


TECHNICAL LIBRARY ARCHIVES
TECHNICAL INFORMATION DEPARTMENT

This Publication is released for
distribution

By Ellen R. Shyne
Date 11/18/04

NAWCWD TP 8592
NAV  AIR

MODELING AN INTERFEROMETRIC FIBER OPTIC GYROSCOPE

by
Jorge M. Martin
Research Department

OCTOBER 2004

NAVAL AIR WARFARE CENTER WEAPONS DIVISION
CHINA LAKE, CA 93555-6100



Approved for public release; distribution is unlimited.

Naval Air Warfare Center Weapons Division

FOREWORD

This report presents the computations involved in the development of a general Interferometric Fiber Optic Gyroscope model. The work was done at the Naval Air Warfare Center Weapons Division, China Lake, California, from October 2001 to September 2004. It was funded by the Precision Strike Navigator project sponsored by the Office of Naval Research.

Approved by
C. F. MARKARIAN, *Head*
Research Department
21 September 2004

Under authority of
D. J. VENLET
RADM, U.S. Navy
Commander

Released for publication by
KAREN HIGGINS
Director for Research and Engineering

NAWCWPNS Technical Publication 8592

Published by Technical Information Division
Collation Cover, 60 leaves
First printing 30 copies

REPORT DOCUMENTATION PAGE

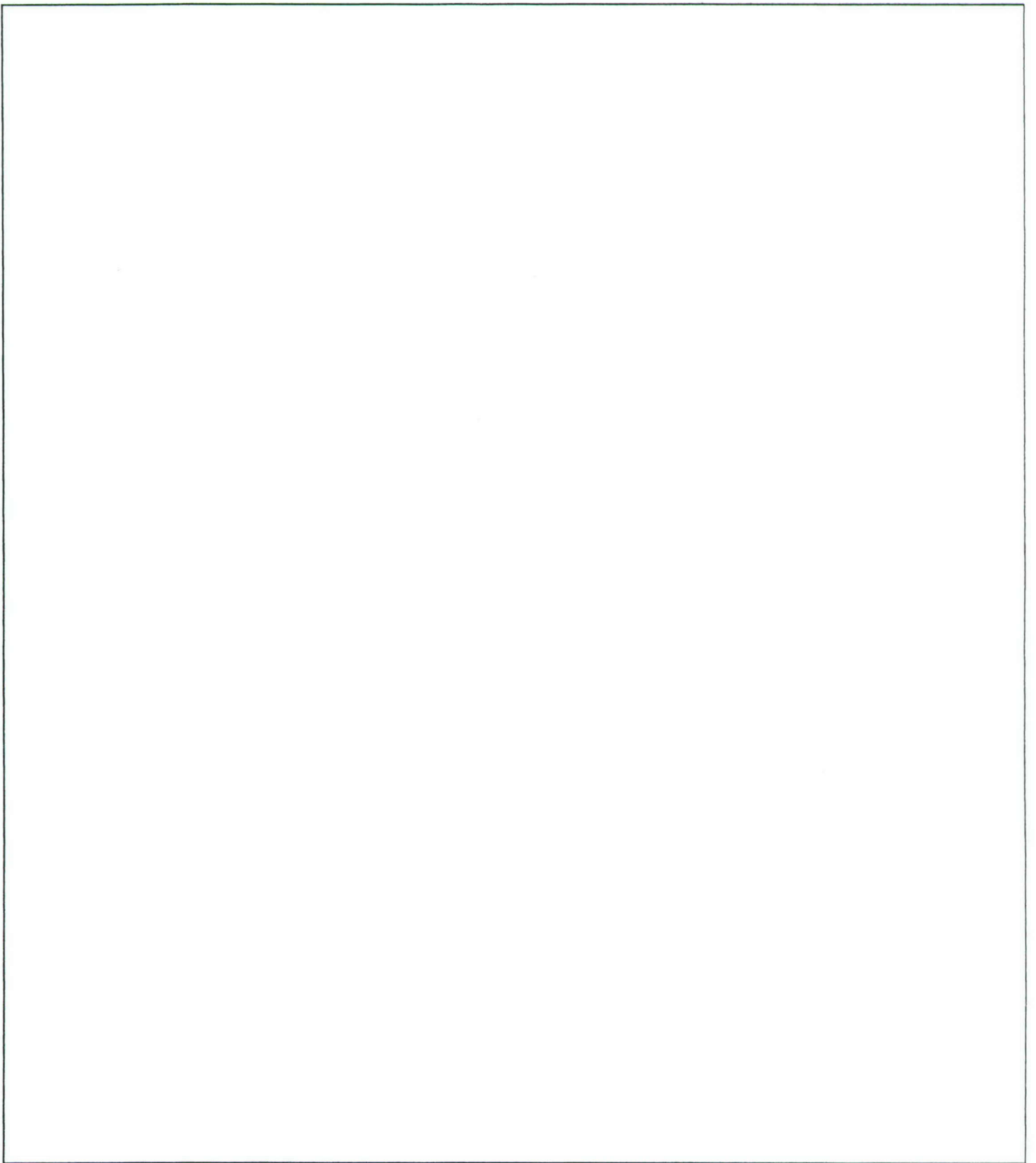
Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, D.C. 20503.

1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE <p style="text-align: center;">October 2004</p>	3. REPORT TYPE AND DATES COVERED <p style="text-align: center;">October 2001-September 2004</p>	
4. TITLE AND SUBTITLE <p style="text-align: center;">Modeling an Interferometric Fiber Optic Gyroscope (U)</p>			5. FUNDING NUMBERS <p style="text-align: center;">0603114N</p>	
6. AUTHOR(S) <p style="text-align: center;">Jorge M. Martin</p>			8. PERFORMING ORGANIZATION REPORT NUMBER <p style="text-align: center;">NAWCWD TP 8592</p>	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <p style="text-align: center;">Naval Air Warfare Center Weapons Division China Lake, CA 93555-6100</p>				
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) <p style="text-align: center;">Office of Naval Research, Code 351 800 N. Quincy Street Arlington, VA 22217</p>			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT <p style="text-align: center;">A Statement; public release; distribution unlimited.</p>			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) <p>(U) In this paper we derive an expression for the output intensity of a general Interferometric Fiber Optic Gyroscope (IFOG). The light source is assumed to be non-monochromatic with an arbitrary frequency distribution function belonging to $L^2(R)$. This result is expanded and expressed in terms of the parameters involved in the definition of each of the four entries of the clockwise Jones matrix of a generic IFOG with an even frequency distribution. We obtain an expression for the Sagnac phase shift error and compute it for a Gaussian, rectangular, and triangular frequency distribution function. The clockwise loop Jones matrix of a fairly general IFOG model is computed. Matlab programs for computing the output intensity and Sagnac phase shift error are also included.</p>				
14. SUBJECT TERMS <p style="text-align: center;">Interferometric Fiber Optic Gyroscope, Model, Jones Matrices, Clockwise Loop Matrix, Output Intensity, Coherence, Sagnac Error, Matlab Programs</p>			15. NUMBER OF PAGES <p style="text-align: center;">120</p>	
17. SECURITY CLASSIFICATION OF REPORT <p style="text-align: center;">UNCLASSIFIED</p>			16. PRICE CODE	
18. SECURITY CLASSIFICATION OF ABSTRACT <p style="text-align: center;">UNCLASSIFIED</p>		19. SECURITY CLASSIFICATION OF THIS PAGE <p style="text-align: center;">UNCLASSIFIED</p>		20. LIMITATION OF ABSTRACT <p style="text-align: center;">SAR</p>

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)



CONTENTS

1. Introduction	3
2. Output Intensity	4
3. Output Intensity in terms of the Matrix T and in terms of the parameters defining the IFOG	10
3.1. The Sagnac Phase Shift Error	15
3.2. Gaussian Function g	17
3.3. Step Function g	18
3.4. Tent Function g	20
4. An Elementary IFOG Model	22
4.1. The Model, Output Intensity, and Sagnac Error	22
4.2. The Intensity as a function of l_5 ; The Output Coherence	28
4.3. Modulation	32
5. A More General IFOG Model	33
5.1. The Jones Matrices of the Components	34
5.2. The Clockwise and Counter-Clockwise Loops	37
References	41
Appendixes:	
A. The Clockwise-Path Matrix T	A-1
B. Matlab Programs	A-61

1. INTRODUCTION

The importance of polarization control is increasingly being recognized in the development of optical fiber gyroscopes. In Reference 1, Kintner presented a concise analysis of a very basic interferometric fiber optic gyroscope (IFOG) arrangement that demonstrates how the lack of perfect polarizers contributes to an error in the estimate of the Sagnac phase shift due to polarization. His design is an idealized design that does not include a model of any of the possible misalignments in the various optical fiber junctions involved. This simplification leads to a very simple model that allows him to derive a close form expression for the output intensity where the ideal gyro output and the error term due to imperfect polarizers are readily identifiable. Thus, he is able to estimate the most unfavorable error in the Sagnac phase shift due to polarization.

In this paper we consider a more general and realistic design that includes a model of the misalignment errors at the various optical fiber junctions involved. It also includes a model of the integrated optics chips and depolarizers as well as a general model for the coil. We obtain a closed form expression for the output intensity that includes the effects of the light source coherence. From this expression the ideal gyro output can be deduced and an expression for the Sagnac phase shift error due to imperfect polarizers is derived.

In Section 2 we obtain an expression for the output intensity of a general IFOG. The light source is assumed to be non-monochromatic with an arbitrary frequency distribution function belonging to $L^2(R)$. This result is expanded in Section 3 and expresses in terms of the four entries of the 2×2 complex matrix representing the Jones matrix of the clockwise loop and further, in terms of the parameters involved in the definition of each of the four entries of the clockwise Jones matrix for a generic IFOG with an even frequency distribution function. At this point the Sagnac phase shift error is derived in Section 3.1. In Sections 3.2, 3.3, and 3.4 we compute the output intensity for three

particular frequency distributions: a Gaussian distribution, a rectangular distribution, and a triangular distribution. The theory developed in these sections is illustrated with a simple example in Section 4 for the case of a rectangular frequency distribution.

The effects of the light source coherence on the Sagnac error are demonstrated in Section 4.2 by varying the length of a particular section of PM-fiber. We show plots of the Sagnac error angle as a function of fiber length for five different models of the coil. Section 4.3 includes a short discussion on how one might model modulation effects.

The Jones matrices of the components of a more general IFOG model are described in Section 5. The clockwise and counter clockwise loop matrices are defined in terms of products of the Jones matrices of the components involved. The complete computation of the clockwise loop matrix is carried out in Appendix A. The end result are the four arrays of 256 coefficients and four arrays of 256 sets of indices needed to define the four entries of the clockwise IFOG Jones matrix as defined by Equation 3.3.

We wrote Matlab programs to compute the output intensities given by Equations 3.14, 3.15, and 3.19 and to compute the Sagnac error given by Equation 3.13 for Gaussian, rectangular, and triangular frequency distribution functions. These programs are described in Appendix B where we include their listings. The plots in Figures 7, 8, and 9 were produced using these programs.

2. OUTPUT INTENSITY

Each component of the gyro has a matrix representation, it's Jones matrix (Reference 2). The clockwise loop of the gyro is represented by a matrix T which is obtained by multiplying the Jones matrices of all the components of the gyro in the proper order. The counter clockwise loop of the gyro is represented by the matrix $T_c = e^{i\phi} \cdot T^T$, where T^T is the transpose of the clockwise loop matrix T and ϕ is the

Sagnac phase shift due to rotation. Since some of the optical components of the gyro are frequency dependent, the matrix T is also frequency dependent. The sum of the clockwise and counter-clockwise loop matrices is denoted by Ω . Thus, $\Omega(\omega) = T(\omega) + e^{i\phi}T^T(\omega)$, where ω denotes the frequency dependence of the optical components of the gyro.

We will consider an input of the form

$$e(t) = \text{Re} \left[\bar{a} \int_{-\infty}^{\infty} g(\omega - \omega_0) e^{-i\omega t} d\omega \right] = \text{Re} \left[\bar{a} \int_{-\infty}^{\infty} g(\omega) e^{-i(\omega + \omega_0)t} d\omega \right]$$

where Re denotes the real part, $\bar{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ is a two dimensional complex vector of norm 1;

that is, $|a_1|^2 + |a_2|^2 = 1$. In the first part of our analysis, the frequency distribution function g can be any function in $L^2(R)$. Later we specialize g to be a step function, a tent function, or a Gaussian function of the form $g(\omega) = \frac{1}{\sigma\sqrt{\pi}} e^{-\omega^2/\sigma^2}$ that has been normalized so that $\int_{-\infty}^{\infty} g(\omega) d\omega = 1$.

Let $\Theta(t) = \int_{-\infty}^{\infty} \Omega(\omega) \bar{a} g(\omega) e^{-i(\omega + \omega_0)t} d\omega$. Then, the output of the gyro is given by

$\text{Re}\Theta(t)$.

Theorem.

(a) If the output intensity is given by $I_C = \int_{-\infty}^{\infty} \Theta^*(t) \Theta(t) dt$, then

$$I_C = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega.$$

(b) If the output intensity is given by $I_R = \int_{-\infty}^{\infty} [\text{Re}\Theta(t)]^T \text{Re}\Theta(t) dt$ and g is a real valued function, then

$$I_R = \frac{1}{2} I_C + \pi \int_{-\infty}^{\infty} \text{Re}[\bar{a}^T \Omega^T(u - \omega_o) \Omega(-u - \omega_o) \bar{a}] g(u - \omega_o) g(-u - \omega_o) du.$$

Proof.

Part (a) is based on the following identity satisfied by any function $f \in L^2(R)$.

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s) e^{\pm i(s-t)\omega} ds d\omega. \quad (2.1)$$

which is simply the Fourier inversion formula.

Substituting $\Theta(t)$ in the Integral defining I_C gives

$$I_C = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \bar{a}^* \Omega^*(\omega) g(\omega) e^{i(\omega+\omega_o)t} d\omega \right] \left[\int_{-\infty}^{\infty} \Omega(s) \bar{a} g(s) e^{-i(s+\omega_o)t} ds \right] dt.$$

Combining the two inner integrals as a double integral gives

$$I_C = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{a}^* \Omega^*(\omega) g(\omega) \Omega(s) \bar{a} g(s) e^{-i(s-\omega)t} d\omega ds \right] dt.$$

Writing I as a triple integral and switching the order of integration gives

$$I_C = \int_{-\infty}^{\infty} \bar{a}^* \Omega^*(\omega) g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Omega(s) \bar{a} g(s) e^{-i(s-\omega)t} ds dt \right] d\omega.$$

Applying the Identity 2.1 to the inner double integral (note that t and ω are switched) gives

$$I_C = 2\pi \int_{-\infty}^{\infty} \bar{a}^* \Omega^*(\omega) g(\omega) \Omega(\omega) \bar{a} g(\omega) d\omega = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega) \bar{a}\|^2 g^2(\omega) d\omega.$$

Part (b) is based on the following identity satisfied by any function $f: R \rightarrow R$ belonging to $L^2(R)$.

$$f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s) \cos(\omega - s)t ds dt. \quad (2.2)$$

Note that Equation 2.2 is simply the Fourier inversion formula for a real valued function f .

First we compute $\text{Re}\Theta(t)$.

$$\text{Re}\Theta(t) = \int_{-\infty}^{\infty} \text{Re}[\Omega(s)\bar{a}]g(s) \cos(s + \omega_o)t ds - \int_{-\infty}^{\infty} \text{Im}[\Omega(s)\bar{a}]g(s) \sin(s + \omega_o)t ds$$

$$[\text{Re}\Theta(t)]^T = \int_{-\infty}^{\infty} \text{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \cos(\omega + \omega_o)t d\omega - \int_{-\infty}^{\infty} \text{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \sin(\omega + \omega_o)t d\omega$$

$$\begin{aligned}
I_R = & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \operatorname{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \cos(\omega + \omega_o)t \, d\omega \right] \left[\int_{-\infty}^{\infty} \operatorname{Re}[\Omega(s)\bar{a}]g(s) \cos(s + \omega_o)t \, ds \right] dt + \\
& - \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \operatorname{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \cos(\omega + \omega_o)t \, d\omega \right] \left[\int_{-\infty}^{\infty} \operatorname{Im}[\Omega(s)\bar{a}]g(s) \sin(s + \omega_o)t \, ds \right] dt + \\
& - \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \operatorname{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \sin(\omega + \omega_o)t \, d\omega \right] \left[\int_{-\infty}^{\infty} \operatorname{Re}[\Omega(s)\bar{a}]g(s) \cos(s + \omega_o)t \, ds \right] dt + \\
& \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \operatorname{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \sin(\omega + \omega_o)t \, d\omega \right] \left[\int_{-\infty}^{\infty} \operatorname{Im}[\Omega(s)\bar{a}]g(s) \sin(s + \omega_o)t \, ds \right] dt .
\end{aligned}$$

If we write the above integrals as a triple integral and integrate with respect to t first, the second and third integrals vanish because $\cos(\omega + \omega_o)t \sin(s + \omega_o)t$ and $\sin(\omega + \omega_o)t \cos(s + \omega_o)t$ are odd functions of t . Thus, we are left with the first and fourth integrals which we write as

$$\begin{aligned}
I_R = & \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(s)\bar{a}]g(s) \cos(\omega + \omega_o)t \cos(s + \omega_o)t \, ds \, dt \right] d\omega + \\
& \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(s)\bar{a}]g(s) \sin(\omega + \omega_o)t \sin(s + \omega_o)t \, ds \, dt \right] d\omega .
\end{aligned}$$

The identities $\cos(\omega + \omega_o)t \cos(s + \omega_o)t = \frac{1}{2}[\cos(\omega - s)t + \cos(\omega + s + 2\omega_o)t]$ and $\sin(\omega + \omega_o)t \sin(s + \omega_o)t = \frac{1}{2}[\cos(\omega - s)t - \cos(\omega + s + 2\omega_o)t]$ yield

$$\begin{aligned}
I_R = & \frac{1}{2} \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(s)\bar{a}]g(s) \cos(\omega - s)t \, ds \, dt \right] d\omega + \\
& \frac{1}{2} \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(s)\bar{a}]g(s) \cos(\omega + 2\omega_o + s)t \, ds \, dt \right] d\omega + \\
& \frac{1}{2} \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(s)\bar{a}]g(s) \cos(\omega - s)t \, ds \, dt \right] d\omega + \\
& - \frac{1}{2} \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(s)\bar{a}]g(s) \cos(\omega + 2\omega_o + s)t \, ds \, dt \right] d\omega .
\end{aligned}$$

The Identity 2.2 gives

$$\begin{aligned}
 I_R = & \pi \int_{-\infty}^{\infty} \text{Re}[\Omega(\omega)\bar{a}]^T \text{Re}[\Omega(\omega)\bar{a}] g^2(\omega) d\omega + \pi \int_{-\infty}^{\infty} \text{Im}[\Omega(\omega)\bar{a}]^T \text{Im}[\Omega(\omega)\bar{a}] g^2(\omega) d\omega + \\
 & \pi \int_{-\infty}^{\infty} \text{Re}[\Omega(\omega)\bar{a}]^T \text{Re}[\Omega(-\omega - 2\omega_o)\bar{a}] g(\omega) g(-\omega - 2\omega_o) d\omega + \\
 & - \pi \int_{-\infty}^{\infty} \text{Im}[\Omega(\omega)\bar{a}]^T \text{Im}[\Omega(-\omega - 2\omega_o)\bar{a}] g(\omega) g(-\omega - 2\omega_o) d\omega.
 \end{aligned}$$

The identities $\|a\|^2 = a^* a = \text{Re} a^T \cdot \text{Re} a + \text{Im} a^T \cdot \text{Im} a$ and $\text{Re}[a^T c] = \text{Re} a^T \cdot \text{Re} c - \text{Im} a^T \cdot \text{Im} c$, which hold for any two complex vectors a and c , yield

$$I_R = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega + \pi \int_{-\infty}^{\infty} \text{Re}[\bar{a}^T \Omega^T(\omega)\Omega(-\omega - 2\omega_o)\bar{a}] g(\omega) g(-\omega - 2\omega_o) d\omega,$$

or

$$I_R = \frac{1}{2} I_C + \pi \int_{-\infty}^{\infty} \text{Re}[\bar{a}^T \Omega^T(u - \omega_o)\Omega(-u - \omega_o)\bar{a}] g(u - \omega_o) g(-u - \omega_o) du,$$

where in the last integral we have made the change of variables $u = \omega + \omega_o$.

Corollary. If g is zero outside an interval $[-\omega_1, \omega_1]$ and $\omega_1 < \omega_o$, then the product $g(u - \omega_o)g(-u - \omega_o) \equiv 0$ for all $u \in R$, consequently,

$$I_R = \frac{1}{2} I_C = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega. \tag{2.3}$$

Proof. If g is zero outside the interval $[-\omega_1, \omega_1]$, then $g(u - \omega_o)$ can be non zero only when $\omega_o - \omega_1 \leq u \leq \omega_o + \omega_1$, and $g(-u - \omega_o)$ can be non zero only when

$-\omega_o - \omega_1 \leq u \leq \omega_1 - \omega_o$. Since these two sets do not intersect, because $\omega_1 - \omega_o < 0 < \omega_o - \omega_1$, the product $g(u - \omega_o)g(-u - \omega_o)$ is zero for all $u \in R$.

3. OUTPUT INTENSITY IN TERMS OF THE MATRIX T AND IN TERMS OF THE PARAMETERS DEFINING THE IFOG

In this section we obtain a closed form expression for the output intensity of the gyro in terms of the four entries of the complex 2×2 matrix T and in terms of the parameters involved in the four entries of the matrix T for three types of function g : a step function, a tent function, and a Gaussian function.

Part (a) of Theorem 1 holds for any function $g \in L^2(R)$ while part (b) of Theorem 1 holds for any *real valued* function $g \in L^2(R)$. When g is a step or a tent function, Corollary 1 will apply and the output intensity will be given by Equation 2.3. When g is the Gaussian function defined in Section 2, $g(u - \omega_o)g(-u - \omega_o) = g(\sqrt{2}\omega_o)g(\sqrt{2}u)$ for all $u \in R$, so we can write I_R as

$$I_R = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega + \pi g(\sqrt{2}\omega_o) \int_{-\infty}^{\infty} \text{Re}[\bar{a}^T \Omega^T(u - \omega_o) \Omega(-u - \omega_o) \bar{a}] g(\sqrt{2}u) du.$$

Since ω_o/σ is a very large number, $g(\sqrt{2}\omega_o)$ is a very small number, thus the second integral above can be neglected. Therefore, by Equation 2.3, in all three cases it will suffice to compute I_C .

We start by defining expressions for the four entries of T and compute $\|\Omega\bar{a}\|^2$.

If we write T as $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$, then

$$\Omega \bar{a} = [T + e^{i\phi} T^T] \bar{a} = \begin{bmatrix} T_{11} + e^{i\phi} T_{11} & T_{12} + e^{i\phi} T_{21} \\ T_{21} + e^{i\phi} T_{12} & T_{22} + e^{i\phi} T_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} (T_{11} + e^{i\phi} T_{11})a_1 + (T_{12} + e^{i\phi} T_{21})a_2 \\ (T_{21} + e^{i\phi} T_{12})a_1 + (T_{22} + e^{i\phi} T_{22})a_2 \end{bmatrix} \quad (3.1)$$

Hence,

$$\begin{aligned} \|\Omega(\omega) \bar{a}\|^2 &= \bar{a}^* \Omega^*(\omega) \Omega(\omega) \bar{a} = \\ &2|a_1|^2 (1 + \cos \phi) |T_{11}|^2 + 2|a_2|^2 (1 + \cos \phi) |T_{22}|^2 + |T_{12}|^2 + |T_{21}|^2 + \\ &2|a_1|^2 \operatorname{Re}[\bar{T}_{21} T_{12} e^{i\phi}] + 2|a_2|^2 \operatorname{Re}[\bar{T}_{12} T_{21} e^{i\phi}] + \\ &2 \operatorname{Re}\{a_1 \bar{a}_2 [\bar{T}_{12} T_{11} + \bar{T}_{12} e^{i\phi} T_{11}]\} + 2 \operatorname{Re}\{a_1 \bar{a}_2 [e^{-i\phi} \bar{T}_{21} T_{11} + \bar{T}_{21} T_{11}]\} + \\ &2 \operatorname{Re}\{a_1 \bar{a}_2 [\bar{T}_{22} T_{21} + e^{-i\phi} \bar{T}_{22} T_{21}]\} + 2 \operatorname{Re}\{a_1 \bar{a}_2 [\bar{T}_{22} e^{i\phi} T_{12} + \bar{T}_{22} T_{12}]\}. \end{aligned} \quad (3.2)$$

Each T_{ij} is a sum of 2^N terms, where N is the number of rotations (miss-alignments) in the fiber splicings. They have the form (see Section 4)

$$T_{ij}(\omega) = \sum_{s=1}^{2^N} C_{ij}(s) \exp i(\omega \sum_{r \in \Lambda_{ij}(s)} n_r \ell_r / c). \quad (3.3)$$

Thus, the factor $\|\Omega(\omega) \bar{a}\|^2$ in Theorem 1 involves *thousands* of terms if $N \geq 4$. Each C_{ij} is a 2^N -array of coefficients and each $\Lambda_{ij}(s)$ is set of indices, ($s = 1, 2, \dots, 2^N$; $i, j = 1, 2$). Let $\beta_{ij}(s)$ be given by

$$\beta_{ij}(s) \equiv \sum_{r \in \Lambda_{ij}(s)} n_r \ell_r / c, \text{ for } s = 1, 2, \dots, 2^N \text{ and } i, j = 1, 2 \quad (3.4)$$

and let $T_{uv}(\omega) = \sum_{p=1}^{2^N} C_{uv}(p) e^{i\beta_{uv}(p)\omega}$ and $a_1 \bar{a}_2 = |a_1 \bar{a}_2| e^{i\alpha}$. Then,

$$a_1 \bar{a}_2 e^{\pm i\phi} \bar{T}_{ij}(\omega) T_{uv}(\omega) = |a_1 \bar{a}_2| \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{uv}(p) e^{i\{\alpha \pm \phi + [\beta_{uv}(p) - \beta_{ij}(s)]\omega\}}$$

$$\text{Re}[a_1 \bar{a}_2 e^{\pm i\phi} \bar{T}_{ij}(\omega) T_{uv}(\omega)] = |a_1 \bar{a}_2| \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{uv}(p) \cos\{\alpha \pm \phi + [\beta_{uv}(p) - \beta_{ij}(s)]\omega\}$$

and

$$|T_{ij}(\omega)|^2 = \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{ij}(p) \cos[\beta_{ij}(p) - \beta_{ij}(s)]\omega.$$

Thus,

$$\int_{-\infty}^{\infty} |T_{ij}(\omega)|^2 g^2(\omega) d\omega = \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{ij}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos\{[\beta_{ij}(p) - \beta_{ij}(s)]\omega\} d\omega \quad (3.5)$$

and

$$\int_{-\infty}^{\infty} \text{Re}[a_1 \bar{a}_2 e^{\pm i\phi} \bar{T}_{ij}(\omega) T_{uv}(\omega)] g^2(\omega) d\omega =$$

$$|a_1 \bar{a}_2| \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{uv}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos\{\alpha \pm \phi + [\beta_{uv}(p) - \beta_{ij}(s)]\omega\} d\omega, \quad (3.6)$$

where g in Equations 3.5 and 3.6 can be any function in L^2 .

