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Many structures are hierarchical in nature and are made up of substructures distributed across several length scales. Examples include aircraft wings made from fiber reinforced laminates and naturally occurring structures like bone. From the perspective of failure initiation it is crucial to quantify the interaction between stress concentrations due to abrupt charges in structural geometry and local stress fluctuations at the level of the microstructure. The presence of geometrically induced stress or strain singularities at either the structural or sub-structural scale can have influence across length scales and initiate non-linear phenomena that result in overall structural failure.

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COMPUTATIONAL DESIGN of HETEROGENEOUS STRUCTURAL, ELECTRIC and OPTICAL COMPONENTS

F49620-02-1-0041

January 1, 2002 through December 31, 2004

Final report.

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Abstract

Many structures are hierarchical in nature and are made up of substructures distributed across several length scales. Examples include aircraft wings made from fiber reinforced laminates and naturally occurring structures like bone. From the perspective of failure initiation it is crucial to quantify the interaction between stress concentrations due to abrupt changes in structural geometry and local stress fluctuations at the level of the microstructure. The presence of geometrically induced stress or strain singularities at either the structural or substructural scale can have influence across length scales and initiate nonlinear phenomena that result in overall structural failure.

During the course of the supported research this investigator has pursued two complementary sets of questions related to failure initiation in composites. The first part of the investigation focuses on quantifying the extent of overstressed zones inside composite structures due to reentrant corners, bolt holes, rivets and other stress risers. New mathematical objects beyond the well known effective elastic tensor have been introduced that deliver upper bounds on the overstressed zones near stress risers in hierarchical structures. These quantities dubbed macro stress modulation functions provide upper bounds on the distribution function of the Von Mises equivalent stress inside microstructured materials. The macro stress modulation functions can be employed in numerical methods for the optimal design of graded microsturcture. A new numerical methodology is presented that enables one to design graded microstructures in order to minimize the effect of stress concentrations due to reentrant corners within a composite structure. The second line of investigation develops optimal lower bounds on the point wise maximum local stress or strain fields inside random microstructure due to macroscopic loading. These bounds are given in terms of the elastic properties of the constituent materials and can be used to quantify the local stress and strain amplification as loads are passed from macroscopic to microscopic length scales.

In parallel work, new extremal microstructures and isoperimetric inequalities have been obtained for nonlinear multi-phase dielectrics and for linear elastic coated fiber reinforced shafts.

Contents

1 Overview

Over the last century significant progress has been made in the characterization of effective constitutive laws for random composite media. However an equally important set of problems are those related to the strength of composites. Investigations into failure initiation require one to assess the macroscopic implications of extreme field behavior at the level of the microstructure. These questions can not be resolved through the knowledge of effective constitutive laws alone.

During the course of the supported research this investigator has pursued two complementary sets of questions. The first part of the investigation focuses on quantifying the extent of overstressed zones inside composite structures due to reentrant corners, bolt holes, rivets and other stress risers. The second line of investigation initiated during the latter part of the support period develops optimal lower bounds on the point wise maximum of local fields inside random microstructure due to macroscopic loading. These bounds are given in terms of the elastic properties of the constituent materials and can be used to quantify the local amplification as loads are passed from macroscopic to microscopic scales in statistically defined microstructures.

We start by outlining the progress made during the support period on the assessment of overstressed zones in composites due to the presence of stress risers. In order to quantify the effect of stress risers in microstructured materials one is required to go beyond effective constitutive laws that relate average stress to average strain. To this end new multiscale quantities have been identified by this investigator and are reported in [9, 8, 10, 11]. These quantities are given by the higher moments of corrector fields [8, 11]. To fix ideas the first moment of the corrector field contains averaged information on microscopic fields and corresponds to the effective elasticity tensor. The higher order moments contain information on the stress fluctuations about the average. During the course of this supported research it has been shown that these quantities characterize the aggregate effects of small scale stress fluctuations generated by the interaction between the macroscopic loading and the geometry of the heterogeneous microstructure [8, 11]. These quantities are used to identify overstressed regions generated by reentrant corners inside structures made from composite materials [9, 11].

