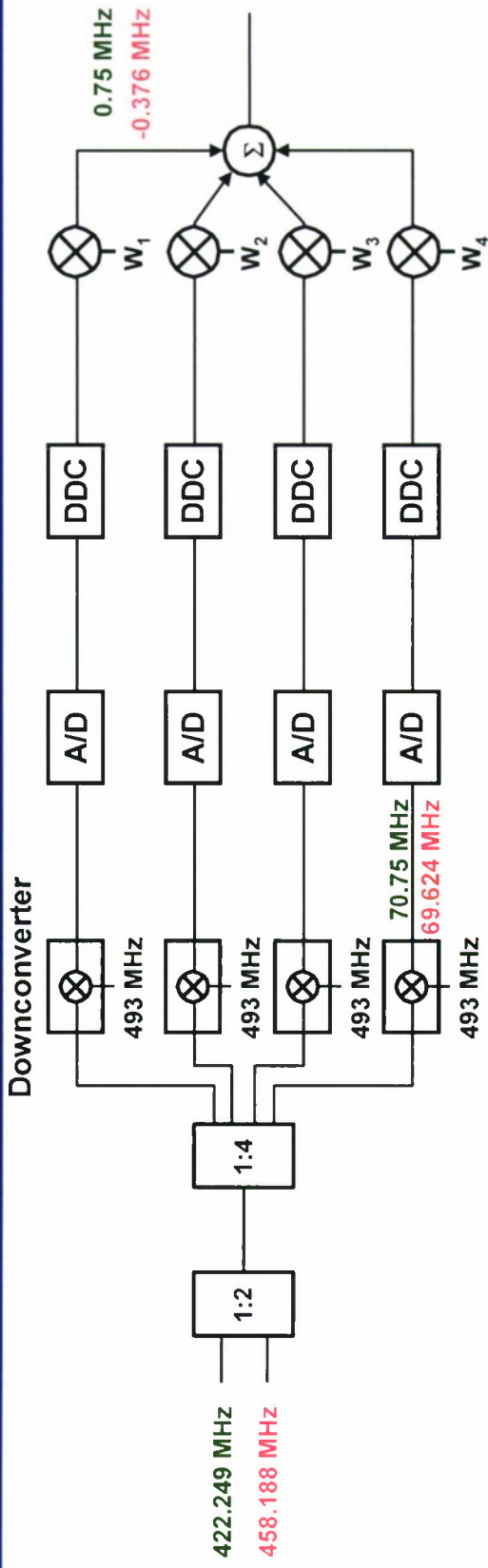
# -2 x 2 Mixer Intermodulation Product Correlation



