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Resolution and algorithmic influences on the baroclinic pressure gradient in finite element-based hydrodynamic models

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In 3D shallow water models, the baroclinic pressure gradient (BPG) can become unstable or physically unrealistic in areas of rapidly changing topography and/or density fields. By using a 2D finite element x-z model, whose formulation uses the generalized wave continuity equation, we assess the impact on accuracy of three methods used to compute the BPG: the sigma coordinate method, the $z-$coordinate (level coordinate) method and a hybrid method that switches from sigma coordinates to $z-$coordinates at a prescribed depth. Resolution studies then look at horizontal and vertical resolution independently, and the interplay of the horizontal and vertical resolutions for these three methods of computing the BPG. Numerical experiments are carried out on several domains, from constant to rapidly changing bathymetry, with two different density fields, which vary only in the horizontal or vertical directions. Results thus far indicate that the $z-$coordinate method provides the least amount of error in the solution.

1. INTRODUCTION

In areas where the topography changes rapidly, such as a seamount or continental rise region, many three-dimensional hydrodynamic models have problems computing a stable and realistic BPG. The main topic of this paper is the calculation of the BPG term. The motivation for this paper stems from anomalous results observed by Blain [4] in baroclinic Arabian Gulf simulations using a wave-continuity based FE model. Figure 1 illustrates the problem, where the source of error was identified as an unrealistic (and unstable) BPG computed by the model in a region of steep bathymetry and density gradients.

Four common coordinate systems are utilized in ocean models: sigma coordinates, which are terrain-following; $z-$coordinates (also called level coordinates), which follow a fixed depth [3]; isopycnal, which follow lines of constant density; and hybrid, which combines the sigma and $z-$coordinates. Advantages and disadvantages exist for all of these coordinate systems, which several investigators have mentioned in their studies (e.g., [3,6,16]). To summarize, sigma coordinates provide more resolution in the shallower parts of the domain and capture the bottom and free moving surfaces, thus allowing the boundary conditions to be implemented easily. The disadvantage of sigma coordinate

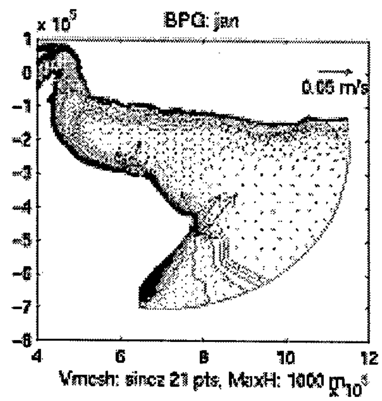

Figure 1. Instabilities caused by errors in the calculation of the BPG during simulations of the Arabian Gulf [4].

system involves the "hydrostatic inconsistency" condition, first discussed in the context of oceanography models by Haney [11]. This condition indicates that in areas of steep sloping topography, there needs to be an accurate horizontal resolution in order to obtain stable and realistic BPG results. However, in some cases the amount of horizontal resolution needed to produce accurate results lead to high computational costs. If the "hydrostatic inconsistency" condition is not met, then spurious modes tend to be introduced into the solution through truncation errors obtained from the transformation of the BPG term to the sigma coordinate system. Thus, in the areas of sloping bathymetry, large truncation errors can mask the true BPG [6]. Suggestions from some researchers have reduced these errors, but the problem has not been completely solved. In some models, this problem has been reduced by subtracting a mean vertical density gradient or an area-averaged density from the initial density field [16–18].

As for $z-$coordinates, they do not suffer from problems with the coordinate transformation. However, the disadvantage of the $z-$coordinate system is its inability to properly resolve the flow around the bottom topography in areas of sloping bathymetry ("stair-step" resolution), and the correct flow at the surface is often not captured [6]. To obtain an accurate BPG where sloping bathymetries come into play for $z-$coordinates, several researchers suggest using extrapolation techniques, e.g. Beckmann and Haidvogel utilized a Chebyshev polynomial [2].

Another method that has been used in global ocean models is isopycnal coordinates, which follow lines of constant density [10]. This type of coordinate system does well in the deeper parts of the ocean because the density profile tends to be stably stratified. However, this coordinate system does not do well in shallower parts of the ocean due to the mixed stratified nature and due to the mixing and advection processes that tend to be dominant in this part of the ocean [6]. It also has the same "stair-step" resolution problem as the $z-$coordinate method because the lines of constant density do not follow the topography changes [5].

