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Overview of Reduction Methods and Their Implementation Into Finite-Element Local-to-Global Techniques

by Michael A. Minnicino II and David A. Hopkins

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14. ABSTRACT
Reduction methods are used to reduce the number of degrees of freedom in a finite-element (FE) model, typically at the expense of high-fidelity solutions. The reduced solution's fidelity depends on the reduction method implemented. Most of the reduction methods described in this report were developed to efficiently determine the natural frequencies and mode shapes of large FE models. It is the goal of this report to briefly review these reduction methods and describe their implementation into feasible FE local-to-global analysis techniques. Local-to-global techniques are also part of the family of model reduction methods, but are normally used for structural analysis purposes.

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1. Introduction

Reduction methods are used to reduce the number of degrees of freedom (DOF) in a finite-element (FE) model, typically at the expense of high-fidelity solutions. The reduced solution's fidelity depends on the reduction method implemented. Most of the reduction methods described in this report were developed to efficiently determine the natural frequencies and mode shapes of large FE models. It is the goal of this report to briefly review these reduction methods and describe their implementation into feasible FE local-to-global analysis techniques. Local-to-global techniques are also part of the family of model reduction methods, but are normally used for structural analysis purposes.

2. Motivation

Explicit time domain FE solution techniques are attractive for simulations of many high-g loaded structures because of the increased computational speed and reduced memory requirements of these methods compared to implicit solution techniques. However, explicit methods require using small time-steps, compared to implicit methods, in order to solve the governing equations because explicit solvers are only conditionally stable. Each element's time-step is determined by its size and its material properties. The conditional stability requirement is dependent on the explicit solver employed in the solution. The maximum value of this stable time-step is proportional to the size of the smallest element used in the model. This time-step restriction is commonly known as the Courant condition. Conversely, most implicit methods are unconditionally stable, thereby allowing an arbitrarily large time-step, but implicit methods also requires matrix inversion, which is computationally expensive. Accuracy considerations further limit the size of the allowable time-step in implicit methods. Finally, explicit techniques are faster and more reliable in obtaining results for most high-g FE simulation models. However, FE models of these structures can require hundreds of thousands of elements to accurately capture the desired structural response. This is especially true when modeling electronic packages to determine operational failure. If solutions are desired in a timeframe where the FE method can be used as a design tool for electronic packages, a significant reduction of the computational effort for a given CPU architecture (i.e., a set number of processors, processor speed, and amount of memory) is required.

An FE model, whether of a cell phone or a smart projectile, can have large differences in the scales of the geometry that must be modeled. For instance, structural features in high-g launch applications may have characteristic dimensions of meters, while the electronic components and/or electronic packages can have characteristic dimensions of millimeters. Such large differences in scale lengths present significant issues for generating appropriate FE models of

these structures. Some possible meshing choices include using a uniform mesh, using disparate meshes with constraints, and using global/local modeling techniques.

First, a uniform mesh for the entire FE model with the characteristic element size based on the electronic component's geometry can be generated. While the resultant mesh has element sizing that can accurately capture the electronic component's response, larger features of the geometry may have an overly refined mesh. This approach can yield accurate response data for the entire model, but at the expense of solution time due to having more elements and therefore more DOF than needed. The solution time is further increased because of the size restriction of the solution time-step by the inclusion of the small structural features into the FE model. This reduction in the time-step size can also result in inaccurate results if numerical round-off errors accumulate in the solution.

A second approach is to mesh the individual components of an FE model separately. This approach reduces the total number of elements, but the resultant mesh has inconsistent element sizing. Subsequently, interface constraints, such as contact, are defined at the interfaces between the noncongruent meshes. Material properties excluded, the solution time-step is controlled by the smallest element in the model and thus long analysis times can still be issue. Also, depending on the type of interface used to couple the disparate meshes, results near the interfaces may be unsatisfactory. For very complex structures, it is possible that properly defining the interface can be a serious issue as well, since multiple interfaces may share some nodes or element faces (*1*).

Finally, a third approach is to perform separate local and global analyses. The global/local approach is advantageous when disparate length scales must be modeled, such as when modeling the response on electronics that are embedded in projectiles. The local model normally consists of all those components whose inclusion would either lead to an excessive number of elements and/or a significant reduction in the time-step used in the solution. Determination of the local model is thus a subjective process. The global model then consists of all the remaining components. The goal in the selection of the global and local models is to allow larger elements to be employed in the global model, thus increasing the size of the stable time-step and consequently reducing the computational time needed. Also, because of the reduction in the number of elements and nodes there are fewer DOF to solve. The correct selection of the local and global models therefore reduces the computational effort in two ways: (1) increasing the characteristic time-step and (2) reducing the total number of elements in the model.

A global/local approach is sometimes referred to as a submodeling or substructuring approach, depending upon how the model is formulated and solved. However, in the context of this report, we will reserve these terms for specific modeling approaches discussed later and use global/local to refer to the more general case where disparate length scales and corresponding mesh scales are required to obtain a representative model of the structure. The global/local approach can also allow the use of multiple local models, depending on the global-local modeling method used.

However, the correct representation of the local model's structural response or the proper representation of the effects of the local model upon the global model's response can be difficult to determine and verify. This report reviews several possible approaches to global/local modeling and discusses their applicability to modeling high-g environments.

3. Traditional Global/Local Process

First, though, it is advantageous to review the typical global/local modeling process, which is shown in figure 1. Conceptually, this is very straightforward. As mentioned, models are typically constructed such that the smaller components of the model are modeled with high detail due to the need of hi-fidelity measurements/results. When this is done, the smaller elements of the highly detailed component are selected to form the local model and the larger elements form the global model. The resulting FE models each retain consistent element sizes. In this approach, the highly detailed subscale local FE model and the larger scale global FE model can be developed simultaneously. The global model's loading, both magnitude and location, is assumed known a priori. It is also assumed that the time histories for the global model, e.g., acceleration-time history, can be applied to the local structure at the local-to-global interfaces. The local model's structural properties are homogenized via a qualitative local-to-global method, since the applied local load is not the true load seen by the local model. This homogenized local model is then incorporated into the global model. The response of the resulting model is then determined and a global solution is obtained. The global nodal solution at the local-to-global interfaces is then used as the applied load to the detailed local model for the local analysis. Also, for all reduction methods, the choice of the reduction method leads to different approximations to the true response of the structure.