Since

$$\begin{aligned} \cos\{\alpha \pm \phi + [\beta_{uv}(p) - \beta_{ij}(s)]\omega\} = \\ \cos(\alpha \pm \phi) \cos[\beta_{uv}(p) - \beta_{ij}(s)]\omega - \sin(\alpha \pm \phi) \sin[\beta_{uv}(p) - \beta_{ij}(s)]\omega, \end{aligned}$$

when the function g is an *even* function Equation 3.6 simplifies to

$$\begin{aligned} \int_{-\infty}^{\infty} \text{Re}[a_1 \bar{a}_2 e^{\pm i\phi} \bar{T}_{ij}(\omega) T_{uv}(\omega)] g^2(\omega) d\omega = \\ |a_1 \bar{a}_2| \cos(\alpha \pm \phi) \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{uv}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{uv}(p) - \beta_{ij}(s)]\omega d\omega, \quad (3.7) \end{aligned}$$

which implies that when g is *even*

$$\int_{-\infty}^{\infty} \text{Re}[a_1 \bar{a}_2 e^{\pm i\phi} \bar{T}_{ij}(\omega) T_{uv}(\omega)] g^2(\omega) d\omega = |a_1 \bar{a}_2| \cos(\alpha \pm \phi) \int_{-\infty}^{\infty} \text{Re}[\bar{T}_{ij}(\omega) T_{uv}(\omega)] g^2(\omega) d\omega. \quad (3.8)$$

We can now put together Part (a) of Theorem 1 with Equations 3.2 and 3.8 to obtain an expression for the output intensity in terms of $\int_{-\infty}^{\infty} \text{Re}[\bar{T}_{ij}(\omega) T_{uv}(\omega)] g^2(\omega) d\omega$, for $i, j, u, v = 1$ or 2 when the function g is an even function.

$$\begin{aligned}
 I_C = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega = \\
 2\pi \{ & 2|a_1|^2(1 + \cos\phi) \int_{-\infty}^{\infty} |T_{11}|^2 g^2 d\omega + 2|a_2|^2(1 + \cos\phi) \int_{-\infty}^{\infty} |T_{22}|^2 g^2 d\omega + \\
 & \int_{-\infty}^{\infty} |T_{12}|^2 g^2 d\omega + \int_{-\infty}^{\infty} |T_{21}|^2 g^2 d\omega + 2\cos\phi \int_{-\infty}^{\infty} \text{Re}[\bar{T}_{21}T_{12}] g^2 d\omega + \\
 & 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha + \phi)] \int_{-\infty}^{\infty} \text{Re}(\bar{T}_{12}T_{11}) g^2 d\omega + \\
 & 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha - \phi)] \int_{-\infty}^{\infty} \text{Re}(\bar{T}_{21}T_{11}) g^2 d\omega + \\
 & 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha - \phi)] \int_{-\infty}^{\infty} \text{Re}(\bar{T}_{22}T_{21}) g^2 d\omega + \\
 & 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha + \phi)] \int_{-\infty}^{\infty} \text{Re}(\bar{T}_{22}T_{12}) g^2 d\omega \}.
 \end{aligned}$$

More explicitly, in terms of the integrals

$$\int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega, \tag{3.9}$$

when the function g is an even function using Equations 3.5 and 3.7 we get,

$$\begin{aligned}
 I_C = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega = \\
 2\pi \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} \{ 2|a_1|^2(1 + \cos\phi) C_{11}(s)C_{11}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{11}(p) - \beta_{11}(s)] \omega d\omega +
 \end{aligned}$$

$$\begin{aligned}
 & 2|a_2|^2(1 + \cos \phi)C_{22}(s)C_{22}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{22}(p) - \beta_{22}(s)]\omega \, d\omega + \\
 & C_{12}(s)C_{12}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{12}(p) - \beta_{12}(s)]\omega \, d\omega + \\
 & C_{21}(s)C_{21}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{21}(p) - \beta_{21}(s)]\omega \, d\omega + \\
 & 2 \cos \phi C_{12}(s)C_{21}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{12}(p) - \beta_{21}(s)]\omega \, d\omega + \\
 & 2|a_1\bar{a}_2|[\cos \alpha + \cos(\alpha + \phi)]C_{12}(s)C_{11}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{12}(p) - \beta_{11}(s)]\omega \, d\omega + \\
 & 2|a_1\bar{a}_2|[\cos \alpha + \cos(\alpha - \phi)]C_{21}(s)C_{11}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{21}(p) - \beta_{11}(s)]\omega \, d\omega + \\
 & 2|a_1\bar{a}_2|[\cos \alpha + \cos(\alpha - \phi)]C_{22}(s)C_{21}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{22}(p) - \beta_{21}(s)]\omega \, d\omega + \\
 & 2|a_1\bar{a}_2|[\cos \alpha + \cos(\alpha + \phi)]C_{22}(s)C_{12}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{22}(p) - \beta_{12}(s)]\omega \, d\omega \}.
 \end{aligned}
 \tag{3.10}$$

3.1. THE SAGNAC PHASE SHIFT ERROR

In this Section we derive the Sagnac phase shift error due to polarization. Let $I_o(\phi)$ denote the output intensity of the gyro with perfect polarizers (attenuating coefficient $\varepsilon = 0$) as a function of the Sagnac phase shift ϕ and let $I_\varepsilon(\phi)$ denote the output intensity of the gyro with imperfect polarizers ($\varepsilon > 0$). When the polarizers are perfect, $T_{12} = 0$, $T_{21} = 0$, and $T_{22} = 0$ (see the examples in Sections 4 and 5). Thus,

$$I_o(\phi) = 4\pi |a_1|^2(1 + \cos \phi) \int_{-\infty}^{\infty} |T_{11}|^2 g^2 d\omega.$$

Since $I_o(-\pi/2) = 4\pi |a_1|^2 \int_{-\infty}^{\infty} |T_{11}|^2 g^2 d\omega$, we can write $I_o(\phi)$ as

$$I_o(\phi) = I_o(-\pi/2) (1 + \cos \phi). \quad (3.11)$$

The difference between $I_e(\phi)$ and $I_o(\phi)$ is the error induced by having an imperfect polarizer. This error induces an error in the estimate of the Sagnac phase shift ϕ that we will call ϕ_ϵ . It can be calculated by comparing $I_e(-\pi/2)$ with $I_o(-\pi/2 + \phi_\epsilon)$. The idea is that there is a rotation angle ϕ_ϵ such that

$$I_e(-\pi/2) = I_o(-\pi/2 + \phi_\epsilon). \quad (3.12)$$

By Equation 3.11, $I_o(-\pi/2 + \phi_\epsilon) = I_o(-\pi/2) [1 + \cos(-\pi/2 + \phi_\epsilon)]$. Substituting in Equation 3.12 and solving for ϕ_ϵ we get,

$$\begin{aligned} I_e(-\pi/2) &= I_o(-\pi/2) [1 + \cos(-\pi/2 + \phi_\epsilon)] \\ &= I_o(-\pi/2) [1 + \cos(\pi/2) \cos \phi_\epsilon + \sin(\pi/2) \sin \phi_\epsilon] \\ &= I_o(-\pi/2) [1 + \sin \phi_\epsilon] \end{aligned}$$

$$\Rightarrow \phi_\epsilon = \sin^{-1} \left(\frac{I_e(-\pi/2)}{I_o(-\pi/2)} - 1 \right). \quad (3.13)$$

3.2. GAUSSIAN FUNCTION g

According to Equation 3.10, all we need to obtain the intensity I_C for an even function g are the integrals $\int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega$.

When the function g is the Gaussian function defined in Section 2, which is an even function, we use the following identity (Reference 3).

$$\int_{-\infty}^{\infty} e^{-x^2/q^2} \cos[p(x+\lambda)] dx = q\sqrt{\pi} e^{-p^2 q^2/4} \cos p\lambda,$$

from which we get,

$$\begin{aligned} \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega &= \\ \frac{1}{\sigma^2 \pi} \int_{-\infty}^{\infty} e^{-2\omega^2/\sigma^2} \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega &= \frac{1}{\sigma\sqrt{2\pi}} e^{-[\beta_{ij}(s) - \beta_{uv}(p)]^2 \sigma^2/8}, \end{aligned}$$

thus, by Equation 3.10

$$I_C = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega =$$

$$\begin{aligned}
 & \frac{\sqrt{2\pi}}{\sigma} \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} \{ 2|a_1|^2 (1 + \cos\phi) C_{11}(s) C_{11}(p) e^{-[\beta_{11}(s) - \beta_{11}(p)]^2 \sigma^2 / 8} + \\
 & \quad 2|a_2|^2 (1 + \cos\phi) C_{22}(s) C_{22}(p) e^{-[\beta_{22}(s) - \beta_{22}(p)]^2 \sigma^2 / 8} + \\
 & \quad C_{12}(s) C_{12}(p) e^{-[\beta_{12}(s) - \beta_{12}(p)]^2 \sigma^2 / 8} + \\
 & \quad C_{21}(s) C_{21}(p) e^{-[\beta_{21}(s) - \beta_{21}(p)]^2 \sigma^2 / 8} + \\
 & \quad 2 \cos\phi C_{12}(s) C_{21}(p) e^{-[\beta_{12}(s) - \beta_{21}(p)]^2 \sigma^2 / 8} + \\
 & \quad 2|a_1 \bar{a}_2| [\cos\alpha + \cos(\alpha + \phi)] C_{12}(s) C_{11}(p) e^{-[\beta_{12}(s) - \beta_{11}(p)]^2 \sigma^2 / 8} + \\
 & \quad 2|a_1 \bar{a}_2| [\cos\alpha + \cos(\alpha - \phi)] C_{21}(s) C_{11}(p) e^{-[\beta_{21}(s) - \beta_{11}(p)]^2 \sigma^2 / 8} + \\
 & \quad 2|a_1 \bar{a}_2| [\cos\alpha + \cos(\alpha - \phi)] C_{22}(s) C_{21}(p) e^{-[\beta_{22}(s) - \beta_{21}(p)]^2 \sigma^2 / 8} + \\
 & \quad 2|a_1 \bar{a}_2| [\cos\alpha + \cos(\alpha + \phi)] C_{22}(s) C_{12}(p) e^{-[\beta_{22}(s) - \beta_{12}(p)]^2 \sigma^2 / 8} \}.
 \end{aligned}
 \tag{3.14}$$

3.3. STEP FUNCTION g

In this Section we let the frequency distribution function be a step function g_s . Let g_s be defined as

$$g_s(\omega) = \begin{cases} \frac{1}{2\omega_1} & \text{for } |\omega| \leq \omega_1, \\ 0 & \text{otherwise} \end{cases}, \quad \text{with } 0 < \omega_1 < \omega_o.$$

Since g_s is an even function, we only need the integrals $\int_{-\infty}^{\infty} g_s^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega$. We use the following result.

$$\int_{-\infty}^{\infty} g_s^2(x) \cos \beta x dx = \frac{1}{4\omega_1^2} \int_{-\omega_1}^{\omega_1} \cos \beta x dx = \frac{1}{2\omega_1} \frac{\sin \beta \omega_1}{\beta \omega_1},$$

from which we get,

$$\int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega =$$

$$\frac{1}{4\omega_1^2} \int_{-\infty}^{\infty} \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega = \frac{1}{2\omega_1} \frac{\sin[\beta_{ij}(s) - \beta_{uv}(p)] \omega_1}{[\beta_{ij}(s) - \beta_{uv}(p)] \omega_1},$$

thus, by Equation 3.10

$$I_C = \frac{\pi}{2\omega_1} \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} \left\{ \begin{aligned} & 2|a_1|^2 (1 + \cos \phi) C_{11}(s) C_{11}(p) \frac{\sin[\beta_{11}(s) - \beta_{11}(p)] \omega_1}{[\beta_{11}(s) - \beta_{11}(p)] \omega_1} + \\ & 2|a_2|^2 (1 + \cos \phi) C_{22}(s) C_{22}(p) \frac{\sin[\beta_{22}(s) - \beta_{22}(p)] \omega_1}{[\beta_{22}(s) - \beta_{22}(p)] \omega_1} + \\ & C_{12}(s) C_{12}(p) \frac{\sin[\beta_{12}(s) - \beta_{12}(p)] \omega_1}{[\beta_{12}(s) - \beta_{12}(p)] \omega_1} + \\ & C_{21}(s) C_{21}(p) \frac{\sin[\beta_{21}(s) - \beta_{21}(p)] \omega_1}{[\beta_{21}(s) - \beta_{21}(p)] \omega_1} + \\ & 2 \cos \phi C_{12}(s) C_{21}(p) \frac{\sin[\beta_{12}(s) - \beta_{21}(p)] \omega_1}{[\beta_{12}(s) - \beta_{21}(p)] \omega_1} + \\ & 2|a_1 \bar{a}_2| [\cos \alpha + \cos(\alpha + \phi)] C_{11}(s) C_{12}(p) \frac{\sin[\beta_{11}(s) - \beta_{12}(p)] \omega_1}{[\beta_{11}(s) - \beta_{12}(p)] \omega_1} + \\ & 2|a_1 \bar{a}_2| [\cos \alpha + \cos(\alpha - \phi)] C_{11}(s) C_{21}(p) \frac{\sin[\beta_{11}(s) - \beta_{21}(p)] \omega_1}{[\beta_{11}(s) - \beta_{21}(p)] \omega_1} + \\ & 2|a_1 \bar{a}_2| [\cos \alpha + \cos(\alpha - \phi)] C_{21}(s) C_{22}(p) \frac{\sin[\beta_{21}(s) - \beta_{22}(p)] \omega_1}{[\beta_{21}(s) - \beta_{22}(p)] \omega_1} + \\ & 2|a_1 \bar{a}_2| [\cos \alpha + \cos(\alpha + \phi)] C_{22}(s) C_{12}(p) \frac{\sin[\beta_{22}(s) - \beta_{12}(p)] \omega_1}{[\beta_{22}(s) - \beta_{12}(p)] \omega_1} \end{aligned} \right\}. \quad (3.15)$$

3.4. TENT FUNCTION g

The analysis above can be carried out when the function g is a "tent" function g_T defined as

$$g_T(x) \equiv \frac{1}{\omega_1} \begin{cases} 0 & \text{for } |x| > \omega_1 \\ 1 + \frac{1}{\omega_1}x & \text{for } -\omega_1 \leq x \leq 0 \\ 1 - \frac{1}{\omega_1}x & \text{for } 0 \leq x \leq \omega_1 \end{cases}$$

Since g_T is even, we only need $\int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)] \omega d\omega$. We will use the next three formulas:

$$\int 1 \cdot \cos \beta x dx = \frac{1}{\beta} \sin \beta x$$

$$\int x \cdot \cos \beta x dx = \frac{1}{\beta^2} \cos \beta x + \frac{1}{\beta} x \sin \beta x$$

$$\int x^2 \cdot \cos \beta x dx = -\frac{2}{\beta^3} \sin \beta x + \frac{2}{\beta^2} x \cos \beta x + \frac{1}{\beta} x^2 \sin \beta x.$$

From the definition of g_T we have

$$g_T^2(x) \equiv \frac{1}{\omega_1^2} \begin{cases} 0 & \text{for } |x| > \omega_1 \\ 1 + \frac{2}{\omega_1}x + \frac{1}{\omega_1^2}x^2 & \text{for } \omega_1 \leq x \leq 0 \\ 1 - \frac{2}{\omega_1}x + \frac{1}{\omega_1^2}x^2 & \text{for } 0 \leq x \leq \omega_1 \end{cases}$$

When $\beta \neq 0$

$$\begin{aligned} \int_{-\infty}^{\infty} g_T^2(x) \cos \beta x \, dx &= \frac{2}{\omega_1^2} \int_0^{\omega_1} \left(1 - \frac{2}{\omega_1}x + \frac{1}{\omega_1^2}x^2\right) \cos \beta x \, dx = \\ &= \frac{2}{\omega_1^2 \beta} \left[1 - \frac{2x}{\omega_1} + \frac{x^2}{\omega_1^2} - \frac{2}{\omega_1^2 \beta^2}\right] \sin \beta x + \frac{2}{\omega_1^2 \beta^2} \left[-\frac{2}{\omega_1} + \frac{2x}{\omega_1^2}\right] \cos \beta x \Bigg|_{x=0}^{\omega_1} \\ &= \frac{4}{\beta^2 \omega_1^3} \left[1 - \frac{\sin \beta \omega_1}{\beta \omega_1}\right]. \end{aligned} \quad (3.16)$$

If $\beta = 0$, then

$$\int_{-\infty}^{\infty} g_T^2(x) \, dx = \frac{2}{\omega_1^2} \int_0^{\omega_1} \left(1 - \frac{2}{\omega_1}x + \frac{1}{\omega_1^2}x^2\right) \, dx = \frac{1}{\omega_1^2} \left(x - \frac{x^2}{\omega_1} + \frac{x^3}{3\omega_1^2}\right) \Bigg|_0^{\omega_1} = \frac{2}{3\omega_1}. \quad (3.17)$$

Let $Z_{ij,uv} \equiv \{(s,p) : \beta_{ij}(p) - \beta_{uv}(s) = 0\}$ and define

$$\begin{aligned} \Lambda_{ij,uv} &\equiv \frac{2}{3\omega_1} \sum_{(s,p) \in Z_{uv,ij}} C_{ij}(s) C_{uv}(p) + \\ &\frac{4}{\omega_1^3} \sum_{(s,p) \in Z_{uv,ij}} \frac{C_{ij}(s) C_{uv}(p)}{[\beta_{ij}(p) - \beta_{uv}(s)]^2(s,p)} \left[1 - \frac{\sin\{\omega_1[\beta_{ij}(p) - \beta_{uv}(s)]\}}{\omega_1[\beta_{ij}(p) - \beta_{uv}(s)]}\right]. \end{aligned} \quad (3.18)$$

Note that $\Lambda_{ij,uv} = \int_{-\infty}^{\infty} \text{Re}[\bar{T}_{ij}(\omega)T_{uv}(\omega)]g_T^2(\omega) d\omega$ by Equations 3.7, 3.16, and 3.17.

Then, by Equations 3.10, 3.16, 3.17, and the definition of $\Lambda_{ij,uv}$

$$\begin{aligned}
 I_C = 2\pi \{ & 2|a_1|^2(1 + \cos\phi)\Lambda_{11,11} + 2|a_2|^2(1 + \cos\phi)\Lambda_{22,22} + \Lambda_{12,12} + \Lambda_{21,21} + \\
 & 2\cos\phi\Lambda_{12,21} + 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha + \phi)]\Lambda_{12,11} + \\
 & 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha - \phi)]\Lambda_{21,11} + 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha - \phi)]\Lambda_{22,21} + \\
 & 2|a_1\bar{a}_2|[\cos\alpha + \cos(\alpha + \phi)]\Lambda_{22,12} \}. \tag{3.19}
 \end{aligned}$$

4. AN ELEMENTARY IFOG MODEL

As a first example we consider the simplest possible gyroscope. It consists of an integrated optics chip (IOC) connected to a length of polarization maintaining (PM) fiber at a 45° angle. The PM fiber is connected to a coil of single mode (SM) fiber which is then connected back to the IOC.

4.1. THE MODEL, OUTPUT INTENSITY, AND SAGNAC ERROR

The IOC is modeled by the Jones matrix

$$M = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \epsilon e^{ik_6 l_3} \end{bmatrix}, \text{ with } k_5 = \frac{\omega}{c} n_5 \text{ and } k_6 = \frac{\omega}{c} n_6.$$

The PM fiber of length l_5 and the 45° angle connection are modeled, respectively, by the Jones matrices

$$U = \begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix}, \text{ with } k_9 = \frac{\omega}{c} n_9, k_{10} = \frac{\omega}{c} n_{10}, \text{ and } R(45^\circ) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

The coil of SM fiber will be modeled by the matrix

$$C = \begin{bmatrix} t_{11} e^{i\tau_{11}} & t_{12} e^{i\tau_{12}} \\ t_{21} e^{i\tau_{21}} & t_{22} e^{i\tau_{22}} \end{bmatrix}, \text{ with } \tau_{ij} = \frac{\omega}{c} n_{ij} l_o = k_{ij} l_o.$$

The clockwise loop matrix T is modeled by the product of the corresponding Jones matrices.

$$T(\omega) = \frac{1}{2\sqrt{2}} \begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \epsilon e^{ik_6 l_3} \end{bmatrix} \begin{bmatrix} t_{11} e^{i\tau_{11}} & t_{12} e^{i\tau_{12}} \\ t_{21} e^{i\tau_{21}} & t_{22} e^{i\tau_{22}} \end{bmatrix} \begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \epsilon e^{ik_6 l_3} \end{bmatrix}.$$

The product of the first three matrices is

$$\begin{bmatrix} e^{ik_5l_3} & 0 \\ 0 & \epsilon e^{ik_6l_3} \end{bmatrix} \begin{bmatrix} t_{11}e^{i\tau_{11}} & t_{12}e^{i\tau_{12}} \\ t_{21}e^{i\tau_{21}} & t_{22}e^{i\tau_{22}} \end{bmatrix} \begin{bmatrix} e^{ik_9l_5} & 0 \\ 0 & e^{ik_{10}l_5} \end{bmatrix} = \begin{bmatrix} t_{11}e^{i(\tau_{11}+k_5l_3+k_9l_5)} & t_{12}e^{i(\tau_{12}+k_5l_3+k_{10}l_5)} \\ \epsilon t_{21}e^{i(\tau_{21}+k_6l_3+k_9l_5)} & \epsilon t_{22}e^{i(\tau_{22}+k_6l_3+k_{10}l_5)} \end{bmatrix}.$$

Notice that the diagonal matrices modeling the IOC and the PM fiber do not increase the number of terms in the resulting matrix, they contribute to the number of terms in the complex exponential. However, a rotation in general will double the number of terms in the resulting matrix as can be seen next with the 45° rotation. This explains why, if the coil is modeled by a full Jones matrix, each T_{ij} is a sum of 2^N terms, where N is the number of rotations (missalignments) in the fiber splicings along the clockwise loop.

$$\begin{bmatrix} t_{11}e^{i(\tau_{11}+k_5l_3+k_9l_5)} & t_{12}e^{i(\tau_{12}+k_5l_3+k_{10}l_5)} \\ \epsilon t_{21}e^{i(\tau_{21}+k_6l_3+k_9l_5)} & \epsilon t_{22}e^{i(\tau_{22}+k_6l_3+k_{10}l_5)} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} t_{11}e^{i(\tau_{11}+k_5l_3+k_9l_5)} - t_{12}e^{i(\tau_{12}+k_5l_3+k_{10}l_5)} & [t_{11}e^{i(\tau_{11}+k_5l_3+k_9l_5)} + t_{12}e^{i(\tau_{12}+k_5l_3+k_{10}l_5)}] \\ \epsilon [t_{21}e^{i(\tau_{21}+k_6l_3+k_9l_5)} - t_{22}e^{i(\tau_{22}+k_6l_3+k_{10}l_5)}] & \epsilon [t_{21}e^{i(\tau_{21}+k_6l_3+k_9l_5)} + t_{22}e^{i(\tau_{22}+k_6l_3+k_{10}l_5)}] \end{bmatrix}.$$

Finally, multiplying by the matrix modeling the IOC gives

$$T(\omega) = \frac{1}{2\sqrt{2}} \cdot$$

$$\begin{bmatrix} t_{11}e^{i(\tau_{11}+2k_5l_3+k_9l_5)} - t_{12}e^{i(\tau_{12}+2k_5l_3+k_{10}l_5)} & \epsilon [t_{11}e^{i(\tau_{11}+k_5l_3+k_6l_3+k_9l_5)} + t_{12}e^{i(\tau_{12}+k_5l_3+k_6l_3+k_{10}l_5)}] \\ \epsilon [t_{21}e^{i(\tau_{21}+k_5l_3+k_6l_3+k_9l_5)} - t_{22}e^{i(\tau_{22}+k_5l_3+k_6l_3+k_{10}l_5)}] & \epsilon^2 [t_{21}e^{i(\tau_{21}+2k_6l_3+k_9l_5)} + t_{22}e^{i(\tau_{22}+2k_6l_3+k_{10}l_5)}] \end{bmatrix}. \quad (4.1)$$

Thus, for the simple configuration of this example we have

$$\begin{aligned}
 T_{11}(\omega) &= \frac{1}{2\sqrt{2}} t_{11} e^{i(\tau_{11} + 2k_5 l_3 + k_9 l_5)} - \frac{1}{2\sqrt{2}} t_{12} e^{i(\tau_{12} + 2k_5 l_3 + k_{10} l_5)}, \\
 T_{12}(\omega) &= \frac{1}{2\sqrt{2}} \mathcal{E} [t_{11} e^{i(\tau_{11} + k_5 l_3 + k_6 l_3 + k_9 l_5)} + t_{12} e^{i(\tau_{12} + k_5 l_3 + k_6 l_3 + k_{10} l_5)}], \\
 T_{21}(\omega) &= \frac{1}{2\sqrt{2}} \mathcal{E} [t_{21} e^{i(\tau_{21} + k_5 l_3 + k_6 l_3 + k_9 l_5)} - t_{22} e^{i(\tau_{22} + k_5 l_3 + k_6 l_3 + k_{10} l_5)}], \\
 T_{22}(\omega) &= \frac{1}{2\sqrt{2}} \mathcal{E}^2 [t_{21} e^{i(\tau_{21} + 2k_6 l_3 + k_9 l_5)} + t_{22} e^{i(\tau_{22} + 2k_6 l_3 + k_{10} l_5)}],
 \end{aligned}$$

and the $C_{ij}(s)$ and $\beta_{ij}(s)$ for $s=1, 2$ representing the coefficients and the phases in the expressions for $T_{ij}(\omega)$ (see Equations 3.3 and 3.4) are

$$\begin{aligned}
 C_{11}(1) &= \frac{1}{2\sqrt{2}} t_{11}, & C_{11}(2) &= -\frac{1}{2\sqrt{2}} t_{12}, \\
 C_{12}(1) &= \frac{1}{2\sqrt{2}} \mathcal{E} t_{11}, & C_{12}(2) &= \frac{1}{2\sqrt{2}} \mathcal{E} t_{12}, \\
 C_{21}(1) &= \frac{1}{2\sqrt{2}} \mathcal{E} t_{21}, & C_{21}(2) &= -\frac{1}{2\sqrt{2}} \mathcal{E} t_{22}, \\
 C_{22}(1) &= \frac{1}{2\sqrt{2}} \mathcal{E}^2 t_{21}, & C_{22}(2) &= \frac{1}{2\sqrt{2}} \mathcal{E}^2 t_{22},
 \end{aligned}$$

$$\begin{aligned}
 \omega \beta_{11}(1) &= \tau_{11} + 2k_5 l_3 + k_9 l_5, & \omega \beta_{11}(2) &= \tau_{12} + 2k_5 l_3 + k_{10} l_5, \\
 \omega \beta_{12}(1) &= \tau_{11} + k_5 l_3 + k_6 l_3 + k_9 l_5, & \omega \beta_{12}(2) &= \tau_{12} + k_5 l_3 + k_6 l_3 + k_{10} l_5, \\
 \omega \beta_{21}(1) &= \tau_{21} + k_5 l_3 + k_6 l_3 + k_9 l_5, & \omega \beta_{21}(2) &= \tau_{22} + k_5 l_3 + k_6 l_3 + k_{10} l_5, \\
 \omega \beta_{22}(1) &= \tau_{21} + 2k_6 l_3 + k_9 l_5, & \omega \beta_{22}(2) &= \tau_{22} + 2k_6 l_3 + k_{10} l_5.
 \end{aligned}$$

Since we only wish to keep terms of first order in \mathcal{E} or lower, any term in the expression for $\|\Omega(\omega)\bar{a}\|^2$ involving T_{22} or products involving T_{12} and/or T_{21} twice will be ignored. Thus, Equation 3.2 leads to the simpler expression

$$\|\Omega(\omega)\bar{a}\|^2 \equiv 2|a_1|^2(1+\cos\phi)|T_{11}|^2 + 2\operatorname{Re}\{a_1\bar{a}_2[\bar{T}_{12}T_{11} + \bar{T}_{12}e^{i\phi}T_{11}]\} + 2\operatorname{Re}\{a_1\bar{a}_2[e^{-i\phi}\bar{T}_{21}T_{11} + \bar{T}_{21}T_{11}]\}.$$

When the frequency distribution g of the input is a step function, the results of Section 3 give

$$I_R = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega) d\omega \equiv \frac{\pi}{4\omega_1} \sum_{s=1}^2 \sum_{p=1}^2 \left\{ \begin{aligned} & a_1^2 2(1+\cos\phi)C_{11}(s)C_{11}(p) \frac{\sin[\beta_{11}(s) - \beta_{11}(p)]\omega_1}{[\beta_{11}(s) - \beta_{11}(p)]\omega_1} + \\ & 2|a_1a_2|(1+\cos\phi)C_{11}(s)C_{12}(p) \frac{\sin[\beta_{11}(s) - \beta_{12}(p)]\omega_1}{[\beta_{11}(s) - \beta_{12}(p)]\omega_1} + \\ & 2|a_1a_2|(1+\cos\phi)C_{11}(s)C_{21}(p) \frac{\sin[\beta_{11}(s) - \beta_{21}(p)]\omega_1}{[\beta_{11}(s) - \beta_{21}(p)]\omega_1} \end{aligned} \right\}, \quad (4.2)$$

where we have assumed that both a_1 and a_2 are real, thus $\alpha = 0$.

Next, we compute some of the differences $\omega_1[\beta_{ij}(s) - \beta_{ij}(p)]$ ($s, p = 1, 2$).

$$\omega_1[\beta_{11}(1) - \beta_{11}(2)] = \tau_{11} - \tau_{12} - \Delta_5 l_5, \text{ where } \Delta_5 = k_{10} - k_9$$

$$\omega_1[\beta_{11}(1) - \beta_{12}(1)] = -\Delta_3 l_3, \text{ where } \Delta_3 = k_6 - k_5$$

$$\omega_1[\beta_{11}(1) - \beta_{12}(2)] = \tau_{11} - \tau_{12} - \Delta_3 l_3 - \Delta_5 l_5$$

$$\omega_1[\beta_{11}(2) - \beta_{12}(1)] = \tau_{12} - \tau_{11} - \Delta_3 l_3 + \Delta_5 l_5$$

$$\omega_1[\beta_{11}(2) - \beta_{12}(2)] = -\Delta_3 l_3$$

$$\omega_1[\beta_{11}(1) - \beta_{21}(1)] = \tau_{11} - \tau_{21} - \Delta_3 l_3$$

$$\omega_1[\beta_{11}(1) - \beta_{21}(2)] = \tau_{11} - \tau_{22} - \Delta_3 l_3 - \Delta_5 l_5$$

$$\begin{aligned}\omega_1[\beta_{11}(2) - \beta_{21}(1)] &= \tau_{12} - \tau_{21} - \Delta_3 l_3 + \Delta_5 l_5 \\ \omega_1[\beta_{11}(2) - \beta_{21}(2)] &= \tau_{12} - \tau_{22} - \Delta_3 l_3.\end{aligned}$$

Let $\text{sinc } x \equiv \frac{\sin x}{x}$ and note that $\text{sinc } x = \text{sinc } (-x)$. Equation 4.2 gives the output intensity as a function of the Sagnac phase shift ϕ for any value of the attenuating coefficient ε ; let it be denoted by $I_\varepsilon(\phi)$

$$\begin{aligned}I_\varepsilon(\phi) &\equiv \frac{\pi}{16\omega_1} (1 + \cos \phi) \left\{ a_1^2 [t_{11}^2 + t_{12}^2 - 2t_{11}t_{12} \text{sinc}(\Delta_5 l_5 + \tau_{12} - \tau_{11})] + \right. \\ &\quad |a_1 a_2| \varepsilon [(t_{11}^2 - t_{12}^2) \text{sinc } \Delta_3 l_3 + \\ &\quad t_{11}t_{12} (\text{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{12} - \tau_{11}) - \text{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{12})) + \\ &\quad t_{11}t_{21} \text{sinc}(\Delta_3 l_3 + \tau_{21} - \tau_{11}) - t_{11}t_{22} \text{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) + \\ &\quad \left. - t_{12}t_{21} \text{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{21} - \tau_{12}) + t_{12}t_{22} \text{sinc}(\Delta_3 l_3 + \tau_{22} - \tau_{12})] \right\}. \quad (4.3)\end{aligned}$$

To simplify the notation, define A and B as

$$A \equiv a_1^2 [t_{11}^2 + t_{12}^2 - 2t_{11}t_{12} \text{sinc}(\Delta_5 l_5 + \tau_{12} - \tau_{11})],$$

$$\begin{aligned}B &\equiv |a_1 a_2| \varepsilon [(t_{11}^2 - t_{12}^2) \text{sinc } \Delta_3 l_3 + \\ &\quad t_{11}t_{12} (\text{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{12} - \tau_{11}) - \text{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{12})) + \\ &\quad t_{11}t_{21} \text{sinc}(\Delta_3 l_3 + \tau_{21} - \tau_{11}) - t_{11}t_{22} \text{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) + \\ &\quad - t_{12}t_{21} \text{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{21} - \tau_{12}) + t_{12}t_{22} \text{sinc}(\Delta_3 l_3 + \tau_{22} - \tau_{12})]\end{aligned}$$

so that we can write $I_\varepsilon(\phi) \equiv \frac{\pi}{16\omega_1} (1 + \cos \phi) [A + B]$ and $I_o(\phi) \equiv \frac{\pi}{16\omega_1} (1 + \cos \phi) A$.

