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This paper proposes a number of analytical approximations to the classic and recently measured seawater light scattering phase functions. The three types of analytical phase functions are derived: individual representations for 15 Petzoid, 41 Mankovsky, and 91 Gulf of Mexico phase functions; collective fits to Petzoid phase functions and analytical representations that take into account dependencies between inherent optical properties of seawater. The proposed phase functions may be used for problems of radiative transfer, remote sensing, visibility and image propagation in natural waters of various turbidity.

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Analytical Approximations to Seawater Optical Phase Functions of Scattering

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ABSTRACT

This paper proposes a number of analytical approximations to the classic and recently measured seawater light scattering phase functions. The three types of analytical phase functions are derived: individual representations for 15 Petzold, 41 Mankovsky, and 91 Gulf of Mexico phase functions; collective fits to Petzold phase functions; and analytical representations that take into account dependencies between inherent optical properties of seawater. The proposed phase functions may be used for problems of radiative transfer, remote sensing, visibility and image propagation in natural waters of various turbidity.

Keywords: seawater, scattering, phase function, light, ocean optics

1. INTRODUCTION

After three decades of decline since Petzold published his classic phase functions [1], recent years witnessed unprecedented increase in the *in-situ* measurements of oceanic light scattering phase functions (LSPF). Experiments accomplished under programs of LEO-15 in 2000 and 2001, and *in-situ* measurements of LSPF in the Gulf of Mexico in 2002 [2-4] added more than thousand quality phase functions to the previously available database of 15 Petzold phase functions measured in Pacific and Caribbean. This paper analyzes a number of analytical approximations to the classic and recently measured seawater optical phase functions. The proposed analytical equations may be used for problems of radiative transfer, remote sensing, visibility and image propagation in natural waters of various turbidity. It is suggested that the use of these enhanced approximations may significantly increase precision and reliability of remote sensing- and in-situ-based algorithms of optical data processing.

The seawater phase function of light scattering used in this paper are normalized as it is usually accepted in hydrologic optics, that is

$$0.5\int_0^{\pi} p(\vartheta)\,\sin\vartheta\, d\vartheta = 1, \qquad (1)$$

here ϑ is a scattering angle in radians which is expressed through initial (θ,φ) and scattered (θ',φ') zenith and azimuth angles of light propagation, *i.e.*

$$\vartheta = \cos^{-1}\left(\cos\theta\,\cos\theta' + \sin\theta\,\sin\theta'\cos(\varphi-\varphi')\right), \qquad (2)$$

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2. INDIVIDUAL FITS TO EXPERIMENTAL PHASE FUNCTIONS OF SCATTERING

The experimentally measured phase functions of light scattering by marine water have two extremely prominent peaks in forward and backward direction, with the forward peak much larger than the backward one. Any attempt to represent even logarithms of these phase functions as a polynome of arbitrary order over scattering angle produces very poor representation of a phase function. It was found many years ago among ocean optics experimentalists that regressions with polynomials of square root of an angle ϑ^{γ} give much better results. Numerical analysis of experimentally measured phase functions shows that maximum of regression coefficient is achieved when polynomials are taken from the powers of an angle ϑ^{γ} with γ varying from phase function to phase function in the range of $0.48 - 0.49$. For that reason it is preferable to choose $\gamma = 0.5$, that is to regress a logarithm or any other smoothing procedure of phase function against a square root of a scattering angle.

Because available database of experimental phase functions measured in acceptable range of scattering angles and accompanied by simultaneous measurements of scattering b and absorption a coefficients include measurements by Petzold [1], Mankovsky [2-4], and measurements made in waters of Mobile Bay in 2002 (unpublished), we restrict our analysis only to these data.