Hybrid methods have been suggested that take advantage of the strengths of sigma and $z-$coordinate, which have been used successfully in several models (e.g, [6,16]). The degree of hybridization between the two coordinate systems and the technique of the hybridization can both vary in the model. For example, Beckers [1] examined a hybrid scheme that used only one $z-$coordinate (fixed) with sigma coordinates above and below it. Also, Spall and Robinson [20] analyzed a hybrid scheme that used $z-$coordinates in the upper layers and sigma coordinates in the bottom layers. Another hybrid scheme, which is used by NCOM (Naval Coastal Ocean Model) [16], applies sigma coordinates in the upper layers and the $z-$coordinates in the bottom layers. We note that other types of hybrid models have been developed, such as HYCOM [7], which switches from isopycnal in deep water to sigma or $z-$coordinates in coastal areas.

Several researchers have investigated the unstable or unrealistic results of the BPG term in the context of finite difference models (e.g., [6,11,18]), however, only a few studies have been done in the context of finite element or unstructured grid models. Using a finite element model, Walters and Foreman looked at the influence of resolution on the velocity field using sigma coordinates, first varying the horizontal resolution for a fixed vertical resolution and then vice versa [21]. They determined that the sigma coordinate system produced either second- or first-order accurate solutions, depending on the density profile, for the continental shelf region. From their studies, they indicated that the sigma coordinate system studied should be replaced with either $z-$coordinates or the density field should be post-processed using a density gradient.

Fortunato and Baptista evaluated all of the horizontal gradients in the momentum equation in either sigma coordinates or $z-$coordinates in a 2D barotropic and baroclinic (diagnostic) model [9]. They determined that evaluating all of the horizontal gradients in the sigma coordinate system provided the best approach in most cases; however, in certain cases the $z-$coordinates proved to be better, in particular for the case study presented in Walters and Foreman [21]. They also provided some steps to obtain the proper horizontal resolution for a sigma coordinate model near steep bathymetry gradients [9].

Herein, we build on this earlier work and investigate BPG calculations using several of these coordinate systems in a finite element model. This study will only look at methods to calculate the BPG term, while all other horizontal gradients in the momentum equation will utilize sigma coordinates. In this respect, it differs from the work done by Fortunato and Baptista [9]. The study extends the work done by Walters and Foreman [21] by looking at other methods of calculating the BPG term.

2. BACKGROUND OF THE MODEL

The model utilized in this study is a 2D laterally-averaged shallow water model that uses the finite element method; it follows the same development steps as the 3D ADCIRC model [13]. It employs a mode splitting scheme in which the external mode solves a 1D (depth-averaged) continuity equation for the elevation field and a 2D (x-z) momentum equation to resolve the velocity field. The depth-averaged velocity values utilized in the continuity equation are obtained from the integration of the 2D momentum equation results. We replace the continuity equation with the generalized wave continuity equation to eliminate the spurious modes that occur with primitive finite element models (e.g., [12,13,15]). The generalized wave continuity equation is as follows:

$$\frac{\partial^2\zeta}{\partial t^2}+\tau_0\frac{\partial\zeta}{\partial x}-HU\frac{\partial\tau_0}{\partial x} -\frac{\partial}{\partial x}\left[\frac{\partial(HUU)}{\partial x}+gH\frac{\partial\zeta}{\partial x}-E_l\frac{\partial^2(HU)}{\partial x^2}+\tau_{3D}-\tau_0HU-B_x-D_x\right]=0, \tag{1}$$

where ζ is surface elevation above a datum, H is the total fluid depth, U is the depth-averaged velocity, g is gravity, D_x is the depth-integrated momentum dispersion (momentum transfer due to a non-uniform velocity profile), B_x is the depth-integrated baroclinic forcing, τ_0 is a numerical parameter that allows either a pure wave form of the equation when the parameter is small or the primitive form of the continuity equation if the parameter is large, and $\tau_{3D}=K_{slip}u_b$, where K_{slip} is the linear slip coefficient and u_b is the velocity at the bottom boundary. This code does not include the atmospheric forcing or the Coriolis forcing terms. We currently utilize a constant eddy viscosity coefficient, E_l, in the horizontal direction.