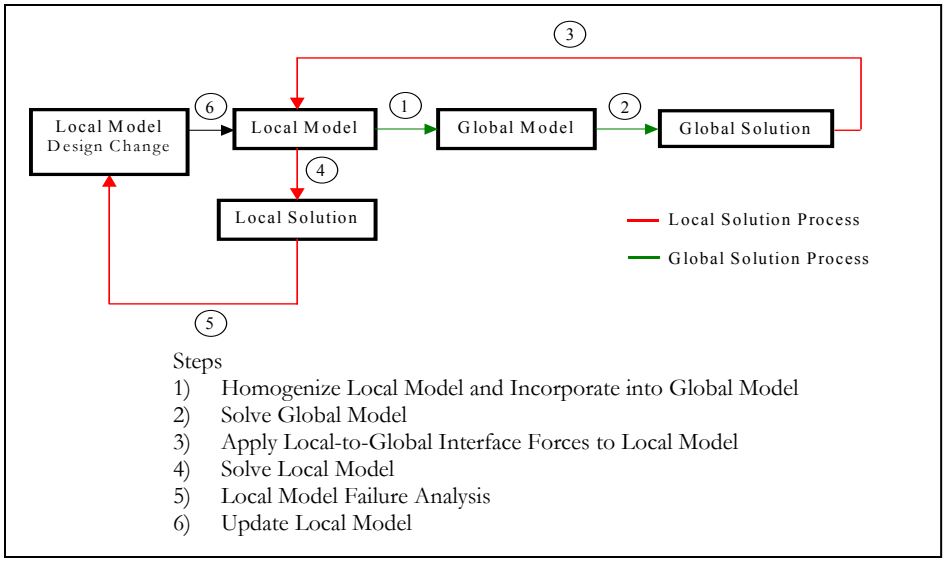

Figure 1. Depiction of global/local process.

The issue of whether the local response affects the global response can be investigated by repeating the global/local process, using various methods to homogenize the properties of the local structure, and then looking for differences in the global response. If the global model's response does not change for the differently homogenized local structures, then it is normally assumed that the local response probably does not affect the global response. Alternatively, if it is found that the global response differs between the differently homogenized local structures, then it is assumed that the local response at least partially drives the global response. The question of how strongly one response affects the other needs to be determined in this latter case. If the dependence is weak, it can be concluded that the global response is largely unaffected by the local response except at or near the local-to-global interface. However, decisions concerning whether the solutions are accurate based on the qualitative response of the local and global models may not be sufficient for determining a viable design. It is precisely these uncertainties that this traditional approach to the analysis fails to answer. Quantitative measures of the accuracy of the analyses are needed if the designer is to make informed decisions regarding potential failure of a design. Several global/local techniques have been proposed which can address this concern.

4. Static, Quasi-Static, Dynamic, and Transient Domains

Before proceeding with discussion of the techniques, it is important to note that some of the discussed reduction methods were developed and implemented for specific time domains. For the remainder of this report, the following terms are defined:

4.1 Static Domain

The applied load is static, or changes so gradually that there are no rate effects and the entire system is in static equilibrium with the applied load at all times. Inertial effects are nonexistent and the model is completely defined by its stiffness matrix.

4.2 Quasi-Static

The applied load is time dependent, yet rate effects are not present. The inertial response of the structure is restricted to the response where the entire structure is subjected to a constant acceleration field. The system deforms and may also translate/rotate in this constant acceleration field. Acceleration loads are normally applied as D'Alembert forces.

4.3 Dynamic Domain

The period of load may be shorter than the quasi-static domain, and like the quasi-static domain, rate effects are not present. However, inertial effects are present such that different parts of the structure are accelerating at different rates. The system is in dynamic equilibrium. Vibratory motion exists, as well as deformation and translation/rotation in a variable acceleration field.

4.4 Transient Domain

The load is applied rapidly, as in a shock event. Rate effects are present. Steady state motion, such as vibratory motion, has not had time to become established. Wave propagation effects must be considered.

5. Qualitative vs. Quantitative Analyses

Currently, the simplest global/local methods to implement provide qualitative information in the sense that the true loading seen by the local structure is unknown and therefore only the trending of the response as a function of design iteration can be determined, similar to a parametric study. A typical qualitative example is applying the solution time-history results from a global model, e.g., an acceleration located at the global model's CG, to the local structure at the global/local models' interface. Thus, the designer can look at the differences in the local structure's responses due to local structural changes. This approach can enable the designer to determine the important structural parameters at the expense of not knowing the true response. For initial design iterations, this level of knowledge is sufficient. These qualitative analyses can still require considerable computational time and effort.

For dynamic events, these qualitative methods cannot be used to accurately predict mechanical failure of an electronics package or component, since they do not include the transmissibility effects that manifest as amplification or attenuation of the transmitted force to the local structure from the global loading through the load path. This is because these qualitative analyses do not account for a coupled response between the local and global models. In a quantitative analysis, the amplification/attenuation of the transmitted force is captured and thus accurately represents the true load seen by the local model. This allows for the accurate determination of failure of an electronics package or component.

6. Reduction Methods Overview and Local-to-Global Implementation

6.1 Overview of Nonmathematical Methods

The first class of global/local methods discussed are not mathematically based, but rather a "best-practices" approach to increase the computationally efficiency at the cost of quantitative results.

The methods discussed in this section include the following:

- uncoupled global/local analysis
- lumped mass approximation
- submodeling

6.1.1 Uncoupled Global/Local Analysis

The simplest reduction method, this qualitative method involves partitioning the original model into a global model and a local model. In this approach, the global model solution is obtained without feedback from the local model, i.e., it is assumed that local model has no effect on the global model's response. Consequently, the local model isn't even considered in obtaining the solution for the global model. The global solution, e.g., displacement or acceleration, at the global model's local-to-global interface is then prescribed as the local model's loading at the interface. The unconstrained explicit solution for the local model is then determined, and a qualitative failure analysis can be conducted as a function of design iteration. A slight refinement to this technique is to model the local model as a rigid body in the global model, thus capturing the rigid body inertial response of the local structure in the global solution. However, modeling the local model as a rigid body will stiffen the global model at the global/local interface. If the global model is actually compliant at this interface, then poor results will be obtained from this enhancement.