The Sagnac phase shift error due to polarization is given by Equation 3.13

$$\phi_\epsilon = \sin^{-1}\left(\frac{I_\epsilon(-\pi/2)}{I_o(-\pi/2)} - 1\right) = \sin^{-1}(B/A) = \sin^{-1}\left(\frac{I_\epsilon(\phi_s)}{I_o(\phi_s)} - 1\right). \quad (4.4)$$

4.2. THE INTENSITY AS A FUNCTION OF l_s ; THE OUTPUT COHERENCE

Next, we analyze the error in Equation 4.3, the difference between $I_\epsilon(\phi)$ and $I_o(\phi)$ as a function of l_s .

$$E \equiv I_\epsilon(\phi) - I_o(\phi) = \frac{\pi}{16\omega_1}(1 + \cos\phi)B$$

The limit of E as $l_s \rightarrow \infty$ is

$$\lim_{l_s \rightarrow \infty} E = \frac{\pi}{16\omega_1}(1 + \cos\phi) a_1 a_2 \mathcal{E} [(t_{11}^2 - t_{12}^2) \text{sinc } \Delta_3 l_3 + t_{11} t_{21} \text{sinc}(\Delta_3 l_3 + \tau_{21} - \tau_{11}) + t_{12} t_{22} \text{sinc}(\Delta_3 l_3 + \tau_{22} - \tau_{12})].$$

Assuming $\tau_{11} = \tau_{21}$ and $\tau_{12} = \tau_{22}$, this limit is zero provided

$$t_{11}^2 - t_{12}^2 + t_{11} t_{21} + t_{12} t_{22} = 0. \quad (4.5)$$

If we assume that the t_{ij} values represent a rotation through an angle θ , so that $t_{11} = \cos\theta$, $t_{12} = \sin\theta$, $t_{21} = -\sin\theta$, and $t_{22} = \cos\theta$, then Equation 4.5 implies $\cos^2\theta - \sin^2\theta = 0$, forcing $\theta = \pm 45^\circ, \pm 135^\circ, \pm 225^\circ, \text{ or } \pm 315^\circ$. If $\theta = 45^\circ$ or -135° , the error E is zero and hence $\phi_\epsilon = 0$. If $\theta = -45^\circ$ or 135° , then Equation 4.8 is satisfied and the t_{ij} values are, respectively, $(t_{11}, t_{12}, t_{21}, t_{22}) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and

$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$. The error angle ϕ_e as a function of l_5 for these two sets of values of t_{ij} is the same and is shown in Figure 1.

Examples of other sets of t_{ij} values satisfying Equation 4.5 that are not rotations include the following: $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm \alpha \cdot (1, -1, 1, 1)$ for any real value α , which generalize the previous sets of solutions. Note that these are not rotations unless $\alpha = \frac{1}{\sqrt{2}}$. The sets of values $(1, 0, -1, 1)$ and $(1, \frac{1+\sqrt{5}}{2}, 0, 1)$ also satisfy Equation 4.5. The error angles ϕ_e as a function of l_5 for these sets of values of t_{ij} are shown in Figures 2, 3, and 4. Figure 5 shows the error angle as a function of l_5 for a rotation through an angle of 2 radians. Note that the error does not converge to zero as $l_5 \rightarrow \infty$.

Whenever Equation 4.5 is satisfied with $\tau_{11} = \tau_{21}$ and $\tau_{12} = \tau_{22}$, the error becomes

$$E = \frac{\pi}{16\omega_1} (1 + \cos \phi) a_1 a_2 \mathcal{E} [t_{11} t_{12} (\text{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) - \text{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{22})) - t_{11} t_{22} \text{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) - t_{12} t_{21} \text{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{22})]$$

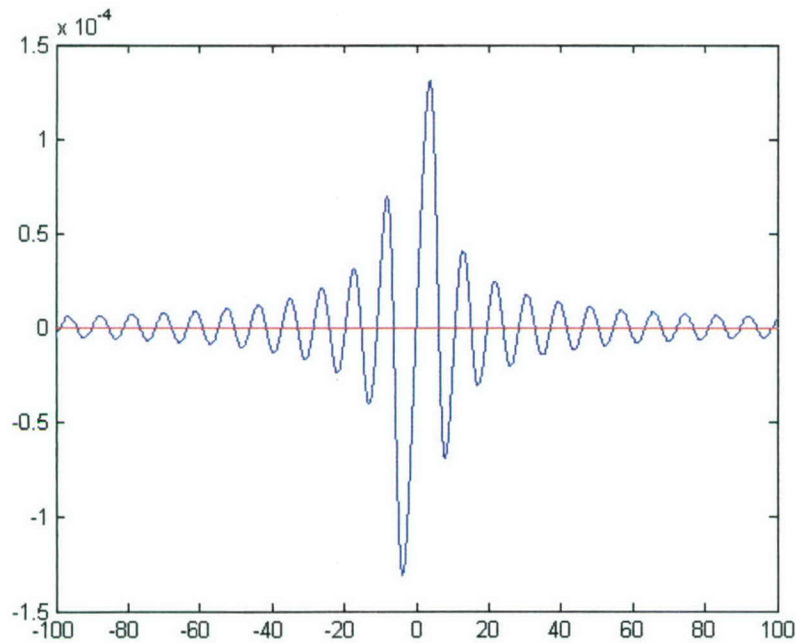


FIGURE 1. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, -45° and 135° Rotations.

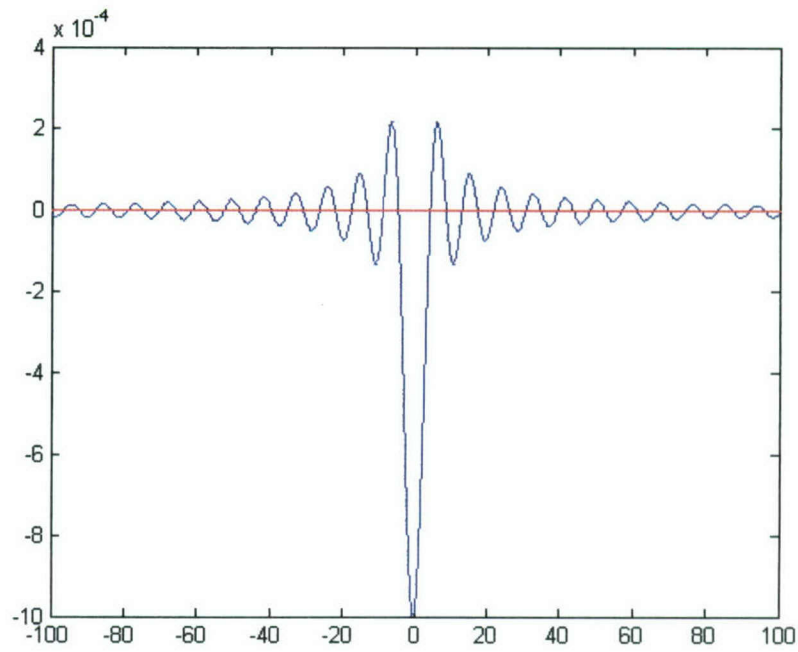


FIGURE 2. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm (1, 0, -1, 1)$, not a Rotation.

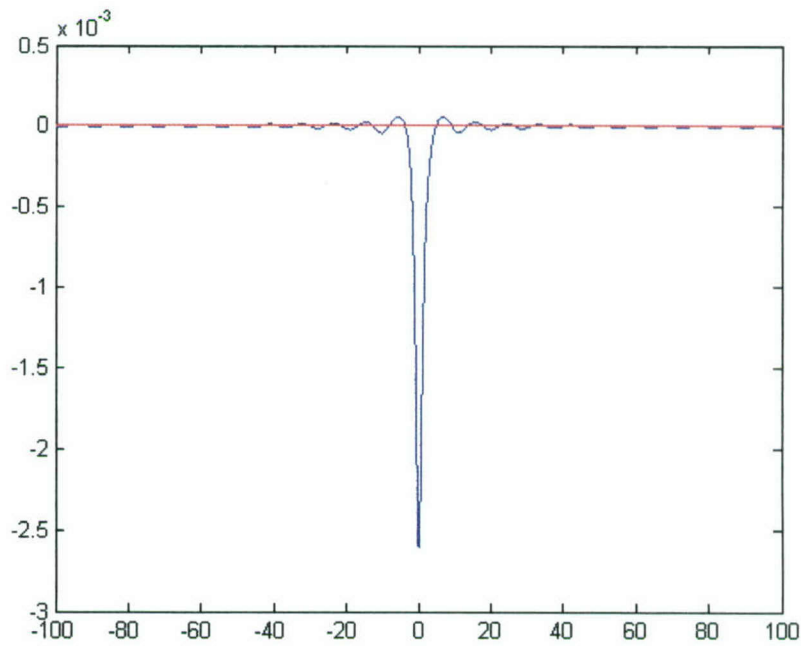


FIGURE 3. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm (1, \frac{1+\sqrt{5}}{2}, 0, 1)$, not a Rotation.

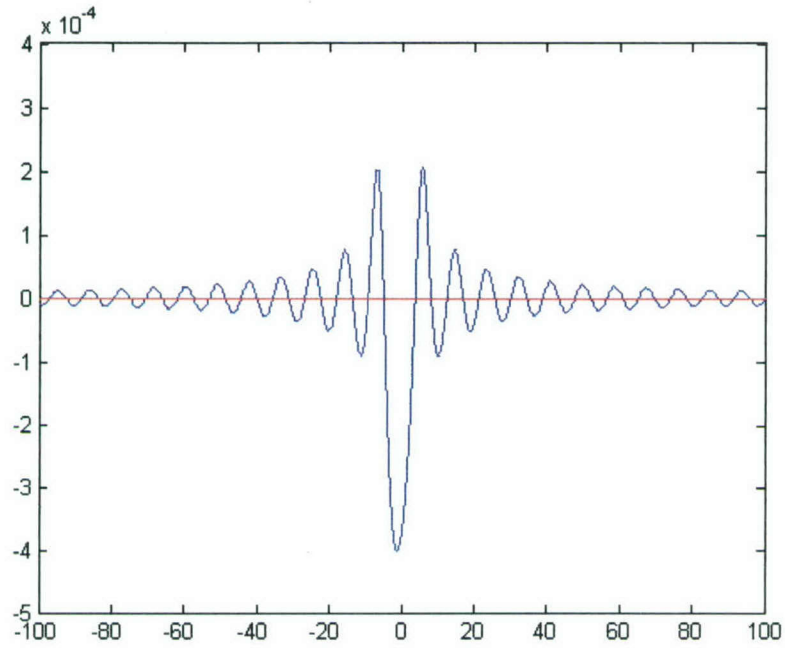


FIGURE 4. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm(1, \frac{1-\sqrt{5}}{2}, 0, 1)$, not a Rotation.

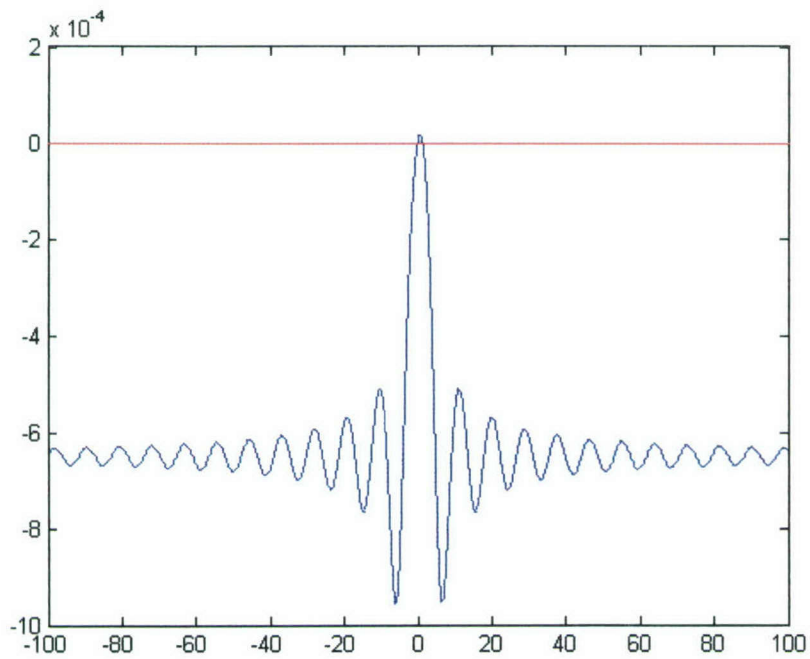


FIGURE 5. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm(\cos 2, \sin 2, -\sin 2, \cos 2)$.

4.3. MODULATION

To model the effects of modulation rewrite Equation 4.3 as

$$I_\varepsilon(\phi) \cong \frac{\pi}{16\omega_1}(A+B) + \frac{\pi}{16\omega_1}(A+B)\cos\phi.$$

We then replace $\cos\phi$ by $\cos(\phi - \phi_p \sin \omega_p t)$ and expand.

$$\begin{aligned} \cos(\phi - \phi_p \sin \omega_p t) &= \cos\phi \cos(\phi_p \sin \omega_p t) + \sin\phi \sin(\phi_p \sin \omega_p t) \\ &= \cos\phi \left[J_0(\phi_p) + 2 \sum_{k=1}^{\infty} J_{2k}(\phi_p) \cos(2k\omega_p t) \right] + \sin\phi \left[2 \sum_{k=0}^{\infty} J_{2k+1}(\phi_p) \sin(2k+1)\omega_p t \right]. \end{aligned}$$

If we replace $\cos\phi$ by $\cos(\phi - \phi_p \sin \omega_p t)$ in the expression for $I_\varepsilon(\phi)$, the only term with frequency $\omega_p t$ is the one with J_1 , therefore the demodulated and filtered intensity will be

$$\begin{aligned} I_\varepsilon(\phi) &\cong \frac{\pi}{8\omega_1} J_1(\phi_p)(A+B)\sin\phi, \text{ or} \\ I_\varepsilon(\phi) &\cong \frac{\pi}{8\omega_1} J_1(\phi_p)\sin\phi \cdot \left\{ \alpha_1^2 [t_{11}^2 + t_{12}^2 - 2t_{11}t_{12} \operatorname{sinc}(\Delta_5 l_5 + \tau_{12} - \tau_{11})] + \right. \\ &\quad \alpha_1 \alpha_2 \varepsilon [(t_{11}^2 - t_{12}^2) \operatorname{sinc} \Delta_3 l_3 + \\ &\quad t_{11} t_{12} (\operatorname{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{12} - \tau_{11}) - \operatorname{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{12})) + \\ &\quad t_{11} t_{21} \operatorname{sinc}(\Delta_3 l_3 + \tau_{21} - \tau_{11}) - t_{11} t_{22} \operatorname{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) + \\ &\quad \left. - t_{12} t_{21} \operatorname{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{21} - \tau_{12}) + t_{12} t_{22} \operatorname{sinc}(\Delta_3 l_3 + \tau_{22} - \tau_{12}) \right] \}. \end{aligned}$$

To obtain an expression for the rotation angle error ϕ_ε when the output of the gyro is demodulated and filtered, write $I_\varepsilon(\phi)$ as $I_\varepsilon(\phi) \cong \frac{\pi}{8\omega_1} J_1(\phi_p)[A+B]\sin\phi$ and $I_o(\phi) \cong \frac{\pi}{8\omega_1} J_1(\phi_p)A\sin\phi$.

If we set $I_\varepsilon(\phi) = I_o(\phi + \phi_\varepsilon)$ and solve for ϕ_ε we get

$$\frac{\pi}{8\omega_1} J_1(\phi_p)[A+B]\sin\phi = \frac{\pi}{8\omega_1} J_1(\phi_p)A\sin(\phi + \phi_\varepsilon)$$

$$\Rightarrow \sin\phi \cdot [A+B] = [\sin\phi \cos\phi_\varepsilon + \cos\phi \sin\phi_\varepsilon] A$$

$$\phi_\varepsilon \cong 0 \Rightarrow \sin\phi \cdot [A+B] \cong [\sin\phi + \sin\phi_\varepsilon \cos\phi] A$$

$$\Rightarrow \phi_\varepsilon = \sin^{-1}\left(\frac{B \sin\phi}{A \cos\phi}\right) \cong \frac{B \sin\phi}{A \cos\phi} \text{ for all } \phi \text{ such that } \cos\phi \neq 0.$$

In particular, if $\phi = 45^\circ$ then $\phi_\varepsilon = \sin^{-1}(B/A) = \sin^{-1}\left(\frac{I_\varepsilon(\phi)}{I_o(\phi)} - 1\right)$ as in Equation 4.4.

5. A MORE GENERAL IFOG MODEL

Our second example is a more general gyroscope. The clockwise loop in the second example consists of an IOC connected to a length l_2 of PM fiber at an angle α_1 which represents a miss-alignment and is therefore nominally small, however, in our analysis and simulation it can have any value. The PM fiber is connected to a second IOC at an angle α_2 representing a miss-alignment but may have arbitrary value. The second IOC is

connected to a length l_4 of PM fiber at an angle α_3 representing a miss-alignment of a zero or a 90 degree splicing. This PM fiber is connected to a length l_5 of PM fiber at a 45 degree angle plus an error angle β_1 representing a miss-alignment. The PM fiber is connected to the coil which is then connected to a length l_6 of PM fiber that is connected at a $(45^\circ + \beta_2)$ angle to a length l_7 of PM fiber. The last piece of fiber is connected to the second IOC at an angle α_4 representing a miss-alignment of a zero or a 90 degree splicing. The path returns back from the second IOC to the first IOC through the same forward path connections. The clockwise loop is depicted symbolically in Figure 6.

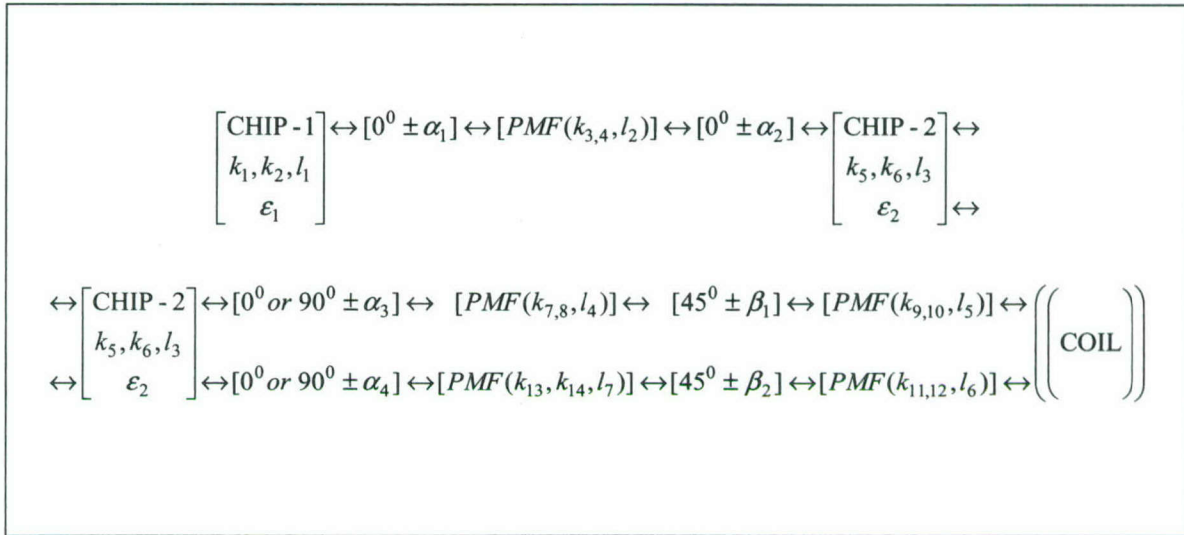


FIGURE 6. Interferometric Fiber Optic Gyroscope.

5.1. THE JONES MATRICES OF THE COMPONENTS

The Jones matrices of the gyro components are as follows. The forward path of the two IOCs have Jones matrices M_1 and M_2 given by

$$M_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_1 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_1 l_1} & 0 \\ 0 & e^{ik_2 l_1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{ik_1 l_1} & 0 \\ 0 & \epsilon_1 e^{ik_2 l_1} \end{bmatrix} = \frac{e^{ik_1 l_1}}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_1 e^{i(k_2 - k_1) l_1} \end{bmatrix}$$

and

$$M_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_2 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & e^{ik_6 l_3} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \epsilon_2 e^{ik_6 l_3} \end{bmatrix} = \frac{e^{ik_5 l_3}}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_2 e^{i(k_6 - k_5) l_3} \end{bmatrix},$$

where $k_s = \frac{\omega}{c} n_s$ for all s and n_s is the index of refraction along the respective path. The return path of the IOCs have Jones matrices M_{1c} and M_{2c} given by

$$M_{1c} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_1 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_1 l_9} & 0 \\ 0 & e^{ik_2 l_9} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{ik_1 l_9} & 0 \\ 0 & \epsilon_1 e^{ik_2 l_9} \end{bmatrix} = \frac{e^{ik_1 l_9}}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_1 e^{i(k_2 - k_1) l_9} \end{bmatrix}$$

and

$$M_{2c} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_2 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_5 l_8} & 0 \\ 0 & e^{ik_6 l_8} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{ik_5 l_8} & 0 \\ 0 & \epsilon_2 e^{ik_6 l_8} \end{bmatrix} = \frac{e^{ik_5 l_8}}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \epsilon_2 e^{i(k_6 - k_5) l_8} \end{bmatrix},$$

The lengths of PM fiber have Jones matrices U_1, U_2, \dots, U_5 given by

$$U_1 = \begin{bmatrix} e^{ik_3 l_2} & 0 \\ 0 & e^{ik_4 l_2} \end{bmatrix}, \quad U_2 = \begin{bmatrix} e^{ik_7 l_4} & 0 \\ 0 & e^{ik_8 l_4} \end{bmatrix}, \quad U_3 = \begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix},$$

$$U_4 = \begin{bmatrix} e^{ik_{11} l_6} & 0 \\ 0 & e^{ik_{12} l_6} \end{bmatrix}, \quad U_5 = \begin{bmatrix} e^{ik_{13} l_7} & 0 \\ 0 & e^{ik_{14} l_7} \end{bmatrix}.$$

These matrices can be written in the form $U = e^{ikl} \begin{bmatrix} 1 & 0 \\ 0 & e^{i(k' - k)l} \end{bmatrix}$ and the factor e^{ikl} can be ignored when computing the output intensity.

The rotations are represented by R_1, R_2, \dots, R_6 , where

$$R_1 = \begin{bmatrix} \cos \alpha_1 & \sin \alpha_1 \\ -\sin \alpha_1 & \cos \alpha_1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} \cos \alpha_2 & \sin \alpha_2 \\ -\sin \alpha_2 & \cos \alpha_2 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} \cos \alpha_3 & \sin \alpha_3 \\ -\sin \alpha_3 & \cos \alpha_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -\sin \alpha_3 & \cos \alpha_3 \\ -\cos \alpha_3 & -\sin \alpha_3 \end{bmatrix},$$

a rotation through and angle $(0^\circ + \alpha_3)$ or $(90^\circ + \alpha_3)$.

$$R_4 = \begin{bmatrix} \cos(45^\circ + \beta_1) & \sin(45^\circ + \beta_1) \\ -\sin(45^\circ + \beta_1) & \cos(45^\circ + \beta_1) \end{bmatrix}, \quad R_5 = \begin{bmatrix} \cos(45^\circ + \beta_2) & \sin(45^\circ + \beta_2) \\ -\sin(45^\circ + \beta_2) & \cos(45^\circ + \beta_2) \end{bmatrix},$$

and

$$R_6 = \begin{bmatrix} \cos \alpha_4 & \sin \alpha_4 \\ -\sin \alpha_4 & \cos \alpha_4 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -\sin \alpha_4 & \cos \alpha_4 \\ -\cos \alpha_4 & -\sin \alpha_4 \end{bmatrix},$$

a rotation through and angle $(0^\circ + \alpha_4)$ or $(90^\circ + \alpha_4)$.

We use a general 2x2 complex matrix A for the Jones matrix of the coil,

$$A = \begin{bmatrix} t_{11}e^{i\tau_{11}} & t_{12}e^{i\tau_{12}} \\ t_{21}e^{i\tau_{21}} & t_{22}e^{i\tau_{22}} \end{bmatrix}.$$

5.2. THE CLOCKWISE AND COUNTER-CLOCKWISE LOOPS

Let $C_1 = R_2 \cdot U_1 \cdot R_1$, $C_2 = U_3 \cdot R_4 \cdot U_2 \cdot R_3$, and $C_3 = R_6 \cdot U_5 \cdot R_5 \cdot U_4$. Then, the clockwise-path is given by

$$T = [(-i) \cdot M_1^T \cdot C_1^T \cdot (-i) \cdot M_{2c}^T] \cdot [C_3 \cdot A \cdot C_2] \cdot [M_2 \cdot C_1 \cdot M_1] = -T_1^T \cdot T_2 \cdot T_1, \quad (5.1)$$

where we have set $T_1 = M_2 \cdot C_1 \cdot M_1$, and $T_2 = C_3 \cdot T \cdot C_2$.

Note that $M_1^T = M_1$ and $M_{2c}^T = M_{2c}$.

The counter clockwise-path T_c is given by

$$T_c = [(-i) \cdot M_1^T \cdot C_1^T \cdot M_2^T] \cdot [C_2^T \cdot A^T \cdot C_3^T] \cdot [(-i) \cdot M_{2c} \cdot C_1 \cdot M_1]. \quad (5.2)$$

Comparing Equations 5.1 and 5.2, we conclude that $T_c = T^T$.

The four entries T_{11} , T_{12} , T_{21} , and T_{22} of the clockwise path matrix T are computed in Appendix A. The end result of that computation are the four arrays C_{11} , C_{12} , C_{21} , C_{22} of 256 coefficients and four arrays Λ_{11} , Λ_{12} , Λ_{21} , Λ_{22} , of 256 sets of indices needed to define the four entries of the clockwise IFOG Jones matrix as defined by Equation 3.3.

We wrote Matlab programs to compute the output intensities given by Equations 3.14, 3.15, and 3.19 and to compute the Sagnac error given by Equation 3.13 for the IFOG model presented here for Gaussian, rectangular, and triangular frequency distribution functions. These programs are described in Appendix B where we include their listings. The plots in Figures 7, 8, and 9 were produced using these programs.

Figures 7, 8, and 9 show the output of the function **SagnacBias** for a Gaussian, step, and triangular distribution function, respectively. The horizontal axis shows the values of

the length l_7 and the vertical axis shows the values of the length l_4 in meters. The color bars show the value of the bias angle in radians for the different colors.

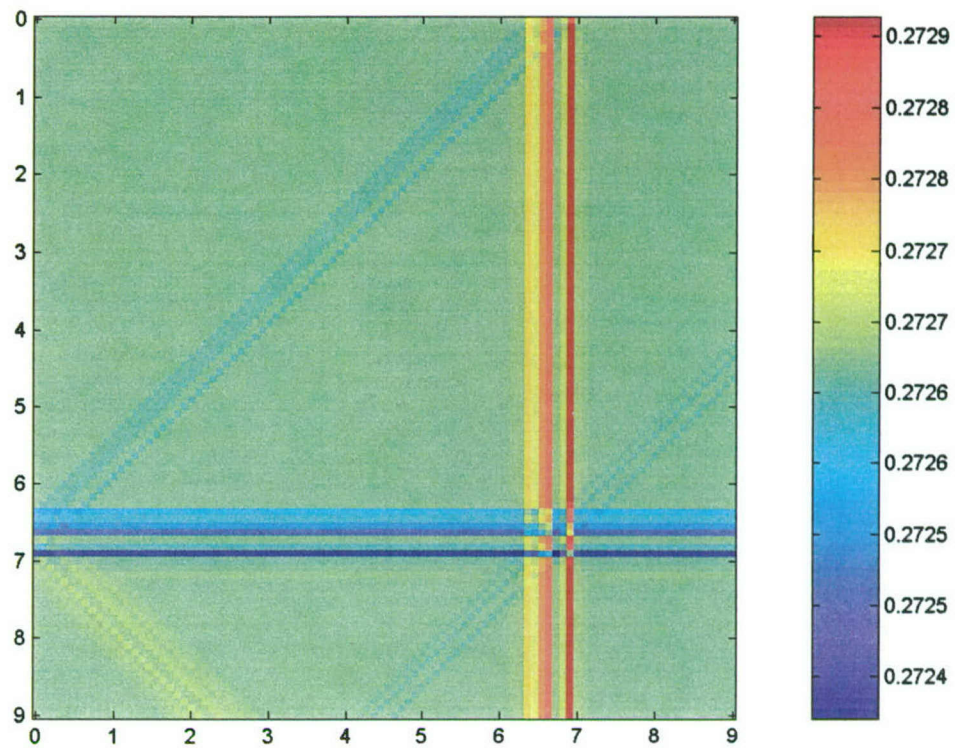


FIGURE 7. Bias Error - Gaussian Distribution.