The Petzold [1] and Mankovsky [2-4] phase functions are related to clear marine waters with a beam attenuation coefficient $c = a + b$ that is less than 2 m^{-1}. The Mobile Bay measurements are related to typical coastal waters with beam attenuation coefficient c lying in the range up to 18 m^{-1}. According to Ref. [2] both Petzold and Mankovsky phase functions can be represented as the following analytic equation:

$$p(\vartheta) = \exp\left(c_0 + c_1\sqrt{\vartheta} + c_2\vartheta + c_3\vartheta\sqrt{\theta} + c_4\vartheta^2 + c_5\vartheta^2\sqrt{\vartheta} + c_6\vartheta^3\right), \quad r^2 \sim 0.999, \tag{3}$$

with coefficients c_i $(i = 0,...,6)$ that are given below in Tables 1 and 2.

The Mobile Bay phase functions can be represented in the similar to Eq. (3) manner,

$$p(\vartheta) = \frac{A_m}{\left(1 + m_1\sqrt{\vartheta} + m_2\vartheta + m_3\vartheta\sqrt{\theta} + m_4\vartheta^2 + m_5\vartheta^2\sqrt{\vartheta} + m_6\vartheta^3\right)^{10}}, \tag{4}$$

Table 1. Inherent optical properties and coefficients to Eq. (3) for 15 Petzold phase functions of light scattering measured off Southern California coast, San Diego Harbor, and near Bahama Islands [1].

No.	$B \equiv b_B / b$	a	b	c_0	c_1	c_2	c_3	c_4	c_5	c_6
P01	0.025	0.082	0.117	7.4978	-55.043	134.23	-193.23	149.02	-57.929	9.0317
P02	0.044	0.114	0.037	5.4968	-45.768	104.56	-145.18	106.71	-38.639	5.5094
P03	0.038	0.122	0.043	5.9592	-48.635	111.97	-153.75	111.37	-39.755	5.5927
P04	0.014	0.195	0.275	8.4228	-53.032	126.42	-185.22	146.91	-59.155	9.6300
P05	0.013	0.179	0.219	7.9517	-50.705	118.95	-172.19	134.04	-52.659	8.3485
P06	0.019	0.337	1.583	10.425	-50.155	110.20	-151.59	115.70	-45.639	7.3571
P07	0.020	0.366	1.824	10.480	-48.072	100.25	-131.05	94.482	-34.872	5.2289
P08	0.018	0.125	1.205	9.5746	-46.257	100.16	-138.28	105.08	-40.762	6.3982
P09	0.119	0.093	0.009	2.2073	-32.619	77.634	-128.15	113.01	-48.471	8.0665
P10	0.018	0.138	0.547	6.2440	-24.362	34.734	-40.675	25.605	-7.2741	0.74276
P11	0.017	0.764	0.576	6.3907	-25.709	41.649	-56.117	41.540	-14.874	2.1002
P12	0.015	0.196	1.284	9.2428	-36.955	54.016	-44.470	13.073	2.3826	-1.3574
P13	0.017	0.188	0.407	8.6129	-49.121	108.22	-145.70	105.87	-38.904	5.7801
P14	0.025	0.093	0.081	6.8610	-49.667	111.72	-151.09	108.03	-37.991	5.2398
P15	0.146	0.085	0.008	1.7708	-41.835	134.13	-243.06	221.35	-97.049	16.448

with coefficients A_m and m_i $(i = 1,\dots,6)$ presented in Table 3. The scattering angle in both Eqs. (3) and (4) is in radians. The geographical locations, time and depth of measurements, as well as full description of measuring device are given in Refs. [2-4].

Table 2. Inherent optical properties and coefficients to Eq. (3) for 41 Mankovsky phase functions of light scattering measured in Mediterranean and Black Seas, Atlantic and Indian Oceans, and in Lake Baikal [2-4].

