Fabri-Perot spectral filter that preserves image

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Abstract: Special arrangement of confocal Fabri-Perot interferometer is suggested as a spectral filter, which preserves image. Post-paraxial approximation is developed for the modes. Number of resolvable image pixels is estimated based on corrections to the eigenfrequencies.

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1. Introduction

In some cases of active illumination of the target, high-resolution spectral selection may be used for discrimination between targets moving with different velocities and of different chemical composition. Most monochromators and various spectrographs limit spatial and angular spread of input light, [1, 2], and thus distort image. Volume diffraction grating at normal incidence may be used for spectral selection. However, spectral resolution $\delta\nu$ (1/cm) is about $1/L$, i.e. limited by the longitudinal thickness L of the grating. Number of available image pixels $N_x * N_y \approx$ (Area)$*(\delta\theta/\lambda)^2$ is limited by the acceptance angle of the volume grating, $\delta\theta \approx 1/(2\pi L/\lambda)$, [3].

Fabri-Perot interferometers (FPI) yield better spectral resolution, $\delta\nu \approx (2 - |r_1|^2 - |r_2|^2)/L$, [1]. FPI with flat mirrors trades angle to the wavelength and thus distorts image. Pierre Connes suggested in 1956 using Fabri-Perot interferometer with spherical mirrors, whose foci are coincident, for the spectral selection of broad area and angles of input radiation, [3]. However, the image is also scrambled by confocal FPI (CFPI).

We suggest here an arrangement of CFPI, which yields high spectral selectivity and preserving the image.

2. Layout of the suggested device

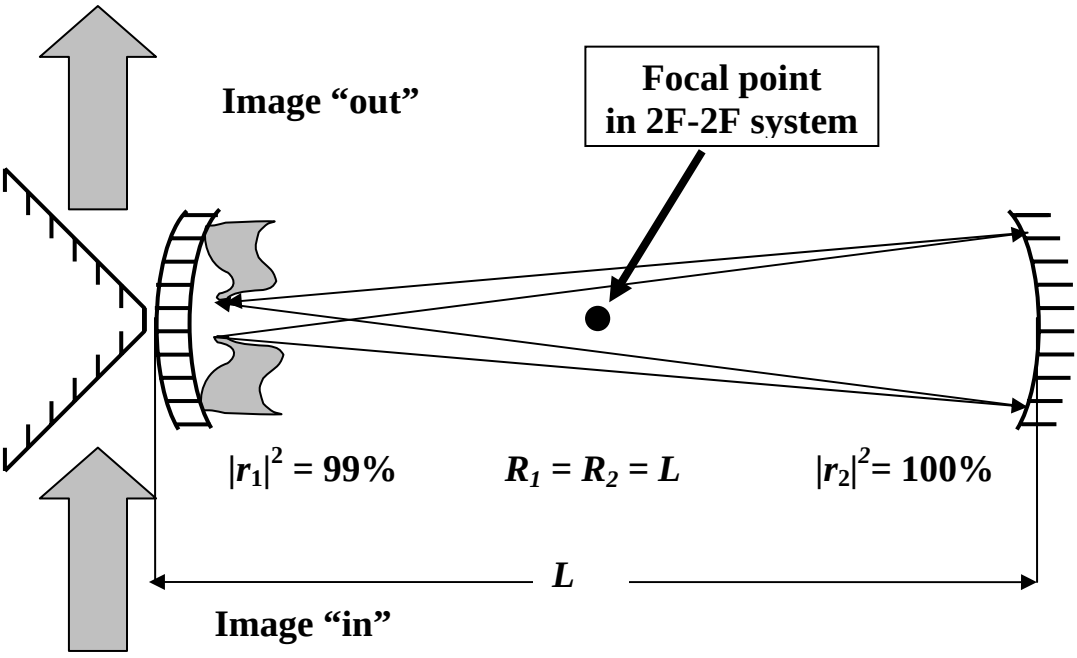

Fig. 1. This confocal FPI has highly reflective input/output mirror and 100% back mirror.

Input radiation illuminates the lower half of the first mirror. This light is imaged by the second mirror back to the upper half of the first mirror (2F-2F system!) From there spectrally filtered image is outputted.