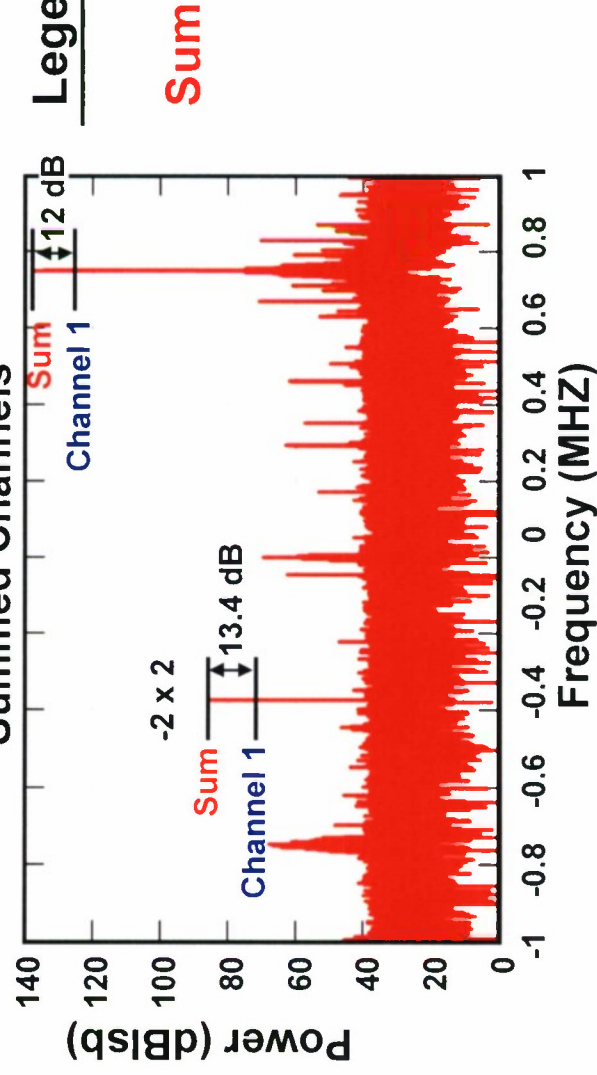
Legend  
Channel 1



# -2 x 2 Mixer Intermodulation Product Correlation



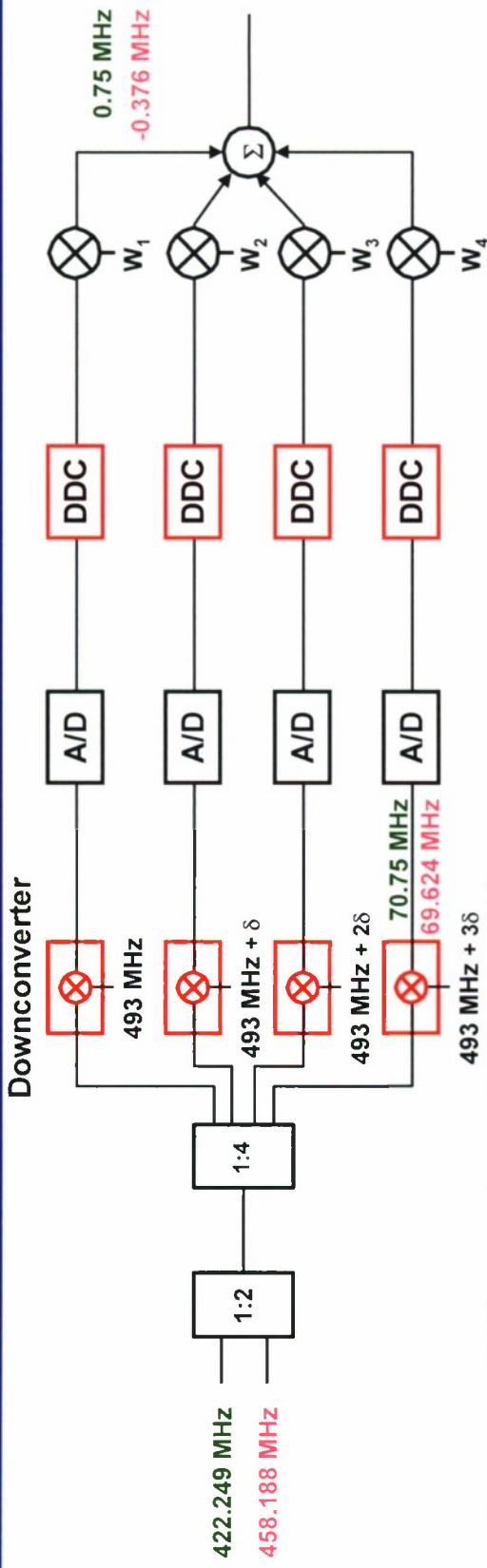
Summed Channels Legend



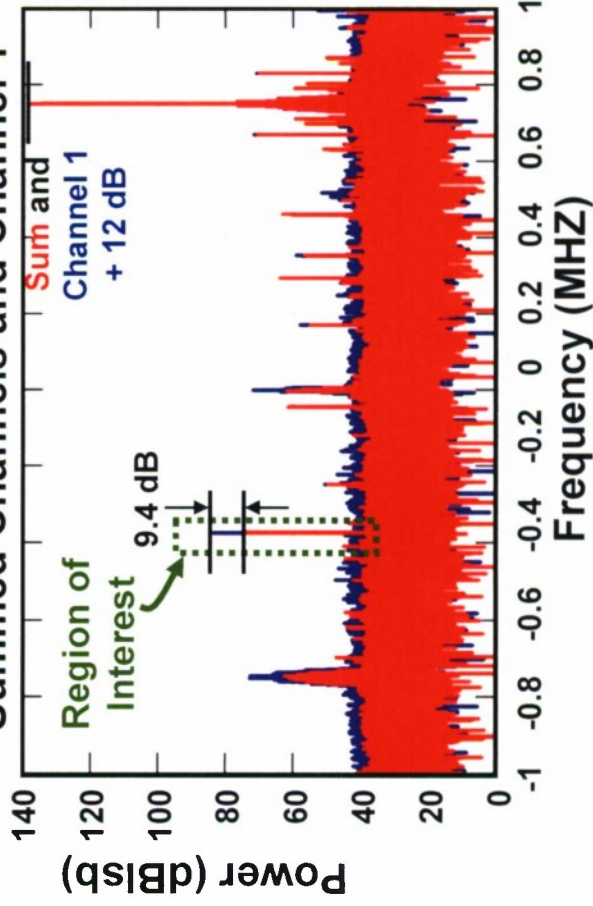




# LO Offset Mitigation Applied to -2 x 2 Mixer Intermodulation Product Correlation



Summed Channels and Channel 1



Legend

Channel 1 + 12 dB

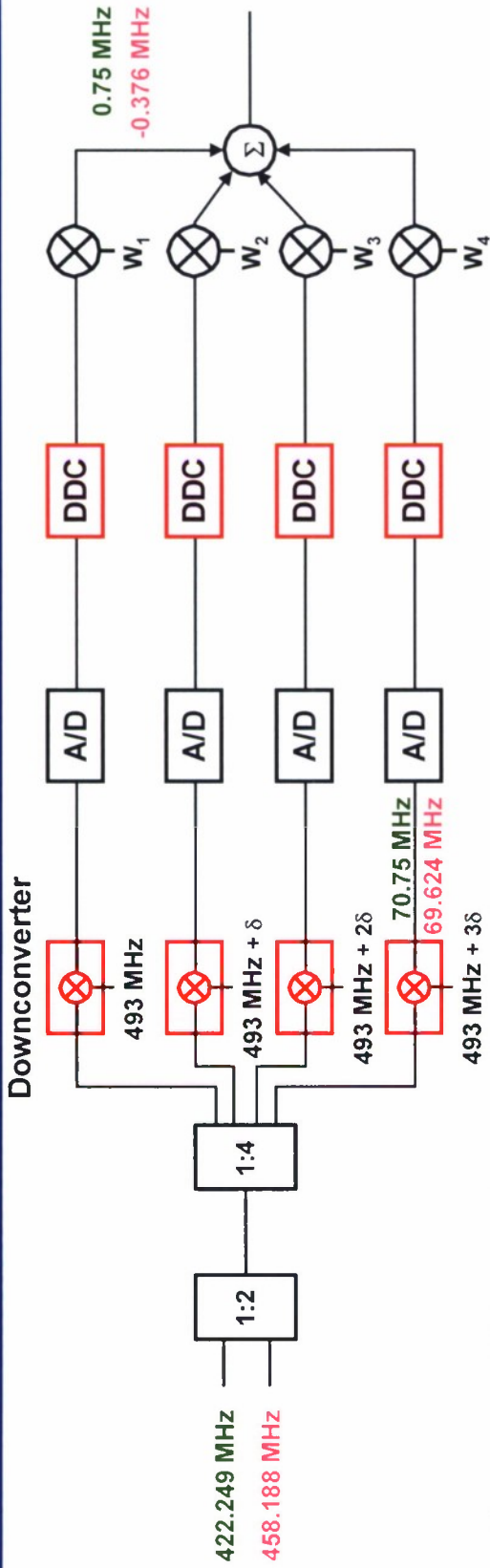
Sum

$\delta = 684 \text{ Hz}$

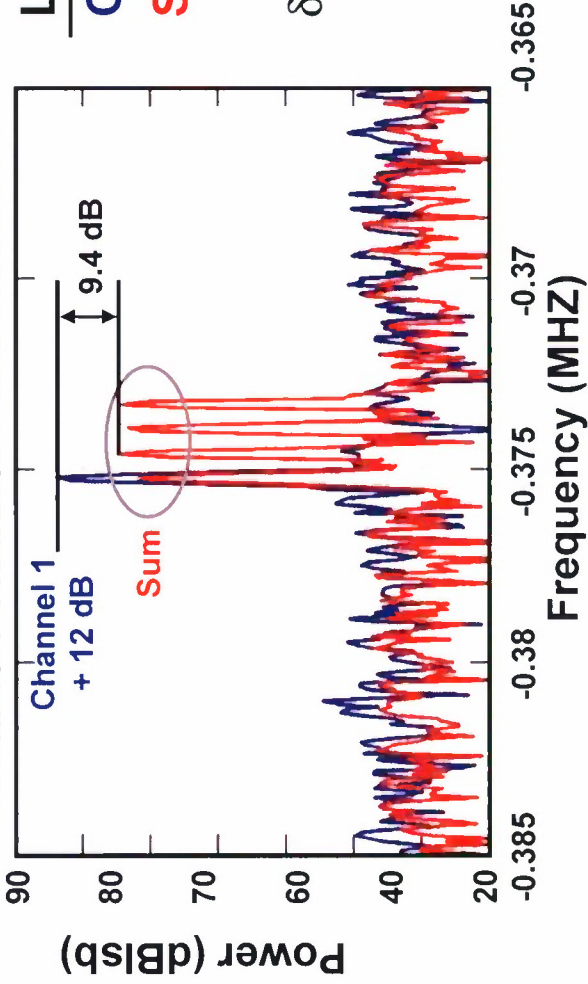
LO Offset Mitigation Technique  
Decorrelates -2 x 2 Spur