No.	$B \equiv b_B / b$	a	b	c_0	c_1	c_2	c_3	c_4	c_5	c_6
M01	0.00781	0.14737	0.40986	8.7523	-45.099	62.778	-37.338	-13.128	22.983	-6.3979
M02	0.01429	0.10361	0.29934	6.4515	-32.012	37.607	-14.917	-19.179	20.440	-5.1816
M03	0.01786	0.11973	0.15658	5.8812	-31.494	34.853	-16.519	-9.3988	12.653	-3.3721
M04	0.02128	0.05526	0.10592	9.7354	-66.274	112.19	-86.477	12.345	16.031	-5.6094
M05	0.03704	0.08059	0.06217	7.7753	-55.438	90.605	-67.294	5.6820	15.804	-5.1711
M06	0.01370	0.10822	0.51808	4.3780	-11.706	-15.955	54.837	-67.644	37.508	-7.5588
M07	0.02041	0.14506	0.29013	3.3909	-8.5579	-24.545	65.429	-73.632	39.087	-7.7336
M08	0.01010	0.11744	0.46742	6.3595	-23.204	-6.3717	70.489	-96.181	53.038	-10.455
M09	0.01786	0.06908	0.16809	7.3020	-41.132	50.489	-24.352	-9.3636	12.934	-3.1985
M10	0.02222	0.05296	0.11513	9.2313	-70.099	165.45	-226.06	166.75	-62.170	9.2855
M11	0.01163	0.16344	0.94176	7.1389	-26.698	10.652	43.957	-77.833	47.219	-9.7692
M12	0.01333	0.14970	0.40986	7.9753	-44.577	83.727	-100.57	63.020	-17.898	1.6602
M13	0.01577	0.02763	0.17960	6.6247	-35.700	43.529	-33.044	13.188	-1.9456	-0.013128
M14	0.01348	0.03684	0.23026	6.5968	-35.314	49.886	-48.025	24.508	-4.9875	0.13011
M15	0.02865	0.02303	0.09210	8.3182	-60.498	125.68	-154.53	103.69	-34.929	4.6379
M16	0.02703	0.11604	0.11513	8.3295	-55.874	99.135	-97.451	47.060	-8.7793	0.10543
M17	0.01818	0.05756	0.15888	4.9707	-21.637	-4.4404	52.645	-66.715	34.590	-6.4741
M18	0.03030	0.03915	0.11743	6.4081	-38.767	52.491	-37.966	7.7029	4.5656	-1.7821
M19	0.01675	0.03224	0.19111	3.8511	-6.5846	-57.663	131.96	-124.76	55.332	-9.3715
M20	0.02415	0.02072	0.13125	5.5298	-27.380	8.0476	41.470	-62.707	34.333	-6.5317
M21	0.01164	0.11407	0.09788	8.1411	-47.271	77.913	-82.443	48.195	-13.513	1.3736
M22	0.01755	0.11024	0.06441	6.2176	-35.377	47.567	-41.307	17.422	-1.5600	-0.51063
M23	0.02050	0.11347	0.09262	7.0676	-44.777	97.065	-144.47	118.77	-48.747	7.9003
M24	0.01270	0.11348	0.09272	7.3521	-40.201	55.904	-50.344	24.590	-5.0571	0.20331
M25	0.01816	0.11490	0.10520	8.6258	-51.687	87.710	-82.427	36.197	-4.7874	-0.47199
M26	0.01752	0.11376	0.09517	6.4233	-29.504	10.992	40.269	-64.817	36.852	-7.2746
M27	0.01928	0.11446	0.10128	5.0769	-19.225	-6.8060	41.964	-45.463	21.214	-3.6608
M28	0.02076	0.11085	0.06968	6.9968	-39.126	47.690	-27.090	-1.9527	8.4500	-2.3639
M29	0.01059	0.11250	0.08419	9.9691	-63.606	113.05	-110.38	52.259	-8.7756	-0.28293
M30	0.01918	0.11117	0.07249	8.5080	-53.213	89.329	-82.860	35.893	-4.4527	-0.56560
M31	0.01323	0.11449	0.10159	10.784	-73.056	158.03	-196.95	132.86	-45.218	6.1593
M32	0.01442	0.11087	0.06991	7.2763	-37.753	25.367	30.381	-63.142	37.765	-7.5693
M33	0.02096	0.10888	0.05248	7.6712	-48.616	75.774	-61.950	17.423	4.1788	-2.1799
M34	0.01388	0.11085	0.06966	8.1720	-49.001	72.357	-56.152	15.459	3.1388	-1.6609
M35	0.02168	0.10759	0.04117	8.4495	-58.468	106.19	-104.60	46.523	-4.8440	-1.2519
M36	0.02862	0.10820	0.04651	6.5951	-43.008	72.702	-79.227	49.067	-15.714	2.1129
M37	0.02937	0.10673	0.03362	5.2386	-29.334	13.371	35.163	-61.069	35.971	-7.2738
M38	0.01816	0.11490	0.10520	8.6258	-51.687	87.710	-82.427	36.197	-4.7874	-0.47199
M39	0.01752	0.11376	0.09517	6.4233	-29.504	10.992	40.269	-64.817	36.852	-7.2746
M40	0.01396	0.12993	0.23674	5.7277	-12.068	-55.117	158.08	-169.36	81.449	-14.570
M41	0.01981	0.11123	0.07306	7.1366	-39.714	47.563	-23.768	-6.0577	10.032	-2.4921