The higher moments depend on the local microgeometry and can be obtained computationally. It is shown by myself and in joint work with my Ph.D. student Michael Stuebner that these homogenized quantities provide a good parameterization of the microgeometry over which the numerical optimization of graded microstructures can be carried out [5, 6, 7]. These methods are distinctive in that they provide for the first time a rigorous method for control of the stress distribution within a composite through the tailoring of microscopic properties across the structure [5, 6, 7]. Currently these methods are being extended in joint work with my Ph.D. student Tim Breitzman to more complicated situations involving the effects of prestress, viscoelasticity and elastic to plastic behavior inside the composite [2]. Both of my Ph.D. students Michael Stuebner and Tim Brietzman have been supported in part by this grant.

This approach has stimulated strong interdisciplinary collaboration with computational scientists and experimentalists at the Materials Directorate of the Air Force Research Laboratories at Wright Patterson AFB. In joint work with Dr. Endle Iarve and Dr. Greg Schoeppner we are developing a fast and robust computational stress assessment methodology for fiber–epoxy composite structures used in aircraft. These methods are being applied to the problem of designing composite scarf repair patches for high performance aircraft. Part of this work includes a three year Summer internship for my Ph.D. student Tim Brietzman at the Materials Directorate.

During the latter part of the support period this investigator began a new area of investigation aimed towards quantifying load transfer from macroscopic to microscopic scales. This inquiry is motivated by the fact that an applied field can be locally amplified near a small defect or heterogeneity at the level of the microstructure [4]. The basic problem studied here is described in the context of a two-phase linear dielectric material. We impose a homogeneous electric field $\overline{E}$ on a sample of two-phase composite material for which only a partial statistical description of the microstructure is available. For this case we consider the point wise maximum of the local electric field intensity inside the composite denoted by $|E_{max}|$. The point wise maximum of the electric field intensity is normalized by the applied field intensity $|\overline{E}|$ and we consider the ratio $|E_{max}|/|\overline{E}|$. The basic question is to find an optimal lower bound L on the ratio $|E_{max}|/|\overline{E}|$ given in terms of the partial statistical description of the microstructure and the dielectric properties of the constituent materials, ie.,

$$L \leq |E_{max}|/|\overline{E}|. \tag{1}$$

The lower bound L provides a quantitative measure of the minimum amount of electric field amplification that can be expected from the ensemble of microstructures consistent with the given statistical information. This information can be used to predict the applied field $\overline{E}$ sufficient for failure initiation for statistically defined dielectric materials. Indeed if it is known that dielectric breakdown occurs when the local field exceeds a value β then one sees that dielectric breakdown will occur for applied fields such that $|\overline{E}| > \beta/L$. During the course of the supported research this investigator has obtained an optimal lower bound L for two-phase dielectric mixtures specified by volume fractions and two point statistics. This is reported in [12]. This work has been extended to the case of two phase elastic composites subjected to imposed hydrostatic stress and strain, see [13], [14].

During the course of the supported research this investigator identified new optimal material configurations and new isoperimetric inequalities that naturally arise in the study of heterogeneous materials. In the context of nonlinear dielectric materials a rigorous methodology has been developed for constructing optimal configurations of nonlinear dielectric materials that minimize the L^p norm $(2 \leq p < \infty)$ of the electric field inside a domain with prescribed potential at the boundary [15]. Recently in collaboration with T. Chen the problem of selecting and arranging coated fibers in a prismatic shaft for maximum torsional rigidity is addressed. Here the cross section of the coated fibers are assumed to be circular and optimal configurations are identified and isoperimetric inequalities are obtained [18]. In this context the criteria for identifying optimal configurations is shown to depend on the effective shear modulus and torsional rigidity of each coated fiber.