As is typical for local/global approaches, this modeling technique reduces the FE computational cost in two ways. First, the total number of DOF is reduced for the solution by eliminating the elements associated with the local model. Secondly, the elimination of the smaller elements that constitute the local model increases the size of the characteristic time-step to be proportional to the smallest retained element of the global model. The computational savings of the global model is realized by taking larger time-steps and by solving fewer equations.

One may consider the reduction in the number of DOF in the global model superficial because these DOF are subsequently solved for in the separate local model analysis. However, given the local excitation obtained from the global solution and the assumption that the local model response does not significantly affect the global model response, then, as a design tool, this FE approach is suitable because it allows for expedient local model solutions and analyses due to local model design modifications. Specifically, since the response of the global model is independent of the local model, different local models can be analyzed using the same loading as specified by the global model. This approach provides the fastest solution iterations, but at the expense of assuming there is no coupling between the global and local models' responses. This approach is equivalent, both in spirit and practice, to the qualitative approach discussed earlier.

6.1.2 Mass Element Approximation

This method replaces the local model with equivalent point masses distributed at the local-to-global interface in the global model. This method thus accounts for the local model's translational inertial loading upon the global model's response. The local model's mass is distributed to the nodes on the interface surface of the global model's elements. Using this method may allow the modeler to remove contact interfaces, thereby further decreasing the number of equations to solve. This effectively reduces the model's computational complexity by eliminating/truncating the local model DOF, yet it includes the inertial feedback that may be

important in high-g loading scenarios. Also, the use of mass elements means that the global solution's time-step size is dictated by the smallest global model element for an explicit analysis. The global solution is obtained and the nodal results corresponding to the local-to-global interface are then applied as the loading of the local model. Errors due to the misrepresentation of the moments of inertia may be significant at high-g loadings, thereby limiting its implementation into more complex loading scenarios. This approach assumes that the stiffness of the local model has no effect on the global response. This modeling technique is reasonable when the acceleration of the global system is dominated by a uniaxial response, and the stiffness of the local model is low compared to the global model.

Figure 2 presents results for an axially accelerated disk with electronic components attached. The baseline response, figure 2a, is the computed response for the full model shown in figure 2d. Figure 2a shows contours of the axial displacement for the baseline model. Figure 2b shows the computed response when the components are replaced by lumped mass approximations. As can be seen, the axial response for the baseline model is very similar to the lumped mass approximation at the same solution time. This is more clearly shown in the plot in figure 2c, which plots the time history of the axial displacement for each model. However, while the axial displacement compares very well, the stress levels, as shown in figure 3, are very different. This difference is probably due to the stiffening effect of the actual components. Examining figure 3a closely, it is seen that the actual location of the components are clearly evident in the contour plots. The attachment points for the chips, capacitors, and resistors show up as either isolated high or low stress areas at these points.

6.1.3 Submodeling

Submodeling is used to analyze a local region of an FE model with a refined mesh based on the interpolated results of the coarsely meshed global solution. Submodeling makes use of St. Venant's Principle; thus, the submodeling results are accurate if both St. Venant's Principle is followed and if the homogenized properties for the local model are reasonable. Both of these conditions are vague; consequently, the validity of a submodel is highly dependent upon the skill of the modeler.

In the submodeling approach, a global model is developed and its solution determined. The displacements at each node that enclose the local model's boundary are determined from the global solution. These nodal displacements are then interpolated and applied to the local model at the local model's boundary as the forced response boundary conditions (*2*). The accuracy of the local model depends on how closely the global model captures the solution at the local model's boundary. The computational savings is achieved by forcing element size consistency and by reducing the total number of elements in the global model. The success of this approach is highly dependent upon the determination of the homogenized properties used in the global analysis.