The model employs the non-conservative form of the momentum equation, which is as follows:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-g\frac{\partial\zeta}{\partial x}+\frac{\partial}{\partial z}\left(\frac{\tau_{sx}}{\rho_0}\right)-b_x+m_x, \tag{2}$$

where $\tau_{sx}/\rho_0=E_z\partial u/\partial x$ is the vertical stress gradient with an eddy viscosity parameterization, $m_x=\partial/\partial x(E_l\partial(u)/\partial x)$ is the lateral stress gradient also with an eddy viscosity parameterization and

$$b_x=g\frac{\partial}{\partial x}\int_z^\zeta\frac{(\rho-\rho_0)}{\rho_0}dz \tag{3}$$

is the BPG. To evaluate Equation 2, we mapped the terms onto a sigma coordinate system, where σ ranges from a at the surface ($a=1$) to b at the bottom ($b=-1$). The equation in the sigma coordinate system is as follows:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x_\sigma}+w_\sigma\left(\frac{a-b}{H}\right)\frac{\partial u}{\partial\sigma}=-g\frac{\partial\zeta}{\partial x}+\left(\frac{a-b}{H}\right)\frac{\partial}{\partial\sigma}\left(\frac{\tau_{sx}}{\rho_0}\right)-b_x+m_x \tag{4}$$

where $m_x=\partial/\partial x_\sigma(E_l\partial u/\partial x_\sigma)$ and w_σ incorporates terms from the variable transformation (see [13] for more details). The evaluation of the m_x term occurs along the stretched surfaces directly with no coordinate transformation.

These equations use C^0 linear finite elements for the spatial discretization with the exact quadrature rules. For the temporal discretization, a three time-level scheme centered at k is used in Equation (1) and a two time-level scheme centered at $k+1/2$ is used in Equation (4). Except for the BPG, all the horizontal derivatives utilize the sigma coordinate system; the BPG term uses different coordinate systems as described below.

In this work, we have isolated the BPG term (b_x in Equation (4)) and only implement this term differently in accordance with the different coordinate systems. Initially, we calculate a buoyancy term (Equation (5)), which then is used to evaluate the horizontal gradient:

$$bouyancy = g\int_{z}^{\zeta}\frac{(\rho-\rho_0)}{\rho_0}dz. \tag{5}$$

Then in $z-$coordinates, the BPG is given as

$$b_x = \frac{\partial}{\partial x_z}(bouyancy), \tag{6}$$

while in the sigma coordinate system, the BPG is given as

$$b_x = \frac{\partial}{\partial x_\sigma} - \frac{\partial z}{\partial x}\frac{\partial(bouyancy)}{\partial z}. \tag{7}$$

In this study, we utilize both of these equations along with a combination of the two to develop the hybrid scheme. The hybrid scheme employed in our study follows the method used in NCOM [16], which switches from sigma coordinates to $z-$coordinates as the depth increases. In the $z-$coordinate BPG implementation, we utilize the framework of sigma coordinates for all terms except the BPG, which computes horizontal gradients by interpolating the values between adjacent vertical nodes [2,8]. Near bottom boundaries, a linear extrapolation technique is used in regions where gradients based on $z-$coordinates "run into the ground". Also note that in this study, all test cases were conducted utilizing the diagnostic baroclinic mode of the model.

3. NUMERICAL EXPERIMENTS

Three domains and two different density profiles serve as the test cases for this study. In order to evaluate each of the methods for calculating the BPG term, we compare the error behavior of the BPG, the horizontal velocity field, and the vertical velocity field. We utilize a L_2 norm to determine the errors at several stations. Point comparisons at twelve locations are made by linearly interpolating results from neighboring nodes. These twelve errors are then averaged to produce one value for each grid resolution. In all of the test cases, we evaluated the horizontal and vertical resolutions independently, and then the interplay between both resolutions. In each test case, we compared the results to a "true solution", which is obtained either from an analytical solution or from grid refinement studies. We indicate below what the "true solution" is for each test case.

