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Daniel J. Gauthier
Associate Professor

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ROBUST CONTROL AND SYNCHRONIZATION OF CHAOS

Final Report

Daniel J. Gauthier

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1. Forward

In many cases of practical importance, chaotic instabilities limit the performance of devices, such as lasers and electronic oscillators, and hence it is desirable to devise schemes to control the instabilities. During the period of this program, we have developed novel algorithms that efficiently control and synchronize chaotic dynamics of high-speed and high-dimensional systems using small perturbations. We determined the sensitivity of the schemes to noise and slow variation in the system parameters and found that robust controllers and synchronizers can be constructed, but care must be applied in their design: in some situations, dynamical systems can amplify noise by extreme amounts even under conditions when the system is stable. The research has uncovered several fundamental issues related to the control and synchronization of nonlinear systems, which we anticipate will lead to improved performance of devices that are based on nonlinear systems and to the development of chaos-based communication systems, for example. In addition, we have compared experimental observations with detailed theoretical models of the nonlinear dynamical systems to further enhance our understanding of the control and synchronization processes.

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4. Statement of the problem

One major area of research of the nonlinear dynamics community is the investigation of novel techniques for controlling and synchronizing the dynamics of systems using only small perturbations. The goals of this project include improving the performance of devices whose behavior is degraded by instabilities and chaos, and developing new devices that take advantage of the unique capabilities of the control and synchronization schemes.

The primary purpose of the proposed research was to test experimentally the conditions under which extreme amplification of noise occurs in controlled and synchronized chaotic systems. Simple moderate-dimension electronic and optical chaotic devices were designed and constructed, their stability was be characterized, accurate mathematical models of the devices were devised and analyzed, and a detailed comparison between experimental observations and theoretical predictions were undertaken. One outcome of this research is the identification of generic design procedures for the robust control and synchronization of chaotic systems. Hence, the research is not specific to the particular electronic and optical devices investigated as part of this program; rather, it should impact our understanding of stability of a wide class of dynamical systems.

5. Summary of the most important results

5.1. Synchronization of hyperchaotic oscillators

During this research period, we completed our analysis of our experiments on synchronizing coupled hyperchaotic oscillators. We were invited to describe our observations and analyses in a special Focus Issue on Chaotic Control and Synchronization in the International Journal of Bifurcations and Chaos. In our experiments, we investigated attractor bubbling in a system of two coupled hyperchaotic electronic circuits. The degree of synchronization over a range of coupling strengths for two different coupling schemes was measured to identify bubbling. The circuits displayed regimes of both attractor bubbling and high-quality synchronization. For the coupling scheme where high-quality synchronization was observed, the transition to bubbling is “soft” and its scaling with coupling strength near the transition point does not fit into the known categories of transition types. We also compared the observed behavior to several proposed criteria for estimating the regime of high-quality synchronization. It is found that none of these methods is completely satisfactory for predicting accurately the regimes of attractor bubbling and high-quality synchronization.

As an example of our experimental finding, we show the quality of synchronization for a particular choice of a coupling scheme of the oscillators. Figure 5.1 shows the experimentally observed degree of synchronization and the numerically determined largest transverse Lyapunov exponent. From Fig. 5.1(b), it is seen that the synchronization manifold is asymptotically stable for all coupling strengths larger than 0.26 since the exponent crosses zero at this point and remains negative for larger coupling strengths. Experimentally, we observe a small region of attractor bubbling occurring for the range of coupling strengths between 0.25 and 0.32. No desynchronization events greater than the noise level ($|\mathbf{x}_\perp(t)|_{MAX}^2 < 1.0$

V^2 or 0.5% of the maximum possible value of $|\mathbf{x}_\perp(t)|^2$ on the attractor) are observed for coupling strengths greater than 0.32. Thus, there is a large range of coupling strengths where high-quality synchronization can be achieved despite the hyperchaotic nature of the system. Our observations are consistent with those of previous researchers who found that the transition to high-quality synchronization for this coupling scheme occurred for a coupling strength slightly higher than that expected based on the negativity of the transverse Lyapunov exponents.