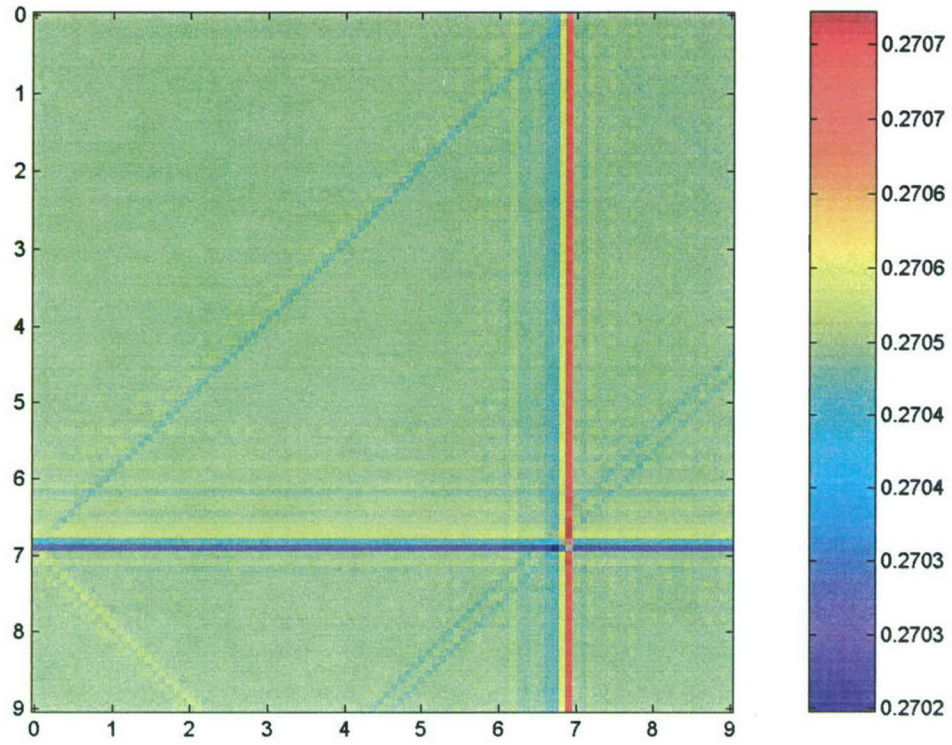


FIGURE 8. Bias Error - Step Distribution.

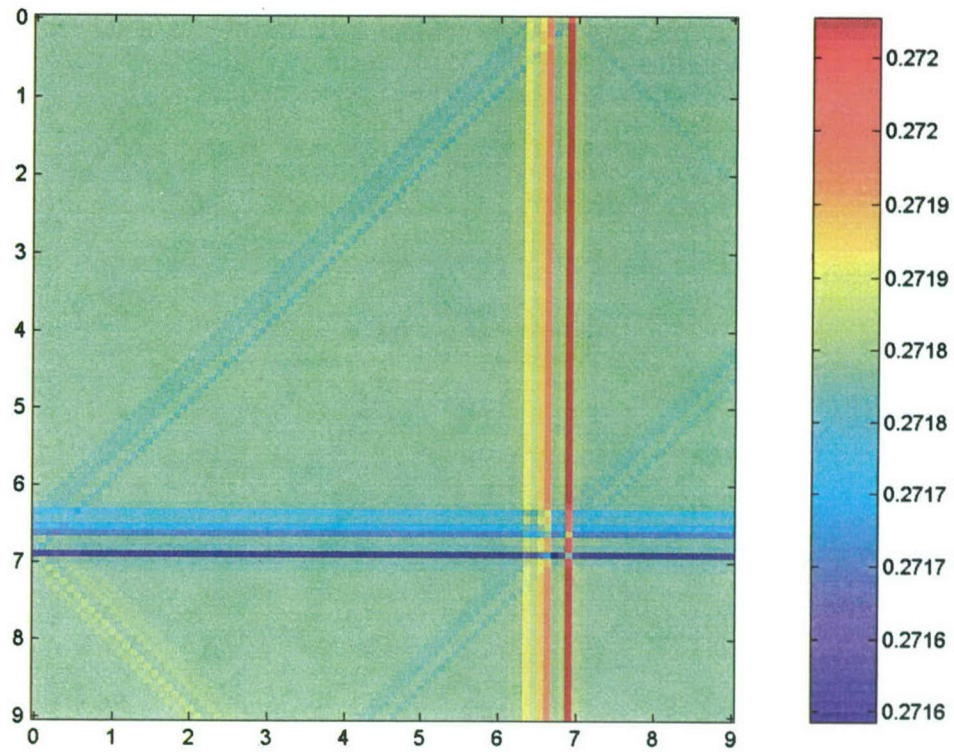


FIGURE 9. Bias Error - Triangular Distribution.

REFERENCES

1. E. C. Kintner. "Polarization Control in Optical-Fiber Gyroscopes," *Opt. Lett.*, Vol. 6 (1981), pp. 154-6.
2. Grant R. Fowles, *Introduction to Modern Optics*, 2nd ed., Holt, Rinehart and Winston, 1975.
3. I. S. Gradshteyn and I. M. Ryzhik. *Table of Integrals, Series, and Products*, ed. by Alan Jeffrey. 5th ed. Boston, Academic Press, 1994, p. 514.

Appendix A
THE CLOCKWISE-PATH MATRIX T

In this Appendix we compute the matrix T defined by Equation 5.1. As before, we write T as $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$. Since there are 8 rotations along the clockwise path, each T_{ij} is a sum of $2^8 = 256$ terms. They have the form (see Example 1, Section 4)

$$T_{ij}(\omega) = \sum_{s=1}^{256} C_{ij}(s) \exp(i \sum_{r \in \Lambda_{ij}(s)} \phi_r),$$

where each C_{ij} is a 256-array of coefficients, each $\Lambda_{ij}(s)$ is a set of 11 indices, ($s = 1, 2, \dots, 256$; $i, j = 1, 2$), and $\phi_1 = k_1 l_1$, $\phi_2 = k_2 l_1$, $\phi_3 = k_3 l_2$, $\phi_4 = k_4 l_2$, ..., $\phi_r = k_r l_{\frac{r+1}{2}}$ for r odd, $\phi_r = k_r l_{\frac{r}{2}}$ for r even, where $k_r = n_r \omega / c$ for $r = 1, 2, 3, \dots, 14$, and $r = 19, 20, 21$, and 22.

Let θ_r denote the angle in rotation R_r and let $s_r = \sin \theta_r$ and $c_r = \cos \theta_r$ for $r = 1, 2, \dots, 6$. The four taus, τ_{11} , τ_{12} , τ_{21} , and τ_{22} representing the coil phase shifts will receive the indices 15, 16, 17, and 18, respectively; that is $\tau_{11} = \phi_{15}$, $\tau_{12} = \phi_{16}$, $\tau_{21} = \phi_{17}$, and $\tau_{22} = \phi_{18}$.

An exponential of the form $e^{i(\phi_1 + \phi_2 + \dots + \phi_s)}$ will be represented by a row of indices $[r_1, r_2, \dots, r_s]$, thus $[2, 3, 5]$ represents $e^{i(\phi_2 + \phi_3 + \phi_5)}$ and a product $e^{i(\phi_2 + \phi_3 + \phi_5)} e^{i\phi_6}$ will be represented by $[2,3,5][6] = [2,3,5,6] = e^{i(\phi_2 + \phi_3 + \phi_5 + \phi_6)}$.

The product of the first r Jones matrices will be denoted by $P_r = \begin{bmatrix} P_{r,11} & P_{r,12} \\ P_{r,21} & P_{r,22} \end{bmatrix}$.

$$P_2 = \begin{bmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi_1} & 0 \\ 0 & \epsilon_1 e^{i\phi_2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_1 e^{i\phi_1} & s_1 \epsilon_1 e^{i\phi_2} \\ -s_1 e^{i\phi_1} & c_1 \epsilon_1 e^{i\phi_2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[1] & s_1 \epsilon_1[2] \\ (-s_1)[1] & c_1 \epsilon_1[2] \end{bmatrix}.$$

$$P_4 = R_2 U_1 P_2 = \begin{bmatrix} c_2 & s_2 \\ -s_2 & c_2 \end{bmatrix} \begin{bmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[1] & s_1 \epsilon_1[2] \\ (-s_1)[1] & c_1 \epsilon_1[2] \end{bmatrix} =$$

$$= \begin{bmatrix} c_2[3] & s_2[4] \\ (-s_2)[3] & c_2[4] \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[1] & s_1 \epsilon_1[2] \\ (-s_1)[1] & c_1 \epsilon_1[2] \end{bmatrix} =$$

$$P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} c_2 c_1[1,3] + s_2 (-s_1)[1,4] & c_2 s_1 \epsilon_1[2,3] + s_2 c_1 \epsilon_1[2,4] \\ (-s_2) c_1[1,3] + c_2 (-s_1)[1,4] & (-s_2) s_1 \epsilon_1[2,3] + c_2 c_1 \epsilon_1[2,4] \end{bmatrix}.$$

Thus,

$$P_{4,11} = \frac{1}{\sqrt{2}} \{ c_2 c_1[1,3] + s_2 (-s_1)[1,4] \}, \quad P_{4,12} = \frac{1}{\sqrt{2}} \{ c_2 s_1 \epsilon_1[2,3] + s_2 c_1 \epsilon_1[2,4] \},$$

$$P_{4,21} = \frac{1}{\sqrt{2}} \{ (-s_2) c_1[1,3] + c_2 (-s_1)[1,4] \}, \quad P_{4,22} = \frac{1}{\sqrt{2}} \{ (-s_2) s_1 \epsilon_1[2,3] + c_2 c_1 \epsilon_1[2,4] \},$$

where the red symbols indicate small numbers.

$$P_6 = R_3 M_2 P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} c_3 & s_3 \\ -s_3 & c_3 \end{bmatrix} \begin{bmatrix} e^{i\phi_5} & 0 \\ 0 & \epsilon_2 e^{i\phi_6} \end{bmatrix} \cdot P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} c_3[5] & s_3 \epsilon_2[6] \\ (-s_3)[5] & c_3 \epsilon_2[6] \end{bmatrix} \cdot P_4.$$

Thus,

$$P_{6,11} = \frac{1}{\sqrt{2}} c_3 [5] P_{4,11} + \frac{1}{\sqrt{2}} s_3 \epsilon_2 [6] P_{4,21}$$

$$P_{6,12} = \frac{1}{\sqrt{2}} c_3 [5] P_{4,12} + \frac{1}{\sqrt{2}} s_3 \epsilon_2 [6] P_{4,22}$$

$$P_{6,21} = \frac{1}{\sqrt{2}} (-s_3)[5] P_{4,11} + \frac{1}{\sqrt{2}} c_3 \epsilon_2 [6] P_{4,21}$$

$$P_{6,22} = \frac{1}{\sqrt{2}} (-s_3)[5] P_{4,12} + \frac{1}{\sqrt{2}} c_3 \epsilon_2 [6] P_{4,22}$$

$$P_{6,11} = \frac{1}{2} \{ c_3 c_2 c_1[1,3,5] + c_3 s_2 (-s_1)[1,4,5] + s_3 \epsilon_2 (-s_2) c_1[1,3,6] + s_3 \epsilon_2 c_2 (-s_1)[1,4,6] \}$$

$$P_{6,12} = \frac{1}{2} \{ c_3 c_2 s_1 \epsilon_1[2,3,5] + c_3 s_2 c_1 \epsilon_1[2,4,5] + s_3 \epsilon_2 (-s_2) s_1 \epsilon_1[2,3,6] + s_3 \epsilon_2 c_2 c_1 \epsilon_1[2,4,6] \}$$

$$P_{6,21} = \frac{1}{2} \{ (-s_3) c_2 c_1[1,3,5] + (-s_3) s_2 (-s_1)[1,4,5] + c_3 \epsilon_2 (-s_2) c_1[1,3,6] + c_3 \epsilon_2 c_2 (-s_1)[1,4,6] \}$$

$$P_{6,22} = \frac{1}{2} \{ (-s_3) c_2 s_1 \epsilon_1[2,3,5] + (-s_3) s_2 c_1 \epsilon_1[2,4,5] + c_3 \epsilon_2 (-s_2) s_1 \epsilon_1[2,3,6] +$$

$$c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6] \},$$

where the blue symbols indicate numbers that could be small.

$$P_8 = R_4 U_2 P_6 = \begin{bmatrix} c_4 & s_4 \\ -s_4 & c_4 \end{bmatrix} \begin{bmatrix} e^{i\phi_7} & 0 \\ 0 & e^{i\phi_8} \end{bmatrix} \cdot P_6 = \begin{bmatrix} c_4[7] & s_4[8] \\ (-s_4)[7] & c_4[8] \end{bmatrix} \cdot P_6$$

Thus,

$$\begin{aligned} P_{8,11} &= c_4[7] P_{6,11} + s_4[8] P_{6,21} \\ P_{8,12} &= c_4[7] P_{6,12} + s_4[8] P_{6,22} \\ P_{8,21} &= (-s_4)[7] P_{6,11} + c_4[8] P_{6,21} \\ P_{8,22} &= (-s_4)[7] P_{6,12} + c_4[8] P_{6,22} \end{aligned}$$

$$\begin{aligned} P_{8,11} &= \frac{1}{2} \{ c_4 c_3 c_2 c_1 [1,3,5,7] + c_4 c_3 s_2 (-s_1) [1,4,5,7] + c_4 s_3 \epsilon_2 (-s_2) c_1 [1,3,6,7] + \\ & c_4 s_3 \epsilon_2 c_2 (-s_1) [1,4,6,7] + s_4 (-s_3) c_2 c_1 [1,3,5,8] + s_4 (-s_3) s_2 (-s_1) [1,4,5,8] + \\ & s_4 c_3 \epsilon_2 (-s_2) c_1 [1,3,6,8] + s_4 c_3 \epsilon_2 c_2 (-s_1) [1,4,6,8] \} \end{aligned}$$

$$\begin{aligned} P_{8,12} &= \frac{1}{2} \{ c_4 c_3 c_2 s_1 \epsilon_1 [2,3,5,7] + c_4 c_3 s_2 c_1 \epsilon_1 [2,4,5,7] + c_4 s_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,7] + \\ & c_4 s_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,7] + s_4 (-s_3) c_2 s_1 \epsilon_1 [2,3,5,8] + s_4 (-s_3) s_2 c_1 \epsilon_1 [2,4,5,8] + \\ & s_4 c_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,8] + s_4 c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,8] \} \end{aligned}$$

$$\begin{aligned} P_{8,21} &= \frac{1}{2} \{ (-s_4) c_3 c_2 c_1 [1,3,5,7] + (-s_4) c_3 s_2 (-s_1) [1,4,5,7] + (-s_4) s_3 \epsilon_2 (-s_2) c_1 [1,3,6,7] + \\ & (-s_4) s_3 \epsilon_2 c_2 (-s_1) [1,4,6,7] + c_4 (-s_3) c_2 c_1 [1,3,5,8] + c_4 (-s_3) s_2 (-s_1) [1,4,5,8] + \\ & c_4 c_3 \epsilon_2 (-s_2) c_1 [1,3,6,8] + c_4 c_3 \epsilon_2 c_2 (-s_1) [1,4,6,8] \} \end{aligned}$$

$$\begin{aligned} P_{8,22} &= \frac{1}{2} \{ (-s_4) c_3 c_2 s_1 \epsilon_1 [2,3,5,7] + (-s_4) c_3 s_2 c_1 \epsilon_1 [2,4,5,7] + (-s_4) s_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,7] + \\ & (-s_4) s_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,7] + c_4 (-s_3) c_2 s_1 \epsilon_1 [2,3,5,8] + c_4 (-s_3) s_2 c_1 \epsilon_1 [2,4,5,8] + \\ & c_4 c_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,8] + c_4 c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,8] \}. \end{aligned}$$

$$P_{11} = U_4 T U_3 P_8 = \begin{bmatrix} e^{i\phi_{11}} & 0 \\ 0 & e^{i\phi_{12}} \end{bmatrix} \cdot \begin{bmatrix} t_{11} e^{i\tau_{11}} & t_{12} e^{i\tau_{12}} \\ t_{21} e^{i\tau_{21}} & t_{22} e^{i\tau_{22}} \end{bmatrix} \begin{bmatrix} e^{i\phi_9} & 0 \\ 0 & e^{i\phi_{10}} \end{bmatrix} \cdot P_8$$

$$P_{11} = \begin{bmatrix} t_{11} [9,15,11] & t_{12} [10,16,11] \\ t_{21} [9,17,12] & t_{22} [10,18,12] \end{bmatrix} \cdot P_8$$

Thus,

$$\begin{aligned}
 P_{11,11} &= t_{11} [9,15,11] P_{8,11} + t_{12} [10,16,11] P_{8,21} \\
 P_{11,12} &= t_{11} [9,15,11] P_{8,12} + t_{12} [10,16,11] P_{8,22} \\
 P_{11,21} &= t_{21} [9,17,12] P_{8,11} + t_{22} [10,18,12] P_{8,21} \\
 P_{11,22} &= t_{21} [9,17,12] P_{8,12} + t_{22} [10,18,12] P_{8,22}
 \end{aligned}$$

$$\begin{aligned}
 P_{11,11} &= \frac{1}{2} \{ t_{11} \{ c_4 c_3 c_2 c_1 [1,3,5,7,9,15,11] + c_4 c_3 s_2 (-s_1) [1,4,5,7,9,15,11] + \\
 & c_4 s_3 \epsilon_2 (-s_2) c_1 [1,3,6,7,9,15,11] + c_4 s_3 \epsilon_2 c_2 (-s_1) [1,4,6,7,9,15,11] + \\
 & s_4 (-s_3) c_2 c_1 [1,3,5,8,9,15,11] + s_4 (-s_3) s_2 (-s_1) [1,4,5,8,9,15,11] + \\
 & s_4 c_3 \epsilon_2 (-s_2) c_1 [1,3,6,8,9,15,11] + s_4 c_3 \epsilon_2 c_2 (-s_1) [1,4,6,8,9,15,11] \} + \\
 & t_{12} \{ (-s_4) c_3 c_2 c_1 [1,3,5,7,10,16,11] + (-s_4) c_3 s_2 (-s_1) [1,4,5,7,10,16,11] + \\
 & (-s_4) s_3 \epsilon_2 (-s_2) c_1 [1,3,6,7,10,16,11] + (-s_4) s_3 \epsilon_2 c_2 (-s_1) [1,4,6,7,10,16,11] + \\
 & c_4 (-s_3) c_2 c_1 [1,3,5,8,10,16,11] + c_4 (-s_3) s_2 (-s_1) [1,4,5,8,10,16,11] + \\
 & c_4 c_3 \epsilon_2 (-s_2) c_1 [1,3,6,8,10,16,11] + c_4 c_3 \epsilon_2 c_2 (-s_1) [1,4,6,8,10,16,11] \} \}
 \end{aligned}$$

$$\begin{aligned}
 P_{11,12} &= \frac{1}{2} \{ t_{11} \{ c_4 c_3 c_2 s_1 \epsilon_1 [2,3,5,7,9,15,11] + c_4 c_3 s_2 c_1 \epsilon_1 [2,4,5,7,9,15,11] + \\
 & c_4 s_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,7,9,15,11] + c_4 s_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,7,9,15,11] + \\
 & s_4 (-s_3) c_2 s_1 \epsilon_1 [2,3,5,8,9,15,11] + s_4 (-s_3) s_2 c_1 \epsilon_1 [2,4,5,8,9,15,11] + \\
 & s_4 c_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,8,9,15,11] + s_4 c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,8,9,15,11] \} + \\
 & t_{12} \{ (-s_4) c_3 c_2 s_1 \epsilon_1 [2,3,5,7,10,16,11] + (-s_4) c_3 s_2 c_1 \epsilon_1 [2,4,5,7,10,16,11] + \\
 & (-s_4) s_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,7,10,16,11] + (-s_4) s_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,7,10,16,11] + \\
 & c_4 (-s_3) c_2 s_1 \epsilon_1 [2,3,5,8,10,16,11] + c_4 (-s_3) s_2 c_1 \epsilon_1 [2,4,5,8,10,16,11] + \\
 & c_4 c_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,8,10,16,11] + c_4 c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,8,10,16,11] \} \}
 \end{aligned}$$

$$\begin{aligned}
 P_{11,21} &= \frac{1}{2} \{ t_{21} \{ c_4 c_3 c_2 c_1 [1,3,5,7,9,17,12] + c_4 c_3 s_2 (-s_1) [1,4,5,7,9,17,12] + \\
 & c_4 s_3 \epsilon_2 (-s_2) c_1 [1,3,6,7,9,17,12] + c_4 s_3 \epsilon_2 c_2 (-s_1) [1,4,6,7,9,17,12] + \\
 & s_4 (-s_3) c_2 c_1 [1,3,5,8,9,17,12] + s_4 (-s_3) s_2 (-s_1) [1,4,5,8,9,17,12] + \\
 & s_4 c_3 \epsilon_2 (-s_2) c_1 [1,3,6,8,9,17,12] + s_4 c_3 \epsilon_2 c_2 (-s_1) [1,4,6,8,9,17,12] \} + \\
 & t_{22} \{ (-s_4) c_3 c_2 c_1 [1,3,5,7,10,18,12] + (-s_4) c_3 s_2 (-s_1) [1,4,5,7,10,18,12] + \\
 & (-s_4) s_3 \epsilon_2 (-s_2) c_1 [1,3,6,7,10,18,12] + (-s_4) s_3 \epsilon_2 c_2 (-s_1) [1,4,6,7,10,18,12] + \\
 & c_4 (-s_3) c_2 c_1 [1,3,5,8,10,18,12] + c_4 (-s_3) s_2 (-s_1) [1,4,5,8,10,18,12] + \\
 & c_4 c_3 \epsilon_2 (-s_2) c_1 [1,3,6,8,10,18,12] + c_4 c_3 \epsilon_2 c_2 (-s_1) [1,4,6,8,10,18,12] \} \}
 \end{aligned}$$

$$\begin{aligned}
 P_{11,22} = & \frac{1}{2} \{ t_{21} \{ c_4 c_3 c_2 s_1 \epsilon_1 [2,3,5,7,9,17,12] + c_4 c_3 s_2 c_1 \epsilon_1 [2,4,5,7,9,17,12] + \\
 & c_4 s_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,7,9,17,12] + c_4 s_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,7,9,17,12] + \\
 & s_4 (-s_3) c_2 s_1 \epsilon_1 [2,3,5,8,9,17,12] + s_4 (-s_3) s_2 c_1 \epsilon_1 [2,4,5,8,9,17,12] + \\
 & s_4 c_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,8,9,17,12] + s_4 c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,8,9,17,12] \} + \\
 & t_{22} \{ (-s_4) c_3 c_2 s_1 \epsilon_1 [2,3,5,7,10,18,12] + (-s_4) c_3 s_2 c_1 \epsilon_1 [2,4,5,7,10,18,12] + \\
 & (-s_4) s_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,7,10,18,12] + (-s_4) s_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,7,10,18,12] + \\
 & c_4 (-s_3) c_2 s_1 \epsilon_1 [2,3,5,8,10,18,12] + c_4 (-s_3) s_2 c_1 \epsilon_1 [2,4,5,8,10,18,12] + \\
 & c_4 c_3 \epsilon_2 (-s_2) s_1 \epsilon_1 [2,3,6,8,10,18,12] + c_4 c_3 \epsilon_2 c_2 c_1 \epsilon_1 [2,4,6,8,10,18,12] \}
 \end{aligned}$$

(16 terms each $P_{11,ij}$)

$$P_{13} = U_5 R_5 P_{11} = \begin{bmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{bmatrix} \begin{bmatrix} c_5 & s_5 \\ -s_5 & c_5 \end{bmatrix} P_{11} = \begin{bmatrix} c_5[13] & s_5[13] \\ -s_5[14] & c_5[14] \end{bmatrix} P_{11}$$

Thus,

$$\begin{aligned}
 P_{13,11} &= [13] \{ c_5 P_{11,11} + s_5 P_{11,21} \} \\
 P_{13,12} &= [13] \{ c_5 P_{11,12} + s_5 P_{11,22} \} \\
 P_{13,21} &= [14] \{ -s_5 P_{11,11} + c_5 P_{11,21} \} \\
 P_{13,22} &= [14] \{ -s_5 P_{11,12} + c_5 P_{11,22} \} .
 \end{aligned}$$

Remark. According to Equations 3.3 and 3.4, to define the four entries T_{11} , T_{12} , T_{21} , T_{22} , of the transfer function T and to compute the output intensities for various g functions we need the four arrays of coefficients C_{11} , C_{12} , C_{21} , C_{22} , and the four 256×11 arrays of indices Λ_{11} , Λ_{12} , Λ_{21} , Λ_{22} . At this point we start writing the coefficients and the phase indices in array form as we continue to compute them. When we finish, they will be in a format that is ready to be scanned or copied for numerical computations. For example, instead of writing $c_5 c_4 c_3 s_2 (-s_1)$ we will write $c_5 * c_4 * c_3 * s_2 * (-s_1)$.

Arrays of coefficients and phase indices for $P_{13,11}$. (32 terms each $P_{13,ij}$)

$$P_{13,11} = [13] \{ c_5 P_{11,11} + s_5 P_{11,21} \}$$

$$\begin{aligned}
 \frac{1}{2} * [t_{11} * [& c_5 * c_4 * c_3 * c_2 * c_1 & [1,3,5,7,9,15,11,13] \\
 & c_5 * c_4 * c_3 * s_2 * (-s_1) & [1,4,5,7,9,15,11,13]
 \end{aligned}$$

$C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,13]
$C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13]
$C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13]
$C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13]
$C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13]
$C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,15,11,13]
$t_{12} * [$	
$C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13]
$C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13]
$C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13]
$C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13]
$C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13]
$C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13]
$C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13]
$C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,13]
$t_{21} * [$	
$S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13]
$S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13]
$S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13]
$S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13]
$S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13]
$S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13]
$S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13]
$S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,13]
$t_{22} * [$	
$S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13]
$S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13]
$S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13]
$S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13]
$S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13]
$S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13]
$S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13]
$S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]]	[1,4,6,8,10,18,12,13]

Arrays of coefficients and phase indices for $P_{13,12}$.

$$P_{13,12} = [13] \{ c_5 P_{11,12} + s_5 P_{11,22} \}$$

$\frac{1}{2} * [t_{11} * [$	
$C_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,13]
$C_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,13]
$C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,13]
$C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,13]

$C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,13]
$C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,13]
$C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,13]
$C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,8,9,15,11,13]
$h * [C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,13]
$C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,13]
$C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,13]
$C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,13]
$C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,13]
$C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,13]
$C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,13]
$C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,10,16,11,13]
$h * [S_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,13]
$S_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,13]
$S_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,13]
$S_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,13]
$S_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,13]
$S_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,13]
$S_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,13]
$S_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,9,17,12,13]
$S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,13]
$S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,13]
$S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,13]
$S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,13]
$S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,13]
$S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,13]
$S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,13]
$S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,10,18,12,13]

Arrays of coefficients and phase indices for $P_{13,21}$.

$$P_{13,21} = [14] \{ -s_5 P_{11,11} + c_5 P_{11,21} \}$$

$\frac{1}{2} * [t_{11} * [(-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14]
$(-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14]
$(-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14]
$(-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14]
$(-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14]
$(-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14]

$(-S_5)*S_4*C_3*E_2*(-S_2)*C_1$	[1,3,6,8,9,15,11,14]
$(-S_5)*S_4*C_3*E_2*C_2*(-S_1)$	[1,4,6,8,9,15,11,14]
$h*[(-S_5)*(-S_4)*C_3*C_2*C_1$	[1,3,5,7,10,16,11,14]
$(-S_5)*(-S_4)*C_3*S_2*(-S_1)$	[1,4,5,7,10,16,11,14]
$(-S_5)*(-S_4)*S_3*E_2*(-S_2)*C_1$	[1,3,6,7,10,16,11,14]
$(-S_5)*(-S_4)*S_3*E_2*C_2*(-S_1)$	[1,4,6,7,10,16,11,14]
$(-S_5)*C_4*(-S_3)*C_2*C_1$	[1,3,5,8,10,16,11,14]
$(-S_5)*C_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,10,16,11,14]
$(-S_5)*C_4*C_3*E_2*(-S_2)*C_1$	[1,3,6,8,10,16,11,14]
$(-S_5)*C_4*C_3*E_2*C_2*(-S_1)$]	[1,4,6,8,10,16,11,14]
$h*[C_5*C_4*C_3*C_2*C_1$	[1,3,5,7,9,17,12,14]
$C_5*C_4*C_3*S_2*(-S_1)$	[1,4,5,7,9,17,12,14]
$C_5*C_4*S_3*E_2*(-S_2)*C_1$	[1,3,6,7,9,17,12,14]
$C_5*C_4*S_3*E_2*C_2*(-S_1)$	[1,4,6,7,9,17,12,14]
$C_5*S_4*(-S_3)*C_2*C_1$	[1,3,5,8,9,17,12,14]
$C_5*S_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,9,17,12,14]
$C_5*S_4*C_3*E_2*(-S_2)*C_1$	[1,3,6,8,9,17,12,14]
$C_5*S_4*C_3*E_2*C_2*(-S_1)$]	[1,4,6,8,9,17,12,14]
$C_5*(-S_4)*C_3*C_2*C_1$	[1,3,5,7,10,18,12,14]
$C_5*(-S_4)*C_3*S_2*(-S_1)$	[1,4,5,7,10,18,12,14]
$C_5*(-S_4)*S_3*E_2*(-S_2)*C_1$	[1,3,6,7,10,18,12,14]
$C_5*(-S_4)*S_3*E_2*C_2*(-S_1)$	[1,4,6,7,10,18,12,14]
$C_5*C_4*(-S_3)*C_2*C_1$	[1,3,5,8,10,18,12,14]
$C_5*C_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,10,18,12,14]
$C_5*C_4*C_3*E_2*(-S_2)*C_1$	[1,3,6,8,10,18,12,14]
$C_5*C_4*C_3*E_2*C_2*(-S_1)$]	[1,4,6,8,10,18,12,14]

Arrays of coefficients and phase indices for $P_{13,22}$.