Table 3. Inherent optical properties and coefficients to Eq. (4) for 91 experimental phase functions of light scattering measured at 555 nm in waters of Gulf of Mexico (at Mobile Bay, 2002).

$B \equiv b_B / b$	a	b	A_m	m_1	m_2	m_3	m_4	m_5	m_6	r^2
0.0081	0.537	8.332	20166.1	2.81511	-1.57870	3.59271	-3.75734	1.81021	-0.38390	0.997
0.0086	0.537	4.341	13164.8	2.02022	2.57003	-6.57819	7.62542	-4.06341	0.75343	0.997
0.0092	0.494	3.769	13721.2	2.05432	1.82010	-3.85160	4.26357	-2.36224	0.44775	0.997
0.0092	0.494	3.830	19976.9	2.70075	-0.39207	-0.42736	1.70826	-1.42784	0.31113	0.997
0.0093	0.592	3.912	22382.5	2.94263	-0.88480	0.33987	0.92205	-1.02453	0.23337	0.997
0.0093	0.855	5.282	6082.5	1.30758	2.99307	-5.19837	5.07576	-2.55306	0.45301	0.997
0.0096	0.483	3.566	39991.3	3.35478	-0.47752	-1.41837	2.71360	-1.77855	0.34667	0.997
0.0096	1.345	16.058	3830.7	0.79222	4.29119	-6.98882	5.93679	-2.50981	0.36666	0.997
0.0097	0.804	5.192	11019.8	1.82593	2.33341	-4.88688	5.07837	-2.54781	0.43646	0.997
0.0098	0.865	5.637	13853.5	1.99611	2.46380	-5.86365	6.38942	-3.24493	0.56724	0.997
0.0099	0.655	4.404	9758.9	2.11915	0.01892	-0.10161	0.80713	-0.81708	0.17667	0.997
0.0099	1.075	7.812	8155.0	0.97856	6.66560	-13.64106	13.20199	-6.08722	1.02491	0.997
0.0099	1.269	8.539	2860.9	1.01223	1.77430	-1.69513	1.27244	-0.64172	0.08650	0.997
0.0099	1.283	8.730	4126.7	1.07419	2.87780	-4.52742	4.19542	-2.02292	0.33225	0.997
0.0100	1.015	6.779	12193.4	1.74309	3.54121	-7.78319	7.91693	-3.79436	0.63909	0.997
0.0101	0.655	4.412	19827.0	2.39559	1.15504	-1.32017	-0.12977	0.60548	-0.23137	0.997
0.0101	0.658	4.278	7431.9	1.49434	2.54182	-4.25433	3.93441	-1.91765	0.32497	0.997
0.0101	0.769	5.073	14023.6	2.17853	1.22078	-2.85109	3.18367	-1.69915	0.28981	0.997
0.0101	1.147	7.704	8487.4	1.08135	6.04784	-11.87353	10.90291	-4.75870	0.74665	0.997
0.0102	0.764	4.674	10465.5	2.04328	0.84777	-1.80212	2.28062	-1.40891	0.26773	0.997
0.0102	1.158	7.864	9936.7	1.80274	1.98371	-4.06994	4.25218	-2.13797	0.35556	0.997
0.0102	1.273	8.221	3839.4	1.33887	1.06225	-1.16494	1.34976	-0.85932	0.14586	0.997
0.0103	1.664	10.382	4437.9	1.42286	1.06357	-1.18059	1.25859	-0.75148	0.11219	0.997
0.0104	0.880	10.948	6298.9	1.48559	1.96311	-2.93519	2.51499	-1.14969	0.16327	0.997
0.0104	1.162	7.685	10861.7	1.97262	1.40706	-3.31814	3.98931	-2.24020	0.41149	0.997
0.0105	0.405	5.065	20014.6	2.44941	1.84182	-5.64590	6.78720	-3.68235	0.69054	0.996