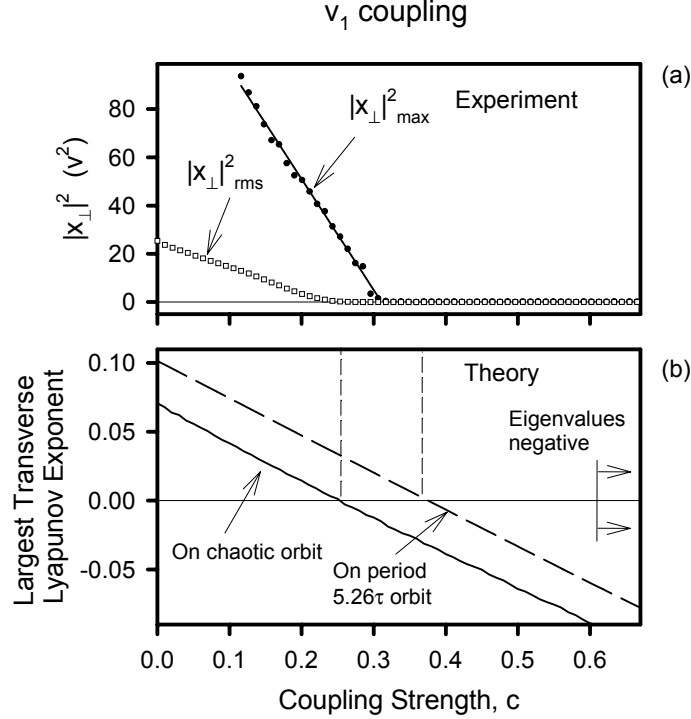


Figure 5.1: (a) Quality of synchronization as a function of coupling strength for coupled hyperchaotic oscillators. (b) Theoretical measure of the stability of the synchronized state as a function of the coupling strength. The systems should be synchronized when the curves pass below zero.

Additional information concerning the bubbling transition can be obtained by observing the scaling of $|\mathbf{x}_\perp(t)|^2_{MAX}$ in the vicinity of the transition. Venkataramani *et al.* predict that the transition can be “hard” (the bursts appear abruptly with large amplitude as the coupling strength is varied) or “soft” (the maximum burst amplitude increases continuously from zero), and that the symmetry of the coupling has a fundamental effect on the transition. From Fig. 5.1(a), it is seen that the transition is soft. For such a transition and the one-way (asymmetric) coupling scheme used in our experiment, they predict that $|\mathbf{x}_\perp(t)|^2_{MAX} \sim (c_b - c)^2$, where c_b is the coupling strength at which the transition occurs. On the contrary, we observe that $|\mathbf{x}_\perp(t)|^2_{MAX} \sim (c_b - c)$, as illustrated by the solid straight line shown in the figure. The fit between our data and the straight line is good in that the deviation from

the straight line is comparable to the observed data point-to-data point variation, which is a reasonable estimate of our experimental error. Our observation indicates the existence of a new type of bubbling transition, or that the lowest-order nonlinear contribution to the transverse dynamics has an even symmetry even though the coupling has an odd symmetry.

5.2. Synchronization of periodic oscillators

In addition to investigating synchronization of chaotic oscillators, we have also conducted two experiments in which we investigated the synchronization of coupled *periodic* oscillators. Each experimental system consisted of two identical coupled electronic periodic oscillators that display bursts of desynchronization events similar to those observed previously in coupled chaotic systems. We measured the degree of synchronization as a function of coupling strength. In the first experiment, high-quality synchronization is achieved for all coupling strengths above a critical value. In the second experiment, no high-quality synchronization is observed. We compared our results to the predictions of the several proposed criteria for synchronization. We find that none of the criteria accurately predict the range of coupling strengths over which high-quality synchronization is observed. The paper describing our research appeared in *Chaos*.

As an example of our results from one of the experiments, the dynamical behavior of a single periodic oscillator in the absence of noise is shown in Fig. 5.2(a) (solid line). A brief, sufficiently large perturbation to the system when the trajectory is in the vicinity of $V = -V_o/G$ (dashed line) causes it to undergo a large excursion away from the orbit before returning, as shown by the dotted line in Fig. 5.2(a). Once the trajectory crosses the threshold, the growth rate of the perturbation is very large. Thus, a perturbation during the brief interval when the trajectory is in the neighborhood of the threshold can be amplified significantly. This behavior resembles the bursting observed in coupled chaotic double scroll oscillators evolving near the saddle point at the origin of their phase space, as discussed previously.