# LO Offset Mitigation Applied to -2 x 2 Mixer Intermodulation Product Correlation



Summed Channels and Channel 1



Legend

Channel 1 + 12 dB

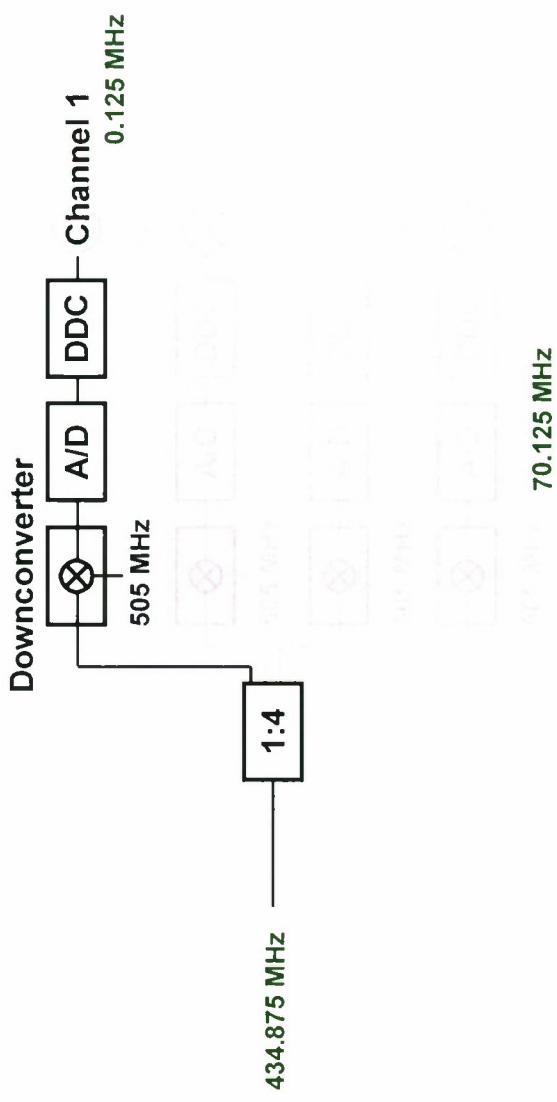
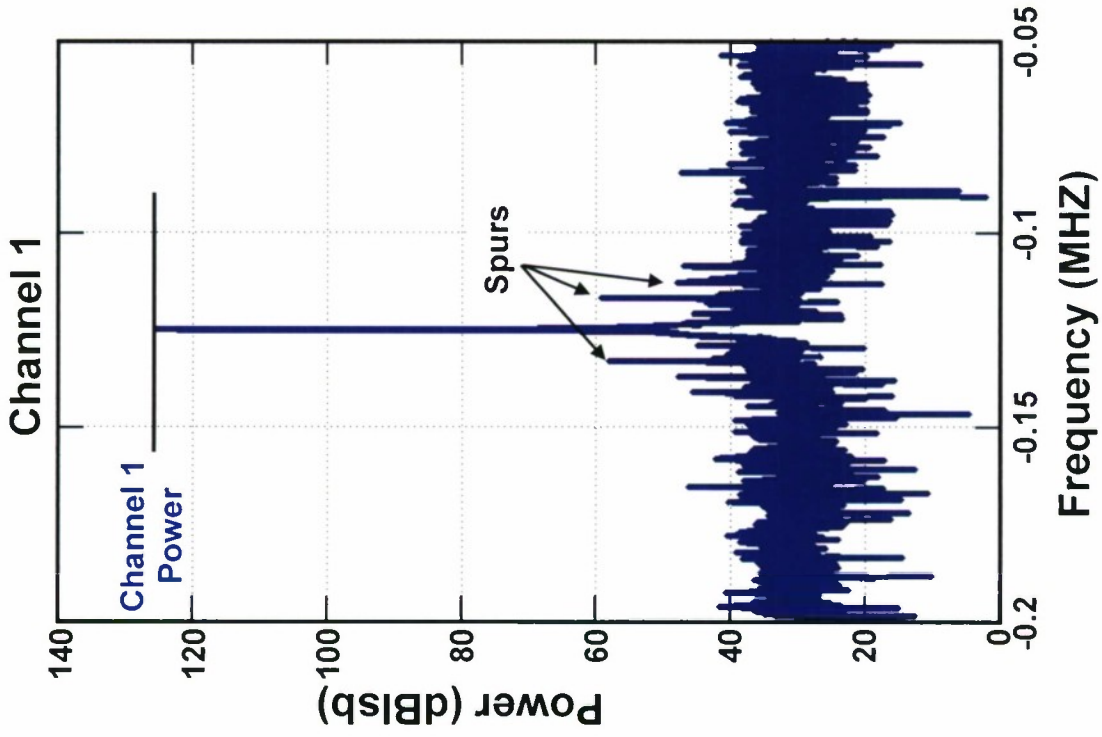
Sum

$\delta = 684$  Hz

LO Offset Moves Spur in Each Channel to a Different Frequency

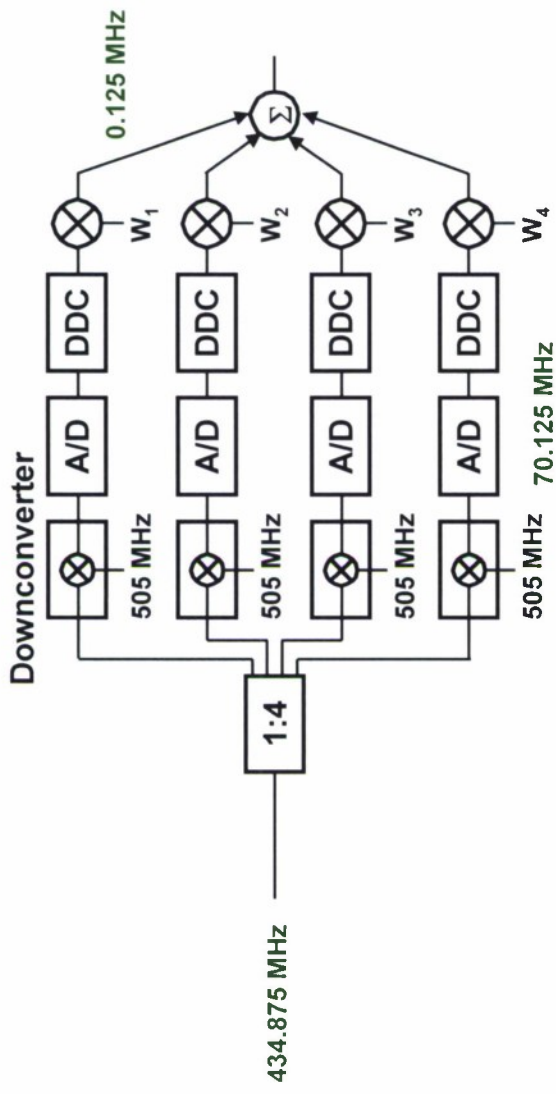
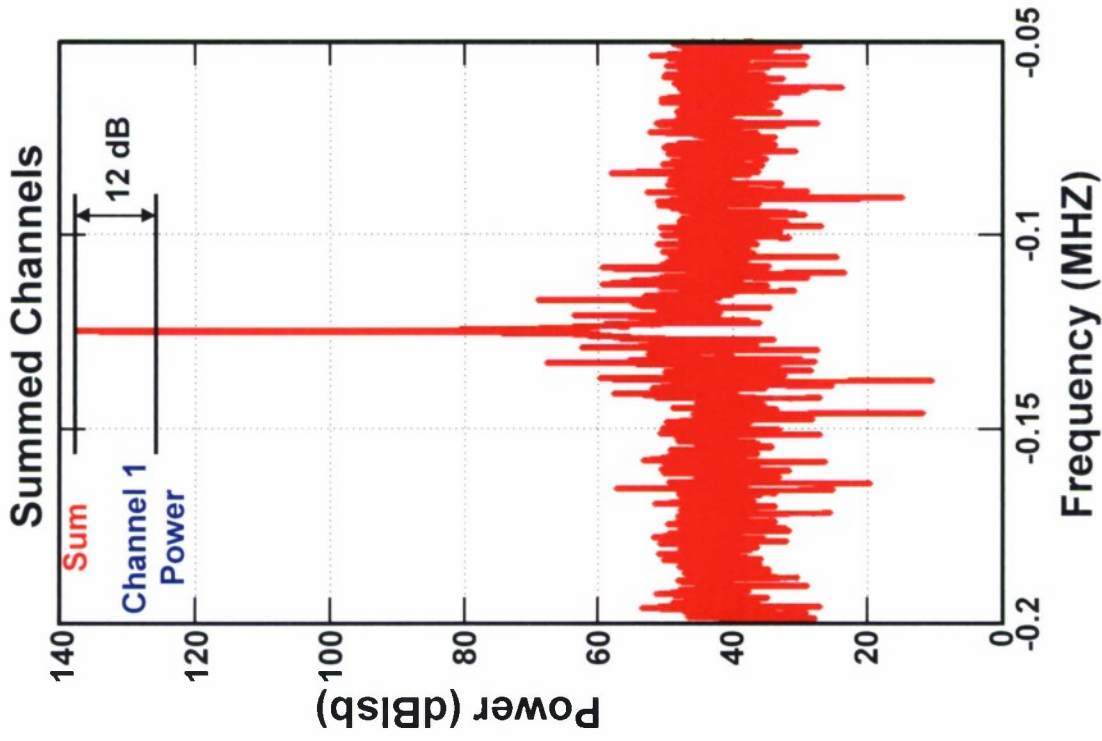