0.0105	0.639	4.350	14214.5	2.30183	0.31183	-0.28844	0.04564	-0.01989	-0.03279	0.996
0.0105	1.242	8.523	8741.1	1.73088	1.72087	-2.79058	2.34238	-0.96763	0.10185	0.997
0.0106	0.762	4.611	15208.8	1.83034	3.78222	-7.41041	6.43808	-2.66529	0.37898	0.997
0.0106	0.766	4.801	8687.1	1.30071	4.75443	-9.53362	9.26638	-4.38291	0.75527	0.997
0.0106	0.883	10.848	4040.5	0.79090	4.66423	-8.20589	7.48778	-3.38587	0.54929	0.997
0.0107	0.645	4.046	13291.2	2.29731	0.32199	-1.10601	1.71957	-1.16359	0.22400	0.997
0.0108	0.277	1.872	11704.6	1.81041	2.90566	-6.73786	7.81418	-4.37929	0.87545	0.997
0.0109	0.713	4.629	21960.4	3.35580	-5.32188	13.19058	-15.01997	7.80701	-1.55920	0.997
0.0110	0.779	4.638	10617.2	1.99191	0.94198	-1.13646	0.72469	-0.31794	0.02108	0.996
0.0111	0.522	3.075	4816.4	0.66183	6.44321	-12.75077	12.49883	-5.95545	1.05039	0.997
0.0111	0.889	5.590	11606.5	1.52835	4.75447	-10.27561	10.28402	-4.90184	0.84682	0.996
0.0111	0.897	5.734	7151.6	1.37473	3.16095	-5.36387	4.61038	-2.00104	0.29856	0.997
0.0113	0.405	2.474	15637.6	1.91492	3.69004	-8.23536	8.37221	-4.10848	0.73118	0.997
0.0114	0.475	2.398	59143.6	3.76200	2.02022	-10.71471	13.92433	-7.59361	1.45586	0.996
0.0119	0.495	3.573	2662.2	0.60164	4.00251	-5.69030	4.38188	-1.79408	0.26241	0.997
0.0119	0.503	2.997	30791.4	2.96351	0.20290	-1.14217	0.73693	-0.13481	-0.05289	0.996
0.0119	0.573	4.020	3037.9	0.79765	3.20193	-4.03987	2.74126	-1.01621	0.12018	0.997
0.0120	0.260	1.485	14056.4	1.92502	3.83206	-9.97271	11.54270	-6.22564	1.21217	0.996
0.0120	0.563	3.104	30293.8	3.21042	-0.80816	-0.61018	1.89260	-1.39266	0.28020	0.996
0.0120	1.641	8.765	3729.0	1.35181	0.73385	0.03485	-0.34813	0.12698	-0.05517	0.997
0.0121	0.606	3.649	3958.0	0.82922	4.56121	-8.63841	8.42801	-4.03324	0.70133	0.997
0.0121	0.906	4.709	11427.6	1.67676	3.51424	-6.93073	6.29585	-2.76455	0.42563	0.997
0.0122	0.488	3.423	3491.6	0.97452	2.54997	-2.76816	1.60041	-0.56645	0.05889	0.997
0.0122	0.513	3.575	2994.9	0.54740	4.83301	-7.61333	6.29965	-2.68948	0.42227	0.997
0.0122	0.752	4.189	2637.1	0.86787	2.32220	-2.37255	1.43980	-0.56915	0.06623	0.997
0.0123	0.483	3.376	3029.1	0.56488	4.76004	-7.42159	6.01502	-2.49905	0.37730	0.997
0.0123	0.625	4.199	2619.8	0.52345	4.46405	-6.61555	5.16928	-2.08992	0.30160	0.997
0.0123	0.764	4.223	9835.0	2.02638	0.73574	-1.70201	2.24724	-1.42117	0.27700	0.997
0.0123	1.310	8.221	2637.7	0.38381	5.40914	-8.54527	6.81462	-2.68236	0.36952	0.996