In the experiment, the slave oscillator is coupled to the master by injecting a current $I_{sync} = \gamma C(V_m - V_s)$ into the slave circuit at the same node as the drive signal. We bias both oscillators very close to the threshold. For this bias value and the inherent level of noise in the system, the master oscillator never crosses the threshold and remains on the trajectory shown as the solid line in Fig. 5.2(a). A small Gaussian white noise current (bandwidth from 10 Hz to 1 kHz, RMS current $\sim 0.5\%$ of the characteristic oscillator current) is injected into the slave oscillator. When there is no coupling ($\gamma = 0$), the slave occasionally crosses the threshold and bursts away from the periodic orbit. For the oscillators to be synchronized, the coupling has to be chosen so that the slave never undergoes a burst.

For each of several different values of the coupling strength, we record a long time series of the Euclidean norm $|x_\perp| = |V_m - V_s|$. To quantify the degree of synchronization, we determine from these time series the average distance from the synchronization manifold $|x_\perp|_{\text{rms}}$ and the maximum observed value of the distance from the manifold $|x_\perp|_{\text{max}}$ for each coupling strength, as shown in Fig. 5.2(b). For coupling strengths between 0.6 and $0.8 \times 10^4 \text{ s}^{-1}$, $|x_\perp|_{\text{max}}$ is on the order of the size of the orbit ($\sim 2 \text{ V}$) even though $|x_\perp|_{\text{rms}}$ is very small ($\sim 1\%$ of the orbit size), implying that there exist large, brief, occasional desynchronization

events even when the oscillators are synchronized on average. From the figure, it is seen that the large desynchronization events only cease for $\gamma \gtrsim 1.3 \times 10^4 \text{ s}^{-1}$, as indicated by the large drop in $|x_{\perp}|_{\max}$.

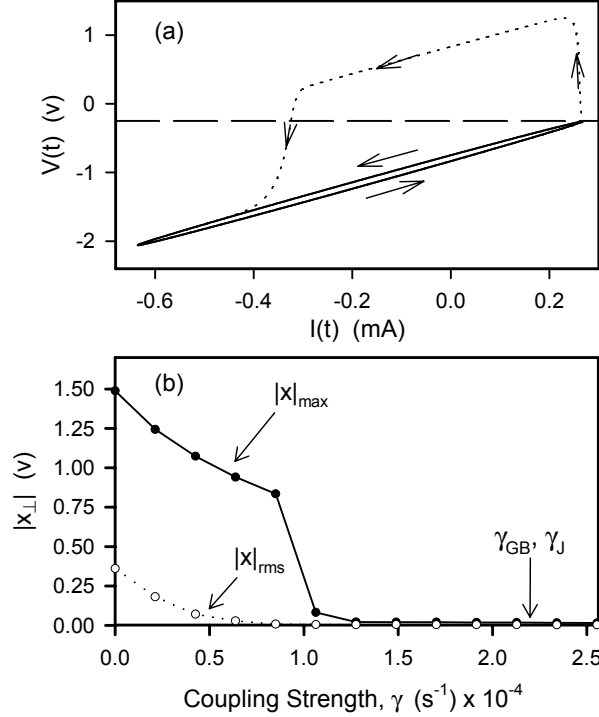


Figure 5.2: Dynamics of coupled periodic oscillators. (a) Phase-space portrait. (b) Quality of synchronization as a function of the coupling strength between the oscillators.

Our observations cannot be explained by any of the current theories, strongly suggesting that our understanding of the synchronization process is incomplete.

5.3. A new source of ultra-high-speed optical chaos

During the period of this grant, we have developed an ultra-high speed chaotic laser system that has been used at a test-bed to investigate the fundamental aspects of chaos control and synchronization on fast time scales, which will serve as the building block of a chaotic communication system. It consists of a semiconductor laser with incoherent feedback.

In this device, shown schematically in Fig. 5.3, light generated by the laser (any commercial semiconductor laser will work in this application) is sent to an unequal-path Mach-Zehnder interferometer. The interferometer converts variations in the frequency of the laser (which is sensitive to the injection current) into variations in the intensity of the light. These intensity variations are detected by a silicon photodiode, converted to a voltage, amplified, delayed in time by an amount τ using a transmission line, and summed with the dc injection current i_{DC} using a bias-T.