# Noise Correlation





# Noise Correlation

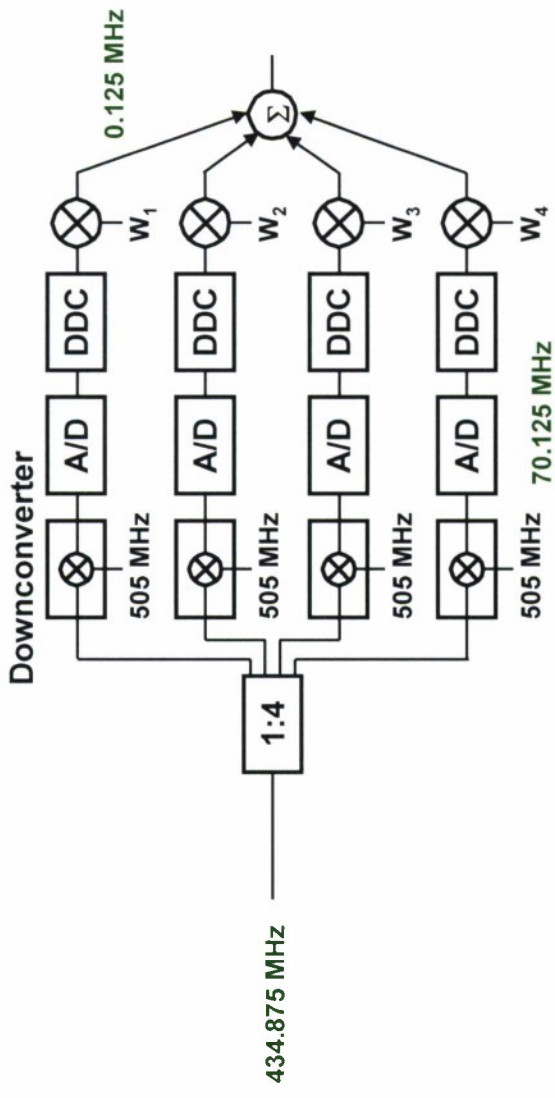
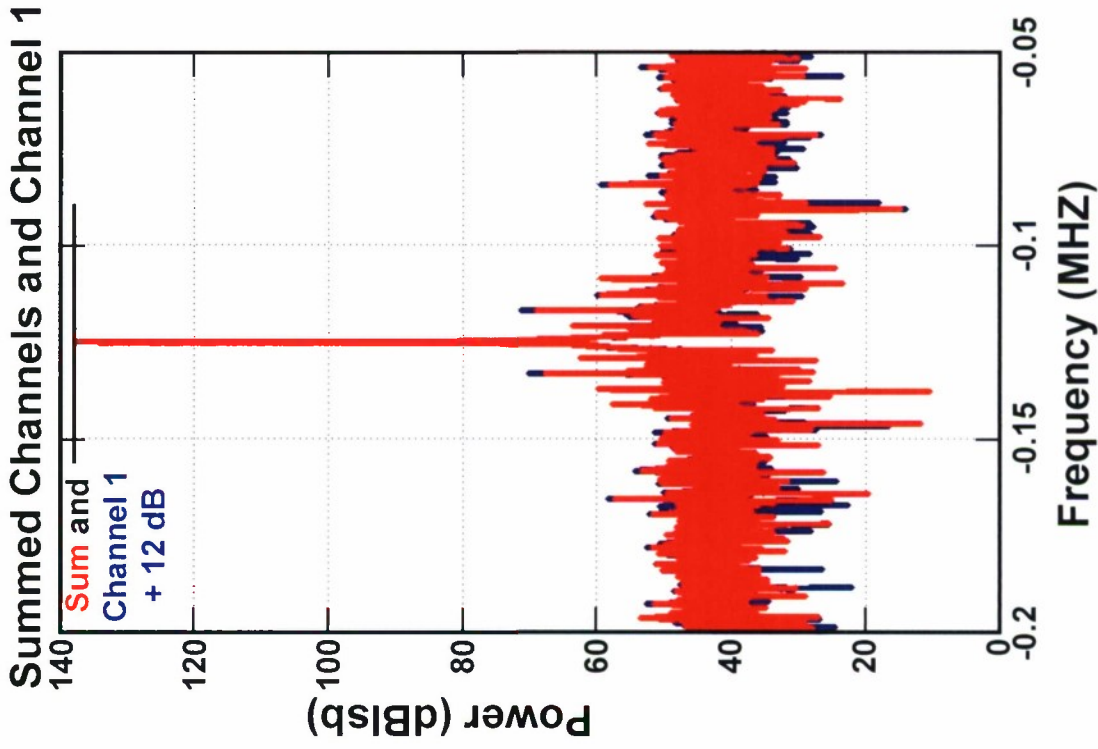


Legend

Sum of 4 Channels



# Noise Correlation



## Legend

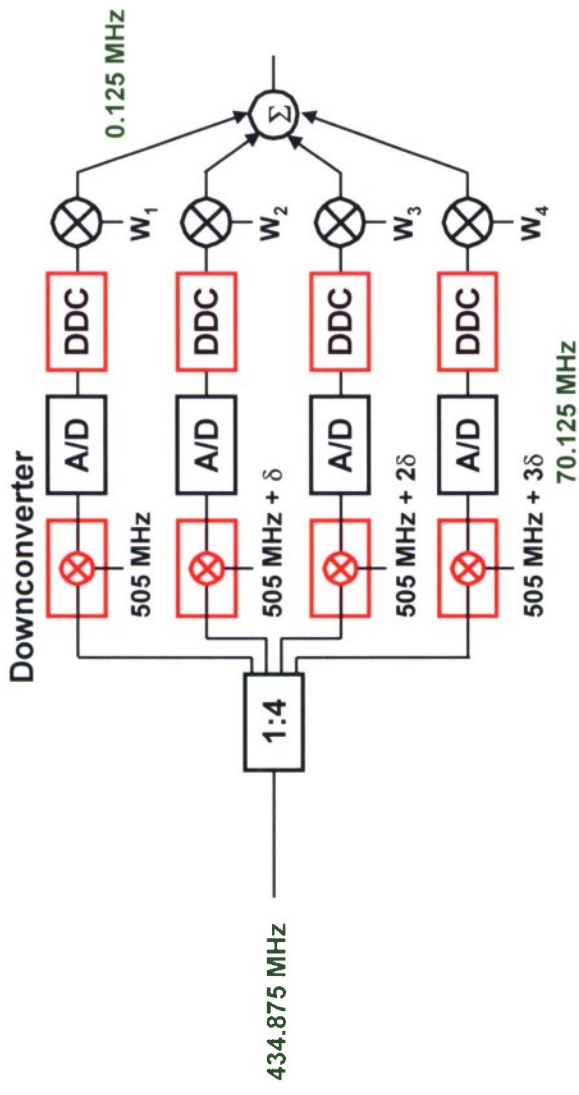
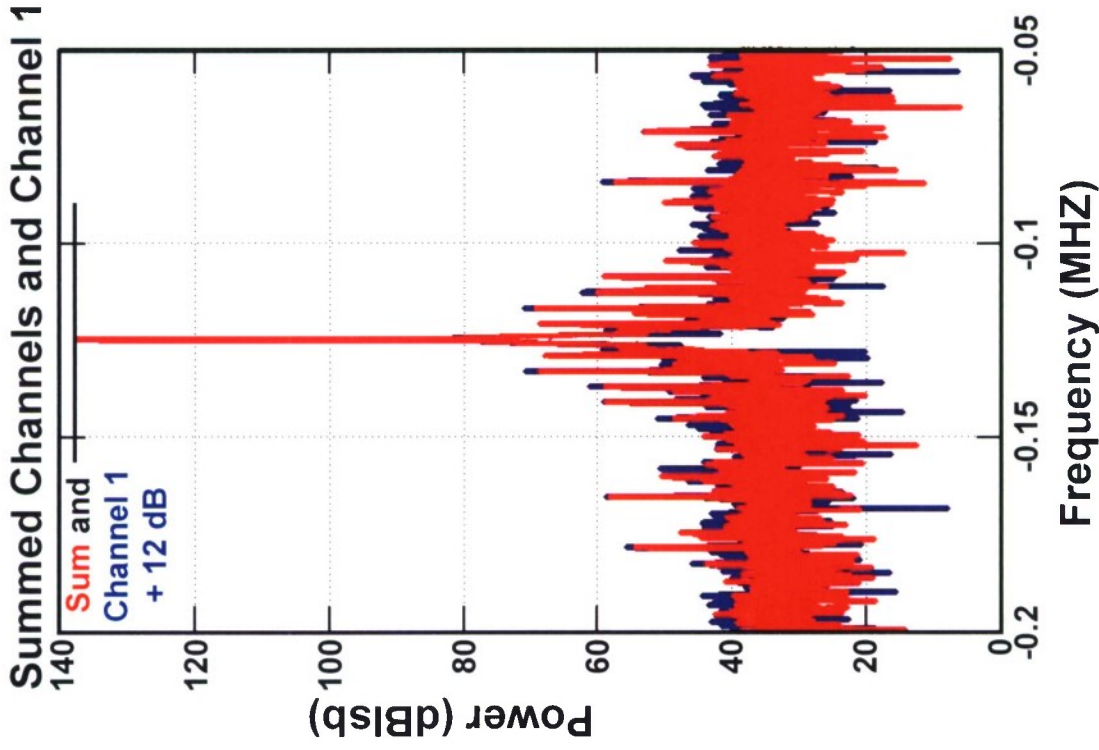
Channel 1 + 12 dB

Sum of 4 Channels

SNR of 4 Channels only 0.8 dB higher  
(vs 6 dB expected)  
⇒ Phase Noise is correlated!