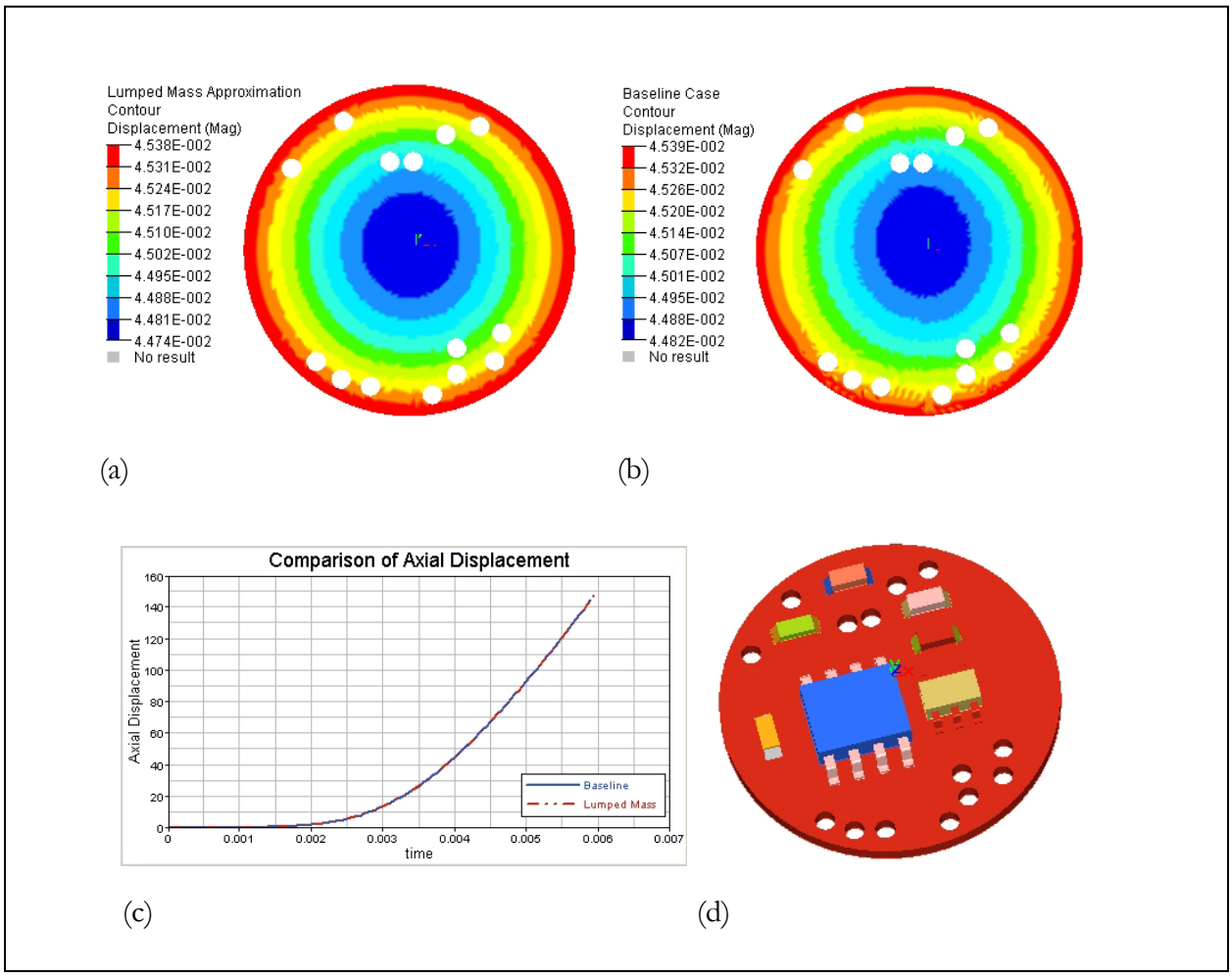

Figure 2. Comparison of axial displacement of baseline and lumped mass for a simple chip with components.

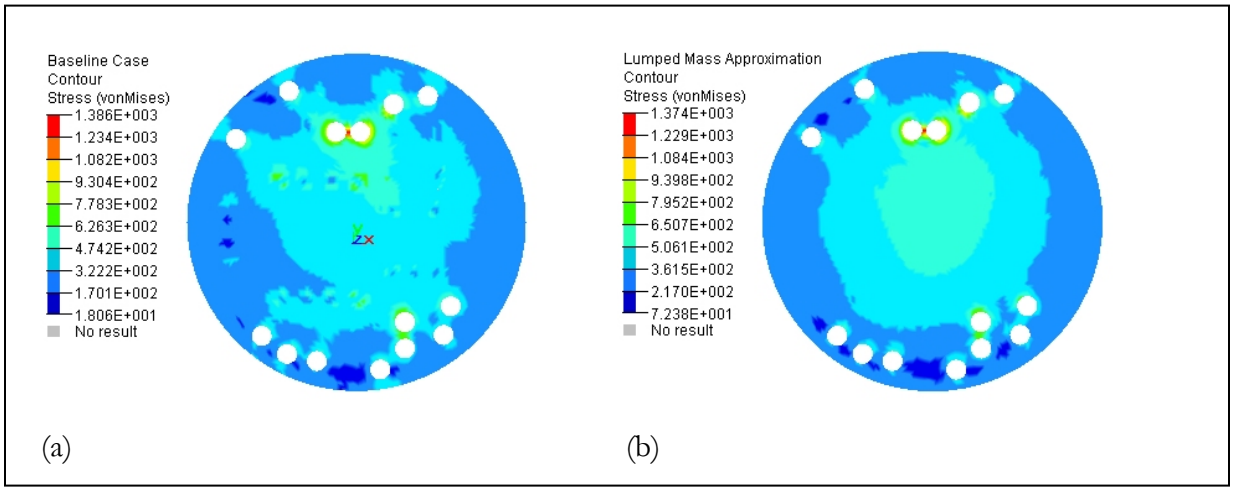

Figure 3. Comparison of von Mises stress of baseline and lumped mass for a simple chip with components.

Once the global solution is determined, the nodal displacements enclosing the local-global boundary are interpolated and are used as a prescribed displacement boundary condition of the local model. In figure 4, solid black lines depict three out of many possible local-global boundaries for a submodeling analysis. For local-global boundary 2, the "x" symbols denote the nodes of which the global solution nodal displacements are interpolated to effectively drive the local solution. Note that the local model mesh does not need to be congruent with the global mesh. The local results for the three boundaries are seen in figure 5. It is seen that the error increases as the boundary moves into the region affected by the local structure. This is due to St. Venant's Principle.

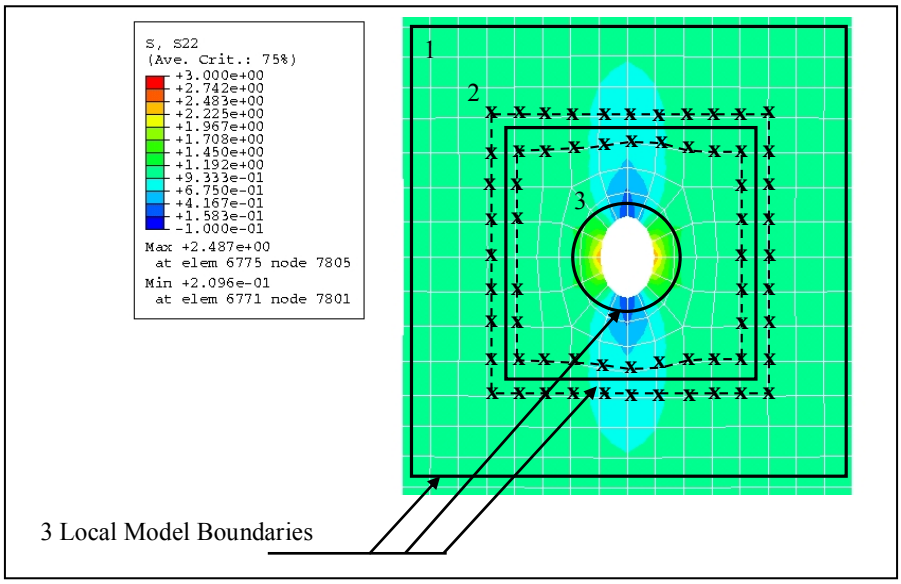

Figure 4. Global model with three possible local-global boundaries. The "x" symbols denote the nodes from which results will be interpolated for boundary 2 and subsequently prescribed on the boundary of the local model to effectively drive the local solution.