2 Summary of Research Projects

1 Multiscale stress assessment inside composite structures

Macroscopic quantities beyond effective elastic tensors are presented that can be used to assess the local state of stress within a composite in the linear elastic regime. These are introduced the general homogenization context of G-convergence. It is shown that the gradient of the effective elastic property (the G-limit) can be used to develop a lower bound on the maximum point wise equivalent stress in the fine scale limit. Upper bounds are more sensitive and are correlated with the distribution of states of the equivalent stress in the fine scale limit. The upper bounds are given in terms of the macrostress modulation function. This function gages the magnitude of the actual stress. For $1 \leq p < \infty$ upper bounds are found on the limit superior of the sequence of L^p norms of the equivalent stress associated with the microstructure in the fine scale limit. Conditions are given for which upper bounds can be found on the limit superior of the sequence of L^∞ norms of the equivalent stress associated with the microstructure in the fine scale limit. For microstructure with oscillation on a sufficiently fine scale we are able to give point wise bounds on the actual equivalent stress in terms of the macrostress modulation function. These results are reported in [8].

Next we focus on quantifying the distribution of stress within a composite structure in the presence of a reentrant corner. The composite is composed of N anisotropic linearly elastic materials and the length scale of the microstructure relative to the loading is denoted by ε. The stress distribution function inside the composite $\lambda^\varepsilon(t)$ gives the volume of the set where the norm of the stress exceeds the value t. The analysis focuses on the case when $0 < \varepsilon << 1$. A rigorous upper bound on $\lim_{\varepsilon \to 0} \lambda^\varepsilon(t)$ is found. The bound is given in terms of the macrostress modulation function. We use the macrostress modulation function to provide a rigorous assessment of the volume of over stressed regions near stress concentrators generated by reentrant corners or by an abrupt change of boundary loading. This work is reported in [9].

2 Multiscale assessment of the electric field inside random composite structures

In this work we extend the multiscale theory for field assessment to the context of random media. To fix ideas the problem of multiphase dielectric materials are considered. Suitable macroscopic quantities are identified and used to assess the electric field distribution within a composite specimen of finite size with random microstructure. Composites made of N anisotropic dielectric materials are considered. The characteristic length scale of the microstructure relative to the length scale of the specimen is denoted by ε and realizations of the random composite microstructure are labeled by ω. Consider any cube C_0 located inside the composite. The function $P^\varepsilon(t, C_0, \omega)$ gives the proportion of C_0 where the square of the electric field intensity exceeds t. The analysis focuses on the case when $0 < \varepsilon << 1$. Rigorous upper bounds on $\lim_{\varepsilon \to 0} P^\varepsilon(t, C_0, \omega)$ are found. They are given in terms of the macrofield modulation functions. The macrofield modulation functions capture the excursions of the local electric field fluctuations about the homogenized or macroscopic electric field. Information on the regularity of the macrofield modulations translate into bounds on $\lim_{\varepsilon \to 0} P^\varepsilon(t, C_0, \omega)$. Sufficient conditions are given in terms of the macrofield modulation functions that guarantee polynomial and exponential decay of $\lim_{\varepsilon \to 0} P^\varepsilon(t, C_0, \omega)$ with respect to t. For random microstructure with oscillation on a sufficiently small scale we demonstrate that a point wise bound on the macrofield modulation function provides a point wise bound on the actual electric field intensity. These results are applied to assess the distribution of extreme electric field intensity for an L shaped domain filled with a random laminar microstructure. This work is reported in [11].

3 Optimal design subject to point wise stress constraints through inverse homogenization

Modern design practice increasingly incorporates the use of load bearing components made from composite materials. Composites are now used in structural geometries that involve abrupt dimensional changes within structural components, such as skins connected to ribs, panel reinforcements and junctions of struts. Associated with these geometries are stress concentrations and the potential for failure. In this project new higher order homogenization results are developed and applied in an inverse homogenization procedure to identify graded microstructures that provide desirable structural response while confining the effects of stress concentrations generated along joints or junctions between structural elements.

The design method is based the macrostress modulation functions introduced in [8]. These quantities are applied to develop a rigorous theory for the design of continuously graded locally periodic microstructures in [5]. The design method for continuously graded locally layered microstructures is worked out in collaboration with my Ph.D. student Michael Stuebner in [6].