# LO Offset Mitigation Applied to Noise Correlation Data



Legend

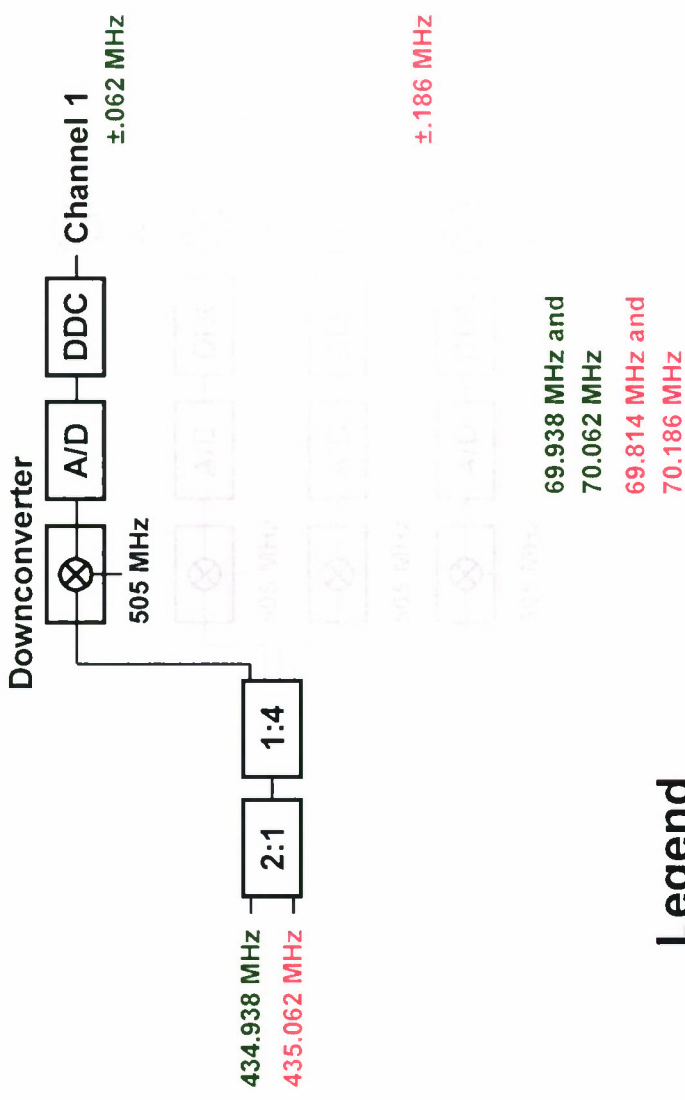
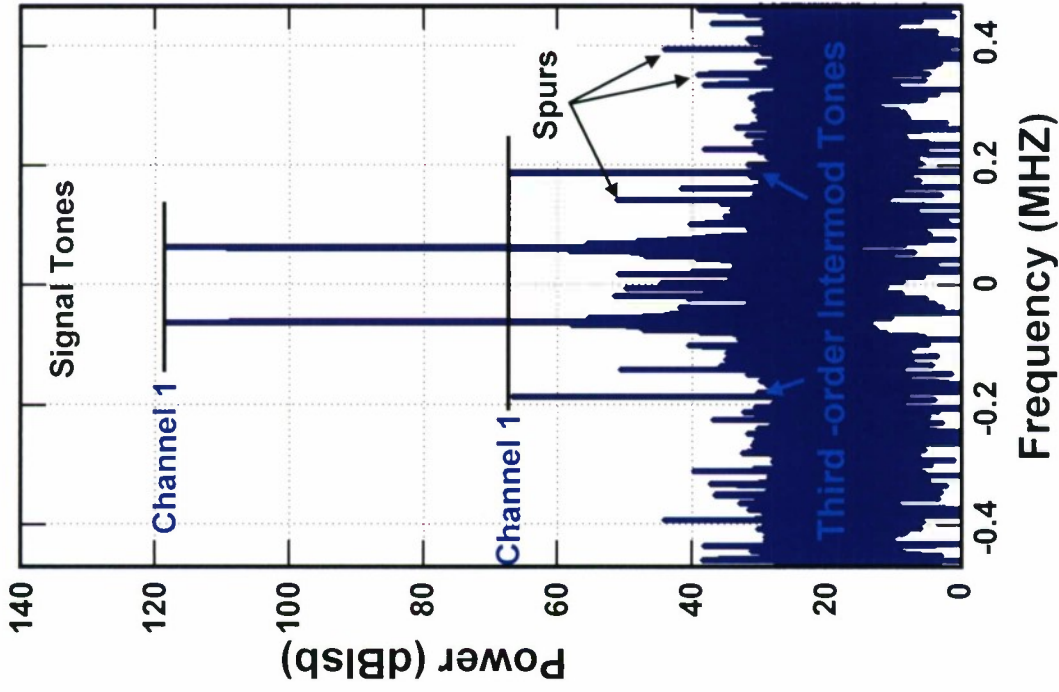
Channel 1 + 12 dB

Sum of 4 Channels

SNR Gain on Beamforming is Now  
4.1 dB (vs 0.8 dB)