6.2 Overview of Mathematical Methods

In general, an FE analytical model leads to a system of equations consisting of mass, damping, and stiffness matrices M, C, and K, respectively. Usually, the model is considered to have proportional or zero damping if a linear analysis is conducted. In a nonlinear analysis, advanced constitutive models are typically employed which account for material damping directly in the constitutive law. For simplicity, in the following discussion, the damping is set to zero. The resulting equations of motion are given by

$$M\ddot{x} + Kx = f(t)\,. \tag{1}$$

The dimensionality of the mass and stiffness matrices is N × N, where N is the length of the coordinate vector x, which represents the DOF of the model.

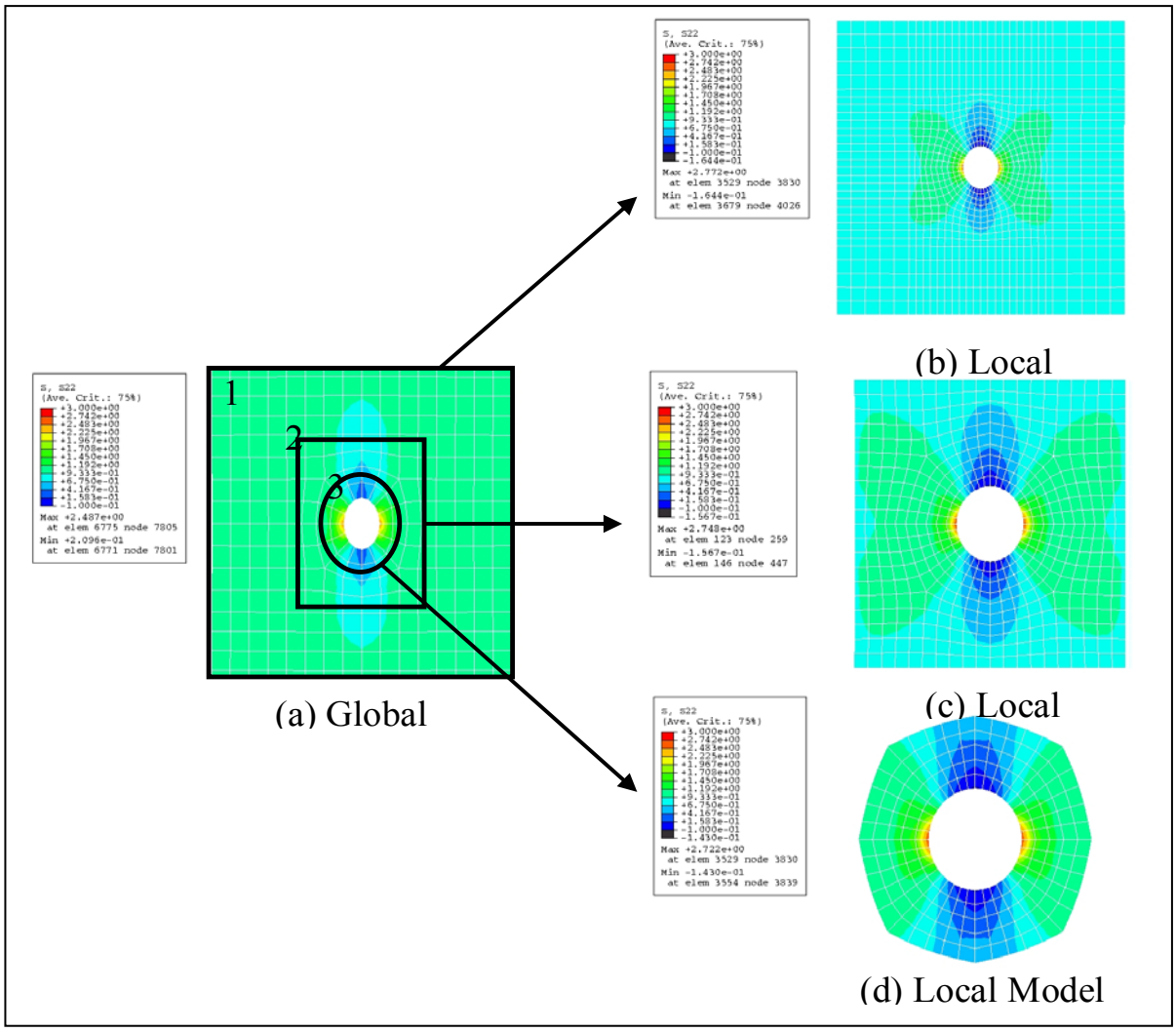

Figure 5. (a) Global model with three boundaries, (b) local model results corresponding to boundary 1, (c) local model results corresponding to boundary 2, and (d) local model results corresponding to boundary 3.

Reduction techniques often utilize a projection matrix T, sometimes referred to as a condensation or expansion matrix, depending on its usage, to relate the retained DOF and the truncated DOF of the original model. To clarify, the original full set of N DOF, x_n, is partitioned into the retained "r" DOF and the truncated "t" DOF. Thus, x_n can be expressed as

$$x_n = \begin{Bmatrix} x_r \\ x_t \end{Bmatrix} = T x_r . \tag{2}$$

The form and calculation of the projection matrix, T, depends on the reduction method implemented. Note that, whereas the global/local methods described in subsection 6.1 do not have clearly defined projection matrices, the methods in this section are clearly defined. For

these methods, the reduced mass and stiffness matrices are found by pre- and postmultiplying by the projection matrix T,

$$\begin{aligned} M_{rr} &= T^T M T, \text{ and} \\ K_{rr} &= T^T K T, \end{aligned} \tag{3}$$

where the reduced mass and stiffness matrices, M_{rr} and K_{rr}, are of size r x r.

The following methods have been used to generate representations of the local model in the global model:

- substructuring, superelements, and guyan reduction
- dynamic reduction
- summed of weighted accelerations technique (SWAT)
- system equivalent reduction-expansion process (SEREP)

6.3 Superelements/Substructuring and Static Reduction

Generally, substructuring is the process of decomposing a large FE model into smaller, component-based models. These component models are the substructures of the full system. A definitive distinction between superelements and substructures is difficult to define. The ABAQUS/Standard User's Manual (*3*) makes the following delineation between substructures and superelements: a substructure retains its response data at internal DOF locations within the local model; whereas, a superelement does not. Therefore, the implementation of substructures and superelements are equivalent with the recording of results being the difference. Substructure will refer to both substructure and superelement for the remainder of this report

Static reduction, sometimes referred to as Guyan reduction or static condensation, is the most popular reduction method, and is the basis for several FE substructuring techniques. This reduction method applied in FE techniques reduces the FE model by condensing (truncating) internal DOF. Specifically, the technique removes the DOF not located at the substructure's boundary, the local-to-global interface. These remaining DOF, located at the boundary, retain the stiffness of the local structure, but omit the inertial terms to create a more compact and thus more efficient representation, at the cost of accuracy for non-static loading conditions. This discussion is similar to another overview paper discussing Guyan reduction, as well as the discussion for dynamic reduction, improved reduced system (IRS), and SEREP (*4, 5*), where the focus is modal estimation. The difference here is the application of these reduction techniques for efficient structural analysis rather than efficient modal analysis.