The methodology is illustrated for long cylindrical shafts reinforced with stiff cylindrical elastic fibers with generators parallel to the shaft. The local fiber geometry can change across the shaft cross section. The methodology is implemented numerically for a cross–sectional shape that possesses reentrant corners typically seen in lap joints and junctions of struts. Graded locally layered microgeometies are identified that provide the required structural rigidity with respect to torsion loading while at the same time minimizing the spatial extent of high stress zones generated by stress concentrations at the reentrant corners. Several numerical examples are provided that demonstrate the capability of the inverse homogenization procedure. The numerical results are obtained jointly with my Ph.D. student Michael Stuebner who is supported on this grant. This work has been submitted for publication in Quarterly Journal of Mechanics and Applied Mathematics [6]. The numerical design method is also implemented by myself and Michael Stuebner for graded locally periodic circular fiber reinforced geometries and will be submitted for publication in Structural Optimization [7].

4 Optimal lower bounds on the electric field concentration in composite media

This project treats a random medium composed of two isotropic dielectric materials. A constant electric field is imposed on the random medium and one is interested in the assessment of the local electric field when only a partial statistical description of the composite geometry is available. During the course of the supported research an optimal lower bound on the L^∞ norm of the electric field for random two phase dielectric materials has been derived. The lower bound depends explicitly upon the volume fractions and the two point correlation function of the random media. Optimal lower bounds on the L^p norm ($p \geq 2$) of the electric field inside each component phase are also derived. All of the bounds are attained by suitably constructed confocal ellipsoid assemblages. These bounds have also been shown to hold for composites with complex dielectric constants. However the regimes of optimality for the complex case remains to be worked out. These bounds significantly extend the previous lower bounds which were known only for the case $p = 2$. These results are reported in [12].

5 Optimal lower bounds on the dilatational strain inside random two-phase composites subjected to hydrostatic loading

Failure initiation in composite materials is a multi-scale phenomena. Central to the analysis is the assessment of the local stress and strain fields generated by macroscopic forces. Quantities sensitive to local field behavior include higher order moments of the stress and strain fields inside the composite. These quantities have seen extensive application in the theoretical analysis of material failure, see Kelly and Macmillan [4]. Failure criteria are often associated with the deviatoric part of the elastic strain tensor. However critical dilatational deformation can precede critical deviatoric deformation in polymers, see Asp, Berglund and Talerja [1]. The dilatational strain has recently been incorporated into failure criteria for epoxy matrix composites seen in aircraft, see Gosse and Christensen [3].

Composites made from two linear isotropic elastic materials are considered. It is assumed that only the volume fraction of each elastic material is known. The composite is subjected to a uniform hydrostatic strain. For this case lower bounds on all r^{th} moments of the dilatational strain field inside each phase are obtained for $r \geq 2$. A lower bound on the maximum value of the dilatational strain field is also obtained. These bounds are given in terms of the volume fractions of the component materials. All of these bounds are shown to be the best possible as they are attained by the dilatational strain field inside the Hashin–Shtrikman coated sphere assemblage. The bounds provide a new opportunity for the assessment of the local dilatational strain in terms of a statistical description of the microstructure. These results are reported in [14].

6 Load transfer and stress amplification inside random two-phase elastic composites

Many composite structures are hierarchical in nature and are made up of substructures distributed across several length scales. From the perspective of failure initiation it is crucial to quantify the load transfer between length scales. It is common knowledge that the load transfer can result in local stresses that are significantly greater than the applied macroscopic stress, see for example Kelly and Macmillan [4]. Quantities useful for the study of load transfer include the higher order moments of the local stress. The higher moments are sensitive to local stress concentrations generated by the interaction between the microstructure and the macroscopic load. In this project we use higher order moments to gage the effect of the microstructure on the amplification of the macroscopic stress inside random composites made from two isotropic elastic materials in prescribed proportions.

Composites made from two linear isotropic elastic materials are subjected to a uniform hydrostatic stress. It is assumed that only the volume fraction of each elastic material is known. Lower bounds on all r^{th} moments of the hydrostatic stress field inside each phase are obtained for $r \geq 2$. These are given in terms of the volume fractions of the constituent materials. A lower bound on the maximum value of the hydrostatic stress field is obtained when only the volume fractions of the constituents are known. All of these bounds are shown to be the best possible as they are attained by the hydrostatic stress field inside the Hashin–Shtrikman coated sphere assemblage.