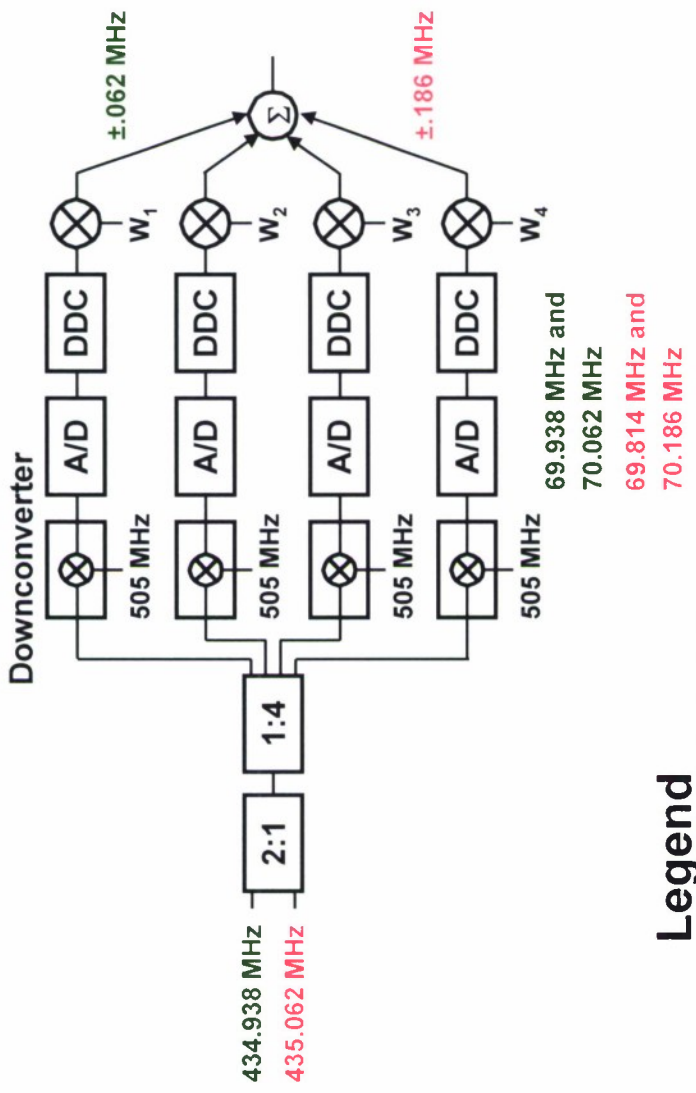
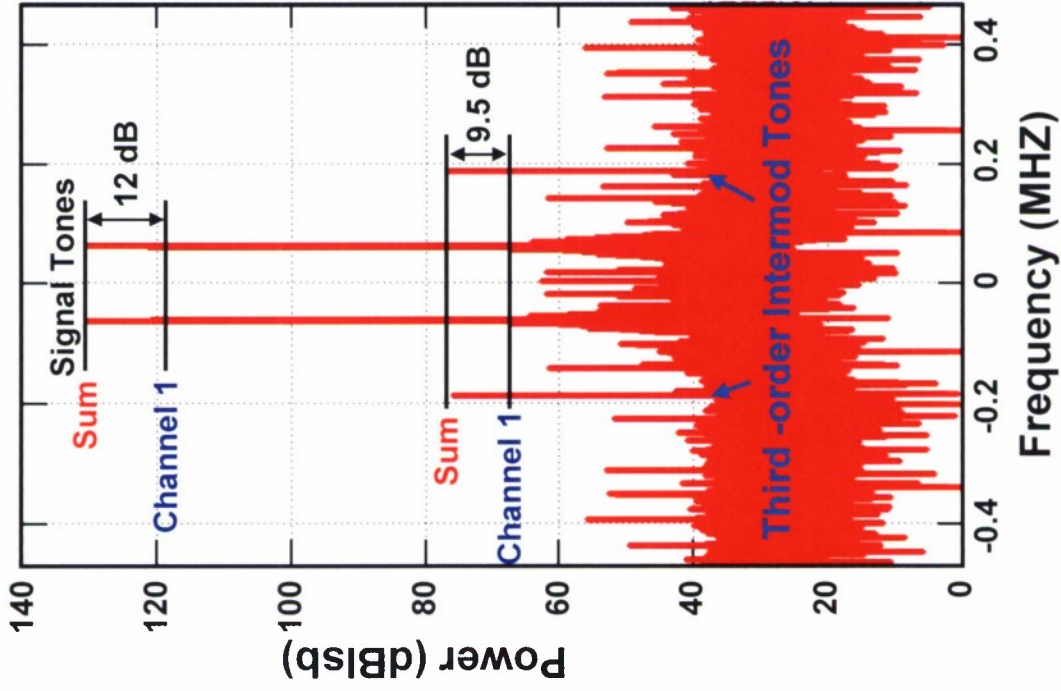


# Third-order Intermodulation Correlation





# Third-order Intermodulation Correlation



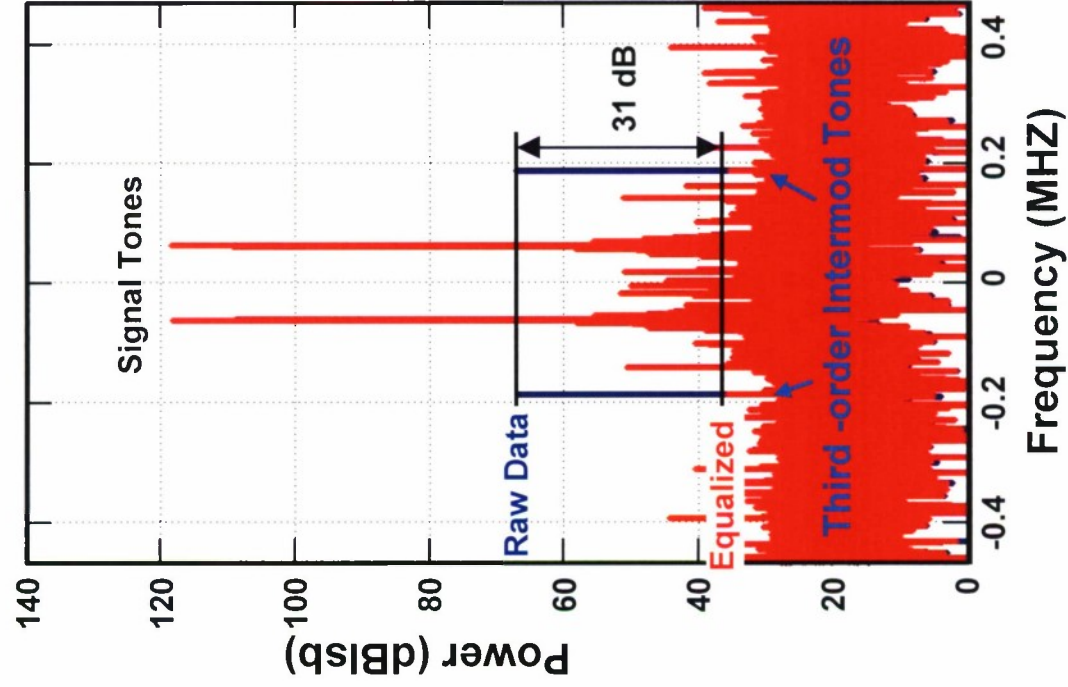
## Legend

Sum of 4 Channels

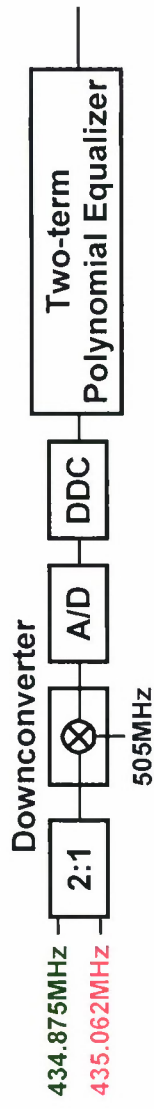
Third-order intermodulation distortion is correlated



# Third-order Intermodulation Mitigation



*Approach: Mitigate most undesired terms by decorrelating them, as described above. Then use a simple, low-order nonlinear equalizer to cancel the rest.*