3. Post-paraxial approximation for Gaussian beam and Hermit-Gaussian modes

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Number of undistorted image pixels equals half of the modes' number, which possess eigenfrequencies within CFPI's linewidth Γ. Since eigenfrequencies are coincident within the paraxial approximation, we explore post-paraxial corrections, both in the solutions of Helmholtz equation,

$$(\nabla\cdot\nabla + k^2)E(x,y,z) = 0, \quad k = 2\pi/\lambda, \qquad (1)$$

and in the boundary conditions at the mirrors. We use these notations:

$$R_1 = R_2 = L, \qquad z_0 = L/2, \quad a = \sqrt{z_0/k} \equiv \sqrt{L\lambda/4\pi} \equiv W_0/\sqrt{2}, \quad kz_0 \equiv kL/2; \qquad (2)$$

z_0, waist length (HWHM), a, waist width (HWe^{-1}M); W_0 = (HWe^{-2}M). A relevant exact solution of Helmholtz equation is

$$E(x,y,z) = \frac{\exp(ikR_{\text{compl}})}{R_{\text{compl}}}, \qquad R_{\text{compl}} = \sqrt{(z - iz_0)^2 + \rho^2}, \qquad \rho^2 = x^2 + y^2 \qquad (3)$$

This solution is a good start to describe paraxial Gaussian beam. It possesses singularity at $z = 0$, $\rho = z_0$, i.e. at large distance from the work space of this FPI. We use the expansion of the beam (3) up to corrections $\approx(\rho/a)^4/(kL)$ including:

$$G_{00}(x,y,z) = \frac{1}{1 + i(z/z_0) - \rho^2/[2a^2kz_0(1 + i(z/z_0))]} \cdot \exp\left\{ikz - \frac{\rho^2}{2a^2[1 + i(z/z_0)]} - \frac{\rho^4}{8a^4[1 + i(z/z_0)]^3 kz_0}\right\} \qquad (4)$$

This should be accompanied by the mirrors' shape equation:

$$z_{mirror}(\rho) \approx \pm\left[z_0 - \frac{\rho^2}{4z_0^2} - \mu\frac{\rho^4}{z_0^4}\right], \qquad (5)$$

μ = (1/16) for spherical mirrors. Envelope of the paraxial Gaussian beam has constant phase at the mirrors (also considered in paraxial approximation); phase is constant for Hermit-Gaussian modes as well, so that

$$\omega_{m,p,q} = ck_{m,p,q} = \frac{c}{2L}\big(2\pi m + \pi(p + q + 1)\big) - i\Gamma + c\delta k_{m,p,q}. \qquad (6)$$

Deviation $\delta\psi(x,y)$ of the phase at the mirror's surface, as it happens in post-paraxial approximation, leads to eigenfrequency correction,

$$M_{m,p,q}\big(z(\rho),x,y\big) = \pm\big|M_{m,p,q}\big(z(\rho),x,y\big)\big|\exp\big[i\delta\psi(x,y)\big], \qquad (7)$$

$$\delta k_{m,p,q} = -\left[\iint \delta\psi(x,y)\cdot\big|M_{m,p,q}\big(z(\rho),x,y\big)\big|^2 dxdy\right] \Big/ \left[\iiint \big|M_{m,p,q}(x,y,z)\big|^2 dxdydz\right]. \qquad (8)$$

Higher Hermit-Gaussian beams may be obtained via generating function:

$$F(u,w)\cdot G_{00}(x-u, y-w, z) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}(u)^p(w)^q M_{p,q}(x,y,z); \qquad F(u,w) = \exp[(u^2 + w^2)/4a^2] \qquad (9)$$

Lengthy calculations based on (6-9) result in limitation on the modes' transverse indexes p, q, for which the condition $\delta\omega < \Gamma$ is satisfied:

$$\big|c\delta k_{m,p,q}\big| \le \Gamma, \quad \Gamma \approx (2 - |r_1|^2 - |r_2|^2)\cdot(c/4L), \qquad N = \frac{1}{2}\delta p\cdot\delta q \approx \frac{4}{3}\cdot\frac{L}{\lambda}\cdot(2 - |r_1|^2 - |r_2|^2) \qquad (10)$$

This estimation was done for purely spherical mirrors; it constitutes the main result of this work.

Note that an ellipsoidal shape of the mirrors exists, which exactly coincides with the wavefront of Helmholtz equation's exact solution (3):

$$\mathrm{Re}\big[R_{\text{compl}}(x,y,z)\big] = 0 \quad \rightarrow \quad \left(\frac{z}{z_0}\right)^2 + \left(\frac{\rho}{z_0\sqrt{2}}\right)^2 = 1, \qquad (11)$$

and here $\mu = +(1/32)$.

Numerical estimation for L=1 m, λ=500 nm, $(2-|r_1|^2 - |r_2|^2)$=0.01 yields N≈30,000.

4. References

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