The bounds provide a new opportunity for the assessment of load transfer between macroscopic and microscopic scales for statistically defined microstructures. For example lower bounds on the maximum point wise hydrostatic stress provide explicit conditions on the applied stress for which the local stress will lie out side the strength domain of the matrix phase inside a fiber-epoxy composite. For other composite systems requiring a Weibull-type failure analysis, lower bounds on the moments deliver lower bounds on the failure probability of the composite material. These results are reported in [13].

7 A rigorous method for design of multi-phase nonlinear dielectric composites

The problem of design of multi-phase nonlinear dielectric materials for minimum dc-electric field amplitude is addressed. The methodology is given for boundary value problems in two dimensions. The potential field U_0 is prescribed on the boundary of the domain and the domain contains N nonlinear dielectrics with energy densities $\gamma_i|E|^p$, $i = 1, \ldots, N$, where γ_i is the nonlinear susceptibility of the i^{th} phase and E is the electric field. The area fraction of each dielectric is prescribed and the optimal configuration that gives the minimum value for $\int |E|^p \, dx$ is sought. It is shown that the optimal configuration is attained by arranging the dielectrics to lie between the level curves of the function that is the p-harmonic congugate of the potential field ϕ that solves $\Delta_p \phi = 0$ and $\phi = U_0$ on the boundary. Here Δ_p denotes the p Laplace operator. This work is reported in [15]. Future work will apply this method to the design of capacitors and varistors to hedge against dielectric breakdown.

8 Maximum torsional rigidity for coated fiber reinforced shafts

While supported by AFOSR grant F49620-96-1-0055 this investigator identified extremal fiber configurations for imperfectly bonded fiber reinforced shafts that deliver maximum torsional rigidity [16, 17]. The criteria used for identifying optimal configurations is shown to depend upon the Steklov eigenvalue associated with the cross section of each fiber [16]. These results form the basis for new isoperimetric inequalities for imperfectly bonded fiber reinforced shafts that generalize the classic results of B. de Saint Venant (1859) and Polya and Weinstein (1950). Recently in collaboration with T. Chen we build on these results and consider the problem of reinforcement in the presence of an inter-phase between fiber and matrix. Inter-phases between the fiber and matrix occur during processing and may be intentionally introduced to enhance structural properties. The fiber together with the inter-phase layer is called a coated fiber. In this project the problem of selecting and arranging long coated fibers in a prismatic shaft for maximum torsional rigidity is addressed. Here the cross section of the coated fibers are assumed to be circular and the coating is represented by an annular region surrounding the fiber cross section. The fiber diameter and relative coating thickness with respect to the fiber diameter are design variables and can be different for each coated fiber. New bounds on the torsional rigidity are obtained. The bounds are used to assess the optimality or suboptimality of fiber reinforcement configurations. It is shown how the effective shear modulus and torsional rigidity of each coated fiber can be used to determine whether a configuration provides reinforcement above or below that of a homogeneous shaft containing no coated fibers. Simply connected shaft cross sections of arbitrary shape reinforced with any configuration of coated fibers are considered. Precise conditions on the effective anti plane shear stiffness and torsional rigidity of each coated fiber are given under which the circular shaft reinforced with a single centered circular coated fiber is either optimal or suboptimal. This work is reported in [18].

3 Publications resulting from the supported research

1. Lipton R., "Homogenization and design of functionally graded composites for stiffness and strength," in *Nonlinear Homogenization and its Application to Composites, Polycrystals, and Smart Materials.* P. Ponte Castaneda, et. al. (Eds.), Springer Verlag, Berlin, NATO Science series II Mathematics, Physics, and Chemestry, **170**, 2004, pp. 169–192.

2. Lipton R. and Stuebner, M., "Optimal design of graded microstructure through inverse homogenization for control of point wise stress." Submitted to *Quarterly Journal of Mechanics and Applied Mathematics*, December 2004.