## Legend

Raw Data

Equalized

Nonlinear Equalization Reduces Third-order Intermods by 31 dB



# Outline

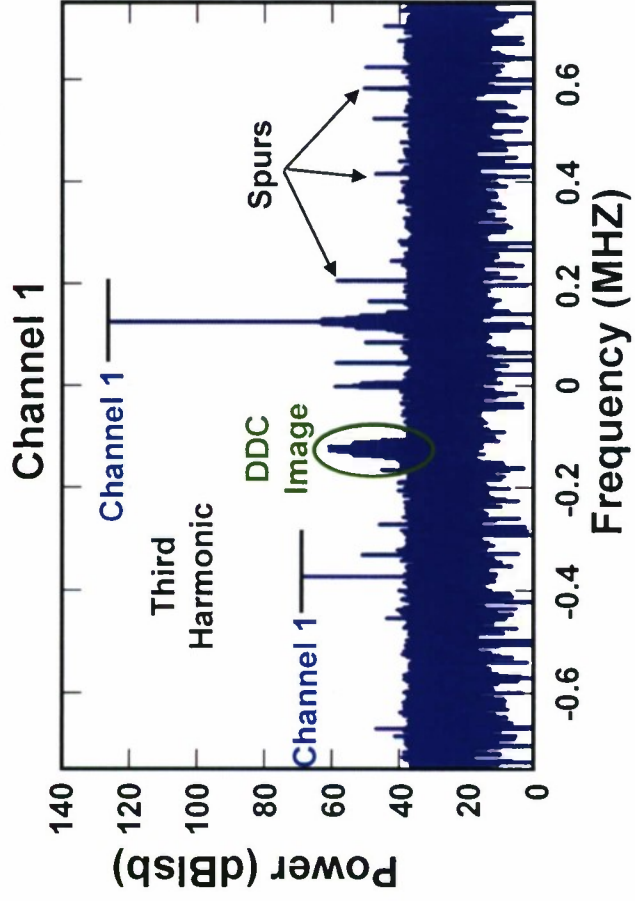
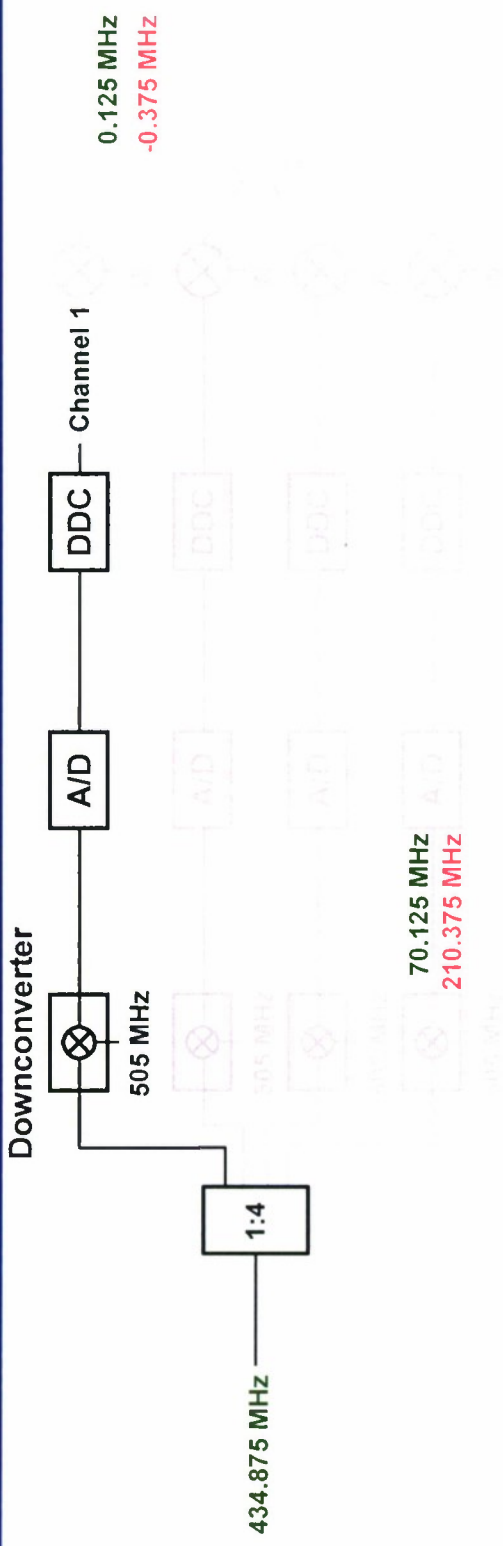
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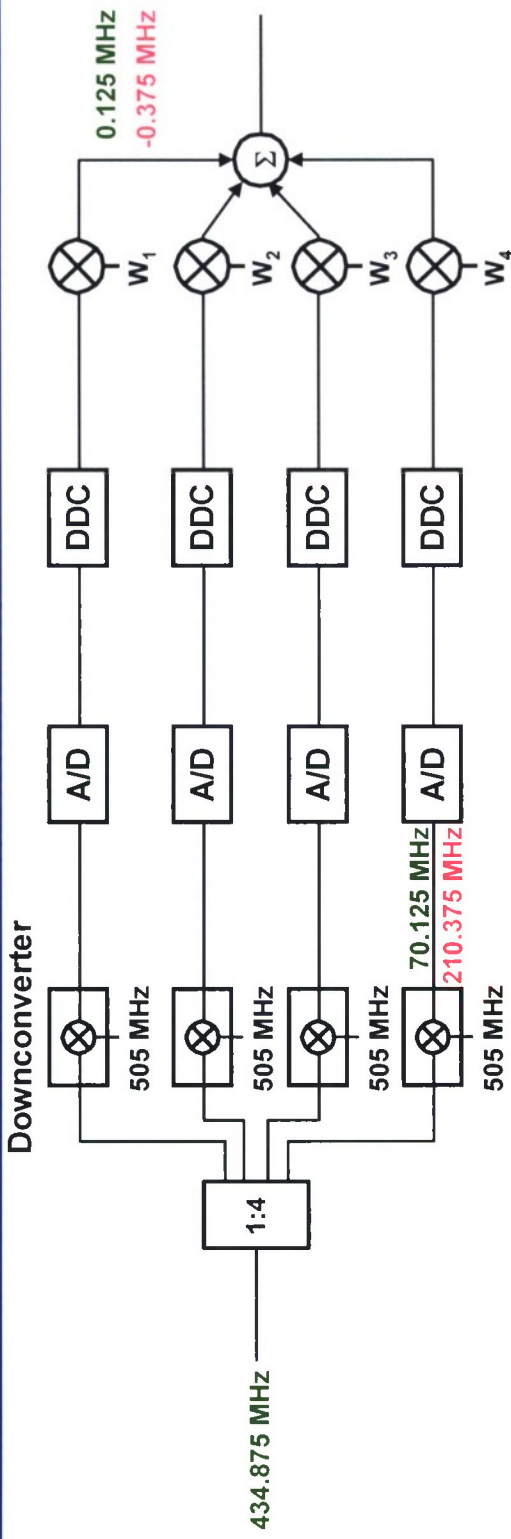
# Third Harmonic Correlation