The static reduction method partitions the DOF, force vectors, and the mass and stiffness matrices into subvectors and submatrices relating to the retained DOF, x_r, and the truncated DOF, x_t. Ignoring the inertial terms of equation 1, the partitioned stiffness matrix K is

$$\begin{bmatrix} K_{rr} & K_{rt} \\ K_{tr} & K_{tt} \end{bmatrix} \begin{Bmatrix} x_r \\ x_t \end{Bmatrix} = \begin{Bmatrix} f_r(t) \\ f_t(t) \end{Bmatrix} . \tag{4}$$

The assumption is then made that no external forces act on the truncated DOF, thus

$$\begin{bmatrix} K_{rr} & K_{rt} \\ K_{tr} & K_{tt} \end{bmatrix} \begin{Bmatrix} x_r \\ x_t \end{Bmatrix} = \begin{Bmatrix} f_r(t) \\ 0 \end{Bmatrix} . \tag{5}$$

Equation 5 can be solved for the truncated DOF; thus,

$$\begin{aligned} &K_{tr} x_r + K_{tt} x_t = 0, \text{and} \\ &x_t = -K_{tt}^{-1} K_{tr} x_r . \end{aligned} \tag{6}$$

From equation 6, the relationship between the retained DOF and the truncated DOF is established as

$$T_s = \begin{bmatrix} I \\ t_s \end{bmatrix} = \begin{bmatrix} I \\ -K_{tt}^{-1} K_{tr} \end{bmatrix} , \tag{7}$$

where T_s is the static reduction projection matrix and the subscript "s" denotes static condensation. Therefore, the Guyan reduced DOF vector, x_n, as well as the system's reduced mass and stiffness matrices, is as given in equations 2 and 3, respectively, where T is defined as T_s.

This reduced representation is only exact for static events and the error increases as the effects of the inertial terms become more significant, e.g., as the event transitions from the static to the quasi-static and the dynamic time domains.

By reducing the total number of elements, the FE model is more efficiently solved, but at the cost of losing high-fidelity data of nonstatic loading conditions. The solution's fidelity decreases with decreasing loading periods, i.e., higher frequency content in the loading time history. Implementation of the static reduction method into a local-to-global technique thus leads to a qualitative analysis due to the reduction process that retains the stiffness of the local model and neglects the inertial terms.

6.4 Dynamic Reduction

Dynamic reduction attempts to accurately represent the system for specific frequencies corresponding to the system's eigenvalues (*4, 6*). For the undamped system equation, equation 1, the dynamic reduction method uses the eigenvalue problem associated with free vibrations at frequency ω, so that

$$[K - \omega^2 M]x = 0 . \tag{8}$$

Forming the dynamic stiffness matrix $B(\omega^2)$,

$$B(\omega^2) = [K - \omega^2 M], \tag{9}$$

such that $B(\omega^2)\, x = 0$. If $B(\omega^2)$ is partitioned in the same manner as in static reduction, with retained DOF and truncated DOF, the characteristic equation is

$$\begin{bmatrix} B_{rr}(\omega^2) & B_{rt}(\omega^2) \\ B_{tr}(\omega^2) & B_{tt}(\omega^2) \end{bmatrix} \begin{Bmatrix} x_r \\ x_t \end{Bmatrix} = 0. \tag{10}$$

The relationship between the retained and truncated DOF is given by

$$T_d(\omega^2) = \begin{bmatrix} I \\ t_d(\omega^2) \end{bmatrix} = \begin{bmatrix} I \\ -B_{tt}(\omega^2)^{-1} B_{tr}(\omega^2) \end{bmatrix}. \tag{11}$$

The parameter ω^2 explicitly states that the projection, or transformation, matrix T_d, and hence B, depend on ω^2. The dynamic reduction process is only exact when ω^2 equals one of the systems eigenvalues, Ω^2, and, consequently, is only exact at that specific eigenvalue. This is evident by realizing that equation 10 is an eigenvalue problem expressed in a partitioned form. Therefore, if $\omega^2 \neq \Omega$, then $B(\omega^2)\, x \neq 0$, and equation 10 is not satisfied. Additionally, notice that the dynamic reduction method maps itself into the static reduction method when $\omega^2 = 0$. Lastly, the dynamically reduced DOF vector, x_n, as well as the system's reduced mass and stiffness matrices, is as given in equations 2 and 3, respectively, where T is defined as T_d.

In addition to the dynamic reduction method described above, O'Callaghan introduced the IRS method (*7*) as an approach for improving the static reduction method by including dynamic effects. This method considers the inertial terms as so called quasi-static forces. Thus, the IRS method is an attempt to represent the low frequency response of the full system more accurately than the static reduction method. Accuracy of this approach decreases as the excitation frequency increases beyond the first few structural resonances. The IRS method works well for low frequency response, but loses accuracy as the inertial effects become more significant. This is equivalent to assuming that the higher frequency response is unimportant (*5*). This is not the case for gun dynamics and other high-g events; therefore, the IRS method will not be considered further.