3. Lipton R. and Stuebner, M., "Design of graded microstructure for optimal stiffness subject to point wise stress constraints." To be submitted to *Structural Optimization.*

4. Lipton R. Bounds on the distribution of extreme values for the stress in composite materials. *Journal of the Mechanics and Physics of Solids*, 52, 2004, pp. 1053–1069.

5. Lipton R. Assessment of the local stress state by macroscopic variables, Philosophical Transactions of the Royal Society of London A, 361, 2003, pp. 921-946.

6. Lipton R. "Stress constrained G closure and relaxation of structural design problems," *Quarterly of Applied Mathematics*, **62**, 2004, pp. 295–321.

7. Lipton R. "Homogenization theory and the assessment of extreme field values in composites with random microstructure." *SIAM J. Applied Mathematics,* **65**, 2004, pp. 475–493.

8. Lipton R. "Optimal lower bounds on the electric-field concentration in composite media". *Journal of Applied Physics*, **96**, 2004, pp. 2821–2827.

9. Lipton R. "Load transfer and stress amplification inside random two-phase elastic composites." Submitted to *Journal of the Mechanics and Physics of Solids.*

10. Lipton R. "Optimal lower bounds on the dilatational strain inside random two-phase elastic composites subjected to hydrostatic loading." Submitted to *Mechanics of Materials.*

11. Lipton R. Configurations of nonlinear materials with electric fields that minimize L^p norms. *Physica B.*, 338, 2003, pp. 48–53.

12. Lipton R. and Chen T. "Bounds and extremal configurations for the torsional rigidity of coated fiber reinforced shafts." *SIAM J. Applied Mathematics,* **65**, 2004, pp. 299–315.

13. Breitzman, T., Iarve, E., and Lipton, R. "Stress assessment in prestressed composites." In preparation.

4 Interactions/Transitions

1 Interactions with Government Laboratories

Since May of 2000 I have been interacting with the research team in the Laboratory at Wright Patterson Air Force Base lead by Dr. Tia Benson Tolle. Since Spring of 2001 I have been collaborating with Dr. Greg Schoeppner (WPAFB) and Dr. Endel Iarve (UDRI contractor with WPAFB). One of the principle objectives of this interaction is the characterization of stress concentrations in composite materials. Together with Dr. Endle Iarve and Dr. Greg Schoeppner we have embarked on the development of a fast multiscale numerical method for accurate stress assessment in prestressed composite materials used in aircraft. This methodology is being applied to the design of composite repair patches for high performance aircraft. My graduate student Tim Breitzman has been working on this project on site at the Materials Directorate at WPAFB. Tim has spent the Summers of 2003 and 2004 and the Fall of 2004 working at the Materials Directorate. I have visited the Labs once during the Summer of 2002 and twice each Summer of 2003 and 2004 in support of the project. Tim Breitzman is running extensive numerical tests, carried out under the direction of Dr. Endel Iarve, for the validation of the macroscopic failure criteria for composite laminates used in applications.

2 Pleanary Talks

- "Composite Properties and Microstructure Part I: Effective Properties; Part II: Strength," at the IMA Tutorial/Workshop: Composites: Where Mathematics Meets Industry, February, 2005, Institute for Mathematics and its Applications University of Minnesota, Minneapolis, MN.
  http://www.ima.umn.edu/matter/winter/t2.html
- "Multi-scale Stress Analysis," at the Third DOE Workshop on Multiscale Methods, Portland, Oregon, September, 2004
  http://multiscalemath.pnl.gov
- " Stress Assessment in Composite Materials," Society for Natural Philosophy Meeting/IMA PI Conference, Multiscale Effects in Material Microstructures and Defects, University of Kentucky, September 2003,
  http://www.ms.uky.edu/~mclxyh/snpmeeting.html
- "Optimal Design of Microstructure for Control of Stress and Stiffness," NATO Advanced Research Workshop on Nonlinear Homogenization and Applications, Kazimierz Dolny, Poland, July 2003, http://www.ippt.gov.pl/NATO-ARW/

3 Fellowships, Sabbatical and Short Term Invited Visits

- J. T. Oden Research Faculty Fellowship at the ICES at the University of Texas, August 2004.
- IMA Workshop: Singularities in Materials, October, 2004.
- IMA Workshop: Future Challenges in Multiscale Modeling and Simulation, November, 2004.