3.1. Test case 1

In this test case, a 48-km domain has a constant bottom slope between 10 m at the shallow end to 100 m at the deep end (approximately a 2% slope), and the density profile varies linearly from 1026 kg/m^3 (shallow) to 1028 kg/m^3 (deep) in the horizontal direction, with no variation in the vertical direction. Boundary conditions were no-flux land boundaries on both sides of the domain. All nonlinear terms were evaluated in the equations. The eddy viscosity parameters in both the lateral and vertical directions are kept constant at 0 and 0.051 m^2/s, respectively, and bottom friction is evaluated with a linear slip condition that utilizes a K_{slip} value of 0.05 m/s. In this test case, the GWC equation numerical parameter, τ_0, was set to 0.001 sec^{-1} and a time step of 0.1 sec was

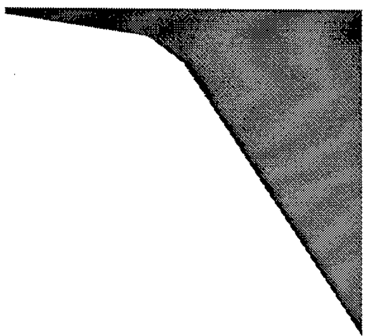

Figure 2. Bathymetry and density profiles for Test Case 2. Density values, which vary from 1000 to 1002 kg/m^3, are shown changing from blue in lighter water to red in the heavier waters. Bathymetry ranges from 10 to 600 m.

Figure 3. Bathymetry and density profiles for Test Case 3. Density values, which vary from 1000 to 1002 kg/m^3, are shown changing from blue in lighter water to red in the heavier water. Bathymetry ranges from 10 to 300 m.

utilized for a simulation of a day. Results were recorded 90 times over the course of the simulation.

A "true solution" for this test case was obtained by refining the grid in the horizontal direction until the L_2 error changes were within machine accuracy and had reached convergence, which occurred with a constant nodal spacing of approximately 180 m. This served as the "true solution" for the horizontal resolution study. Similarly for the vertical resolution study, we refined the grid in the vertical until the L_2 error changes showed that the solution had converged, which occurred with 129 nodes in the vertical. For the interplay study, a nodal spacing of approximately 180 m in the horizontal with 129 nodes in the vertical was used for the "true solution."

3.2. Test case 2

The second test case is taken from both Walters and Foreman [21] and Fortunato and Baptista [9], which provide a test case that mimics the shelf break region. The bathymetry varies linearly in three different areas along a 50 km slice as shown in Figure 2. Density varies only in the vertical and depends on depth, as shown in Figure 2. The density field above 100 m comes from the following relationship: $1001 - \cos(0.01\pi z)$, while below 100 m, a constant density value of 1002 kg/m^3 is used. Boundary conditions were no-flux land boundaries on both sides of the domain. All nonlinear terms were evaluated in the equations. The eddy viscosity parameters in both the lateral and vertical directions are kept constant at 0 and 0.051 m^2/s, respectively, and bottom friction is evaluated with a linear slip condition that utilizes a K_{slip} value of 0.001 m/s. In this test case, the GWC equation numerical parameter, τ_0, was set to 0.001 sec^{-1} and a time step of 0.1 sec was utilized for a simulation of a day. Results were recorded 90 times over the course of the simulation.

For this test case, we compared the results to an analytical solution. As noted by

Walters and Foreman [21] and Fortunato and Baptista [9], the analytical solution for this test case is zero because there are no boundary forcings and the density varies only in the vertical direction (stable stratification) and not horizontally.

3.3. Test case 3

In this test case, we analyzed a different bathymetry with the same density profile as the second test case. For the bathymetry, we developed a domain that includes a seamount, along with a change in topography that mimics the continental shelf region (see Figure 3). Boundary conditions were a no-flux land boundary and elevation boundary of zero on the opposite side of the domain. All nonlinear terms were evaluated in the equations. Eddy viscosity parameters in both the lateral and vertical directions are kept constant at 0 and 0.051 m^2/s, respectively. The bottom friction was evaluated with a linear slip condition that utilized a K_{slip} value of 0.001 m/s. Here again, the GWC equation numerical parameter, τ_0, was set to 0.001 sec^{-1} and a time step of 0.1 sec was utilized for a simulation of a day. Results were recorded 90 times over the course of the simulation.