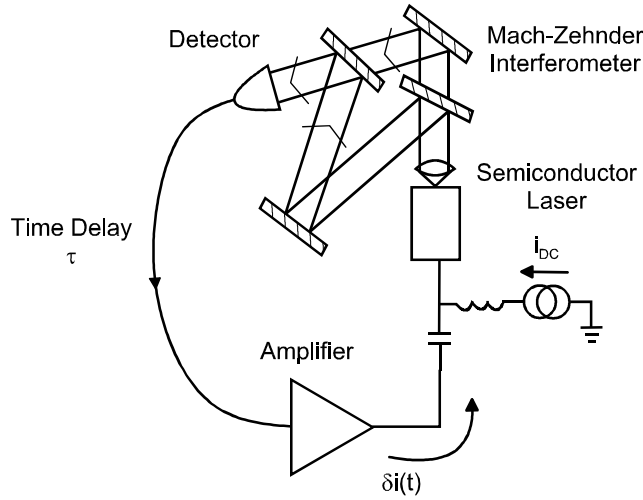


Figure 5.3: Experimental setup of the new source of optical chaos consisting of a semiconductor laser in the presence of incoherent feedback.

For very low amplifier gain, the laser operates in a stable fashion. As the gain is increased, the dynamics switch to a periodic pattern at a well defined and reproducible value of the gain, as shown in Fig. 5.4a. The corresponding power spectrum of the oscillations is shown in Fig. 5.4b. Note that the oscillations are related to an oscillation in the *frequency* of the laser and not the laser intensity since we detect that output of the Mach-Zehnder interferometer. The observation of a transition from a steady to a periodic dynamical pattern indicates that the laser system has passed through a Hopf bifurcation.

For sufficiently high amplifier gain, this new device displays chaotic dynamics whose time scale is of the order of 3 ns, as shown in Fig. 5.4c and 5.4d. We find that the complexity of the oscillations can be controlled by adjusting the time delay. The time scale of the dynamics is also controllable by a bandwidth-limiting filter contained in the loop and can be as small as a ~ 300 ps. We have conducted a preliminary characterization of this new dynamical system. Future plans are to control the ultra-high-speed dynamics using controlling chaos methods, encoding messages within the chaotic dynamics, and synchronizing two chaotic lasers.

We have also developed a detailed mathematical model of an ultra-high speed chaotic laser system in collaboration with Mr. Ilan Harrington of Prof. Socolar's theoretical nonlinear dynamics group. The model development starts from the first principles of semiconductor laser theory and accounts for all pertinent details of the experiment. Parameters appearing in this model, such as the semiconductor linewidth enhancement parameter and decay rates, for example, have been determined by auxiliary experiments that are sensitive to only one or a few of the parameters. We find very good agreement between theoretical simulations of the model and the experimental observations.

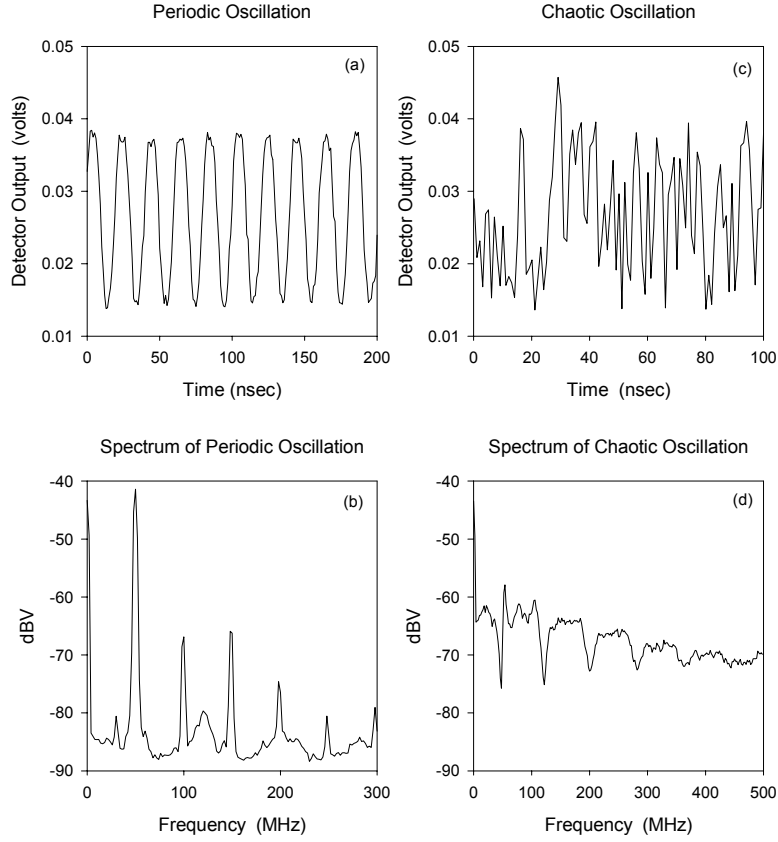


Figure 5.4: Dynamical behavior of the new source of chaotic optical frequency fluctuations. For low amplifier gain, the frequency of the laser undergoes periodic oscillations as seen in the (a) time domain and (b) frequency domain. For higher amplifier gain, the system display chaotic fluctuation in the frequency of the light generated by the laser as seen in the (c) time domain and (d) frequency domain.