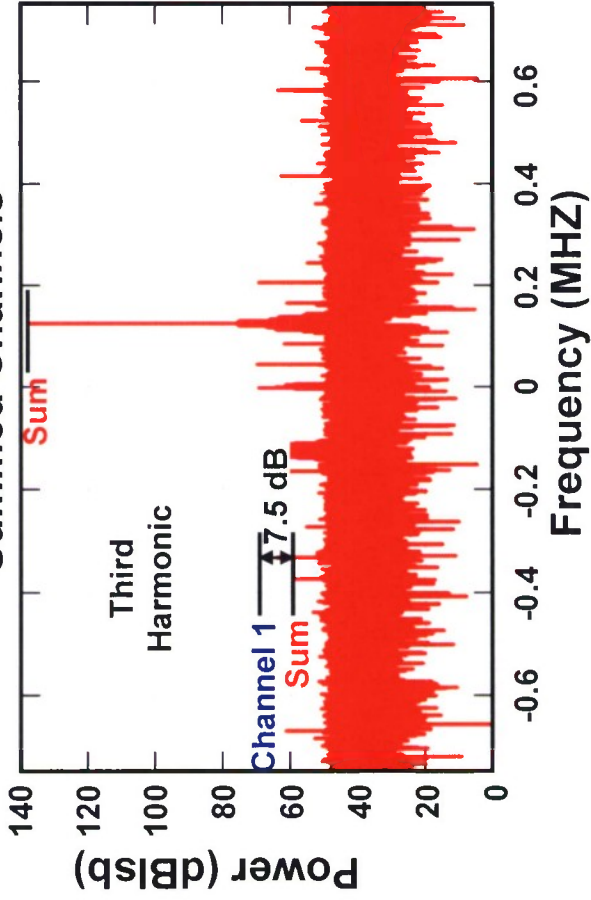
Legend  
Channel 1



# Third Harmonic Correlation



Summed Channels

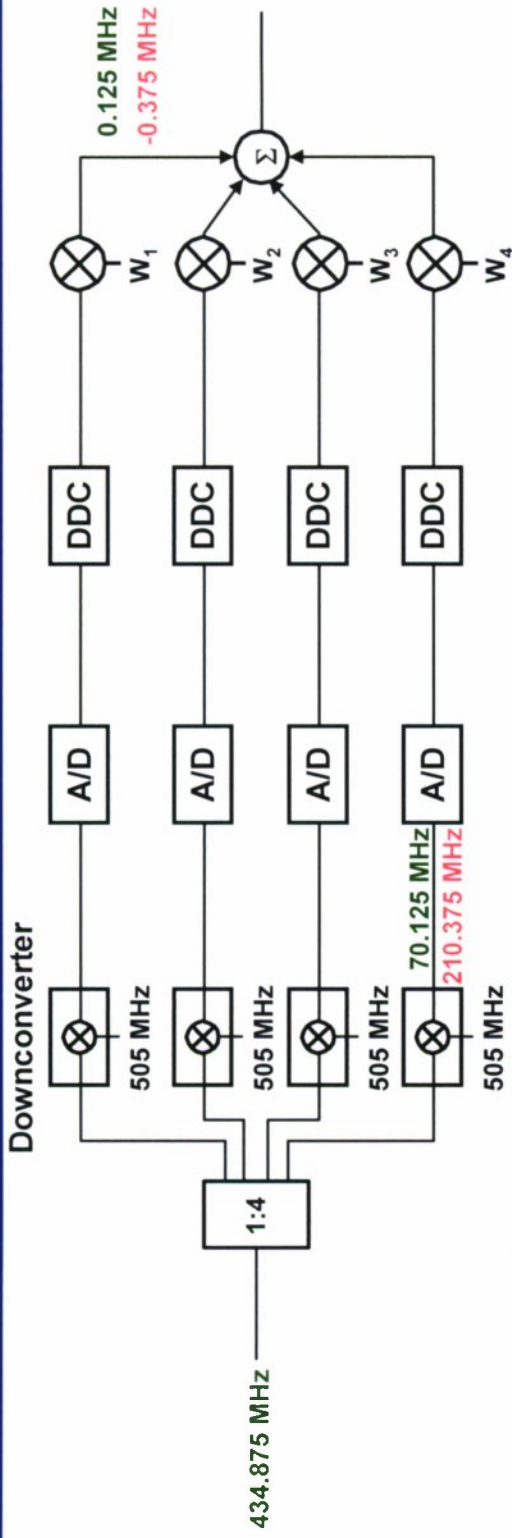


Legend

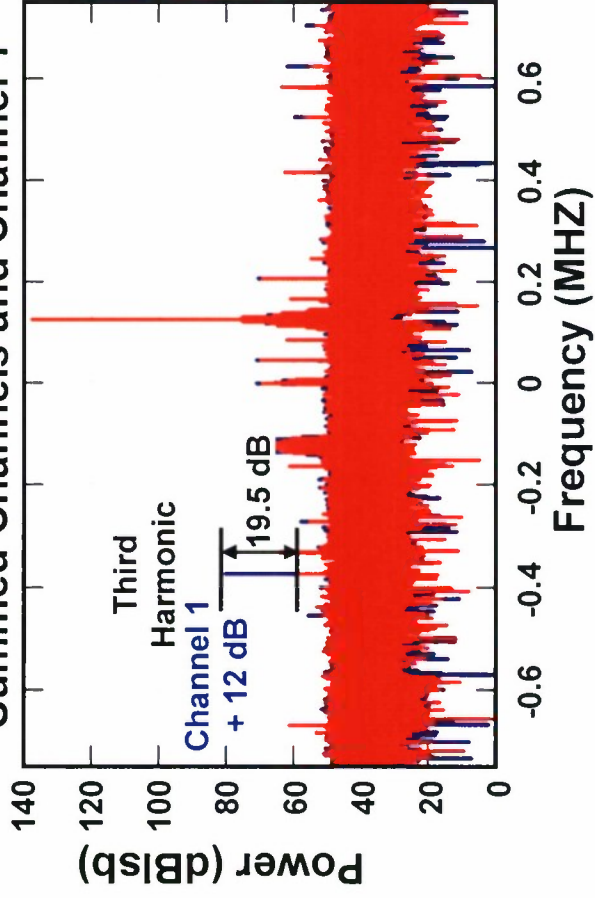
Sum



# Third Harmonic Correlation



Summed Channels and Channel 1



Legend

Channel 1


Sum

Third Harmonic is Uncorrelated



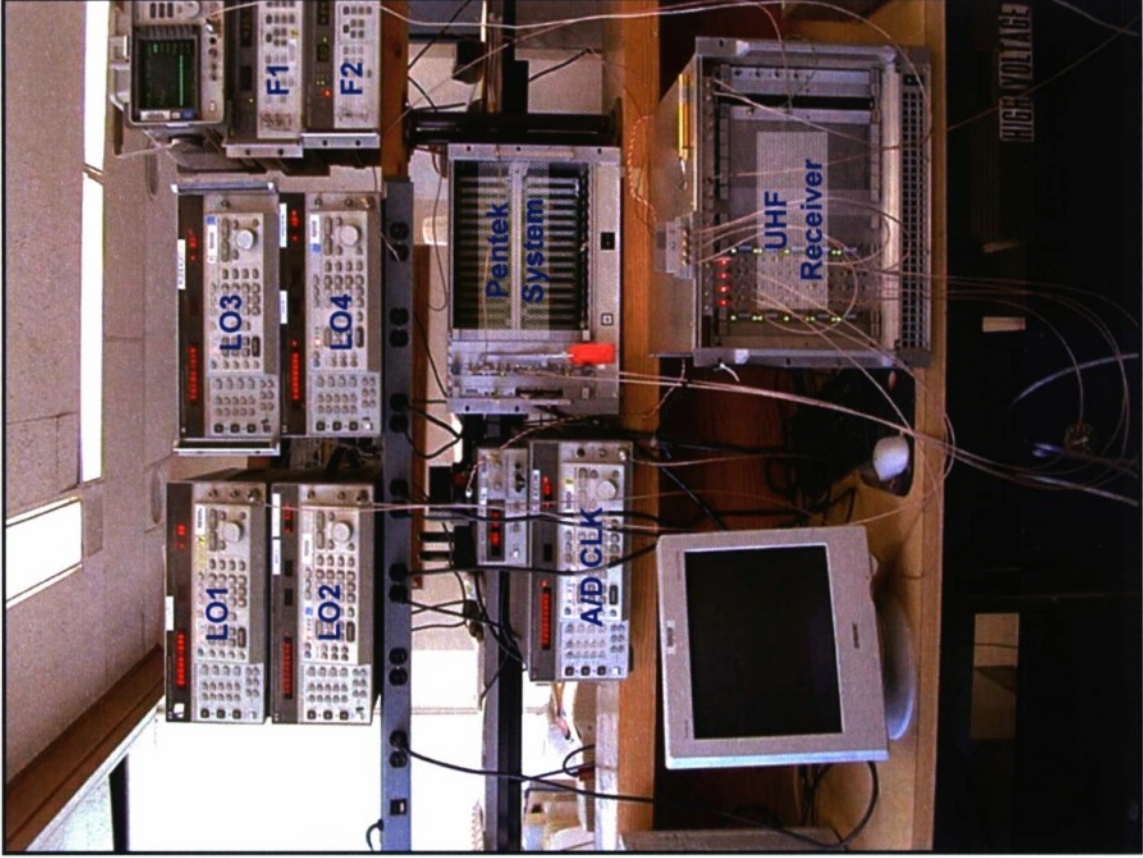
# Outline

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- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
-  Future work and summary



# Future Work



- Shrink synthesizers
  - DDS
  - Phase-locked sources
- Other transformations
- Investigate more channels when Linear Array Upgrade hardware is available



# Summary

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- Many sources of nonlinearities in a digital TR module
- Correlation of nonlinearities will reduce system dynamic range
  - Measurements show that some distortions are strongly correlated, while others are not
- Proposed method “Digital Array Nonlinearity Reduction” to make nonlinearities appear uncorrelated and/or incoherent from channel to channel
  - Several manifestations described
  - Simulations used to examine the effect on some specific nonlinearities
  - Technique has been verified experimentally

# REPORT DOCUMENTATION PAGE

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<b>14. ABSTRACT</b>  In a digital array, each receiver performs analog-to-digital-conversion (ADC), with the resulting digital data later combined via digital beamforming techniques. Since ADC is performed prior to beamforming, it is thus theoretically possible to enhance the dynamic-range (DR) of the individual receivers through post-ADC array integration gain. In practice, however, DR enhancement is limited by correlation of the nonlinearities (from receiver to receiver). Worse still, little published data exists on this subject (i.e., suitable for quantitatively assessing the correlation coefficients). This makes it difficult to predict how much DR enhancement will be achieved on real digital arrays.  This presentation describes the results of recent experiments involving a four-channel digital receiver system. The system was used to measure the correlation (between receivers) of different types of nonlinear distortion. The measurements quantitatively demonstrate that some nonlinearities are highly correlated. Next, the system was used to evaluate a recently proposed method for decorrelating nonlinear distortion in digital arrays. The measurements show that the mitigation technique is successful in decorrelating some nonlinear signal components.					
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