As with the second test case, we compared the results from this test case to an analytical solution of zero, because there are no boundary forcings and the density varies only in the vertical direction (stable stratification) and not horizontally.

4. EXPERIMENTAL RESULTS

4.1. Test case 1

The results of the horizontal resolution study for the linearly sloping bathymetry showed that the only significant difference between the three methods for calculating the BPG was in the value of the BPG itself. For all the refinements in the horizontal, the $z-$coordinate method exhibits a continual decrease in the BPG error; however, sigma coordinates and the hybrid scheme show that the BPG error decreases rapidly until the nodal spacing is approximately 6000 m and then decrease slowly for the more refined grids. The averaged L_2 errors for the horizontal and vertical velocity fields do not show any appreciable changes based on the different methods for calculating the BPG. However, when looking at time series of horizontal velocity results, they indicate that there are subtle changes between the methods, yet these changes are masked when computing the average error over space and time. Similar results were found for the L_∞ norm.

Results of the vertical resolution study are nearly identical for the different ways of evaluating the BPG, as was expected since the variation of the density field is only based in the horizontal direction. Similarly, for this model problem, there are no observable changes between the different methods when looking at the interplay of the horizontal and vertical resolution.

4.2. Test case 2

Figures 4 and 5 illustrate the results of the horizontal resolution study for the second test case for the BPG and horizontal velocity. As can be seen, the evaluation of the BPG with $z-$coordinates produced the best results, while sigma coordinates produced greater errors. This greater sigma error is expected because sigma coordinates are more prone to errors in evaluating the BPG when the density field only varies in the vertical direction.

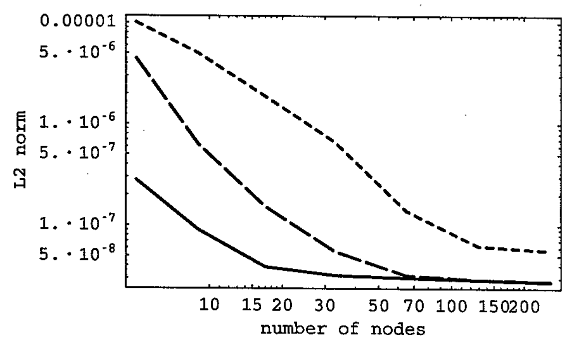

Figure 4. BPG error for Test Case 2 for horizontal resolution (vertical resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

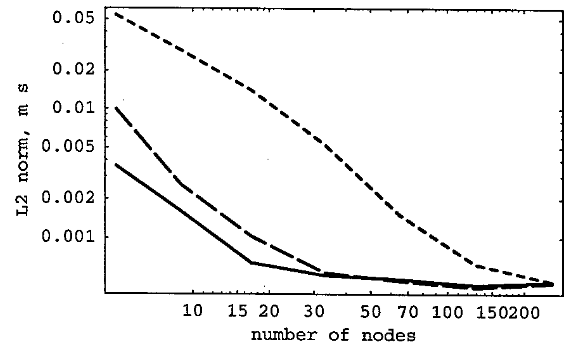

Figure 5. Horizontal velocity error for Test Case 2 for horizontal resolution (vertical resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

The stretching of the coordinate system in the vertical causes there to be two different density values between the two adjacent sigma nodes [9]. The hybrid scheme shows that as refinement increases in the horizontal direction the error decreases and approaches the errors that are found using $z-$coordinates. The hybrid scheme in this case switches from the sigma coordinates to $z-$coordinates in the upper layer (at $z = -25\,m$), where there are changes in the density. These results follow that of Fortunato and Baptista [9] in indicating that $z-$coordinates provide the best solution to this test case; however, the hybrid scheme shows promising results. Also note that errors in the BPG produced corresponding errors in the horizontal velocity fields (as shown in Figure 5). Similar error behavior is seen in the vertical velocity field (not shown).

We also evaluated the influence of the vertical resolution on the BPG and velocity fields. Figures 6 and 7 show the results for the BPG and horizontal velocity. As can be seen, the errors for $z-$coordinates and hybrid scheme decrease when vertical resolution is added; however, we find that, as further vertical resolution is added, the sigma coordinate error starts to separate further from the other two BPG calculation methods and appears to reach an asymptotic value. Similar error behavior is seen in the vertical velocity field (not shown).