5.4. An array of couple chaotic electronic elements

We have also developed an array of coupled chaotic electronic circuits, a paradigm of a relatively simple system displaying spatiotemporal complexity. The individual circuit elements operate in the chaotic regime characterized by a single positive Lyapunov exponent; the chaotic attractor is shown in Fig. 5.5.

The circuits are based on a design by Rulkov because the strange attractor characterizing this system contains unstable sets that are capable of causing extreme amplification of perturbations and we have found that it is easier to match the components of this system. We constructed 64 oscillators and have coupled them in various configurations. The dynamics of all three variables for each oscillator can be recorded simultaneously and the deviations away from the desired state have been determined. We have conducted preliminary experiments to ascertain the robustness of the control for increasing number of oscillators (from 1 to 64)

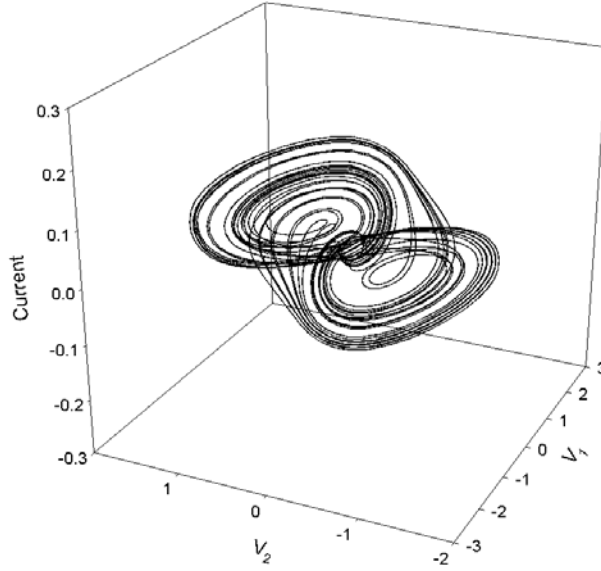


Figure 5.5: Chaotic attractor characterizing a single element in the array of coupled chaotic oscillators.

for various amounts of white noise injected into the collection of oscillators.

5.5. Wide-aperture nonlinear optical resonator

To fully explore the control and synchronization of very complex dynamics, bordering on the state of turbulence found in some fluid flows, for example, we are investigating the behavior of a wide-aperture nonlinear optical resonator. The device is based on a nonlinear optical element known as a liquid crystal light valve (LCLV) that is placed in a simple optical feedback configuration. LCLV's are optically addressable spatial light modulators designed for state-of-the-art display applications; our application uses a LCLV whose reflection characteristics depend on the intensity of light illuminating each pixel. A time sequence of the observed dynamics is shown in Fig. 5.6. The goal of the project is to determine whether the complex spatio-temporal dynamics can be controlled over the entire aperture by measuring and feeding back to a single spatial location.

We have developed a detailed mathematical model for the dynamics observed in the nonlinear resonator and performed experiments to determine all of the model parameters. In addition, we have set up a two-axis acousto-optic laser beam deflector that will be used to deliver control perturbations to the system at any desired location within the field of view. In the future, we intend to control the dynamics of the nonlinear resonator and determine the minimum number and pattern of spots that must be adjusted to effect control.