Table 3 (continuation). Inherent optical properties and coefficients to Eq. (4) for 91 experimental phase functions of light scattering measured at 555 nm in waters of Gulf of Mexico (at Mobile Bay, 2002).

$B \equiv b_B / b$	a	b	A_m	m_1	m_2	m_3	m_4	m_5	m_6	r^2
0.0124	0.622	4.172	2559.5	0.56206	4.10341	-5.79423	4.37674	-1.74677	0.24718	0.997
0.0125	0.902	4.893	17389.0	2.02087	3.50457	-7.63460	7.07895	-3.08777	0.47106	0.996
0.0126	0.610	4.039	2125.8	0.19999	5.67882	-8.66905	6.68639	-2.56207	0.34692	0.996
0.0126	0.896	5.650	2467.7	0.27074	6.01616	-10.11531	8.67351	-3.70212	0.58043	0.996
0.0127	0.607	3.974	2103.5	0.21484	5.62877	-8.81080	7.11135	-2.89574	0.42960	0.996
0.0127	0.609	3.996	2420.8	0.40629	4.97036	-7.72723	6.30433	-2.63503	0.40207	0.996
0.0127	1.249	6.694	12315.9	1.57030	4.80380	-10.12670	9.63196	-4.35368	0.70921	0.996
0.0129	0.643	4.104	2649.3	0.32811	5.79114	-9.36150	7.60700	-3.07439	0.45075	0.996
0.0130	0.458	3.067	2331.9	0.07876	6.78523	-10.91401	8.56243	-3.25841	0.44156	0.996
0.0130	0.985	5.019	17453.8	2.25631	2.06164	-5.22946	5.47246	-2.67435	0.44946	0.996
0.0132	0.464	3.051	3999.0	0.88378	3.85039	-5.90988	4.69609	-1.94346	0.28887	0.996
0.0133	0.937	9.225	6816.6	1.52287	2.06759	-3.10255	2.25932	-0.81391	0.07169	0.996
0.0134	0.433	2.821	3652.7	0.79384	3.94876	-5.67944	4.10616	-1.52300	0.19176	0.996
0.0135	0.386	2.571	3370.1	0.61273	4.63760	-6.55176	4.39270	-1.41444	0.13569	0.996
0.0136	0.514	2.208	58403.4	3.71681	1.50917	-8.54780	10.77884	-5.72101	1.06296	0.996
0.0136	0.528	2.505	13929.2	2.10591	1.50648	-2.85722	2.49812	-1.15969	0.17851	0.996
0.0138	0.361	2.154	44302.2	3.42282	-0.80376	0.67340	-1.17878	0.80184	-0.21806	0.996
0.0138	0.587	2.538	65629.4	4.25401	-1.91878	-0.71331	2.83955	-2.02113	0.41391	0.996
0.0138	0.688	3.410	24959.0	3.15158	-1.52234	1.48848	-0.50136	-0.20714	0.06860	0.996
0.0139	0.641	3.078	6063.6	1.87071	-0.97190	4.02289	-4.78890	2.30156	-0.43536	0.997
0.0140	0.435	2.720	3501.9	0.72873	4.43673	-7.21022	6.07011	-2.62988	0.41995	0.996
0.0140	0.784	3.645	12806.5	1.69867	4.50847	-10.47325	10.73794	-5.16582	0.90108	0.996
0.0149	0.518	2.056	9941.7	1.74911	3.01476	-7.12231	7.72431	-3.97726	0.73292	0.996
0.0150	0.461	1.911	8035.6	1.65258	1.72399	-2.07237	1.11973	-0.37087	0.02840	0.996
0.0151	0.741	3.445	25585.8	2.93066	-0.31652	-0.99738	1.71510	-1.09354	0.19988	0.996
0.0152	0.793	1.739	5703.6	0.62664	7.76158	-16.21658	15.83021	-7.33816	1.25849	0.996
0.0155	0.411	2.350	3835.1	0.97428	3.24227	-4.91133	3.98174	-1.74066	0.27735	0.996
0.0169	0.271	1.105	16293.1	2.15957	2.57000	-6.65533	7.43287	-3.97848	0.76958	0.996
0.0181	1.120	4.833	4035.8	0.70107	5.76344	-11.16237	10.27528	-4.53317	0.72988	0.996
0.0187	0.334	1.186	7916.1	1.31306	4.68173	-10.16981	10.45176	-5.21987	0.96523	0.996
0.0196	0.704	2.371	69469.0	4.50665	-2.25320	-2.21146	5.41042	-3.41779	0.66850	0.995
0.0220	0.365	1.093	33693.5	3.35309	-0.61835	-2.23924	3.87951	-2.41115	0.48410	0.996
0.0230	0.314	0.868	19013.9	2.27632	2.98503	-8.16864	8.79597	-4.43843	0.81652	0.996
0.0233	0.541	1.616	51460.7	3.78843	-1.18491	-1.74304	3.30522	-1.95334	0.35517	0.995
0.0237	0.318	0.808	23438.9	2.61034	1.62694	-4.91083	5.12582	-2.57087	0.46737	0.996
0.0259	0.721	1.827	10225.6	2.07968	1.24792	-3.32385	3.20150	-1.42808	0.20746	0.996