Next, we examined the interplay of the horizontal and vertical resolution and their effect on the BPG and velocity errors. Results (not shown) indicate that the hybrid scheme and $z-$coordinates had similar errors as refinement occurred in both horizontal and vertical directions, while the sigma coordinate error is higher by approximately one-half log cycle than the other methods.

Lastly for this test case, we evaluated the placement of the depth at which the hybrid method switches coordinate systems. Results show that the depth should be between 20 m and 40 m to provide the lowest error. This depth range corresponds to the region above the rapid change in the density field.

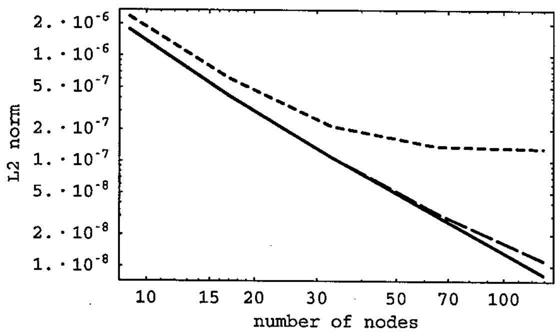

Figure 6. BPG error for Test Case 2 for vertical resolution (horizontal resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

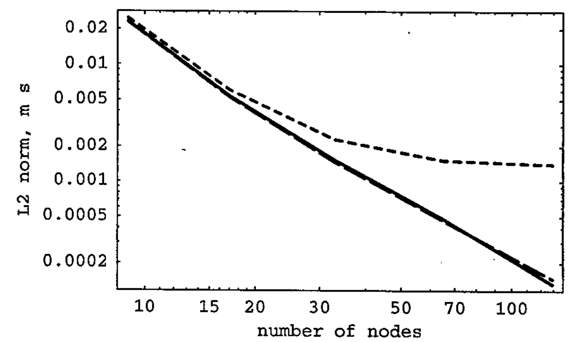

Figure 7. Horizontal velocity error for Test Case 2 for vertical resolution (horizontal resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

4.3. Test case 3

For this test case, the horizontal resolution study showed similar results as the second test case with the sigma coordinate error being higher than the other two BPG calculation methods (Figures 8 and 9). BPG results indicate that errors in the hybrid scheme (which switches from sigma coordinates to $z-$coordinates at $z = -25\ m$) coincide with those of the $z-$coordinate method as the grid is refined, while the sigma coordinate error is approximately one-half log cycle higher. These BPG errors translate to similar errors in the horizontal velocity for all the methods (cf. Figures 8 and 9). We found similar results in the error behavior for the vertical velocity (not shown).

Figures 10 and 11 indicate how vertical resolution affects the error in calculating the BPG and horizontal velocity. The behavior of the three BPG calculation methods exhibits the same trends as in the previous test case with $z-$coordinates and the hybrid scheme having less error than sigma coordinates. Also, the sigma coordinate error tends toward an asymptote for both the BPG and horizontal velocity. Results for the vertical velocity show similar error behavior. In comparing the vertical resolution results from the second test case to the results from this test, one noticeable difference is that the sigma coordinate error tends to separate itself more from other two BPG calculation methods. More specifically in Test Case 2, we find that as the vertical resolution is relaxed the errors approach the same value for each of the BPG calculation method but not in Test Case 3 (cf. Figures 7 and 11).

Next, we analyzed the interplay of the horizontal and vertical resolution. BPG errors (not shown) indicate that in the grids of lower resolution (both horizontal and vertical directions) the three methods of calculating the BPG approach similar values; however, as more resolution is added in both directions the hybrid scheme and $z-$coordinates show similar errors, while the sigma coordinate error is higher than the other methods. In the horizontal velocity results, we find that at the lower resolutions the sigma coordinate error is slightly lower than the other two methods of calculating the BPG; however, as