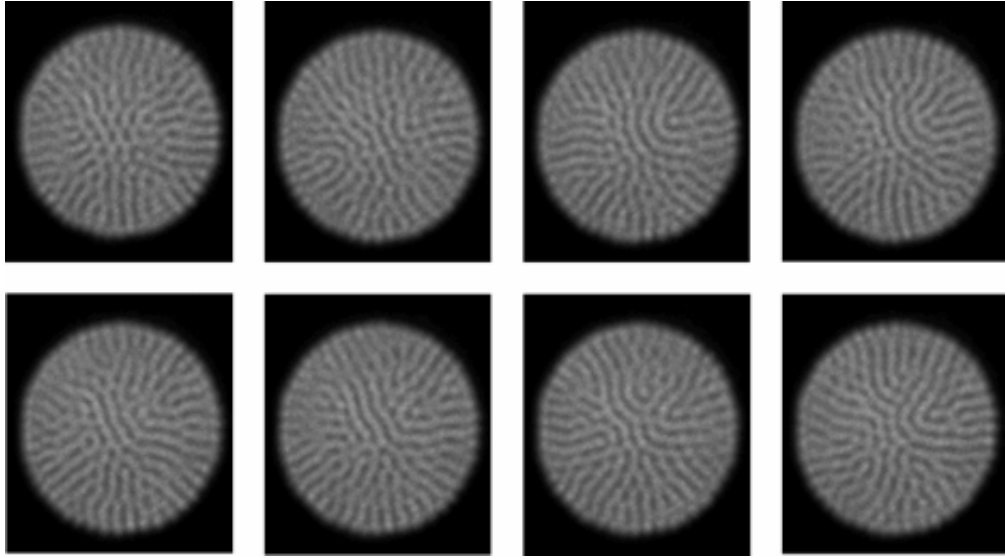


Figure 5.6: A temporal sequence of the observed intensity of light circulating in the broad-area nonlinear optical resonator.

6. List of Publications

6.1. Papers published in peer-reviewed journals

J.N. Blakely, D.J. Gauthier, G. Johnson, T.L. Carroll, and L.M. Pecora, ‘Experimental investigation of high-quality synchronization of coupled oscillators,’ *Chaos* **10**, 738 (2000).

J.N. Blakely and D.J. Gauthier, ‘Attractor bubbling in coupled hyperchaotic oscillators,’ an Invited article in the Theme Issue of Control and Synchronization of Chaos, *Int. J. Bifurcation Chaos* **10**, 835 (2000).

D.W. Sukow and D.J. Gauthier, ‘Entraining power-dropout events in an external cavity semiconductor laser using weak modulation of the injection current,’ *IEEE J. Quantum Electron.* **QE-36**, 175 (2000).

6.2. Papers published in non-peer reviewed journals or in conference proceedings

None to report.

6.3. Papers presented at meetings, but not published in conference proceedings

* denotes invited presentation

D.J. Gauthier, * ‘A Flexible Source of Optical Chaos for Use in Communications,’ Center for Engineering Science Advanced Research, Advances in Computational Nonlinear Science Seminar, Oak Ridge National Laboratory, Oak Ridge, TN, May 1, 2001.

D.J. Gauthier, 'A Flexible Source of Optical Chaos for Use in Communications,' Weapons Sciences Directorate, AMSAM-RD-WS-ST Missile Research, Development and Engineering Center, U. S. Army Aviation and Missile Command, Redstone Arsenal, AL, April 23, 2001.

J.N. Blakely, 'Flexible Source of Optical Chaos for Use in Communications,' Dynamics Days 2001, Chapel Hill, NC, January 4, 2001.

J.N. Blakely, 'Flexible Source of Optical Chaos for Use in Communications,' OPTO-Southeast Meeting on Optoelectronics, Photonics, and Imaging, Charlotte, North Carolina, September 19, 2000.

D.J. Gauthier, * 'Delay-Induced Instabilities in Semiconductor Lasers,' IEEE Colloquium and Nonlinear Systems Program Seminar, School of Electrical Engineering, Cornell University, Ithaca, NY, February 1, 2000.

J.N. Blakely, 'Observation of a New Scaling Relation in the Transition from Synchronized Chaos to Attractor Bubbling,' 66th Annual Meeting Southeastern Section Meeting of the APS, Chapel Hill, NC, November 8, 1999.

J.N. Blakely, 'Experimental Evaluation of Several Proposed Criteria for Synchronization,' 5th SIAM Conference on Applications of Dynamical Systems, May 23, 1999, Snowbird, UT.

6.4. Manuscripts submitted, but not published

None to report.

6.5. Technical reports submitted to ARO

This research was discussed in the US ARO Research Highlights 2000.

7. Scientific Personnel

Dr. O. Pfister (Research Associate), Mr. Jonathan Blakely and Mr. Seth Boyd (Graduate Research Assistants), and the PI have been partially supported by this project.

Dr. Pfister is now an Assistant Professor in Physics at the University of Virginia.

Mr. Blakely received the M.A. degree during the period of the grant and is expected to complete the Ph.D. degree in Spring 2003.

Mr. Boyd received a terminal M.A. degree during the period of the grant and is working for an internet start-up company in Rhode Island.

8. Inventions

No inventions or patent disclosures have been filed during the period of this grant.