3. COLLECTIVE FITS TO EXPERIMENTAL PHASE FUNCTIONS OF SCATTERING

An empirical representation of the Petzold's experimental scattering phase function [1] are represented by the following equations [5]:

$$p(\vartheta) = \frac{4\pi}{b} \exp\left[q\left(1 - k_1\sqrt{\vartheta} + k_2\vartheta - k_3\vartheta\sqrt{\vartheta} + k_4\vartheta^2 - k_5\vartheta^2\sqrt{\vartheta}\right)\right], \qquad (5)$$

$$q = 2.598 + 17.748\sqrt{b} - 16.722b + 5.932b\sqrt{b}, \qquad r^2 = 0.996, \qquad (6)$$

$$\left.\begin{aligned} k_1 &= 1.188 - 0.688\omega_0, & r^2 &= 0.925, \\ k_2 &= 0.1(3.07 - 1.90\omega_0), & r^2 &= 0.897, \\ k_3 &= 0.01(4.58 - 3.02\omega_0), & r^2 &= 0.893, \\ k_4 &= 0.001(3.24 - 2.25\omega_0), & r^2 &= 0.887, \\ k_5 &= 0.0001(0.84 - 0.61\omega_0), & r^2 &= 0.870, \end{aligned}\right\} \tag{7}$$

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here $\omega_0 = b/(a+b)$ is a single scattering albedo, a is an absorption coefficient

The strong regressions given by Eqs. (4)-(6) can be used as a basis for the empirical model of the phase functions with the coefficients dependent on the absorption and scattering coefficients [10]. The single-scattering albedo ω_0 used here varies from 0.09 to 0.96.