$$P_{13,22} = [14] \{ -S_5 P_{11,12} + C_5 P_{11,22} \}$$

$\frac{1}{2} * [t_{11} * [(-S_5)*C_4*C_3*C_2*S_1*E_1$	[2,3,5,7,9,15,11,14]
$(-S_5)*C_4*C_3*S_2*C_1*E_1$	[2,4,5,7,9,15,11,14]
$(-S_5)*C_4*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,9,15,11,14]
$(-S_5)*C_4*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,9,15,11,14]
$(-S_5)*S_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,9,15,11,14]
$(-S_5)*S_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,9,15,11,14]
$(-S_5)*S_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,9,15,11,14]
$(-S_5)*S_4*C_3*E_2*C_2*C_1*E_1$	[2,4,6,8,9,15,11,14]

$$\begin{aligned}
 h^* [& (-S_5)*(-S_4)*C_3*C_2*S_1*E_1 & [2,3,5,7,10,16,11,14] \\
 & (-S_5)*(-S_4)*C_3*S_2*C_1*E_1 & [2,4,5,7,10,16,11,14] \\
 & (-S_5)*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1 & [2,3,6,7,10,16,11,14] \\
 & (-S_5)*(-S_4)*S_3*E_2*C_2*C_1*E_1 & [2,4,6,7,10,16,11,14] \\
 & (-S_5)*C_4*(-S_3)*C_2*S_1*E_1 & [2,3,5,8,10,16,11,14] \\
 & (-S_5)*C_4*(-S_3)*S_2*C_1*E_1 & [2,4,5,8,10,16,11,14] \\
 & (-S_5)*C_4*C_3*E_2*(-S_2)*S_1*E_1 & [2,3,6,8,10,16,11,14] \\
 & (-S_5)*C_4*C_3*E_2*C_2*C_1*E_1 & [2,4,6,8,10,16,11,14]]
 \end{aligned}$$

$$\begin{aligned}
 h^* [& C_5*C_4*C_3*C_2*S_1*E_1 & [2,3,5,7,9,17,12,14] \\
 & C_5*C_4*C_3*S_2*C_1*E_1 & [2,4,5,7,9,17,12,14] \\
 & C_5*C_4*S_3*E_2*(-S_2)*S_1*E_1 & [2,3,6,7,9,17,12,14] \\
 & C_5*C_4*S_3*E_2*C_2*C_1*E_1 & [2,4,6,7,9,17,12,14] \\
 & C_5*S_4*(-S_3)*C_2*S_1*E_1 & [2,3,5,8,9,17,12,14] \\
 & C_5*S_4*(-S_3)*S_2*C_1*E_1 & [2,4,5,8,9,17,12,14] \\
 & C_5*S_4*C_3*E_2*(-S_2)*S_1*E_1 & [2,3,6,8,9,17,12,14] \\
 & C_5*S_4*C_3*E_2*C_2*C_1*E_1 & [2,4,6,8,9,17,12,14]]
 \end{aligned}$$

$$\begin{aligned}
 & C_5*(-S_4)*C_3*C_2*S_1*E_1 & [2,3,5,7,10,18,12,14] \\
 & C_5*(-S_4)*C_3*S_2*C_1*E_1 & [2,4,5,7,10,18,12,14] \\
 & C_5*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1 & [2,3,6,7,10,18,12,14] \\
 & C_5*(-S_4)*S_3*E_2*C_2*C_1*E_1 & [2,4,6,7,10,18,12,14] \\
 & C_5*C_4*(-S_3)*C_2*S_1*E_1 & [2,3,5,8,10,18,12,14] \\
 & C_5*C_4*(-S_3)*S_2*C_1*E_1 & [2,4,5,8,10,18,12,14] \\
 & C_5*C_4*C_3*E_2*(-S_2)*S_1*E_1 & [2,3,6,8,10,18,12,14] \\
 & C_5*C_4*C_3*E_2*C_2*C_1*E_1 & [2,4,6,8,10,18,12,14]]
 \end{aligned}$$

$$P_{15} = M_2 R_6 P_{13} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi_9} & 0 \\ 0 & \epsilon_2 e^{i\phi_{20}} \end{bmatrix} \begin{bmatrix} c_6 & s_6 \\ -s_6 & c_6 \end{bmatrix} P_{13} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_6[19] & s_6[19] \\ \epsilon_2(-s_6)[20] & \epsilon_2 c_6[20] \end{bmatrix} P_{13}$$

Thus,

$$\begin{aligned}
 P_{15,11} &= \frac{1}{\sqrt{2}} [19] \{ c_6 P_{13,11} + s_6 P_{13,21} \} \\
 P_{15,12} &= \frac{1}{\sqrt{2}} [19] \{ c_6 P_{13,12} + s_6 P_{13,22} \} \\
 P_{15,21} &= \frac{1}{\sqrt{2}} [20] \epsilon_2 \{ (-s_6) P_{13,11} + c_6 P_{13,21} \} \\
 P_{15,22} &= \frac{1}{\sqrt{2}} [20] \epsilon_2 \{ (-s_6) P_{13,12} + c_6 P_{13,22} \}
 \end{aligned}$$

Arrays of coefficients and phase indices for $P_{15,11}$. (64 terms each $P_{15,ij}$)

$$P_{15,11} = [19] \{ c_6 P_{13,11} + s_6 P_{13,21} \}$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [c_6 * c_5 * c_4 * c_3 * c_2 * c_1 \quad [1,3,5,7,9,15,11,13,19]$$

$$c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1) \quad [1,4,5,7,9,15,11,13,19]$$

$$c_6 * c_5 * c_4 * s_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,7,9,15,11,13,19]$$

$$c_6 * c_5 * c_4 * s_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,7,9,15,11,13,19]$$

$$c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1 \quad [1,3,5,8,9,15,11,13,19]$$

$$c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1) \quad [1,4,5,8,9,15,11,13,19]$$

$$c_6 * c_5 * s_4 * c_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,8,9,15,11,13,19]$$

$$c_6 * c_5 * s_4 * c_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,8,9,15,11,13,19]$$

$$t_{12} * [c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1 \quad [1,3,5,7,10,16,11,13,19]$$

$$c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1) \quad [1,4,5,7,10,16,11,13,19]$$

$$c_6 * c_5 * (-s_4) * s_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,7,10,16,11,13,19]$$

$$c_6 * c_5 * (-s_4) * s_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,7,10,16,11,13,19]$$

$$c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1 \quad [1,3,5,8,10,16,11,13,19]$$

$$c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1) \quad [1,4,5,8,10,16,11,13,19]$$

$$c_6 * c_5 * c_4 * c_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,8,10,16,11,13,19]$$

$$c_6 * c_5 * c_4 * c_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,8,10,16,11,13,19]$$

$$h * [c_6 * s_5 * c_4 * c_3 * c_2 * c_1 \quad [1,3,5,7,9,17,12,13,19]$$

$$c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1) \quad [1,4,5,7,9,17,12,13,19]$$

$$c_6 * s_5 * c_4 * s_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,7,9,17,12,13,19]$$

$$c_6 * s_5 * c_4 * s_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,7,9,17,12,13,19]$$

$$c_6 * s_5 * s_4 * (-s_3) * c_2 * c_1 \quad [1,3,5,8,9,17,12,13,19]$$

$$c_6 * s_5 * s_4 * (-s_3) * s_2 * (-s_1) \quad [1,4,5,8,9,17,12,13,19]$$

$$c_6 * s_5 * s_4 * c_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,8,9,17,12,13,19]$$

$$c_6 * s_5 * s_4 * c_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,8,9,17,12,13,19]$$

$$c_6 * s_5 * (-s_4) * c_3 * c_2 * c_1 \quad [1,3,5,7,10,18,12,13,19]$$

$$c_6 * s_5 * (-s_4) * c_3 * s_2 * (-s_1) \quad [1,4,5,7,10,18,12,13,19]$$

$$c_6 * s_5 * (-s_4) * s_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,7,10,18,12,13,19]$$

$$c_6 * s_5 * (-s_4) * s_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,7,10,18,12,13,19]$$

$$c_6 * s_5 * c_4 * (-s_3) * c_2 * c_1 \quad [1,3,5,8,10,18,12,13,19]$$

$$c_6 * s_5 * c_4 * (-s_3) * s_2 * (-s_1) \quad [1,4,5,8,10,18,12,13,19]$$

$$c_6 * s_5 * c_4 * c_3 * e_2 * (-s_2) * c_1 \quad [1,3,6,8,10,18,12,13,19]$$

$$c_6 * s_5 * c_4 * c_3 * e_2 * c_2 * (-s_1) \quad [1,4,6,8,10,18,12,13,19]$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [s_6 * (-s_5) * c_4 * c_3 * c_2 * c_1 \quad [1,3,5,7,9,15,11,14,19]$$

$S_6*(-S_5)*C_4*C_3*S_2*(-S_1)$	[1,4,5,7,9,15,11,14,19]
$S_6*(-S_5)*C_4*S_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,7,9,15,11,14,19]
$S_6*(-S_5)*C_4*S_3*\epsilon_2*C_2*(-S_1)$	[1,4,6,7,9,15,11,14,19]
$S_6*(-S_5)*S_4*(-S_3)*C_2*C_1$	[1,3,5,8,9,15,11,14,19]
$S_6*(-S_5)*S_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,9,15,11,14,19]
$S_6*(-S_5)*S_4*C_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,8,9,15,11,14,19]
$S_6*(-S_5)*S_4*C_3*\epsilon_2*C_2*(-S_1)$]	[1,4,6,8,9,15,11,14,19]
$t_{12} * [$	
$S_6*(-S_5)*(-S_4)*C_3*C_2*C_1$	[1,3,5,7,10,16,11,14,19]
$S_6*(-S_5)*(-S_4)*C_3*S_2*(-S_1)$	[1,4,5,7,10,16,11,14,19]
$S_6*(-S_5)*(-S_4)*S_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,7,10,16,11,14,19]
$S_6*(-S_5)*(-S_4)*S_3*\epsilon_2*C_2*(-S_1)$	[1,4,6,7,10,16,11,14,19]
$S_6*(-S_5)*C_4*(-S_3)*C_2*C_1$	[1,3,5,8,10,16,11,14,19]
$S_6*(-S_5)*C_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,10,16,11,14,19]
$S_6*(-S_5)*C_4*C_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,8,10,16,11,14,19]
$S_6*(-S_5)*C_4*C_3*\epsilon_2*C_2*(-S_1)$]	[1,4,6,8,10,16,11,14,19]
$t_{21} * [$	
$S_6*C_5*C_4*C_3*C_2*C_1$	[1,3,5,7,9,17,12,14,19]
$S_6*C_5*C_4*C_3*S_2*(-S_1)$	[1,4,5,7,9,17,12,14,19]
$S_6*C_5*C_4*S_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,7,9,17,12,14,19]
$S_6*C_5*C_4*S_3*\epsilon_2*C_2*(-S_1)$	[1,4,6,7,9,17,12,14,19]
$S_6*C_5*S_4*(-S_3)*C_2*C_1$	[1,3,5,8,9,17,12,14,19]
$S_6*C_5*S_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,9,17,12,14,19]
$S_6*C_5*S_4*C_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,8,9,17,12,14,19]
$S_6*C_5*S_4*C_3*\epsilon_2*C_2*(-S_1)$]	[1,4,6,8,9,17,12,14,19]
$S_6*C_5*(-S_4)*C_3*C_2*C_1$	[1,3,5,7,10,18,12,14,19]
$S_6*C_5*(-S_4)*C_3*S_2*(-S_1)$	[1,4,5,7,10,18,12,14,19]
$S_6*C_5*(-S_4)*S_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,7,10,18,12,14,19]
$S_6*C_5*(-S_4)*S_3*\epsilon_2*C_2*(-S_1)$	[1,4,6,7,10,18,12,14,19]
$S_6*C_5*C_4*(-S_3)*C_2*C_1$	[1,3,5,8,10,18,12,14,19]
$S_6*C_5*C_4*(-S_3)*S_2*(-S_1)$	[1,4,5,8,10,18,12,14,19]
$S_6*C_5*C_4*C_3*\epsilon_2*(-S_2)*C_1$	[1,3,6,8,10,18,12,14,19]
$S_6*C_5*C_4*C_3*\epsilon_2*C_2*(-S_1)$]]	[1,4,6,8,10,18,12,14,19]

Arrays of coefficients and phase indices for $P_{15,12}$.

$$P_{15,12} = [19] \{ c_6 P_{13,12} + s_6 P_{13,22} \}$$

$\frac{1}{2\sqrt{2}} * [t_{11} * [$	$C_6*C_5*C_4*C_3*C_2*S_1*\epsilon_1$	[2,3,5,7,9,15,11,13,19]
	$C_6*C_5*C_4*C_3*S_2*C_1*\epsilon_1$	[2,4,5,7,9,15,11,13,19]
	$C_6*C_5*C_4*S_3*\epsilon_2*(-S_2)*S_1*\epsilon_1$	[2,3,6,7,9,15,11,13,19]

	$C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,13,19]
	$C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,13,19]
	$C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,13,19]
	$C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,13,19]
	$C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,15,11,13,19]
$t_{12} * [$	$C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,13,19]
	$C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,13,19]
	$C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,13,19]
	$C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,13,19]
	$C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,13,19]
	$C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,13,19]
	$C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,13,19]
	$C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,16,11,13,19]
$t_{21} * [$	$C_6 * S_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,13,19]
	$C_6 * S_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,13,19]
	$C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,13,19]
	$C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,13,19]
	$C_6 * S_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,13,19]
	$C_6 * S_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,13,19]
	$C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,13,19]
	$C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,13,19]
$t_{22} * [$	$C_6 * S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,13,19]
	$C_6 * S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,13,19]
	$C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,13,19]
	$C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,13,19]
	$C_6 * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,13,19]
	$C_6 * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,13,19]
	$C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,13,19]
	$C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]]	[2,4,6,8,10,18,12,13,19]
$\frac{1}{2\sqrt{2}} * [t_{11} * [$	$S_6 * (-S_5) * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,14,19]
	$S_6 * (-S_5) * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,14,19]
	$S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,14,19]
	$S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,14,19]
	$S_6 * (-S_5) * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,14,19]
	$S_6 * (-S_5) * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,14,19]
	$S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,14,19]
	$S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,15,11,14,19]

$$t_{12} * [\begin{array}{l} S_6 * (-S_5) * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1 \\ S_6 * (-S_5) * (-S_4) * C_3 * S_2 * C_1 * \epsilon_1 \\ S_6 * (-S_5) * (-S_4) * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1 \\ S_6 * (-S_5) * (-S_4) * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1 \\ S_6 * (-S_5) * C_4 * (-S_3) * C_2 * S_1 * \epsilon_1 \\ S_6 * (-S_5) * C_4 * (-S_3) * S_2 * C_1 * \epsilon_1 \\ S_6 * (-S_5) * C_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1 \\ S_6 * (-S_5) * C_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1 \end{array}] \begin{array}{l} [2,3,5,7,10,16,11,14,19] \\ [2,4,5,7,10,16,11,14,19] \\ [2,3,6,7,10,16,11,14,19] \\ [2,4,6,7,10,16,11,14,19] \\ [2,3,5,8,10,16,11,14,19] \\ [2,4,5,8,10,16,11,14,19] \\ [2,3,6,8,10,16,11,14,19] \\ [2,4,6,8,10,16,11,14,19] \end{array}$$

$$t_{21} * [\begin{array}{l} S_6 * C_5 * C_4 * C_3 * C_2 * S_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * C_3 * S_2 * C_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1 \\ S_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * \epsilon_1 \\ S_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * \epsilon_1 \\ S_6 * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1 \\ S_6 * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1 \end{array}] \begin{array}{l} [2,3,5,7,9,17,12,14,19] \\ [2,4,5,7,9,17,12,14,19] \\ [2,3,6,7,9,17,12,14,19] \\ [2,4,6,7,9,17,12,14,19] \\ [2,3,5,8,9,17,12,14,19] \\ [2,4,5,8,9,17,12,14,19] \\ [2,3,6,8,9,17,12,14,19] \\ [2,4,6,8,9,17,12,14,19] \end{array}$$

$$t_{22} * [\begin{array}{l} S_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1 \\ S_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * \epsilon_1 \\ S_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1 \\ S_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1 \\ S_6 * C_5 * C_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1 \end{array}] \begin{array}{l} [2,3,5,7,10,18,12,14,19] \\ [2,4,5,7,10,18,12,14,19] \\ [2,3,6,7,10,18,12,14,19] \\ [2,4,6,7,10,18,12,14,19] \\ [2,3,5,8,10,18,12,14,19] \\ [2,4,5,8,10,18,12,14,19] \\ [2,3,6,8,10,18,12,14,19] \\ [2,4,6,8,10,18,12,14,19] \end{array}$$

Arrays of coefficients and phase indices for $P_{15,21}$.

$$P_{15,21} = [20] \epsilon_2 \{ (-S_6) P_{13,11} + C_6 P_{13,21} \}$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [\begin{array}{l} \epsilon_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * C_1 \\ \epsilon_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * (-S_1) \\ \epsilon_2 * (-S_6) * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * C_1 \\ \epsilon_2 * (-S_6) * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * (-S_1) \\ \epsilon_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * C_1 \\ \epsilon_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * (-S_1) \\ \epsilon_2 * (-S_6) * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * C_1 \\ \epsilon_2 * (-S_6) * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * (-S_1) \end{array}] \begin{array}{l} [1,3,5,7,9,15,11,13,20] \\ [1,4,5,7,9,15,11,13,20] \\ [1,3,6,7,9,15,11,13,20] \\ [1,4,6,7,9,15,11,13,20] \\ [1,3,5,8,9,15,11,13,20] \\ [1,4,5,8,9,15,11,13,20] \\ [1,3,6,8,9,15,11,13,20] \\ [1,4,6,8,9,15,11,13,20] \end{array}$$

$$t_{12} * [\begin{array}{l} \mathcal{E}_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * C_1 \\ \mathcal{E}_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * C_5 * (-S_4) * S_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * (-S_6) * C_5 * (-S_4) * S_3 * \mathcal{E}_2 * C_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * C_1 \\ \mathcal{E}_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * C_5 * C_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * (-S_6) * C_5 * C_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,10,16,11,13,20] \\ [1,4,5,7,10,16,11,13,20] \\ [1,3,6,7,10,16,11,13,20] \\ [1,4,6,7,10,16,11,13,20] \\ [1,3,5,8,10,16,11,13,20] \\ [1,4,5,8,10,16,11,13,20] \\ [1,3,6,8,10,16,11,13,20] \\ [1,4,6,8,10,16,11,13,20] \end{array}$$

$$t_{21} * [\begin{array}{l} \mathcal{E}_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * S_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * S_3 * \mathcal{E}_2 * C_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * S_5 * S_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * S_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,9,17,12,13,20] \\ [1,4,5,7,9,17,12,13,20] \\ [1,3,6,7,9,17,12,13,20] \\ [1,4,6,7,9,17,12,13,20] \\ [1,3,5,8,9,17,12,13,20] \\ [1,4,5,8,9,17,12,13,20] \\ [1,3,6,8,9,17,12,13,20] \\ [1,4,6,8,9,17,12,13,20] \end{array}$$

$$t_{22} * [\begin{array}{l} \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * C_3 * C_2 * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * S_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * S_3 * \mathcal{E}_2 * C_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * (-S_6) * S_5 * C_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1) \end{array}]] \quad \begin{array}{l} [1,3,5,7,10,18,12,13,20] \\ [1,4,5,7,10,18,12,13,20] \\ [1,3,6,7,10,18,12,13,20] \\ [1,4,6,7,10,18,12,13,20] \\ [1,3,5,8,10,18,12,13,20] \\ [1,4,5,8,10,18,12,13,20] \\ [1,3,6,8,10,18,12,13,20] \\ [1,4,6,8,10,18,12,13,20] \end{array}$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [\begin{array}{l} \mathcal{E}_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * C_1 \\ \mathcal{E}_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1) \\ \mathcal{E}_2 * C_6 * (-S_5) * C_4 * S_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * C_6 * (-S_5) * C_4 * S_3 * \mathcal{E}_2 * C_2 * (-S_1) \\ \mathcal{E}_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1 \\ \mathcal{E}_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1) \\ \mathcal{E}_2 * C_6 * (-S_5) * S_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * C_6 * (-S_5) * S_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,9,15,11,14,20] \\ [1,4,5,7,9,15,11,14,20] \\ [1,3,6,7,9,15,11,14,20] \\ [1,4,6,7,9,15,11,14,20] \\ [1,3,5,8,9,15,11,14,20] \\ [1,4,5,8,9,15,11,14,20] \\ [1,3,6,8,9,15,11,14,20] \\ [1,4,6,8,9,15,11,14,20] \end{array}$$

$$t_{12} * [\begin{array}{l} \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1 \\ \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1) \\ \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * S_3 * \mathcal{E}_2 * (-S_2) * C_1 \\ \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * S_3 * \mathcal{E}_2 * C_2 * (-S_1) \\ \mathcal{E}_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1 \end{array}] \quad \begin{array}{l} [1,3,5,7,10,16,11,14,20] \\ [1,4,5,7,10,16,11,14,20] \\ [1,3,6,7,10,16,11,14,20] \\ [1,4,6,7,10,16,11,14,20] \\ [1,3,5,8,10,16,11,14,20] \end{array}$$

	$\epsilon_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,20]
	$\epsilon_2 * C_6 * (-S_5) * C_4 * C_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,20]
	$\epsilon_2 * C_6 * (-S_5) * C_4 * C_3 * \epsilon_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,14,20]
$t_{21} * [$	$\epsilon_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,20]
	$\epsilon_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,14,20]
$t_{22} * [$	$\epsilon_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,20]
	$\epsilon_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * C_2 * (-S_1)$]]	[1,4,6,8,10,18,12,14,20]

Arrays of coefficients and phase indices for $P_{15,22}$.

$$P_{15,22} = [20] \epsilon_2 \{ (-S_6) P_{13,12} + C_6 P_{13,22} \}$$

$\frac{1}{2\sqrt{2}} * [t_{11} * [$	$\epsilon_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,9,15,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$]	[2,4,6,8,9,15,11,13,20]
$t_{12} * [$	$\epsilon_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,10,16,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,10,16,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,10,16,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,10,16,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,10,16,11,13,20]
	$\epsilon_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,10,16,11,13,20]

	$\mathcal{E}_2*(-S_6)*C_5*C_4*C_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,8,10,16,11,13,20]
	$\mathcal{E}_2*(-S_6)*C_5*C_4*C_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$]	[2,4,6,8,10,16,11,13,20]
$t_{21} *$ [$\mathcal{E}_2*(-S_6)*S_5*C_4*C_3*C_2*S_1*\mathcal{E}_1$	[2,3,5,7,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*C_3*S_2*C_1*\mathcal{E}_1$	[2,4,5,7,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*S_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,7,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*S_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$	[2,4,6,7,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*S_4*(-S_3)*C_2*S_1*\mathcal{E}_1$	[2,3,5,8,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*S_4*(-S_3)*S_2*C_1*\mathcal{E}_1$	[2,4,5,8,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*S_4*C_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,8,9,17,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*S_4*C_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$]	[2,4,6,8,9,17,12,13,20]
$t_{22} *$ [$\mathcal{E}_2*(-S_6)*S_5*(-S_4)*C_3*C_2*S_1*\mathcal{E}_1$	[2,3,5,7,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*(-S_4)*C_3*S_2*C_1*\mathcal{E}_1$	[2,4,5,7,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*(-S_4)*S_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,7,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*(-S_4)*S_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$	[2,4,6,7,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*(-S_3)*C_2*S_1*\mathcal{E}_1$	[2,3,5,8,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*(-S_3)*S_2*C_1*\mathcal{E}_1$	[2,4,5,8,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*C_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,8,10,18,12,13,20]
	$\mathcal{E}_2*(-S_6)*S_5*C_4*C_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$]]	[2,4,6,8,10,18,12,13,20]
$\frac{1}{2\sqrt{2}} * [t_{11} *$ [$\mathcal{E}_2 * C_6*(-S_5)*C_4*C_3*C_2*S_1*\mathcal{E}_1$	[2,3,5,7,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*C_3*S_2*C_1*\mathcal{E}_1$	[2,4,5,7,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*S_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,7,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*S_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$	[2,4,6,7,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*S_4*(-S_3)*C_2*S_1*\mathcal{E}_1$	[2,3,5,8,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*S_4*(-S_3)*S_2*C_1*\mathcal{E}_1$	[2,4,5,8,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*S_4*C_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,8,9,15,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*S_4*C_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$]	[2,4,6,8,9,15,11,14,20]
$t_{12} *$ [$\mathcal{E}_2 * C_6*(-S_5)*(-S_4)*C_3*C_2*S_1*\mathcal{E}_1$	[2,3,5,7,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*(-S_4)*C_3*S_2*C_1*\mathcal{E}_1$	[2,4,5,7,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*(-S_4)*S_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,7,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*(-S_4)*S_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$	[2,4,6,7,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*(-S_3)*C_2*S_1*\mathcal{E}_1$	[2,3,5,8,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*(-S_3)*S_2*C_1*\mathcal{E}_1$	[2,4,5,8,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*C_3*\mathcal{E}_2*(-S_2)*S_1*\mathcal{E}_1$	[2,3,6,8,10,16,11,14,20]
	$\mathcal{E}_2 * C_6*(-S_5)*C_4*C_3*\mathcal{E}_2*C_2*C_1*\mathcal{E}_1$]	[2,4,6,8,10,16,11,14,20]
$t_{21} *$ [$\mathcal{E}_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * \mathcal{E}_1$	[2,3,5,7,9,17,12,14,20]
	$\mathcal{E}_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * \mathcal{E}_1$	[2,4,5,7,9,17,12,14,20]

$\epsilon_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1$	[2,3,6,7,9,17,12,14,20]
$\epsilon_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1$	[2,4,6,7,9,17,12,14,20]
$\epsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \epsilon_1$	[2,3,5,8,9,17,12,14,20]
$\epsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \epsilon_1$	[2,4,5,8,9,17,12,14,20]
$\epsilon_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1$	[2,3,6,8,9,17,12,14,20]
$\epsilon_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1$]	[2,4,6,8,9,17,12,14,20]
$t_{22} * [\epsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \epsilon_1$	[2,3,5,7,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \epsilon_1$	[2,4,5,7,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1$	[2,3,6,7,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1$	[2,4,6,7,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \epsilon_1$	[2,3,5,8,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \epsilon_1$	[2,4,5,8,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * c_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1$	[2,3,6,8,10,18,12,14,20]
$\epsilon_2 * c_6 * c_5 * c_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1$]]	[2,4,6,8,10,18,12,14,20]

$$P_{17} = U_1 R_2^T P_{15} = \begin{bmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{bmatrix} \begin{bmatrix} c_2 & -s_2 \\ s_2 & c_2 \end{bmatrix} P_{15} = \begin{bmatrix} c_2[3] & -s_2[3] \\ s_2[4] & c_2[4] \end{bmatrix} P_{15}$$

Thus,

$$\begin{aligned} P_{17,11} &= c_2 [3] P_{15,11} + (-s_2) [3] P_{15,21} \\ P_{17,12} &= c_2 [3] P_{15,12} + (-s_2) [3] P_{15,22} \\ P_{17,21} &= s_2 [4] P_{15,11} + c_2 [4] P_{15,21} \\ P_{17,22} &= s_2 [4] P_{15,12} + c_2 [4] P_{15,22} \end{aligned}$$

Arrays of coefficients and phase indices for $P_{17,11}$. (128 terms each $P_{17,ij}$)

$$P_{17,11} = c_2 [3] P_{15,11} + (-s_2) [3] P_{15,21}$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [c_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1 \quad [1,3,5,7,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1) \quad [1,4,5,7,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * (-s_2) * c_1 \quad [1,3,6,7,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * c_2 * (-s_1) \quad [1,4,6,7,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1 \quad [1,3,5,8,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1) \quad [1,4,5,8,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * (-s_2) * c_1 \quad [1,3,6,8,9,15,11,13,19,3]$$

$$c_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * c_2 * (-s_1)] \quad [1,4,6,8,9,15,11,13,19,3]$$

$$t_{12} * [\begin{array}{l} C_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1 \\ C_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ C_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ C_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ C_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1 \\ C_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ C_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 \\ C_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \begin{array}{l} [1,3,5,7,10,16,11,13,19,3] \\ [1,4,5,7,10,16,11,13,19,3] \\ [1,3,6,7,10,16,11,13,19,3] \\ [1,4,6,7,10,16,11,13,19,3] \\ [1,3,5,8,10,16,11,13,19,3] \\ [1,4,5,8,10,16,11,13,19,3] \\ [1,3,6,8,10,16,11,13,19,3] \\ [1,4,6,8,10,16,11,13,19,3] \end{array}$$

$$t_{21} * [\begin{array}{l} C_2 * C_6 * S_5 * C_4 * C_3 * C_2 * C_1 \\ C_2 * C_6 * S_5 * C_4 * C_3 * S_2 * (-S_1) \\ C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ C_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * C_1 \\ C_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * (-S_1) \\ C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 \\ C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \begin{array}{l} [1,3,5,7,9,17,12,13,19,3] \\ [1,4,5,7,9,17,12,13,19,3] \\ [1,3,6,7,9,17,12,13,19,3] \\ [1,4,6,7,9,17,12,13,19,3] \\ [1,3,5,8,9,17,12,13,19,3] \\ [1,4,5,8,9,17,12,13,19,3] \\ [1,3,6,8,9,17,12,13,19,3] \\ [1,4,6,8,9,17,12,13,19,3] \end{array}$$

$$t_{22} * [\begin{array}{l} C_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * C_1 \\ C_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ C_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * C_1 \\ C_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 \\ C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \begin{array}{l} [1,3,5,7,10,18,12,13,19,3] \\ [1,4,5,7,10,18,12,13,19,3] \\ [1,3,6,7,10,18,12,13,19,3] \\ [1,4,6,7,10,18,12,13,19,3] \\ [1,3,5,8,10,18,12,13,19,3] \\ [1,4,5,8,10,18,12,13,19,3] \\ [1,3,6,8,10,18,12,13,19,3] \\ [1,4,6,8,10,18,12,13,19,3] \end{array}$$

$$t_{11} * [\begin{array}{l} C_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * C_1 \\ C_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1) \\ C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ C_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1 \\ C_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1) \\ C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1 \\ C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \begin{array}{l} [1,3,5,7,9,15,11,14,19,3] \\ [1,4,5,7,9,15,11,14,19,3] \\ [1,3,6,7,9,15,11,14,19,3] \\ [1,4,6,7,9,15,11,14,19,3] \\ [1,3,5,8,9,15,11,14,19,3] \\ [1,4,5,8,9,15,11,14,19,3] \\ [1,3,6,8,9,15,11,14,19,3] \\ [1,4,6,8,9,15,11,14,19,3] \end{array}$$

$$t_{12} * [\begin{array}{l} C_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1 \\ C_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1) \\ C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ C_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1 \\ C_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1) \end{array}] \begin{array}{l} [1,3,5,7,10,16,11,14,19,3] \\ [1,4,5,7,10,16,11,14,19,3] \\ [1,3,6,7,10,16,11,14,19,3] \\ [1,4,6,7,10,16,11,14,19,3] \\ [1,3,5,8,10,16,11,14,19,3] \\ [1,4,5,8,10,16,11,14,19,3] \end{array}$$