- IMA Tutorial/Workshop: Composites: Where Mathematics Meets Industry, February, 2005.

- Visiting Scholar, Division of Engineering and Applied Science, Harvard University, September 2004 – June 2005.

4 Supported Graduate Students

- Ph.D. adviser for Michael Stuebner, LSU, supported during the period 1/1/2002-8/20/2004.

- Ph.D. adviser for Timothy Breitzman, LSU, supported during the period 8/21/2004-12/31/2004. It is anticipated that Breitzman will receive his Ph.D. August 2005.

5 Presentations

Invited talks at academic institutions.

- Applied Mechanics Colloquium, Division of Engineering and Applied Sciences, Harvard University, April 2005.

- Mathematics Colloquium, Mathematics Department, University of Kentucky, March 2005.

- Applied Mathematics Seminar, Tulane University, February, 2005.

- Mechanics Seminar, Department of Mechanical Engineering, Massachusetts Institute of Technology, February 2005.

- Department of Mechanical and Aerospace Engineering, University of Florida, February 2005.

- Analysis Seminar, Department of Mathematics, University of Pennsylvania, February 2005.

- PDE Seminar, Department of Mathematics, Brown University, October 2004.

- ICES Seminar, University of Texas at Austin, May 2004.

- Center for Nonlinear Analysis, Carnegie Mellon University, March 2004.

- Tulane University Department of Mathematics, February 2004.

- Wichita State University Lecture Series in Mathematical Science, April 2003

- University of North Carolina Applied Mathematics Colloquium, October 2002

- University of Paris VI Laboratoire Jacques-Louis Lions, European homogenization network Seminar, June 2002

- Texas A & M Department of Mathematics, Spring 2002

- Georgia Institute of Technology Department of Mathematics, Spring 2002

Invited talks at Conferences and Workshops.

- Society for Engineering Science 41st Annual Technical Meeting, Lincoln, Nebraska, October 2004.
- Midnight Sun Conference - Multi Scale Problems and Asymptotic Analysis, Narvik Norway, June 2004.
- 2004 AFOSR Program Review for Computational Mathematics and Applied Mathematics, Dayton, Ohio, June 2004.
- SIAM Conference on Mathematical Aspects of Materials Science, Los Angeles, May 2004.
- Conference on Computational Methods in Multiscale Analysis and Applications, University of Florida, March 2004.
- American Mathematical Society South East Section Meeting, University of North Carolina, October 2003.
- Society of Engineering Science, Annual Technical Meeting, University of Michigan, October 2003.
- Society for Natural Philosophy Meeting, University of Kentucky, September 2003.
- Seventh National US Congress on Computational Mechanics, Albuquerque, New Mexico, July 2003.
- International Congress of Applied Mathematics, Sydney Australia, July 2003.
- NATO Advanced Research Workshop on Nonlinear Homogenization and Applications, Kazimierz Dolny, Poland, July 2003.
- 2003 American Society of Mechanical Engineers, Mechanics and Materials Conference, Scottsdale Arizona, June 2003.
- 2003 AFOSR Program Review for Computational Mathematics and Applied Mathematics, University of Florida, May 2003.
- American Mathematical Society South East Section Meeting, Louisiana State University, March 2003.
- First Joint AMS UMI meeting Pisa Italy, June 2002.
- 2002 AFOSR Program Review for Computational Mathematics and Applied Mathematics, University of Florida, May 2002.
- IUTAM Symposium: "Micromechanics of Fluid Suspensions and Solid Composites", University of Texas, Austin, April, 2002.

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[13] Lipton R. "Load transfer and stress amplification inside random two-phase elastic composites." Submitted to Journal of the Mechanics and Physics of Solids.

[14] Lipton R. "Optimal lower bounds on the dilatational strain inside random two-phase elastic composites subjected to hydrostatic loading." Submitted to Mechanics of Materials.

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