4. FITS BASED ON INTEGRAL PROPERTIES OF PHASE FUNCTIONS OF SCATTERING

Results of experimental measurements published in Refs. [6, 7] show that backscattering probability $B = b_B/b$ strongly correlates with scattering phase function at 140°, and average cosine over phase function of scattering correlates with backscattering probability [8]:

$$B = \eta\, p(140°), \quad \eta = 7.233, \qquad \langle\cos\theta\rangle = 2\frac{1-2B}{2+B}. \tag{8}$$

The last condition in Eqs. (8) was used in Ref. [8] to derive a two-term Henyey-Greenstein phase function to represent scattering by marine water. Numerical analysis shows that it is impossible to force the two-term Henyey-Greenstein phase function to satisfy the first condition in Eqs. (8). This is a consequence of the fact that realistic marine phase functions behave more anisotropically in the range of small scattering angles than can be modeled by Henyey-Greenstein phase function.

In order to match both important empirical criteria (8) the following analytical phase function was proposed in Ref. [9]:

$$p(\vartheta) = f\, p_F(\vartheta) + (1-f)\, p_B(\vartheta), \quad 0 \le f \le 1, \tag{9}$$

The phase function (9) differs from the phase function proposed in Ref. [8] that the first, forward directed term is replaced by more anisotropic exponential function,

$$p_F(\vartheta) = A\exp\left(-\alpha\sqrt{\sin\frac{\vartheta}{2}}\right), \quad A = \frac{\alpha^4}{4\left[6(1-e^{-\alpha}) - \alpha\, e^{-\alpha}(6 + 3\alpha + \alpha^2)\right]}, \quad \alpha > 0\,, \tag{10}$$

that behaves similar to the experimentally measured phase functions [1, 3, 4] $p \propto \exp(-\alpha'\sqrt{\vartheta})$ at small angles $\vartheta \sim 0$, and can be integrated analytically The second, backward directed term, was taken similar to in Ref. [8] in the form of a backwardly-directed Henyey-Greenstein function,

$$p_B(\vartheta) = \frac{1-g^2}{\left(1 + g^2 + 2g\cos\vartheta\right)^{3/2}}, \qquad 0 < g < 1. \tag{11}$$

The condition to satisfy Eqs. (8) gives us the following dependencies of parameters f, α and g from backscattering probability parameter:

$$f = \frac{B_B - \eta\, p_B(140^\circ)}{B_B - B_F + \eta\left[p_F(140^\circ) - p_B(140^\circ)\right]}, \tag{12}$$

where

$$B_B = \frac{1+g}{2g}\left(1 - \frac{1-g}{\sqrt{1+g^2}}\right), \tag{13}$$

and

$$B_F = \frac{e^{-\alpha/2^{1/4}}\left(12 + 6\cdot 2^{3/4}\alpha + 3\sqrt{2}\,\alpha^2 + 2^{1/4}\alpha^3\right) - 2e^{-\alpha}\left(6 + 6\alpha + 3\alpha^2 + \alpha^3\right)}{2\left[6(1-e^{-\alpha}) - \alpha e^{-\alpha}(6 + 3\alpha + \alpha^2)\right]}. \tag{14}$$

$$\alpha = 7.4657\,B^{-0.25458}, \qquad 0 \le B \le 0.5, \tag{15}$$

$$\begin{aligned} g = {} & 0.97847 - 0.01085\,B + 0.63542\,B^2 + 8.3409\,B^3 \\ & -268.24\,B^4 + 2855\,B^5 - 14686\,B^6 + 30187\,B^7. \end{aligned} \tag{16}$$

The set of equations (9)-(16) gives us another one-parameter model of seawater light scattering phase function that is preferable to the two-term Henyey-Greenstein model [8] in the respect that it gives more accurate description of realistic marine phase functions by satisfying both conditions given by Eqs. (8).

5. CONCLUSION

The proposed analytical equations to the individual phase functions as well as the equations that take into account dependence on inherent optical properties of seawater or integral properties of the phase function itself may be used for problems of radiative transfer, remote sensing, visibility and image propagation in natural waters of various turbidity. It is suggested that the use of these enhanced approximations may significantly increase precision and reliability of remote sensing- and *in-situ*-based algorithms of optical data processing.

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† All referenced articles except [1] can be viewed and downloaded in a PDF format from the following web site: <http://haltrin.freeshell.org>.