	$C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,19,3]
	$C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,14,19,3]
$t_{21} * [$	$C_2 * S_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,19,3]
	$C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,14,19,3]
$t_{22} * [$	$C_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,18,12,14,19,3]
$t_{11} * [$	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,13,20,3]
$t_{12} * [$	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,13,20,3]
$t_{21} * [$	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,20,3]

$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * C_4 * S_3 * \mathcal{E}_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * S_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * S_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,13,20,3]
$t_{22} * [(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * S_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * (-S_4) * S_3 * \mathcal{E}_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * C_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,20,3]
$(-S_2) * \mathcal{E}_2 * (-S_6) * S_5 * C_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1)]$	[1,4,6,8,10,18,12,13,20,3]
$t_{11} * [(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * S_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * S_3 * \mathcal{E}_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * S_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * S_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,14,20,3]
$t_{12} * [(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * S_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * (-S_4) * S_3 * \mathcal{E}_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * (-S_5) * C_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,14,20,3]
$t_{21} * [(-S_2) * \mathcal{E}_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * C_4 * S_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * C_4 * S_3 * \mathcal{E}_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * S_4 * C_3 * \mathcal{E}_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,20,3]
$(-S_2) * \mathcal{E}_2 * C_6 * C_5 * S_4 * C_3 * \mathcal{E}_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,14,20,3]

$$t_{22} * [\begin{array}{l} (-S_2)*E_2*C_6*C_5*(-S_4)*C_3*C_2*C_1 \\ (-S_2)*E_2*C_6*C_5*(-S_4)*C_3*S_2*(-S_1) \\ (-S_2)*E_2*C_6*C_5*(-S_4)*S_3*E_2*(-S_2)*C_1 \\ (-S_2)*E_2*C_6*C_5*(-S_4)*S_3*E_2*C_2*(-S_1) \\ (-S_2)*E_2*C_6*C_5*C_4*(-S_3)*C_2*C_1 \\ (-S_2)*E_2*C_6*C_5*C_4*(-S_3)*S_2*(-S_1) \\ (-S_2)*E_2*C_6*C_5*C_4*C_3*E_2*(-S_2)*C_1 \\ (-S_2)*E_2*C_6*C_5*C_4*C_3*E_2*C_2*(-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,10,18,12,14,20,3] \\ [1,4,5,7,10,18,12,14,20,3] \\ [1,3,6,7,10,18,12,14,20,3] \\ [1,4,6,7,10,18,12,14,20,3] \\ [1,3,5,8,10,18,12,14,20,3] \\ [1,4,5,8,10,18,12,14,20,3] \\ [1,3,6,8,10,18,12,14,20,3] \\ [1,4,6,8,10,18,12,14,20,3] \end{array} \quad \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array}$$

Arrays of coefficients and phase indices for $P_{17,12}$.

$$P_{17,12} = c_2 [3] P_{15,12} + (-S_2) [3] P_{15,22}$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [\begin{array}{l} C_2*C_6*C_5*C_4*C_3*C_2*S_1*E_1 \\ C_2*C_6*C_5*C_4*C_3*S_2*C_1*E_1 \\ C_2*C_6*C_5*C_4*S_3*E_2*(-S_2)*S_1*E_1 \\ C_2*C_6*C_5*C_4*S_3*E_2*C_2*C_1*E_1 \\ C_2*C_6*C_5*S_4*(-S_3)*C_2*S_1*E_1 \\ C_2*C_6*C_5*S_4*(-S_3)*S_2*C_1*E_1 \\ C_2*C_6*C_5*S_4*C_3*E_2*(-S_2)*S_1*E_1 \\ C_2*C_6*C_5*S_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \quad \begin{array}{l} [2,3,5,7,9,15,11,13,19,3] \\ [2,4,5,7,9,15,11,13,19,3] \\ [2,3,6,7,9,15,11,13,19,3] \\ [2,4,6,7,9,15,11,13,19,3] \\ [2,3,5,8,9,15,11,13,19,3] \\ [2,4,5,8,9,15,11,13,19,3] \\ [2,3,6,8,9,15,11,13,19,3] \\ [2,4,6,8,9,15,11,13,19,3] \end{array}$$

$$t_{12} * [\begin{array}{l} C_2*C_6*C_5*(-S_4)*C_3*C_2*S_1*E_1 \\ C_2*C_6*C_5*(-S_4)*C_3*S_2*C_1*E_1 \\ C_2*C_6*C_5*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1 \\ C_2*C_6*C_5*(-S_4)*S_3*E_2*C_2*C_1*E_1 \\ C_2*C_6*C_5*C_4*(-S_3)*C_2*S_1*E_1 \\ C_2*C_6*C_5*C_4*(-S_3)*S_2*C_1*E_1 \\ C_2*C_6*C_5*C_4*C_3*E_2*(-S_2)*S_1*E_1 \\ C_2*C_6*C_5*C_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \quad \begin{array}{l} [2,3,5,7,10,16,11,13,19,3] \\ [2,4,5,7,10,16,11,13,19,3] \\ [2,3,6,7,10,16,11,13,19,3] \\ [2,4,6,7,10,16,11,13,19,3] \\ [2,3,5,8,10,16,11,13,19,3] \\ [2,4,5,8,10,16,11,13,19,3] \\ [2,3,6,8,10,16,11,13,19,3] \\ [2,4,6,8,10,16,11,13,19,3] \end{array}$$

$$t_{21} * [\begin{array}{l} C_2*C_6*S_5*C_4*C_3*C_2*S_1*E_1 \\ C_2*C_6*S_5*C_4*C_3*S_2*C_1*E_1 \\ C_2*C_6*S_5*C_4*S_3*E_2*(-S_2)*S_1*E_1 \\ C_2*C_6*S_5*C_4*S_3*E_2*C_2*C_1*E_1 \\ C_2*C_6*S_5*S_4*(-S_3)*C_2*S_1*E_1 \\ C_2*C_6*S_5*S_4*(-S_3)*S_2*C_1*E_1 \\ C_2*C_6*S_5*S_4*C_3*E_2*(-S_2)*S_1*E_1 \\ C_2*C_6*S_5*S_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \quad \begin{array}{l} [2,3,5,7,9,17,12,13,19,3] \\ [2,4,5,7,9,17,12,13,19,3] \\ [2,3,6,7,9,17,12,13,19,3] \\ [2,4,6,7,9,17,12,13,19,3] \\ [2,3,5,8,9,17,12,13,19,3] \\ [2,4,5,8,9,17,12,13,19,3] \\ [2,3,6,8,9,17,12,13,19,3] \\ [2,4,6,8,9,17,12,13,19,3] \end{array}$$

$$t_{22} * [\begin{array}{l} C_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1 \\ C_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1 \\ C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 \\ C_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1 \\ C_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1 \\ C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}] \begin{array}{l} [2,3,5,7,10,18,12,13,19,3] \\ [2,4,5,7,10,18,12,13,19,3] \\ [2,3,6,7,10,18,12,13,19,3] \\ [2,4,6,7,10,18,12,13,19,3] \\ [2,3,5,8,10,18,12,13,19,3] \\ [2,4,5,8,10,18,12,13,19,3] \\ [2,3,6,8,10,18,12,13,19,3] \\ [2,4,6,8,10,18,12,13,19,3] \end{array}$$

$$t_{11} * [\begin{array}{l} C_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * C_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 \\ C_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * C_1 * E_1 \\ C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}] \begin{array}{l} [2,3,5,7,9,15,11,14,19,3] \\ [2,4,5,7,9,15,11,14,19,3] \\ [2,3,6,7,9,15,11,14,19,3] \\ [2,4,6,7,9,15,11,14,19,3] \\ [2,3,5,8,9,15,11,14,19,3] \\ [2,4,5,8,9,15,11,14,19,3] \\ [2,3,6,8,9,15,11,14,19,3] \\ [2,4,6,8,9,15,11,14,19,3] \end{array}$$

$$t_{12} * [\begin{array}{l} C_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * C_1 * E_1 \\ C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * C_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}] \begin{array}{l} [2,3,5,7,10,16,11,14,19,3] \\ [2,4,5,7,10,16,11,14,19,3] \\ [2,3,6,7,10,16,11,14,19,3] \\ [2,4,6,7,10,16,11,14,19,3] \\ [2,3,5,8,10,16,11,14,19,3] \\ [2,4,5,8,10,16,11,14,19,3] \\ [2,3,6,8,10,16,11,14,19,3] \\ [2,4,6,8,10,16,11,14,19,3] \end{array}$$

$$t_{21} * [\begin{array}{l} C_2 * S_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1 \\ C_2 * S_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1 \\ C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 \\ C_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1 \\ C_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1 \\ C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}] \begin{array}{l} [2,3,5,7,9,17,12,14,19,3] \\ [2,4,5,7,9,17,12,14,19,3] \\ [2,3,6,7,9,17,12,14,19,3] \\ [2,4,6,7,9,17,12,14,19,3] \\ [2,3,5,8,9,17,12,14,19,3] \\ [2,4,5,8,9,17,12,14,19,3] \\ [2,3,6,8,9,17,12,14,19,3] \\ [2,4,6,8,9,17,12,14,19,3] \end{array}$$

$$t_{22} * [\begin{array}{l} C_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1 \\ C_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1 \\ C_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ C_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 \end{array}] \begin{array}{l} [2,3,5,7,10,18,12,14,19,3] \\ [2,4,5,7,10,18,12,14,19,3] \\ [2,3,6,7,10,18,12,14,19,3] \\ [2,4,6,7,10,18,12,14,19,3] \end{array}$$

	$C_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,14,19,3]
	$C_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,18,12,14,19,3]
$t_{11} * [$	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,15,11,13,20,3]
$t_{12} * [$	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,13,20,3]
	$(-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,16,11,13,20,3]
$t_{21} * [$	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,13,20,3]
$t_{22} * [$	$(-S_2) * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,13,20,3]
	$(-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,18,12,13,20,3]

$t_{11} * [$	$(-S_2)*E_2 * C_6*(-S_5)*C_4*C_3*C_2*S_1*E_1$	[2,3,5,7,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*C_3*S_2*C_1*E_1$	[2,4,5,7,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*S_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*S_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*S_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,9,15,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*S_4*C_3*E_2*C_2*C_1*E_1]$	[2,4,6,8,9,15,11,14,20,3]
$t_{12} * [$	$(-S_2)*E_2*C_6*(-S_5)*(-S_4)*C_3*C_2*S_1*E_1$	[2,3,5,7,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*(-S_4)*C_3*S_2*C_1*E_1$	[2,4,5,7,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*(-S_4)*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,10,16,11,14,20,3]
	$(-S_2)*E_2*C_6*(-S_5)*C_4*C_3*E_2*C_2*C_1*E_1]$	[2,4,6,8,10,16,11,14,20,3]
$t_{21} * [$	$(-S_2)*E_2*C_6*C_5*C_4*C_3*C_2*S_1*E_1$	[2,3,5,7,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*C_3*S_2*C_1*E_1$	[2,4,5,7,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*S_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*S_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*S_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,9,17,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*S_4*C_3*E_2*C_2*C_1*E_1]$	[2,4,6,8,9,17,12,14,20,3]
$t_{22} * [$	$(-S_2)*E_2*C_6*C_5*(-S_4)*C_3*C_2*S_1*E_1$	[2,3,5,7,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*(-S_4)*C_3*S_2*C_1*E_1$	[2,4,5,7,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*(-S_4)*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,10,18,12,14,20,3]
	$(-S_2)*E_2*C_6*C_5*C_4*C_3*E_2*C_2*C_1*E_1]]$	[2,4,6,8,10,18,12,14,20,3]

Arrays of coefficients and phase indices for $P_{17,21}$.

$$P_{17,21} = s_2 [4] P_{15,11} + c_2 [4] P_{15,21}$$

$$\frac{1}{2\sqrt{2}} * [t_{11} * [\begin{array}{l} S_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1 \\ S_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1) \\ S_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ S_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1 \\ S_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1) \\ S_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,9,15,11,13,19,4] \\ [1,4,5,7,9,15,11,13,19,4] \\ [1,3,6,7,9,15,11,13,19,4] \\ [1,4,6,7,9,15,11,13,19,4] \\ [1,3,5,8,9,15,11,13,19,4] \\ [1,4,5,8,9,15,11,13,19,4] \\ [1,3,6,8,9,15,11,13,19,4] \\ [1,4,6,8,9,15,11,13,19,4] \end{array}$$

$$t_{12} * [\begin{array}{l} S_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1 \\ S_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ S_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ S_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1 \\ S_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ S_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,10,16,11,13,19,4] \\ [1,4,5,7,10,16,11,13,19,4] \\ [1,3,6,7,10,16,11,13,19,4] \\ [1,4,6,7,10,16,11,13,19,4] \\ [1,3,5,8,10,16,11,13,19,4] \\ [1,4,5,8,10,16,11,13,19,4] \\ [1,3,6,8,10,16,11,13,19,4] \\ [1,4,6,8,10,16,11,13,19,4] \end{array}$$

$$t_{21} * [\begin{array}{l} S_2 * C_6 * S_5 * C_4 * C_3 * C_2 * C_1 \\ S_2 * C_6 * S_5 * C_4 * C_3 * S_2 * (-S_1) \\ S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ S_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * C_1 \\ S_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * (-S_1) \\ S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,9,17,12,13,19,4] \\ [1,4,5,7,9,17,12,13,19,4] \\ [1,3,6,7,9,17,12,13,19,4] \\ [1,4,6,7,9,17,12,13,19,4] \\ [1,3,5,8,9,17,12,13,19,4] \\ [1,4,5,8,9,17,12,13,19,4] \\ [1,3,6,8,9,17,12,13,19,4] \\ [1,4,6,8,9,17,12,13,19,4] \end{array}$$

$$t_{22} * [\begin{array}{l} S_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * C_1 \\ S_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ S_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * C_1 \\ S_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 \\ S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,10,18,12,13,19,4] \\ [1,4,5,7,10,18,12,13,19,4] \\ [1,3,6,7,10,18,12,13,19,4] \\ [1,4,6,7,10,18,12,13,19,4] \\ [1,3,5,8,10,18,12,13,19,4] \\ [1,4,5,8,10,18,12,13,19,4] \\ [1,3,6,8,10,18,12,13,19,4] \\ [1,4,6,8,10,18,12,13,19,4] \end{array}$$

$$t_{11} * [\begin{array}{l} S_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * C_1 \\ S_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1) \\ S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ S_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1 \\ S_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1) \end{array}] \quad \begin{array}{l} [1,3,5,7,9,15,11,14,19,4] \\ [1,4,5,7,9,15,11,14,19,4] \\ [1,3,6,7,9,15,11,14,19,4] \\ [1,4,6,7,9,15,11,14,19,4] \\ [1,3,5,8,9,15,11,14,19,4] \\ [1,4,5,8,9,15,11,14,19,4] \end{array}$$

	$S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,19,4]
	$S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,14,19,4]
$t_{12} * [$	$S_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,19,4]
	$S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,14,19,4]
$t_{21} * [$	$S_2 * S_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,14,19,4]
$t_{22} * [$	$S_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,18,12,14,19,4]
$t_{11} * [$	$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,13,20,4]
$t_{12} * [$	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13,20,4]

$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13,20,4]
$C_2 * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13,20,4]
$C_2 * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13,20,4]
$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,20,4]
$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,13,20,4]
$t_{21} * [$	
$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,13,20,4]
$t_{22} * [$	
$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,20,4]
$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,18,12,13,20,4]
$t_{11} * [$	
$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,15,11,14,20,4]
$t_{12} * [$	
$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,20,4]
$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,14,20,4]

$t_{21} * [$	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,14,20,4]
$t_{22} * [$	$C_2 * \epsilon_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,20,4]
	$C_2 * \epsilon_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * C_2 * (-S_1)]]$	[1,4,6,8,10,18,12,14,20,4]

Arrays of coefficients and phase indices for $P_{17,22}$.

$$P_{17,22} = s_2 [4] P_{15,12} + c_2 [4] P_{15,22}$$

$\frac{1}{2\sqrt{2}} * [t_{11} * [$	$S_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,9,15,11,13,19,4]
	$S_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1]$	[2,4,6,8,9,15,11,13,19,4]
$t_{12} * [$	$S_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,10,16,11,13,19,4]
	$S_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1]$	[2,4,6,8,10,16,11,13,19,4]

$$t_{21} * [\begin{array}{l} S_2 * C_6 * S_5 * C_4 * C_3 * C_2 * S_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * C_3 * S_2 * C_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 \\ S_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * S_1 * E_1 \\ S_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * C_1 * E_1 \\ S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}]$$

- [2,3,5,7,9,17,12,13,19,4]
- [2,4,5,7,9,17,12,13,19,4]
- [2,3,6,7,9,17,12,13,19,4]
- [2,4,6,7,9,17,12,13,19,4]
- [2,3,5,8,9,17,12,13,19,4]
- [2,4,5,8,9,17,12,13,19,4]
- [2,3,6,8,9,17,12,13,19,4]
- [2,4,6,8,9,17,12,13,19,4]

$$t_{22} * [\begin{array}{l} S_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1 \\ S_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1 \\ S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}]$$

- [2,3,5,7,10,18,12,13,19,4]
- [2,4,5,7,10,18,12,13,19,4]
- [2,3,6,7,10,18,12,13,19,4]
- [2,4,6,7,10,18,12,13,19,4]
- [2,3,5,8,10,18,12,13,19,4]
- [2,4,5,8,10,18,12,13,19,4]
- [2,3,6,8,10,18,12,13,19,4]
- [2,4,6,8,10,18,12,13,19,4]

$$t_{11} * [\begin{array}{l} S_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * C_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 \\ S_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * C_1 * E_1 \\ S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}]$$

- [2,3,5,7,9,15,11,14,19,4]
- [2,4,5,7,9,15,11,14,19,4]
- [2,3,6,7,9,15,11,14,19,4]
- [2,4,6,7,9,15,11,14,19,4]
- [2,3,5,8,9,15,11,14,19,4]
- [2,4,5,8,9,15,11,14,19,4]
- [2,3,6,8,9,15,11,14,19,4]
- [2,4,6,8,9,15,11,14,19,4]

$$t_{12} * [\begin{array}{l} S_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * C_1 * E_1 \\ S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * C_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 \end{array}]$$

- [2,3,5,7,10,16,11,14,19,4]
- [2,4,5,7,10,16,11,14,19,4]
- [2,3,6,7,10,16,11,14,19,4]
- [2,4,6,7,10,16,11,14,19,4]
- [2,3,5,8,10,16,11,14,19,4]
- [2,4,5,8,10,16,11,14,19,4]
- [2,3,6,8,10,16,11,14,19,4]
- [2,4,6,8,10,16,11,14,19,4]

$$t_{21} * [\begin{array}{l} S_2 * S_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1 \\ S_2 * S_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1 \\ S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 \\ S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 \end{array}]$$

- [2,3,5,7,9,17,12,14,19,4]
- [2,4,5,7,9,17,12,14,19,4]
- [2,3,6,7,9,17,12,14,19,4]
- [2,4,6,7,9,17,12,14,19,4]

	$S_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,14,19,4]
	$S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,14,19,4]
$t_{22} * [$	$S_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,14,19,4]
	$S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,18,12,14,19,4]
$t_{11} * [$	$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,15,11,13,20,4]
$t_{12} * [$	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,13,20,4]
	$C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,16,11,13,20,4]
$t_{21} * [$	$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,13,20,4]

$t_{22} * [$	$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,13,20,4]
	$C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,10,18,12,13,20,4]
$t_{11} * [$	$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,9,15,11,14,20,4]
$t_{12} * [$	$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,14,20,4]
	$C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,10,16,11,14,20,4]
$t_{21} * [$	$C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1]$	[2,4,6,8,9,17,12,14,20,4]
$t_{22} * [$	$C_2 * E_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,14,20,4]
	$C_2 * E_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,14,20,4]

$$\begin{array}{ll}
 c_2 \cdot \epsilon_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot (-s_3) \cdot s_2 \cdot c_1 \cdot \epsilon_1 & [2,4,5,8,10,18,12,14,20,4] \\
 c_2 \cdot \epsilon_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot c_3 \cdot \epsilon_2 \cdot (-s_2) \cdot s_1 \cdot \epsilon_1 & [2,3,6,8,10,18,12,14,20,4] \\
 c_2 \cdot \epsilon_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot c_3 \cdot \epsilon_2 \cdot c_2 \cdot c_1 \cdot \epsilon_1 & [2,4,6,8,10,18,12,14,20,4] \quad \color{yellow}\blacksquare \color{green}\blacksquare \color{blue}\blacksquare
 \end{array}$$

$$T = P_{19} = M_1 R_1^T P_{17} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi_{21}} & 0 \\ 0 & \epsilon_1 e^{i\phi_{22}} \end{bmatrix} \begin{bmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{bmatrix} P_{17} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[21] & -s_1[21] \\ \epsilon_1 s_1[22] & \epsilon_1 c_1[22] \end{bmatrix} P_{17}$$

Thus,

$$\begin{aligned}
 T_{11} &= \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,11} + (-s_1) [21] P_{17,21}\} \\
 T_{12} &= \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,12} + (-s_1) [21] P_{17,22}\} \\
 T_{21} &= \frac{1}{\sqrt{2}} \{\epsilon_1 s_1 [22] P_{17,11} + \epsilon_1 c_1 [22] P_{17,21}\} \\
 T_{22} &= \frac{1}{\sqrt{2}} \{\epsilon_1 s_1 [22] P_{17,12} + \epsilon_1 c_1 [22] P_{17,22}\}
 \end{aligned}$$

Arrays of coefficients and phase indices for T_{11} . (256 terms each T_{ij})

$$T_{11} = \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,11} + (-s_1) [21] P_{17,21}\}$$

$$\frac{1}{4} * \color{yellow}\blacksquare \ t_{11} * \begin{bmatrix} c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot c_3 \cdot c_2 \cdot c_1 & [1,3,5,7,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot c_3 \cdot s_2 \cdot (-s_1) & [1,4,5,7,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot s_3 \cdot \epsilon_2 \cdot (-s_2) \cdot c_1 & [1,3,6,7,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot s_3 \cdot \epsilon_2 \cdot c_2 \cdot (-s_1) & [1,4,6,7,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot s_4 \cdot (-s_3) \cdot c_2 \cdot c_1 & [1,3,5,8,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot s_4 \cdot (-s_3) \cdot s_2 \cdot (-s_1) & [1,4,5,8,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot s_4 \cdot c_3 \cdot \epsilon_2 \cdot (-s_2) \cdot c_1 & [1,3,6,8,9,15,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot s_4 \cdot c_3 \cdot \epsilon_2 \cdot c_2 \cdot (-s_1) & [1,4,6,8,9,15,11,13,19,3,21] \end{bmatrix}$$

$$t_{12} * \begin{bmatrix} c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot (-s_4) \cdot c_3 \cdot c_2 \cdot c_1 & [1,3,5,7,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot (-s_4) \cdot c_3 \cdot s_2 \cdot (-s_1) & [1,4,5,7,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot (-s_4) \cdot s_3 \cdot \epsilon_2 \cdot (-s_2) \cdot c_1 & [1,3,6,7,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot (-s_4) \cdot s_3 \cdot \epsilon_2 \cdot c_2 \cdot (-s_1) & [1,4,6,7,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot (-s_3) \cdot c_2 \cdot c_1 & [1,3,5,8,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot (-s_3) \cdot s_2 \cdot (-s_1) & [1,4,5,8,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot c_3 \cdot \epsilon_2 \cdot (-s_2) \cdot c_1 & [1,3,6,8,10,16,11,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot c_5 \cdot c_4 \cdot c_3 \cdot \epsilon_2 \cdot c_2 \cdot (-s_1) & [1,4,6,8,10,16,11,13,19,3,21] \end{bmatrix}$$

$$t_{21} * \begin{bmatrix} c_1 \cdot c_2 \cdot c_6 \cdot s_5 \cdot c_4 \cdot c_3 \cdot c_2 \cdot c_1 & [1,3,5,7,9,17,12,13,19,3,21] \\
 c_1 \cdot c_2 \cdot c_6 \cdot s_5 \cdot c_4 \cdot c_3 \cdot s_2 \cdot (-s_1) & [1,4,5,7,9,17,12,13,19,3,21] \end{bmatrix}$$

$C_1 * C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,13,19,3,21]
$t_{22} * [C_1 * C_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,18,12,13,19,3,21]
$t_{11} * [C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,15,11,14,19,3,21]
$t_{12} * [C_1 * C_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,14,19,3,21]
$t_{21} * [C_1 * C_2 * S_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,14,19,3,21]

$$\begin{aligned} & c_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 & [1,3,6,8,10,18,12,13,20,3,21] \\ & c_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \] & [1,4,6,8,10,18,12,13,20,3,21] \end{aligned}$$

$$\begin{aligned} t_{11} * [& c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * C_1 & [1,3,5,7,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1) & [1,4,5,7,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1) & [1,4,6,7,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1 & [1,3,5,8,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1) & [1,4,5,8,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1 & [1,3,6,8,9,15,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1) \] & [1,4,6,8,9,15,11,14,20,3,21] \end{aligned}$$

$$\begin{aligned} t_{12} * [& c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1 & [1,3,5,7,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1) & [1,4,5,7,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1) & [1,4,6,7,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1 & [1,3,5,8,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1) & [1,4,5,8,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1 & [1,3,6,8,10,16,11,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1) \] & [1,4,6,8,10,16,11,14,20,3,21] \end{aligned}$$

$$\begin{aligned} t_{21} * [& c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1 & [1,3,5,7,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1) & [1,4,5,7,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) & [1,4,6,7,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1 & [1,3,5,8,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1) & [1,4,5,8,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 & [1,3,6,8,9,17,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \] & [1,4,6,8,9,17,12,14,20,3,21] \end{aligned}$$

$$\begin{aligned} t_{22} * [& c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1 & [1,3,5,7,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1) & [1,4,5,7,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) & [1,4,6,7,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1 & [1,3,5,8,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1) & [1,4,5,8,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 & [1,3,6,8,10,18,12,14,20,3,21] \\ & c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \] & [1,4,6,8,10,18,12,14,20,3,21] \end{aligned}$$

$$\begin{aligned} t_{11} * [& (-S_1) * S_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1 & [1,3,5,7,9,15,11,13,19,4,21] \\ & (-S_1) * S_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1) & [1,4,5,7,9,15,11,13,19,4,21] \\ & (-S_1) * S_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,9,15,11,13,19,4,21] \end{aligned}$$

$(-S_1) * S_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,13,19,4,21]
$t_{12} * [(-S_1) * S_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,19,4,21]
$(-S_1) * S_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,13,19,4,21]
$t_{21} * [(-S_1) * S_2 * C_6 * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,13,19,4,21]
$t_{22} * [(-S_1) * S_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,19,4,21]
$(-S_1) * S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,18,12,13,19,4,21]
$t_{11} * [(-S_1) * S_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,14,19,4,21]
$t_{12} * [(-S_1) * S_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,19,4,21]

$(-S_1) * S_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,19,4,21]
$(-S_1) * S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,14,19,4,21]
$t_{21} * [(-S_1) * S_2 * S_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,14,19,4,21]
$t_{22} * [(-S_1) * S_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,19,4,21]
$(-S_1) * S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,18,12,14,19,4,21]
$t_{11} * [(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,15,11,13,20,4,21]
$t_{12} * [(-S_1) * C_2 * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,20,4,21]

$(-S_1) * C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \mid$	[1,4,6,8,10,16,11,13,20,4,21]
$t_{21} * [(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \mid$	[1,4,6,8,9,17,12,13,20,4,21]
$t_{22} * [(-S_1) * C_2 * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,20,4,21]
$(-S_1) * C_2 * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \mid$	[1,4,6,8,10,18,12,13,20,4,21]
$t_{11} * [(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1) \mid$	[1,4,6,8,9,15,11,14,20,4,21]
$t_{12} * [(-S_1) * C_2 * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1) \mid$	[1,4,6,8,10,16,11,14,20,4,21]
$t_{21} * [(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,20,4,21]
$(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,20,4,21]

$$\begin{aligned}
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) && [1,4,6,7,9,17,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1 && [1,3,5,8,9,17,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1) && [1,4,5,8,9,17,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 && [1,3,6,8,9,17,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \quad] && [1,4,6,8,9,17,12,14,20,4,21]
 \end{aligned}$$

$$\begin{aligned}
 t_{22} * [&(-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1 && [1,3,5,7,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1) && [1,4,5,7,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 && [1,3,6,7,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) && [1,4,6,7,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1 && [1,3,5,8,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1) && [1,4,5,8,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 && [1,3,6,8,10,18,12,14,20,4,21] \\
 &(-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \quad] \quad] && [1,4,6,8,10,18,12,14,20,4,21]
 \end{aligned}$$

Arrays of coefficients and phase indices for T_{12} .

$$T_{12} = \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,12} + (-S_1) [21] P_{17,22}\}$$

$$\begin{aligned}
 \frac{1}{4} * [&t_{11} * [\quad C_1 * C_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1 && [2,3,5,7,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1 && [2,4,5,7,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 && [2,3,6,7,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 && [2,4,6,7,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1 && [2,3,5,8,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1 && [2,4,5,8,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 && [2,3,6,8,9,15,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1 \quad] && [2,4,6,8,9,15,11,13,19,3,21]
 \end{aligned}$$

$$\begin{aligned}
 t_{12} * [&C_1 * C_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1 && [2,3,5,7,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1 && [2,4,5,7,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 && [2,3,6,7,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 && [2,4,6,7,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1 && [2,3,5,8,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1 && [2,4,5,8,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 && [2,3,6,8,10,16,11,13,19,3,21] \\
 &C_1 * C_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 \quad] && [2,4,6,8,10,16,11,13,19,3,21]
 \end{aligned}$$

$$\begin{aligned}
 t_{21} * [&C_1 * C_2 * C_6 * S_5 * C_4 * C_3 * C_2 * S_1 * E_1 && [2,3,5,7,9,17,12,13,19,3,21] \\
 &C_1 * C_2 * C_6 * S_5 * C_4 * C_3 * S_2 * C_1 * E_1 && [2,4,5,7,9,17,12,13,19,3,21] \\
 &C_1 * C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 && [2,3,6,7,9,17,12,13,19,3,21]
 \end{aligned}$$