6.5 SWAT

Developed at Sandia National Laboratory, SWAT (*8, 9*) is a force reconstruction technique that utilizes the sums of the mass-scaled acceleration data. Although SWAT was not developed for FE model reduction, it can be implemented as such by replacing the local model with a SWAT-defined force distributed over the local-to-global interface. The SWAT weighting coefficients are determined from the inverse modal matrix of the system with free boundary conditions, a least squares solution, or from measured frequency response function data. This technique calculates the net force and moment, effectively acting at the selected generalized DOF's center of mass. The spatial distribution of the forces and moments over the structure is not known and is therefore not unique. Although this technique yields non-unique results, for a model

containing a single local-to-global interface, this procedure can be used to determine approximate load transmissibility, including dynamic effects. As with the other reduction methods, SWAT starts from the system's equations of motion in a generalized coordinate system. For an undamped system, the equations of motion are again given by equation 1. The modal matrix U and natural frequencies ω are determined from modal analysis. The coordinate transformation

$$x(t) = Uq(t) \tag{12}$$

defines the mapping of the generalized coordinates to physical coordinates. U includes both rigid body modes and elastic modes. Substitution of this coordinate transform into equation 1 with the premultiplication by a single rigid body mode U_{rigid} yields

$$U_{rigid}^{T} MU\ddot{q} + U_{rigid}^{T} KUq = U_{rigid}^{T} f(t). \tag{13}$$

In rigid body modes, no elastic deformation exists; therefore,

$$KU_{rigid} = 0 \text{ or } U_{rigid}^{T} K = 0. \tag{14}$$

Substituting equation 15 into equation 14 yields

$$m_{rigid}\ddot{q}_{rigid}(t) = U_{rigid}^{T} f(t), \tag{15}$$

where m_{rigid} is 1 x 1 due to orthogonality of eigenvectors and m_{rigid} represents the mass associated with the rigid modal coordinate q_{rigid}. From equation 15, it is apparent that the external net force applied to the system can be determined by the rigid body modal coordinate acceleration-time histories, which is simply Newton's Law. The goal is therefore to determine a specific modal coordinate acceleration-time history from the sum of the generalized coordinate acceleration-time histories. Equation 12 and equation 15 provide insight for the sequence of steps required to reach this goal. To determine the SWAT force, the rigid modal coordinate time history needs to be developed from the vector of generalized acceleration-time histories. Modal analysis states that the acceleration-time response, a(t), can be estimated by a sum of vectors. If the free modes are used as the vectors, then the approximation will be accurate up to a limiting frequency dependent on the number of modes used in the estimation, hence the mode's eigenvalues. The acceleration can be expressed in modal coordinates as

$$a(t) = \Psi\ddot{q}(t), \tag{16}$$

where ψ is the retained free modes used in the approximation of a(t) using the modal coordinate acceleration vector $\ddot{q}(t)$. A SWAT DOF is defined as

$$s(t) = w^{T} a(t), \tag{17}$$

where w is a vector of to be defined weights and a(t) is the measured set of accelerations. Thus, equation 17 becomes

$$s(t) = w^T \Psi \ddot{q}(t). \tag{18}$$

Next, the weights are chosen so that s(t) will be a modal coordinate, e.g., $w^T\psi = [1\ 0\ \ldots\ 0]$, resulting in $s(t) = \ddot{q}_1(t)$. Similarly, the weight vector needed to extract any modal coordinate from acceleration-time histories can be determined by solving

$$W^T\Psi = I \text{ or } \Psi^T W = I, \tag{19}$$

where I is the identity matrix and W is a matrix of which its jth column is the jth weight vector, w_j, for the j^{th} modal coordinate. The weight vectors computed in equation 19 are referred to as the reciprocal modal vectors. In summary, once the desired modal coordinate response has been determined, multiplication by the modal mass for the corresponding modal coordinate yields the net externally applied force.

SWAT, if implemented as a global-to-local FE reduction method, has several advantageous features:

- allows use of a SWAT reconstructed force to determine an accurate approximation of the local loading,
- can be used to check an alternate method's loading on local structure,
- can be used to determine whether or not local response effects global response, and
- can convert experimental acceleration data to numerical force data.

SWAT would be most useful in a single local-to-global interface case for which the local loading is applied from a single point, line, or plane. The SWAT reconstructed force based on the global nodal accelerations would be able to determine the exact local loading force acting through a point, and the net local loading force acting through a line or a plane. If the loading interface were a line or plane, then the net SWAT force could be partitioned by observing the weighting factors of the elastic modes (*7*) or by dividing the net force equally by the number of loading points contained in the line or plane. Although not exact, this method does capture the dynamics of the global structure. Thus, the accuracy of the local loading magnitude is adequate, but the distribution of this local force is not. SWAT is not a reduction technique by condensation, but rather a reduction technique by substitution. SWAT substitutes the local model with an approximately equivalent, time-dependant loading at the local-to-global interface, thereby simultaneously reducing the number of DOF in the FE model and increasing the characteristic time-step. The disadvantage to SWAT is that accurate acceleration-time histories need to exist a priori for SWAT to be utilized. Additionally, for small electronic packages, these accelerations may be difficult or impossible to measure accurately.

6.6 SEREP

SEREP is an attractive approach for FE global/local system reduction because only the DOF located at the local-to-global interface need to be retained, and the method will maintain an accurate representation of the local system. This is advantageous since, like SWAT, the local structure in the global model can then be replaced by an equivalent set of force-time history functions obtained by the reduction process. Also, this method can be modified so that the local model can be completely replaced with a simpler, equivalent representation. Additionally, the local FE model does not need to be developed to get the equivalent set of force-time histories, since a modal model can be experimentally measured to determine the set of equivalent forces. The process is depicted in figure 6.

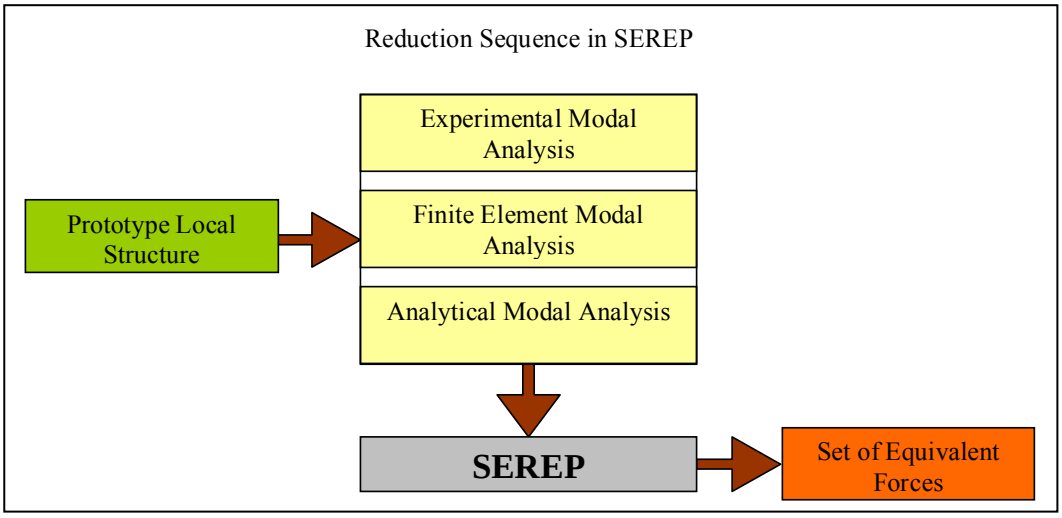

Figure 6. Local-to-global model reduction process for SEREP.