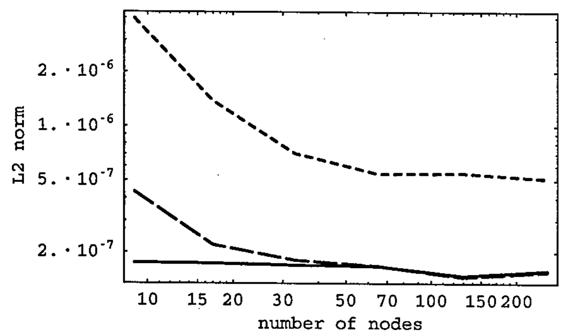

Figure 8. BPG error for Test Case 3 for horizontal resolution (vertical resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

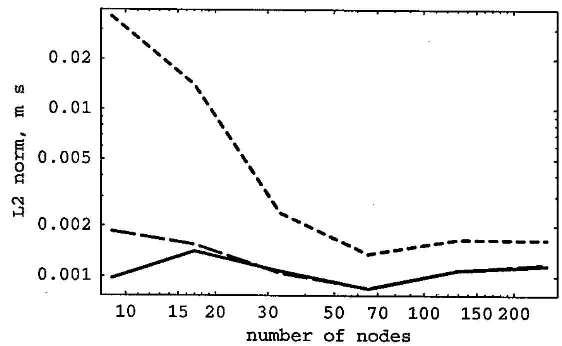

Figure 9. Horizontal velocity error for Test Case 3 for horizontal resolution (vertical resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

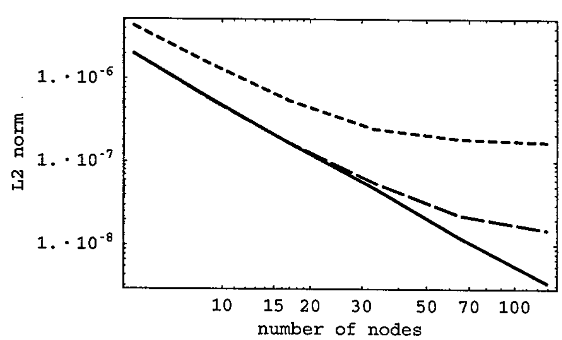

Figure 10. BPG error for Test Case 3 for vertical resolution (horizontal resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

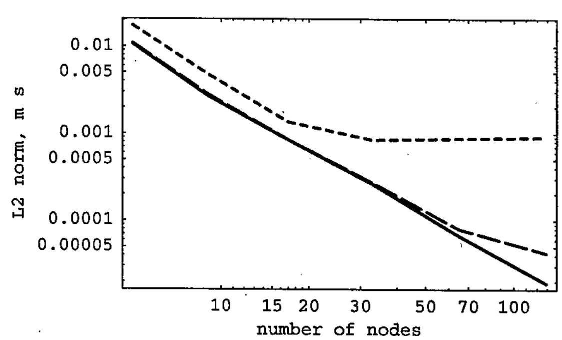

Figure 11. Horizontal velocity error for Test Case 3 for vertical resolution (horizontal resolution held constant) for sigma coordinates (short dashes), hybrid scheme (long dashes) and $z-$coordinates (solid line).

grid refinement occurs the sigma coordinate error is higher than that of the other two methods of calculating the BPG. Similar behavior is found in the vertical velocity results.

Lastly for this test case, we also evaluated the placement of the depth at which the hybrid method switches coordinate systems. Results indicate the depth should be between 10 m to 30 m for minimal error. As with Test Case 2, this depth range again corresponds to the region above the rapid change in the density field.

5. CONCLUSIONS AND FUTURE WORK

Herein, we present some initial results from an assessment of how the coordinate method used to calculate the BPG and resolution impact simulation results. This study uses a 2D laterally-averaged model to investigate these changes. Evidence thus far indicates that the $z-$coordinate method for calculating the BPG provides the best results. However, the three test cases used in this paper are more favorable toward the $z-$coordinate system, so in future work, we will look at other test cases in which the density and bathymetry profiles mimic more "real" world applications, as in the Arabian Gulf problem presented in the introduction. We will also evaluate variable grids with these methods for calculating the BPG. Finally, we plan to look at alternative error measures, such as the Cumulative Area Faction Error (CAFE) plots [14] or the grid convergence index (GCI) [19], which may be less likely to mask errors through averaging procedures.

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