$C_1 * C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,13,19,3,21]
$t_{22} * [C_1 * C_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,13,19,3,21]
$C_1 * C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,18,12,13,19,3,21]
$t_{11} * [C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,15,11,14,19,3,21]
$t_{12} * [C_1 * C_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,14,19,3,21]
$C_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,16,11,14,19,3,21]
$t_{21} * [C_1 * C_2 * S_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,14,19,3,21]
$C_1 * C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,14,19,3,21]

$$t_{22} * [\begin{array}{l} c_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \end{array}]$$

- [2,3,5,7,10,18,12,14,19,3,21]
- [2,4,5,7,10,18,12,14,19,3,21]
- [2,3,6,7,10,18,12,14,19,3,21]
- [2,4,6,7,10,18,12,14,19,3,21]
- [2,3,5,8,10,18,12,14,19,3,21]
- [2,4,5,8,10,18,12,14,19,3,21]
- [2,3,6,8,10,18,12,14,19,3,21]
- [2,4,6,8,10,18,12,14,19,3,21]

$$t_{11} * [\begin{array}{l} c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \end{array}]$$

- [2,3,5,7,9,15,11,13,20,3,21]
- [2,4,5,7,9,15,11,13,20,3,21]
- [2,3,6,7,9,15,11,13,20,3,21]
- [2,4,6,7,9,15,11,13,20,3,21]
- [2,3,5,8,9,15,11,13,20,3,21]
- [2,4,5,8,9,15,11,13,20,3,21]
- [2,3,6,8,9,15,11,13,20,3,21]
- [2,4,6,8,9,15,11,13,20,3,21]

$$t_{12} * [\begin{array}{l} c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * c_1 * \epsilon_1 \\ (c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \end{array}]$$

- [2,3,5,7,10,16,11,13,20,3,21]
- [2,4,5,7,10,16,11,13,20,3,21]
- [2,3,6,7,10,16,11,13,20,3,21]
- [2,4,6,7,10,16,11,13,20,3,21]
- [2,3,5,8,10,16,11,13,20,3,21]
- [2,4,5,8,10,16,11,13,20,3,21]
- [2,3,6,8,10,16,11,13,20,3,21]
- [2,4,6,8,10,16,11,13,20,3,21]

$$t_{21} * [\begin{array}{l} c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \end{array}]$$

- [2,3,5,7,9,17,12,13,20,3,21]
- [2,4,5,7,9,17,12,13,20,3,21]
- [2,3,6,7,9,17,12,13,20,3,21]
- [2,4,6,7,9,17,12,13,20,3,21]
- [2,3,5,8,9,17,12,13,20,3,21]
- [2,4,5,8,9,17,12,13,20,3,21]
- [2,3,6,8,9,17,12,13,20,3,21]
- [2,4,6,8,9,17,12,13,20,3,21]

$$t_{22} * [\begin{array}{l} c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * c_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 \\ c_1 * (-s_2) * \epsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 \end{array}]$$

- [2,3,5,7,10,18,12,13,20,3,21]
- [2,4,5,7,10,18,12,13,20,3,21]
- [2,3,6,7,10,18,12,13,20,3,21]
- [2,4,6,7,10,18,12,13,20,3,21]

$c_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,13,20,3,21]
$c_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,13,20,3,21]
$c_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,13,20,3,21]
$c_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,18,12,13,20,3,21]

$t_{11} * [c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,15,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,15,11,14,20,3,21]

$t_{12} * [c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,16,11,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,16,11,14,20,3,21]

$t_{21} * [c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,9,17,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,9,17,12,14,20,3,21]

$t_{22} * [c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1$	[2,3,5,7,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1$	[2,4,5,7,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,7,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1$	[2,4,6,7,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1$	[2,3,5,8,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1$	[2,4,5,8,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1$	[2,3,6,8,10,18,12,14,20,3,21]
$c_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1$]	[2,4,6,8,10,18,12,14,20,3,21]

$$t_{11} * [\begin{array}{l} (-S_1)*S_2*C_6*C_5*C_4*C_3*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*C_3*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*S_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*S_3*E_2*C_2*C_1*E_1 \\ (-S_1)*S_2*C_6*C_5*S_4*(-S_3)*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*S_4*(-S_3)*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*C_5*S_4*C_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*S_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \begin{array}{l} [2,3,5,7,9,15,11,13,19,4,21] \\ [2,4,5,7,9,15,11,13,19,4,21] \\ [2,3,6,7,9,15,11,13,19,4,21] \\ [2,4,6,7,9,15,11,13,19,4,21] \\ [2,3,5,8,9,15,11,13,19,4,21] \\ [2,4,5,8,9,15,11,13,19,4,21] \\ [2,3,6,8,9,15,11,13,19,4,21] \\ [2,4,6,8,9,15,11,13,19,4,21] \end{array}$$

$$t_{12} * [\begin{array}{l} (-S_1)*S_2*C_6*C_5*(-S_4)*C_3*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*(-S_4)*C_3*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*C_5*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*(-S_4)*S_3*E_2*C_2*C_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*(-S_3)*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*(-S_3)*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*C_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*C_5*C_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \begin{array}{l} [2,3,5,7,10,16,11,13,19,4,21] \\ [2,4,5,7,10,16,11,13,19,4,21] \\ [2,3,6,7,10,16,11,13,19,4,21] \\ [2,4,6,7,10,16,11,13,19,4,21] \\ [2,3,5,8,10,16,11,13,19,4,21] \\ [2,4,5,8,10,16,11,13,19,4,21] \\ [2,3,6,8,10,16,11,13,19,4,21] \\ [2,4,6,8,10,16,11,13,19,4,21] \end{array}$$

$$t_{21} * [\begin{array}{l} (-S_1)*S_2*C_6*S_5*C_4*C_3*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*C_3*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*S_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*S_3*E_2*C_2*C_1*E_1 \\ (-S_1)*S_2*C_6*S_5*S_4*(-S_3)*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*S_4*(-S_3)*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*S_5*S_4*C_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*S_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \begin{array}{l} [2,3,5,7,9,17,12,13,19,4,21] \\ [2,4,5,7,9,17,12,13,19,4,21] \\ [2,3,6,7,9,17,12,13,19,4,21] \\ [2,4,6,7,9,17,12,13,19,4,21] \\ [2,3,5,8,9,17,12,13,19,4,21] \\ [2,4,5,8,9,17,12,13,19,4,21] \\ [2,3,6,8,9,17,12,13,19,4,21] \\ [2,4,6,8,9,17,12,13,19,4,21] \end{array}$$

$$t_{22} * [\begin{array}{l} (-S_1)*S_2*C_6*S_5*(-S_4)*C_3*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*(-S_4)*C_3*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*S_5*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*(-S_4)*S_3*E_2*C_2*C_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*(-S_3)*C_2*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*(-S_3)*S_2*C_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*C_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*C_6*S_5*C_4*C_3*E_2*C_2*C_1*E_1 \end{array}] \begin{array}{l} [2,3,5,7,10,18,12,13,19,4,21] \\ [2,4,5,7,10,18,12,13,19,4,21] \\ [2,3,6,7,10,18,12,13,19,4,21] \\ [2,4,6,7,10,18,12,13,19,4,21] \\ [2,3,5,8,10,18,12,13,19,4,21] \\ [2,4,5,8,10,18,12,13,19,4,21] \\ [2,3,6,8,10,18,12,13,19,4,21] \\ [2,4,6,8,10,18,12,13,19,4,21] \end{array}$$

$$t_{11} * [\begin{array}{l} (-S_1)*S_2*S_6*(-S_5)*C_4*C_3*C_2*S_1*E_1 \\ (-S_1)*S_2*S_6*(-S_5)*C_4*C_3*S_2*C_1*E_1 \\ (-S_1)*S_2*S_6*(-S_5)*C_4*S_3*E_2*(-S_2)*S_1*E_1 \\ (-S_1)*S_2*S_6*(-S_5)*C_4*S_3*E_2*C_2*C_1*E_1 \\ (-S_1)*S_2*S_6*(-S_5)*S_4*(-S_3)*C_2*S_1*E_1 \end{array}] \begin{array}{l} [2,3,5,7,9,15,11,14,19,4,21] \\ [2,4,5,7,9,15,11,14,19,4,21] \\ [2,3,6,7,9,15,11,14,19,4,21] \\ [2,4,6,7,9,15,11,14,19,4,21] \\ [2,3,5,8,9,15,11,14,19,4,21] \end{array}$$

$(-S_1)*S_2*S_6*(-S_5)*S_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,9,15,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*S_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,9,15,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*S_4*C_3*E_2*C_2*C_1*E_1$]	[2,4,6,8,9,15,11,14,19,4,21]
$t_{12} * [(-S_1)*S_2*S_6*(-S_5)*(-S_4)*C_3*C_2*S_1*E_1$	[2,3,5,7,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*(-S_4)*C_3*S_2*C_1*E_1$	[2,4,5,7,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*(-S_4)*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*C_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*C_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*C_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,10,16,11,14,19,4,21]
$(-S_1)*S_2*S_6*(-S_5)*C_4*C_3*E_2*C_2*C_1*E_1$]	[2,4,6,8,10,16,11,14,19,4,21]
$t_{21} * [(-S_1)*S_2*S_6*C_5*C_4*C_3*C_2*S_1*E_1$	[2,3,5,7,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*C_3*S_2*C_1*E_1$	[2,4,5,7,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*S_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*S_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*S_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,9,17,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*S_4*C_3*E_2*C_2*C_1*E_1$]	[2,4,6,8,9,17,12,14,19,4,21]
$t_{22} * [(-S_1)*S_2*S_6*C_5*(-S_4)*C_3*C_2*S_1*E_1$	[2,3,5,7,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*(-S_4)*C_3*S_2*C_1*E_1$	[2,4,5,7,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*(-S_4)*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*(-S_4)*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,10,18,12,14,19,4,21]
$(-S_1)*S_2*S_6*C_5*C_4*C_3*E_2*C_2*C_1*E_1$]	[2,4,6,8,10,18,12,14,19,4,21]
$t_{11} * [(-S_1)*C_2*E_2*(-S_6)*C_5*C_4*C_3*C_2*S_1*E_1$	[2,3,5,7,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*C_4*C_3*S_2*C_1*E_1$	[2,4,5,7,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*C_4*S_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,7,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*C_4*S_3*E_2*C_2*C_1*E_1$	[2,4,6,7,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*S_4*(-S_3)*C_2*S_1*E_1$	[2,3,5,8,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*S_4*(-S_3)*S_2*C_1*E_1$	[2,4,5,8,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*S_4*C_3*E_2*(-S_2)*S_1*E_1$	[2,3,6,8,9,15,11,13,20,4,21]
$(-S_1)*C_2*E_2*(-S_6)*C_5*S_4*C_3*E_2*C_2*C_1*E_1$]	[2,4,6,8,9,15,11,13,20,4,21]
$t_{12} * [(-S_1)*C_2*E_2*(-S_6)*C_5*(-S_4)*C_3*C_2*S_1*E_1$	[2,3,5,7,10,16,11,13,20,4,21]

$$\begin{aligned} & (-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 & [2,3,6,8,10,16,11,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 & [2,4,6,8,10,16,11,14,20,4,21] \end{aligned}$$

$$\begin{aligned} t_{21} * [& (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * E_1 & [2,3,5,7,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * E_1 & [2,4,5,7,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * S_1 * E_1 & [2,3,6,7,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * C_1 * E_1 & [2,4,6,7,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * E_1 & [2,3,5,8,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * E_1 & [2,4,5,8,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 & [2,3,6,8,9,17,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * C_1 * E_1 & [2,4,6,8,9,17,12,14,20,4,21] \end{aligned}$$

$$\begin{aligned} t_{22} * [& (-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * E_1 & [2,3,5,7,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * E_1 & [2,4,5,7,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * S_1 * E_1 & [2,3,6,7,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * C_1 * E_1 & [2,4,6,7,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * E_1 & [2,3,5,8,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * E_1 & [2,4,5,8,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * S_1 * E_1 & [2,3,6,8,10,18,12,14,20,4,21] \\ & (-S_1) * C_2 * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * C_1 * E_1 & [2,4,6,8,10,18,12,14,20,4,21] \end{aligned}$$

Arrays of coefficients and phase indices for T_{21} .



$$T_{21} = \frac{1}{\sqrt{2}} \{ \epsilon_1 s_1 [22] P_{17,11} + \epsilon_1 c_1 [22] P_{17,21} \}$$

$$\frac{1}{4} * [t_{11} * [\begin{aligned} & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1 & [1,3,5,7,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1) & [1,4,5,7,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) & [1,4,6,7,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1 & [1,3,5,8,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1) & [1,4,5,8,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 & [1,3,6,8,9,15,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) & [1,4,6,8,9,15,11,13,19,3,22] \end{aligned}]$$

$$\begin{aligned} t_{12} * [& \epsilon_1 * S_1 * C_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1 & [1,3,5,7,10,16,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1) & [1,4,5,7,10,16,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 & [1,3,6,7,10,16,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) & [1,4,6,7,10,16,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1 & [1,3,5,8,10,16,11,13,19,3,22] \\ & \epsilon_1 * S_1 * C_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1) & [1,4,5,8,10,16,11,13,19,3,22] \end{aligned}$$

$E_1 * S_1 * C_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,13,19,3,22]
$t_{21} * [E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,17,12,13,19,3,22]
$t_{22} * [E_1 * S_1 * C_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,19,3,22]
$E_1 * S_1 * C_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,18,12,13,19,3,22]
$t_{11} * [E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,9,15,11,14,19,3,22]
$t_{12} * [E_1 * S_1 * C_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1)]$	[1,4,6,8,10,16,11,14,19,3,22]
$t_{21} * [E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,14,19,3,22]

$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,14,19,3,22]
$t_{22} * [E_1 * S_1 * C_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,19,3,22]
$E_1 * S_1 * C_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,18,12,14,19,3,22]
$t_{11} * [E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,15,11,13,20,3,22]
$t_{12} * [E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,13,20,3,22]
$t_{21} * [E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * (-S_6) * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,20,3,22]

$E_1 * S_1 * (-S_2) * E_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,14,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,14,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,14,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,14,20,3,22]
$E_1 * S_1 * (-S_2) * E_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,18,12,14,20,3,22] 
$t_{11} * [E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,15,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,15,11,13,19,4,22]
$t_{12} * [E_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,16,11,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,16,11,13,19,4,22]
$t_{21} * [E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * C_3 * C_2 * C_1$	[1,3,5,7,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * C_3 * S_2 * (-S_1)$	[1,4,5,7,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,9,17,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * S_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,9,17,12,13,19,4,22]
$t_{22} * [E_1 * C_1 * S_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * C_1$	[1,3,5,7,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * (-S_4) * C_3 * S_2 * (-S_1)$	[1,4,5,7,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1$	[1,3,6,7,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1)$	[1,4,6,7,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * (-S_3) * C_2 * C_1$	[1,3,5,8,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * (-S_3) * S_2 * (-S_1)$	[1,4,5,8,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * (-S_2) * C_1$	[1,3,6,8,10,18,12,13,19,4,22]
$E_1 * C_1 * S_2 * C_6 * S_5 * C_4 * C_3 * E_2 * C_2 * (-S_1)$]	[1,4,6,8,10,18,12,13,19,4,22] 

$$t_{11} * [\begin{array}{l} \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * C_3 * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * C_3 * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * S_4 * (-S_3) * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * S_4 * (-S_3) * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * S_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}]$$

[1,3,5,7,9,15,11,14,19,4,22]
 [1,4,5,7,9,15,11,14,19,4,22]
 [1,3,6,7,9,15,11,14,19,4,22]
 [1,4,6,7,9,15,11,14,19,4,22]
 [1,3,5,8,9,15,11,14,19,4,22]
 [1,4,5,8,9,15,11,14,19,4,22]
 [1,3,6,8,9,15,11,14,19,4,22]
 [1,4,6,8,9,15,11,14,19,4,22]

$$t_{12} * [\begin{array}{l} \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * (-S_4) * C_3 * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * (-S_4) * C_3 * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * (-S_3) * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * (-S_3) * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * (-S_5) * C_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}]$$

[1,3,5,7,10,16,11,14,19,4,22]
 [1,4,5,7,10,16,11,14,19,4,22]
 [1,3,6,7,10,16,11,14,19,4,22]
 [1,4,6,7,10,16,11,14,19,4,22]
 [1,3,5,8,10,16,11,14,19,4,22]
 [1,4,5,8,10,16,11,14,19,4,22]
 [1,3,6,8,10,16,11,14,19,4,22]
 [1,4,6,8,10,16,11,14,19,4,22]

$$t_{21} * [\begin{array}{l} \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * C_3 * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * C_3 * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * S_4 * (-S_3) * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * S_4 * (-S_3) * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * S_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}]$$

[1,3,5,7,9,17,12,14,19,4,22]
 [1,4,5,7,9,17,12,14,19,4,22]
 [1,3,6,7,9,17,12,14,19,4,22]
 [1,4,6,7,9,17,12,14,19,4,22]
 [1,3,5,8,9,17,12,14,19,4,22]
 [1,4,5,8,9,17,12,14,19,4,22]
 [1,3,6,8,9,17,12,14,19,4,22]
 [1,4,6,8,9,17,12,14,19,4,22]

$$t_{22} * [\begin{array}{l} \epsilon_1 * C_1 * S_2 * S_6 * C_5 * (-S_4) * C_3 * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * (-S_4) * C_3 * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * (-S_4) * S_3 * E_2 * C_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * (-S_3) * C_2 * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * (-S_3) * S_2 * (-S_1) \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * S_2 * S_6 * C_5 * C_4 * C_3 * E_2 * C_2 * (-S_1) \end{array}]$$

[1,3,5,7,10,18,12,14,19,4,22]
 [1,4,5,7,10,18,12,14,19,4,22]
 [1,3,6,7,10,18,12,14,19,4,22]
 [1,4,6,7,10,18,12,14,19,4,22]
 [1,3,5,8,10,18,12,14,19,4,22]
 [1,4,5,8,10,18,12,14,19,4,22]
 [1,3,6,8,10,18,12,14,19,4,22]
 [1,4,6,8,10,18,12,14,19,4,22]

$$t_{11} * [\begin{array}{l} \epsilon_1 * C_1 * C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * C_2 * C_1 \\ \epsilon_1 * C_1 * C_2 * E_2 * (-S_6) * C_5 * C_4 * C_3 * S_2 * (-S_1) \\ \epsilon_1 * C_1 * C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * (-S_2) * C_1 \\ \epsilon_1 * C_1 * C_2 * E_2 * (-S_6) * C_5 * C_4 * S_3 * E_2 * C_2 * (-S_1) \end{array}]$$

[1,3,5,7,9,15,11,13,20,4,22]
 [1,4,5,7,9,15,11,13,20,4,22]
 [1,3,6,7,9,15,11,13,20,4,22]
 [1,4,6,7,9,15,11,13,20,4,22]

$$t_{12} * \begin{bmatrix} \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1 & [1,3,5,7,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1) & [1,4,5,7,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \epsilon_2 * (-s_2) * c_1 & [1,3,6,7,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \epsilon_2 * c_2 * (-s_1) & [1,4,6,7,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1 & [1,3,5,8,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1) & [1,4,5,8,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \epsilon_2 * (-s_2) * c_1 & [1,3,6,8,10,16,11,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \epsilon_2 * c_2 * (-s_1) & [1,4,6,8,10,16,11,14,20,4,22] \end{bmatrix}$$

$$t_{21} * \begin{bmatrix} \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1 & [1,3,5,7,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1) & [1,4,5,7,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * (-s_2) * c_1 & [1,3,6,7,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * c_2 * (-s_1) & [1,4,6,7,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1 & [1,3,5,8,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1) & [1,4,5,8,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * (-s_2) * c_1 & [1,3,6,8,9,17,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * c_2 * (-s_1) & [1,4,6,8,9,17,12,14,20,4,22] \end{bmatrix}$$

$$t_{22} * \begin{bmatrix} \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1 & [1,3,5,7,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1) & [1,4,5,7,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \epsilon_2 * (-s_2) * c_1 & [1,3,6,7,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \epsilon_2 * c_2 * (-s_1) & [1,4,6,7,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1 & [1,3,5,8,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1) & [1,4,5,8,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * c_3 * \epsilon_2 * (-s_2) * c_1 & [1,3,6,8,10,18,12,14,20,4,22] \\ \epsilon_1 * c_1 * c_2 * \epsilon_2 * c_6 * c_5 * c_4 * c_3 * \epsilon_2 * c_2 * (-s_1) & [1,4,6,8,10,18,12,14,20,4,22] \end{bmatrix}$$

Arrays of coefficients and phase indices for T_{22} .

$$T_{22} = \frac{1}{\sqrt{2}} \{ \epsilon_1 s_1 [22] P_{17,12} + \epsilon_1 c_1 [22] P_{17,22} \}$$

$$\frac{1}{4} * \begin{bmatrix} t_{11} * \begin{bmatrix} \epsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \epsilon_1 & [2,3,5,7,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \epsilon_1 & [2,4,5,7,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 & [2,3,6,7,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * s_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 & [2,4,6,7,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \epsilon_1 & [2,3,5,8,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \epsilon_1 & [2,4,5,8,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * (-s_2) * s_1 * \epsilon_1 & [2,3,6,8,9,15,11,13,19,3,22] \\ \epsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * c_3 * \epsilon_2 * c_2 * c_1 * \epsilon_1 & [2,4,6,8,9,15,11,13,19,3,22] \end{bmatrix} \end{bmatrix}$$

$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,9,17,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$]	[2,4,6,8,9,17,12,14,20,3,22]
$t_{22} * [\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,10,18,12,14,20,3,22]
$\epsilon_1 * S_1 * (-S_2) * \epsilon_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$]	[2,4,6,8,10,18,12,14,20,3,22]
$t_{11} * [\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * S_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * S_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,9,15,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * S_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$]	[2,4,6,8,9,15,11,13,19,4,22]
$t_{12} * [\epsilon_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * (-S_4) * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,10,16,11,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * C_5 * C_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$]	[2,4,6,8,10,16,11,13,19,4,22]
$t_{21} * [\epsilon_1 * C_1 * S_2 * C_6 * S_5 * C_4 * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * C_4 * C_3 * S_2 * C_1 * \epsilon_1$	[2,4,5,7,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * C_4 * S_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,7,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * C_4 * S_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$	[2,4,6,7,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * S_4 * (-S_3) * C_2 * S_1 * \epsilon_1$	[2,3,5,8,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * S_4 * (-S_3) * S_2 * C_1 * \epsilon_1$	[2,4,5,8,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * S_4 * C_3 * \epsilon_2 * (-S_2) * S_1 * \epsilon_1$	[2,3,6,8,9,17,12,13,19,4,22]
$\epsilon_1 * C_1 * S_2 * C_6 * S_5 * S_4 * C_3 * \epsilon_2 * C_2 * C_1 * \epsilon_1$]	[2,4,6,8,9,17,12,13,19,4,22]
$t_{22} * [\epsilon_1 * C_1 * S_2 * C_6 * S_5 * (-S_4) * C_3 * C_2 * S_1 * \epsilon_1$	[2,3,5,7,10,18,12,13,19,4,22]

Appendix B MATLAB PROGRAMS.

Our Matlab programs compute the output intensity of the general gyroscope described in Section 5 for three types of frequency distribution functions g ; a Gaussian function, a step function, and a tent function, using Equations 3.11, 3.12, and 3.15, respectively. In order to compute the intensity given by any one of these three equations we need the four arrays of coefficients C_{ij} ($i, j = 1, 2$) and the sets of phase indices $\Lambda_{ij}(s)$ to compute all the phase shifts $\beta_{ij}(s)$ for all $i, j = 1, 2$ and $1 \leq s \leq 256$.

The four arrays of coefficients C_{ij} ($i, j = 1, 2$) are provided by the four functions **Coef11_arr**, **Coef12_arr**, **Coef21_arr**, and **Coef22_arr**. We show part of the function **Coef11_arr** to illustrate its structure. The complete lists of coefficients for T_{11} , T_{12} , T_{21} , and T_{22} can be found in Appendix 1.

```
function Coef11 = Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e1,e2,t)
```

```
% Array of coefficients for T11.
```

```
t11=t(1);  
t12=t(2);  
t21=t(3);  
t22=t(4);
```

```
Coef11 = (1/4) * [
```

```
    t11 * [ c1*c2*c6*c5*c4*c3*c2*c1  
            c1*c2*c6*c5*c4*c3*s2*(-s1)  
            c1*c2*c6*c5*c4*s3*e2*(-s2)*c1  
            c1*c2*c6*c5*c4*s3*e2*c2*(-s1)  
            c1*c2*c6*c5*s4*(-s3)*c2*c1  
            c1*c2*c6*c5*s4*(-s3)*s2*(-s1)  
            c1*c2*c6*c5*s4*c3*e2*(-s2)*c1  
            c1*c2*c6*c5*s4*c3*e2*c2*(-s1) ]
```

$$\begin{aligned}
 t12 * & \begin{bmatrix} c1*c2*c6*c5*(-s4)*c3*c2*c1 \\ c1*c2*c6*c5*(-s4)*c3*s2*(-s1) \\ c1*c2*c6*c5*(-s4)*s3*e2*(-s2)*c1 \\ c1*c2*c6*c5*(-s4)*s3*e2*c2*(-s1) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \\
 t21 * & \begin{bmatrix} (-s1)*c2*e2*c6*c5*c4*c3*c2*c1 \\ (-s1)*c2*e2*c6*c5*c4*c3*s2*(-s1) \\ (-s1)*c2*e2*c6*c5*c4*s3*e2*(-s2)*c1 \\ (-s1)*c2*e2*c6*c5*c4*s3*e2*c2*(-s1) \\ (-s1)*c2*e2*c6*c5*s4*(-s3)*c2*c1 \\ (-s1)*c2*e2*c6*c5*s4*(-s3)*s2*(-s1) \\ (-s1)*c2*e2*c6*c5*s4*c3*e2*(-s2)*c1 \\ (-s1)*c2*e2*c6*c5*s4*c3*e2*c2*(-s1) \end{bmatrix} \\
 t22 * & \begin{bmatrix} (-s1)*c2*e2*c6*c5*(-s4)*c3*c2*c1 \\ (-s1)*c2*e2*c6*c5*(-s4)*c3*s2*(-s1) \\ (-s1)*c2*e2*c6*c5*(-s4)*s3*e2*(-s2)*c1 \\ (-s1)*c2*e2*c6*c5*(-s4)*s3*e2*c2*(-s1) \\ (-s1)*c2*e2*c6*c5*c4*(-s3)*c2*c1 \\ (-s1)*c2*e2*c6*c5*c4*(-s3)*s2*(-s1) \\ (-s1)*c2*e2*c6*c5*c4*c3*e2*(-s2)*c1 \\ (-s1)*c2*e2*c6*c5*c4*c3*e2*c2*(-s1) \end{bmatrix}];
 \end{aligned}$$

The inputs to **Coef11_arr** are $s1, s2, s3, s4, s5, s6, c1, c2, c3, c4, c5, c6, e1, e2, t11, t12, t21,$ and $t22,$ where

$$\begin{aligned}
 s1 &= \sin(a(1)), & c1 &= \cos(a(1)), \\
 s2 &= \sin(a(2)), & c2 &= \cos(a(2)), \\
 s3 &= \sin(A1 + a(3)), & c3 &= \cos(A1 + a(3)), \\
 s4 &= \sin(\frac{\pi}{4} + b(1)), & c4 &= \cos(\frac{\pi}{4} + b(1)), \\
 s5 &= \sin(\frac{\pi}{4} + b(2)), & c5 &= \cos(\frac{\pi}{4} + b(2)), \\
 s6 &= \sin(A2 + a(4)), & c6 &= \cos(A2 + a(4)),
 \end{aligned}$$

$A1 = 0^\circ$ or 90° , $A2 = 0^\circ$ or 90° , and $a(1), a(2), a(3), a(4), b(1), b(2)$ are arbitrary angles. The two polarization extinction ratios are $e1 = \epsilon_1$ and $e2 = \epsilon_2$. The magnitudes of the entries of the coil's 2×2 complex matrix are $t11, t12, t21,$ and $t22.$

The sets of phase indices $\Lambda_{ij}(s)$ ($1 \leq s \leq 256$) for a particular index pair (i, j) have been arranged in an 11×256 array Lij . In our Matlab programs these four arrays are produced by four functions called **Ph_arrCo11, Ph_arrCo12, Ph_arrCo21,**

Ph_arrCo22. We show part of the function **Ph_arrCo11** to illustrate it's structure. The complete lists of phase indices for T_{11} , T_{12} , T_{21} , and T_{22} can be found in Appendix 1.

function L11=Ph_arrCo11

```
L11 = [
    [1,3,5,7,9,15,11,13,19,3,21]
    [1,4,5,7,9,15,11,13,19,3,21]
    [1,3,6,7,9,15,11,13,19,3,21]
    [1,4,6,7,9,15,11,13,19,3,21]
    [1,3,5,8,9,15,11,13,19,3,21]
    [1,4,5,8,9,15,11,13,19,3,21]
    [1,3,6,8,9,15,11,13,19,3,21]
    [1,4,6,8,9,15,11,13,19,3,21]

    [1,3,5,7,10,16,11,13,19,3,21]
    [1,4,5,7,10,16,11,13,19,3,21]
    [1,3,6,7,10,16,11,13,19,3,21]
    [1,4,6,7,10,16,11,13,19,3,21]
    [1,3,5,8,10,16,11,13,19,3,21]
    .
    .
    .
    [1,3,5,7,9,17,12,14,20,4,21]
    [1,4,5,7,9,17,12,14,20,4,21]
    [1,3,6,7,9,17,12,14,20,4,21]
    [1,4,6,7,9,17,12,14,20,4,21]
    [1,3,5,8,9,17,12,14,20,4,21]
    [1,4,5,8,9,17,12,14,20,4,21]
    [1,3,6,8,9,17,12,14,20,4,21]
    [1,4,6,8,9,17,12,14,20,4,21]

    [1,3,5,7,10,18,12,14,20,4,21]
    [1,4,5,7,10,18,12,14,20,4,21]
    [1,3,6,7,10,18,12,14,20,4,21]
    [1,4,6,7,10,18,12,14,20,4,21]
    [1,3,5,8,10,18,12,14,20,4,21]
    [1,4,5,8,10,18,12,14,20,4,21]
    [1,3,6,8,10,18,12,14,20,4,21]
    [1,4,6,8,10,18,12,14,20,4,21] ];
```

For example, $L11(2, :) = [1,4,5,7,9,15,11,13,19,3,21]$ and $L11(s,:)$ contains the eleven indices needed to define the phase shifts

$$\beta_{11}(s)\omega_1 = \omega_1 \sum_{r \in L11(s,:)} \phi_r \quad (1 \leq s \leq 256). \tag{A1}$$

These summations are implemented in Matlab as follows. We define a 22-column vector called $phase = [\omega_1\phi_1 \ \omega_1\phi_2 \ \omega_1\phi_3 \ \dots \ \omega_1\phi_{22}]$ that contains all the phase shifts involved in the clockwise path of the gyro: $\phi_r = 0$ for r odd, $\phi_r = (n_r - n_{r-1})l_{\frac{r}{2}}/c$ for r even, $r = 1, 2, 3, \dots, 14$, and $r = 19, 20, 21, 22$. The four taus, τ_{11} , τ_{12} , τ_{21} , and τ_{22} representing the coil phase shifts receive the indices 15, 16, 17, and 18, respectively; that is $\phi_{15} = \tau_{11}$, $\phi_{16} = \tau_{12}$, $\phi_{17} = \tau_{21}$, and $\phi_{18} = \tau_{22}$. The values of all 256 $\beta_{ij}(s)$ ($1 \leq s \leq 256$) are contained in a 256-row vector called Bij ($i, j = 1, 2$). In order to speed up the program we use the vector/matrix operations available in Matlab. To evaluate the sums in Equation A1 for all s simultaneously we form the matrix

$$Phase_M = phase * ones(1, 256)$$

which is a 22×256 matrix whose columns are all equal to the 22-column vector $phase$. Then we call the function **Ph_arrCoij** and let

$$Lij = \mathbf{Ph_arrCoij}$$

so that Lij is an 11×256 array whose columns contain the indices $\Lambda_{ij}(s)$ ($1 \leq s \leq 256$). The statement

$$Phase_M(Lij)$$

produces an 11×256 array whose s -th column contains the phases ϕ_r for $r \in \Lambda_{ij}(s)$, $1 \leq s \leq 256$. Next, the statement

$$Bij = \text{sum}(Phase_M(Lij))$$

sums the elements of the columns of $Phase_M(Lij)$ to produce the 256-row vector Bij containing the values of $\beta_{ij}(s)$ defined by Equation A1.