Like the prior reduction techniques, excluding SWAT (which was not initially formulated as a reduction technique), SEREP partitions its mass, damping, and stiffness matrices into retained and truncated DOF. In an analytical analysis, the number of modes that can be determined equals the number of DOF. Conversely, in FE analysis, it is typical to have many more DOF than captured modes and thus n>>m, where n is the number of DOF and m is the number of modes. In both cases, however, the desire is to reduce the size of the system to include only the i modes and the j DOF that are necessary to describe the system in the frequency regime of interest.

The derivation of SEREP is fully developed in references (*4, 10*). Once again, the generalized coordinate vector x is partitioned into two parts, x_r and x_t, the retained and truncated coordinates, respectively. Neglecting damping, the mass and stiffness matrices are then reorganized such that

$$\begin{bmatrix} M_{rr} & M_{rt} \\ M_{tr} & M_{tt} \end{bmatrix} \begin{Bmatrix} \ddot{x}_r \\ \ddot{x}_t \end{Bmatrix} + \begin{bmatrix} K_{rr} & K_{rt} \\ K_{tr} & K_{tt} \end{bmatrix} \begin{Bmatrix} x_r \\ x_t \end{Bmatrix} = \begin{Bmatrix} f_r(t) \\ f_t(t) \end{Bmatrix} . \tag{20}$$

The eigensolution of this system generates the eigenvalues Ω^2 and the corresponding eigenvectors U. The coordinate transformation from physical coordinates to modal coordinates is defined as

$$x = Uq. \tag{21}$$

Typically, the calculated modal matrix is a subset of the theoretical full modal matrix, and the coordinate transform becomes

$$x = U_{ac} q_c, \tag{22}$$

where the U_{ac}, the reduced modal matrix, denotes the 'c' captured modes for 'a' all generalized coordinate DOF, and q_c is $c \times 1$. The modal response of the reduced system is directly related to which modes are included. For high-g loads, it is normally desirable to generate a reduced model that is representative up to some upper-frequency limit determined by the frequency content of the anticipated loading. Using the reduced modal matrix, the physical coordinate vector is partitioned as

$$x = \begin{Bmatrix} x_r \\ x_t \end{Bmatrix} = \begin{bmatrix} U_{rc} \\ U_{tc} \end{bmatrix} q_c. \tag{23}$$

Expanding the equation regarding the retained DOF,

$$x_r = U_{rc} q_c. \tag{24}$$

Solving for q_c requires the generalized inverse of the possibly nonsquare matrix U_{rc};

$$U_{rc}^{+} = (U_{rc}^{T} U_{rc})^{-1} U_{rc}^{T}, \tag{25}$$

where + represents that the generalized inverse is taken. Solving for q_c,

$$q_c = U_{rc}^{+} x_r. \tag{26}$$

Substitution of equation 26 into equation 23 gives the relationship between the retained DOF and the full set of DOF and also provides the definition of the SEREP projection matrix, T_u, via

$$x = U_{ac} U_{rc}^{+} x_r = T_u x_r. \tag{27}$$

The eigensolution of the reduced mass and stiffness matrices, M_{rr} and K_{rr}, respectively, produce the reduced eigenvalues ${}^{R}\Omega^2$ and eigenvectors ${}^{R}U$, where the presuperscript "R" denotes reduced. The modal transformation of equation 24 is still valid while

$${}^{R}U = U_{rc}. \tag{28}$$

The modal matrix, U_n, of the full system is obtainable from

$$U_n = T_u \, {}^{R}U. \tag{29}$$

Equation 29 is important because it implies that the SEREP is reversible and exact in the sense that the full eigensolution for all the DOF can be obtained by solving the reduced eigenproblem.

Using the transformation given by equation 27, the SEREP-reduced mass and stiffness matrices can be written as

$$\begin{aligned} M_{rr} &= U_{rc}^{+\,T} U_{rc}^{+}, \text{ and} \\ K_{rr} &= U_{rc}^{+\,T} \Lambda U_{rc}^{+}, \end{aligned} \tag{30}$$

where $\Lambda = \Omega^2$.

These reduced matrices can be used directly in an FE analysis since they are in terms of the physical coordinates. With this method, the dynamic information which is omitted in the FE analysis depends on the number of modes captured in the modal analysis and the number of interface DOF. Usually, the limiting parameter, especially for a distributed system, will be the number of captured modes during the modal analysis of the local structure. Also, the reduced mass matrix is typically a full matrix requiring a matrix inversion technique; consequently, the solution is no longer explicit. However, the method allows such a significant reduction in the number of DOF that must be retained for accurately representing the dynamic response that the increase in computational costs associated with the implicit solution technique may be acceptable. The SEREP reduction technique has several features that are beneficial to an FE analysis:

- the mass and stiffness matrices are in physical coordinates,
- the expansion to the full matrices (the inverse mapping) is exact,
- the resultant equations capture the exact response of the full system for every natural frequency that is retained in the reduction process, and
- the method allows the use of either analytical or experimentally derived modal information.

However, it is worth noting that, because of the reduction process, equation 30a is generally singular. This presents some numerical issues that must be addressed in any implementation of the SEREP method as a general reduction method.

7. Summary

Several model reduction techniques have been introduced and briefly discussed along with their advantages and disadvantages. It has been determined that SEREP is the appropriate reduction techniques to be implemented in the FE method in order to reduce complex models subjected to dynamic loads to smaller, simpler representations while maintaining increased accuracy, as compared with other reduction techniques. Also SEREP, due to its modal based implementation, has the potential of being the most accurate if all modes of the local system are determined.

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