Next, we describe the function **StepIntens** where the above statements appear. This function computes the output intensity of the gyroscope for several values of the lengths

l_4 and l_7 when g is a step function. The inputs to **StepIntens** are Del , l , e , a , b , t , tau , phi , $input$, wl , N , and M .

The differences (deltas) in the index of refraction of the two polarization modes of the PM fibers and IOCs are contained in the 7-vector Del ; $Del(i) = n_{2i} - n_{2i-1}$ for $i=1, 2, 3, \dots, 7$.

The 9-vector l contains the lengths of the PM fibers and the path lengths of the IOCs. The value of l_4 varies from 0 to $l(4)$ meters with increments of $l(4)/(N-1)$, where the input N is the number of values for l_4 . The value of l_7 varies from 0 to $l(7)$ meters with increments of $l(7)/(M-1)$, where the input M is the number of values for l_7 . The output of **StepIntens** is an $N \times M$ array containing the output intensities for the different values of l_4 and l_7 .

The 2-vector e contains the two polarization extinction ratios ϵ_1 and ϵ_2 . The 4-vector a contains the four miss-alignment angles $\alpha_1, \alpha_2, \alpha_3$, and α_4 in radians. The 2-vector b contains the miss-alignment angles β_1, β_2 in radians. The 4-vector t has the magnitudes of the entries of the coil matrix $t = [t_{11} \ t_{12} \ t_{21} \ t_{22}]$, the 4-vector tau has the four phase shifts due to the coil; $tau = [\tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}]$. The input phi is the Sagnac phase, the input $input$ is a 2-column unit vector defining the polarization strengths of the two orthogonal components of the electromagnetic field input, wl is the half width of the step function g and the integers N and M define the size of the output $N \times M$ matrix I .

With these inputs, the function **StepIntens** defines the 22-vector $phase$ described earlier, it computes the six cosines and sines of the six rotations: $c1, c2, c3, c4, c5, c6$, and $s1, s2, s3, s4, s5, c6$, calls the four functions **Coefij_arr** ($ij = 11, 12, 21, 22$) and the four functions **Ph_arrCoij** ($ij = 11, 12, 21, 22$) described above, and computes the coefficients $2|a_1|^2(1 + \cos\phi)$, $2|a_2|^2(1 + \cos\phi)$, $2\cos\phi$, and $2|a_1\bar{a}_2|(1 + \cos\phi)$ involved in Equation 3.10, where we have assumed that the phase $\alpha = 0$ (e.g. a_1 and a_2 real).

Next, there is a double loop where the output intensity is computed for the various values of the lengths l_4 and l_7 . In this loop, the four 256-row vectors $B11, B12, B21,$

B_{22} , are computed as described earlier. The differences $[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1$ are computed for all s and p and stored in the 256×256 matrix Bij_{uv} with the statement

$$Bij_{uv} = Bij' * \text{ones}(1,256) - \text{ones}(256,1) * Buv,$$

so that $Bij_{uv}(s, p) = Bij(s) - Buv(p) = [\beta_{ij}(s) - \beta_{uv}(p)]\omega_1$.

The sums

$$\sum_{s=1}^{256} \sum_{p=1}^{256} C_{ij}(s)C_{uv}(p) \frac{\sin[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1}{[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1} \quad (\text{A2})$$

are computed by a call to the function **StpSuvij** described next. These results are added up according to Equation 3.12 to give the output intensity when g is a step function.

The function **StpSuvij** computes the Sums A2. The inputs to this function are two arrays of coefficients C_{ij} and C_{uv} and a 256×256 array of differences Bij_{uv} . The first step is to find the indices (s, p) such that

$$[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1 = 0, \quad (\text{A3})$$

since for these indices $\frac{\sin[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1}{[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1} = 1$. This is accomplished with the first statement

$$[x, y] = \text{find}(Bij_{uv} == 0).$$

These indices are used in a loop to add the products $C_{ij}(s)C_{uv}(p)$ for all (s, p) such that Equation A3 hold. The statement

$$[i, j] = \text{find}(Bij_{uv});$$

gives all indices (s, p) for which Equation A3 does not hold. A second loop uses these indices to add the products $C_{ij}(s)C_{uv}(p) \frac{\sin[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1}{[\beta_{ij}(s) - \beta_{uv}(p)]\omega_1}$ to the previous sum.

The function **SagnacBias** computes the error in the Sagnac phase shift due to imperfect polarizers. It has two integer inputs N and M which have been described earlier. This function simply defines values for the inputs to the function **StepIntens** in order to compute $I_\epsilon(-\pi/2)$ then defines values for the inputs to the function **StepIntens** to compute $I_o(-\pi/2)$ and computes the Sagnac phase shift error ϕ_ϵ given by Equation 3.13. It then plots the error for the various values of l_4 and l_7 .

When the frequency distribution function g is a triangular function we use the Matlab function **PL_Intens** instead of **StepIntens**. The function **PL_Intens** is identical to **StepIntens** except that it calls the function **PL_Suvij** to compute the sums in Equation 3.18. Also, the phases in **PL_Intens** do not include the factor wl like they do in **StepIntens**. Thus, $Bij_{uv}(s, p) = Bij(s) - Buv(p) = \beta_{ij}(s) - \beta_{uv}(p)$, and $wBij_{uv}(s, p) = wl * Bij_{uv}(s, p) = [\beta_{ij}(s) - \beta_{uv}(p)] \omega_1$.

When the frequency distribution function g is a Gaussian function we use the Matlab function **GausIntens** instead of **StepIntens**. Again, this function is identical to **StepIntens** except that the computation of the sums in Equation 3.14 are taken care of by **GausIntens** with statements of the form

$$\text{sum}(\text{sum}(Cijuv.*\exp(-(Bij*\text{ones}(1,256)-\text{ones}(256,1))*Buv).^2))).$$

The phases in **GausIntens** do not include the factor wl , instead they include the factor $(\sigma/\sqrt{8})$. Thus, $Bij_{uv}(s, p) = Bij(s) - Buv(p) = (\sigma/\sqrt{8}) * [\beta_{ij}(s) - \beta_{uv}(p)]$ as required in Equation 3.14.

PROGRAM LISTINGS

```

function [Err,Bias,I,Io] = SagnacBias(N,M)

%N=10;
%M=10;

% Del=[vector of length 7; index of refraction differences; "Deltas"]; Del_1 = (n_2 - n_1).
% l=[waveguide lengths 1-7]=ones(1,7);
% e=[polarization extinction epsilons 1-2]=0.01*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% tau=[phase shifts due to coil, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a unit 2-column vector=[cos sin]';

z=9; % meters. 12m, 15m.

%Del=.00065*[0 1 .084/.00065 1 1 1 1];
Del=.00065*[0 1 .005/.00065 1 1 1 1];

%l=[.05 0 .05 15 2*z z 15]; % meters
l=[0 .2 .035 6 2*z z 6]; % meters; l(4) and l(7) will vary from -l(4) to l(4) and -l(7) to
% l(7), respectively.
e=[1 .001];
a=[.178 -.032 .032 -.032]; % radians
b=[.032 -.032]; % radians
t=1/sqrt(2)*[1 1 -1 1];
tau=1*[1 1 1 1];
phi=-pi/4;;
input=(1/sqrt(2))*[1 1]';
%input=[1 0]';
%sigma=1.67e+13; % 1/sec
sigma=1.654e+13; % 1/sec lambda_o=1.3e-6 meters, sigma_lambda=15e-9 meters
w1=2*sigma;
%w1=2*sigma/10;

incred_i=1/(N-1);
incred_j=1/(M-1);
i=[1:N];
j=[1:M];
y=l(4)*(i-1)*incred_i; % 0 <= r <= 1.
x=l(7)*(j-1)*incred_j; % 0 <= s <= 1.

tic

```

```

I=GausIntens(Del,l,e,a,b,t,tau,phi,input,w1,sigma,N,M);
% I=StepIntens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);
% I=PL_Intens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);

e=[1 0];
a=[0 0 .032 -.032];
b=[.032 -.032]; % radians;

Io=GausIntens(Del,l,e,a,b,t,tau,phi,input,w1,sigma,N,M);
% Io=StepIntens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);
% Io=PL_Intens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);

%save SagnacBias_10_15_3 I Io; % Io_Step has epsilon1=1 and epsilon2=0.
% Positive quadrant. w1=*/100, 50x50, (100x100=65.22 hours).
%save SagnacBias_10_23_3 I Io; % Positive quadrant. w1=*/100, 100x100, (50x50=16.4
hours).
toc

Err=I-Io;
Bias=asin(Err./Io);

figure(1);
imagesc(x,y,I); axis image;
figure(2);
imagesc(x,y,Io); axis image;
figure(3);
imagesc(x,y,Err); axis image;
figure(4);
imagesc(x,y,Bias); axis image;

```

```

function I = GausIntens(Del,l,e,a,b,t,tau,phi,input,w1,sigma,N,M)

% This is for Gaussian Function g.

% N=60;
% M=60;
% Del=[vector of length 7; index of refraction differences; "Deltas"];
% e.g. Del_1 = (n_2 - n_1).
% l=[waveguide lengths 1-9]=ones(1,9);
% e=[polarization extinction epsilons 1-2]=0.01*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% tau=[phase shifts due to coil T, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a 2-column vector=[1 1];

```

```

c=3.e+8; %m/sec
input=input/sqrt(input(1)^2+input(2)^2);

phase=zeros(22,1);
for i=1:7
    phase(2*i)=Del(i)*l(i)/c;
end
phase(15)=tau(1)/( sigma/sqrt(8) );
phase(16)=tau(2)/( sigma/sqrt(8) );
phase(17)=tau(3)/( sigma/sqrt(8) );
phase(18)=tau(4)/( sigma/sqrt(8) );

phase(19)=phase(5); % 0
phase(20)=phase(6); % Del(3)*l(8)/c;
phase(21)=phase(1); % 0
phase(22)=phase(2); % Del(1)*l(9)/c;

phase=( sigma/sqrt(8) )*phase; % (sigma/sqrt(8))*phase is only for Gaussian g.
% Phases 8 and 14 will vary.
A1=0; % or pi/2 ( 0 or 90 degrees).
A2=0; % or pi/2 ( 0 or 90 degrees).

c1=cos(a(1));
s1=sin(a(1));

c2=cos(a(2));
s2=sin(a(2));

c3=cos(A1+a(3));
s3=sin(A1+a(3));

c4=cos(pi/4+b(1));
s4=sin(pi/4+b(1));

c5=cos(pi/4+b(2));
s5=sin(pi/4+b(2));

c6=cos(A2+a(4));
s6=sin(A2+a(4));

C11=Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C12=Coef12_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C21=Coef21_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C22=Coef22_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);

C1111=C11*C11';
C2222=C22*C22';

```

```

C1212=C12*C12';
C2121=C21*C21';
C1221=C12*C21';
C1112=C11*C12';
C1121=C11*C21';
C2122=C21*C22';
C2212=C22*C12';

L11=Ph_arrCo11';
L12=Ph_arrCo12';
L21=Ph_arrCo21';
L22=Ph_arrCo22';

Err=zeros(N,M);
Rel_Err=Err;
I=Err;

a1a1=input(1)^2;
a2a2=input(2)^2;
cosphi2=2*cos(phi);
cosphi_2=2+2*cos(phi);
a1a1cosphi_2=a1a1*cosphi_2;
a2a2cosphi_2=a2a2*cosphi_2;
a1a2cosphi_2=abs(input(1)*input(2))*cosphi_2;

incred_i=1/(N-1);
incred_j=1/(M-1);

for i=1:N
    for j=1:M
        r=(i-1)*incred_i; % 0 <= r <= 1.
        s=(j-1)*incred_j; % 0 <= s <= 1.

        phase(8)= ( ( sigma/sqrt(8) )/c ) * l(4) * Del(4) * (2*r-1); % To go from -l(4) to l(4).
        phase(14)= ( ( sigma/sqrt(8) )/c ) * l(7) * Del(7) * (2*s-1); % To go from -l(7) to l(7).

        %phase(8)= ( ( sigma/sqrt(8) )/c ) * l(4)/2 * Del(4) * (1+r); % To go from l(4)/2 to l(4).
        %phase(14)= ( ( sigma/sqrt(8) )/c ) * l(7)/2 * Del(7) * (1+s); % To go from l(7)/2 to l(7).

        Phase_M=phase*ones(1,256);

        B11=sum(Phase_M(L11));
        B12=sum(Phase_M(L12));
        B21=sum(Phase_M(L21));
        B22=sum(Phase_M(L22));

        P1=a1a1cosphi_2*...
            sum(sum(C1111.*exp(-(B11'*ones(1,256)-ones(256,1)*B11).^2)));

```

```

P2=a2a2cosphi_2*...
    sum(sum(C2222.*exp(-(B22*ones(1,256)-ones(256,1)*B22).^2)));
P3=sum(sum(C1212.*exp(-(B12*ones(1,256)-ones(256,1)*B12).^2)));
P4=sum(sum(C2121.*exp(-(B21*ones(1,256)-ones(256,1)*B21).^2)));
P5=cosphi2*...
    sum(sum(C1221.*exp(-(B12*ones(1,256)-ones(256,1)*B21).^2)));
P6=a1a2cosphi_2*...
    sum(sum(C1112.*exp(-(B11*ones(1,256)-ones(256,1)*B12).^2)));
P7=a1a2cosphi_2*...
    sum(sum(C1121.*exp(-(B11*ones(1,256)-ones(256,1)*B21).^2)));
P8=a1a2cosphi_2*...
    sum(sum(C2122.*exp(-(B21*ones(1,256)-ones(256,1)*B22).^2)));
P9=a1a2cosphi_2*...
    sum(sum(C2212.*exp(-(B22*ones(1,256)-ones(256,1)*B12).^2)));

I(i,j)=P1+P2+P3+P4+P5+P6+P7+P8+P9;

    end
end

I=(sqrt(2*pi)/sigma)*I;

```

```

function I = StepIntens(Del,l,e,a,b,t,tau,phi,input,w1,N,M)

```

```

% This is for a Step Function g.

```

```

% N=60;
% M=60;
% Del=[vector of length 7; index of refraction differences; "Deltas"];
% e.g. Del_1 = (n_2 - n_1).
% l=[waveguide lengths 1-9]=ones(1,9);
% e=[polarization extinction epsilons 1 and 2]=0.001*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% tau=[phase shifts due to coil, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a 2-column unit vector=[cos sin]';

```

```

c=3.e+8; % m/sec

```

```

input=input/sqrt(input(1)^2+input(2)^2);

```

```

phase=zeros(22,1);

```

```

for i=1:7

```

```

    phase(2*i)=(w1/c)*Del(i)*l(i); % w1*phase is only for Step Function g.

```

```

end

```

```

phase(15)=tau(1);

```

```

phase(16)=tau(2);
phase(17)=tau(3);
phase(18)=tau(4);

```

```

phase(19)=phase(5); % 0
phase(20)=phase(6); % ( w1/c )*Del(3)*l(8);
phase(21)=phase(1); % 0
phase(22)=phase(2); % ( w1/c )*Del(1)*l(9);

```

```

% phases 8 and 14 will vary.

```

```

A1=0; % or pi/2 ( 0 or 90 degrees).
A2=0; % or pi/2 ( 0 or 90 degrees).

```

```

c1=cos(a(1));
s1=sin(a(1));

```

```

c2=cos(a(2));
s2=sin(a(2));

```

```

c3=cos(A1+a(3));
s3=sin(A1+a(3));

```

```

c4=cos(pi/4+b(1));
s4=sin(pi/4+b(1));

```

```

c5=cos(pi/4+b(2));
s5=sin(pi/4+b(2));

```

```

c6=cos(A2+a(4));
s6=sin(A2+a(4));

```

```

C11=Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C12=Coef12_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C21=Coef21_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C22=Coef22_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);

```

```

C1111=C11*C11';
C2222=C22*C22';
C1212=C12*C12';
C2121=C21*C21';
C1221=C12*C21';
C1112=C11*C12';
C1121=C11*C21';
C2122=C21*C22';
C2212=C22*C12';

```

```

L11=Ph_arrCo11';

```

```

L12=Ph_arrCo12';
L21=Ph_arrCo21';
L22=Ph_arrCo22';

Err=zeros(N,M);
Rel_Err=Err;
I=Err;

a1a1=input(1)^2;
a2a2=input(2)^2;
cosphi2=2*cos(phi);
cosphi_2=2+2*cos(phi);
a1a1cosphi_2=a1a1*cosphi_2;
a2a2cosphi_2=a2a2*cosphi_2;
a1a2cosphi_2=abs(input(1)*input(2))*cosphi_2;

incred_i=1/(N-1);
incred_j=1/(M-1);

for i=1:N
    for j=1:M
        r=(i-1)*incred_i; % 0 <= r <= 1.
        s=(j-1)*incred_j; % 0 <= s <= 1.

        %phase(8)=w1*l(4)*(2*r-1)*Del(4)/c; % To go from -l(4) to l(4).
        %phase(14)=w1*l(7)*(2*s-1)*Del(7)/c; % To go from -l(7) to l(7).

        %phase(8)=w1*l(4)*(r-1)*Del(4)/c; % To go from -l(4) to 0.
        %phase(14)=w1*l(7)*(s-1)*Del(7)/c; % To go from -l(7) to 0.

        phase(8)=(w1/c)*Del(4)*l(4)*r; % To go from 0 to l(4).
        phase(14)=(w1/c)*Del(7)*l(7)*s; % To go from 0 to l(7).

        %phase(8)= (l(4)/2)*(1+r)*Del(4)/c; % To go from l(4)/2 to l(4).
        %phase(14)= (l(7)/2)*(1+s)*Del(7)/c; % To go from l(7)/2 to l(7).

    Phase_M=phase*ones(1,256);

    B11=sum(Phase_M(L11));
    B12=sum(Phase_M(L12));
    B21=sum(Phase_M(L21));
    B22=sum(Phase_M(L22));
    %BA=sum(Phase_M(LA));

    B11_11=B11*ones(1,256)-ones(256,1)*B11;
    P1=a1a1cosphi_2*StpSuvij(C1111,B11_11);

    B22_22=B22*ones(1,256)-ones(256,1)*B22;

```

```

P2=a2a2cosphi_2*StpSuvij(C2222,B22_22);

B12_12=B12'*ones(1,256)-ones(256,1)*B12;
P3=StpSuvij(C1212,B12_12);

B21_21=B21'*ones(1,256)-ones(256,1)*B21;
P4=StpSuvij(C2121,B21_21);

B12_21=B12'*ones(1,256)-ones(256,1)*B21;
P5=cosphi2*StpSuvij(C1221,B12_21);

B11_12=B11'*ones(1,256)-ones(256,1)*B12;
P6=a1a2cosphi_2*StpSuvij(C1112,B11_12);

B11_21=B11'*ones(1,256)-ones(256,1)*B21;
P7=a1a2cosphi_2*StpSuvij(C1121,B11_21);

B21_22=B21'*ones(1,256)-ones(256,1)*B22;
P8=a1a2cosphi_2*StpSuvij(C2122,B21_22);

B22_12=B22'*ones(1,256)-ones(256,1)*B12;
P9=a1a2cosphi_2*StpSuvij(C2212,B22_12);

%sum(sum(Cij*Cuv'.*exp(-(Bij*ones(1,256)-ones(256,1)*Buv).^2)));

I(i,j)=P1+P2+P3+P4+P5+P6+P7+P8+P9;

end
end

I=(pi/(2*w1))*I;

```

```

function SS = StpSuvij(Cijuv,Buv_ij)

```

```

% Buv_ij may be Buv-Bij or Buv+Bij. Buv_ij has been multiplied by w1 already.

```

```

SS=0;

```

```

[x,y]=find(Buv_ij == 0);

```

```

for s=1:size(x)
    SS=SS+Cijuv(x(s),y(s));
end

```

```

[i,j]=find(Buv_ij);

```

```

for s=1:size(i)

```

```

SS=SS+Cijuv(i(s),j(s))*( sin( Buv_ij(i(s),j(s))) / Buv_ij(i(s),j(s)) );
end

```

```

function I=PL_Intens(Del,l,e,a,b,t,tau,phi,input,w1,N,M)

```

```

% This is for Triangular Function g.

```

```

%N=60;

```

```

%M=60;

```

```

% Del=[vector of length 7; index of refraction differences; "Deltas"];

```

```

% e.g. Del_1 = (n_2 - n_1).

```

```

% l=[waveguide lengths 1-9]=ones(1,9);

```

```

% e=[polarization extinction epsilons 1-2]=0.01*[1 1];

```

```

% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];

```

```

% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];

```

```

% t=magnitud of entries in the coil matrix, t=[1 1 1 1];

```

```

% tau=[phase shifts due to coil T, 1-4]=[1 1 1 1];

```

```

% phi=Sagnac phase

```

```

% input = a 2-column vector=[1 1]';

```

```

c=3.e+8; %m/sec

```

```

input=input/sqrt(input(1)^2+input(2)^2);

```

```

phase=zeros(22,1);

```

```

for i=1:7

```

```

    phase(2*i)=Del(i)*l(i)/c;

```

```

end

```

```

phase(15)=tau(1)/w1; % tau/w1 is only for PL function g.

```

```

phase(16)=tau(2)/w1;

```

```

phase(17)=tau(3)/w1;

```

```

phase(18)=tau(4)/w1;

```

```

phase(19)=phase(5); % 0

```

```

phase(20)=phase(6); % Del(3)*l(8)/c;

```

```

phase(21)=phase(1); % 0

```

```

phase(22)=phase(2); % Del(1)*l(9)/c;

```

```

% phases 8 and 14 will vary.

```

```

A1=0; % or pi/2 ( 0 or 90 degrees).

```

```

A2=0; % or pi/2 ( 0 or 90 degrees).

```

```

c1=cos(a(1));

```

```

s1=sin(a(1));

```

```

c2=cos(a(2));

```

```
s2=sin(a(2));
```

```
c3=cos(A1+a(3));
```

```
s3=sin(A1+a(3));
```

```
c4=cos(pi/4+b(1));
```

```
s4=sin(pi/4+b(1));
```

```
c5=cos(pi/4+b(2));
```

```
s5=sin(pi/4+b(2));
```

```
c6=cos(A2+a(4));
```

```
s6=sin(A2+a(4));
```

```
C11=Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
```

```
C12=Coef12_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
```

```
C21=Coef21_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
```

```
C22=Coef22_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
```

```
C1111=C11*C11';
```

```
C2222=C22*C22';
```

```
C1212=C12*C12';
```

```
C2121=C21*C21';
```

```
C1221=C12*C21';
```

```
C1112=C11*C12';
```

```
C1121=C11*C21';
```

```
C2122=C21*C22';
```

```
C2212=C22*C12';
```

```
L11=Ph_arrCo11';
```

```
L12=Ph_arrCo12';
```

```
L21=Ph_arrCo21';
```

```
L22=Ph_arrCo22';
```

```
Err=zeros(N,M);
```

```
Rel_Err=Err;
```

```
I=Err;
```

```
a1a1=input(1)^2;
```

```
a2a2=input(2)^2;
```

```
cosphi2=2*cos(phi);
```

```
cosphi_2=2+2*cos(phi);
```

```
a1a1cosphi_2=a1a1*cosphi_2;
```

```
a2a2cosphi_2=a2a2*cosphi_2;
```

```
a1a2cosphi_2=abs(input(1)*input(2))*cosphi_2;
```

```
inrem_i=1/(N-1);
```

```
inrem_j=1/(M-1);
```

```

for i=1:N
  for j=1:M
    r=(i-1)*incred_i; % 0 <= r <= 1.
    s=(j-1)*incred_j; % 0 <= s <= 1.

    phase(8)=l(4)*(2*r-1)*Del(4)/c; % To go from -l(4) to l(4).
    phase(14)=l(7)*(2*s-1)*Del(7)/c; % To go from -l(7) to l(7).

    %phase(8)=(l(4)/2)*(1+r)*Del(4)/c; % To go from l(4)/2 to l(4).
    %phase(14)=(l(7)/2)*(1+s)*Del(7)/c; % To go from l(7)/2 to l(7).

    Phase_M=phase*ones(1,256);

    B11=sum(Phase_M(L11));
    B12=sum(Phase_M(L12));
    B21=sum(Phase_M(L21));
    B22=sum(Phase_M(L22));

    B11_11=B11*ones(1,256)-ones(256,1)*B11;
    wB11_11=w1*B11_11;
    P1=a1a1cosphi_2*PL_Suvij(w1,C1111,B11_11,wB11_11);

    B22_22=B22*ones(1,256)-ones(256,1)*B22;
    wB22_22=w1*B22_22;
    P2=a2a2cosphi_2*PL_Suvij(w1,C2222,B22_22,wB22_22);

    B12_12=B12*ones(1,256)-ones(256,1)*B12;
    wB12_12=w1*B12_12;
    P3=PL_Suvij(w1,C1212,B12_12,wB12_12);

    B21_21=B21*ones(1,256)-ones(256,1)*B21;
    wB21_21=w1*B21_21;
    P4=PL_Suvij(w1,C2121,B21_21,wB21_21);

    B12_21=B12*ones(1,256)-ones(256,1)*B21;
    wB12_21=w1*B12_21;
    P5=cosphi2*PL_Suvij(w1,C1221,B12_21,wB12_21);

    B11_12=B11*ones(1,256)-ones(256,1)*B12;
    wB11_12=w1*B11_12;
    P6=a1a2cosphi_2*PL_Suvij(w1,C1112,B11_12,wB11_12);

    B11_21=B11*ones(1,256)-ones(256,1)*B21;
    wB11_21=w1*B11_21;
    P7=a1a2cosphi_2*PL_Suvij(w1,C1121,B11_21,wB11_21);

    B21_22=B21*ones(1,256)-ones(256,1)*B22;

```

```
wB21_22=w1*B21_22;
P8=a1a2cosphi_2*PL_Suvij(w1,C2122,B21_22,wB21_22);
```

```
B22_12=B22*ones(1,256)-ones(256,1)*B12;
wB22_12=w1*B22_12;
P9=a1a2cosphi_2*PL_Suvij(w1,C2212,B22_12,wB22_12);
```

```
I(i,j)=P1+P2+P3+P4+P5+P6+P7+P8+P9;
```

```
end
end
```

```
I=2*pi*I/w1^2;
```

```
function SS = PL_Suvij(w1,Cijuv,Buv_ij,wBuv_ij)
```

```
w1_x23=2*(w1/3);
w1_4=4/w1;
SS=0;
```

```
[x,y]=find(Buv_ij == 0);
```

```
for s=1:size(x)
    SS=SS+w1_x23*Cijuv(x(s),y(s));
end
```

```
[i,j]=find(Buv_ij);
```

```
for s=1:size(i)
    SS=SS+w1_4*(Cijuv(i(s),j(s))/Buv_ij(i(s),j(s))^2)*...
    (1-sin(wBuv_ij(i(s),j(s)))/(wBuv_ij(i(s),j(s))));
end
```

INITIAL DISTRIBUTION

- 1 Naval War College, Newport, RI (1E22, President)
- 1 Headquarters, 497 INOT, Falls Church, VA (IOG Representative)
- 2 Defense Technical Information Center, Fort Belvoir, VA
- 1 Center for Naval Analyses, Alexandria, VA (Technical Library)

ON-SITE DISTRIBUTION

- 3 Code 4TL000D (2 plus Archives copy)
- 22 Code 4T4100D
 - Chesnut, S. (1)
 - Martin, J. (21)