

TechSat 21: Formation Design, Control, and Simulation¹

Rich Burns
(505) 846-2352
burnsr@plk.af.mil

Craig A. McLaughlin
(505) 853-6545
mclaughc@plk.af.mil

Jesse Leitner
(505) 846-6071
leitnerj@plk.af.mil

Maurice Martin
(505) 853-4118
martinr@plk.af.mil

Air Force Research Laboratory
Space Vehicles Directorate
3550 Aberdeen Avenue
Kirtland Air Force Base, NM 87117-5776

Abstract—The satellite cluster approach to space missions requires science and technology advances in several key areas. Among these challenges is understanding the dynamics of satellites in close proximity to each other so that a formation can be intelligently designed, controlled, and simulated. An overview of on-going research in this area under the TechSat 21 program along with preliminary findings is provided. Included in this overview will be the recent progress made in the design of formations including designs for circular formations, projected circular formations, and J_2 invariant formations. Strategies for formation control are presented as well as the baseline design for the TechSat 21 propulsion system. Fuel expenditure is estimated for various formations using different control strategies. The TechSat 21 mission requires relative position knowledge between satellites to the millimeter level while the radar is transmitting and receiving; concepts for meeting this requirement are also presented.

In order to facilitate mission planning and gain confidence in mission success, the Air Force Research Laboratory (AFRL) is building an end to end simulation testbed for the TechSat 21 mission. An overview of the testbed design and functionality is provided. Focus is centered on the dynamics and control module of the testbed. The dynamics and control module utilizes high fidelity orbit propagation as the basis of the simulation of the formation dynamics. Through this simulation control algorithms, relative navigation techniques, and the effects of errors in initial conditions and control forces are investigated.

TABLE OF CONTENTS

1. INTRODUCTION
2. DYNAMICS AND CONTROL RESEARCH
3. TECHSAT 21 SIMULATION FACILITIES
4. SUMMARY AND CONCLUSIONS

1. INTRODUCTION

The Air Force Research Laboratory (AFRL) has initiated the TechSat 21 program to serve as a proof of concept mission for a new design paradigm for space missions. This paradigm seeks to reduce costs and increase system robustness and maintainability by distributing the functionality over several microsatellites flying in formation. The distributed functionality includes processing, communications, control functions, as well as payload functions. Thus, the system of microsatellites forms a “virtual satellite”, which can be controlled and tasked as a single satellite

Spurred on by the potential of reduced launch costs, increased system robustness, and enhanced maintainability, the topic of formation flying has gained a great deal of interest. Several formation flying missions are planned including NASA's Earth Orbiter 1 [1] and Space Technology 3 and 5, the University Nanosatellite Program [2,3], Cluster [4], Laser Interferometer Space Antenna (LISA) [5], and Discoverer II [6]. Among these systems, TechSat 21 has some of the most stringent requirements for formation flying due to its interferometric based mission in low earth orbit. TechSat 21 will perform the Space Based Radar (SBR) mission for Ground Moving Target Indication (GMTI) and Geolocation missions, which provide a means for comparison to single satellite systems. SBR also provides a mission of significant technological challenges including heavy on-board processing requirements, tight relative navigation accuracy, significant satellite-satellite and satellite-ground communications loads, and precision formation control on tight fuel budgets among others. Das *et al.* [7] and Martin and Stallard [8] give a full description of the TechSat 21 mission. In this paper, the authors will concentrate on those design parameters which have a direct impact on the capabilities and requirements for the formation flying portion of the TechSat 21 mission.

TechSat 21 Satellite Parameters

The TechSat 21 satellites, depicted in Figure 1, each have nominal mass of 100 kg. The satellites are designed to unfold accordion style from their stowed configuration to a gravity gradient stabilized satellite with solar arrays covering the length of the boom. The antenna panels will be on the nadir-pointing end of the boom, while the propulsion module will be situated along the length of the satellites such that it includes the center of mass. The satellite's power module then resides at the opposite end to the antenna panels.

Of course, for formation flying the most important aspect of the satellite design is the propulsion module. An excellent discussion of the technologies considered for this module, including chemical, electromagnetic, and electrostatic technologies, can be found in Schilling *et al.* [9]. Ultimately, a combination of four Pulsed Plasma Thrusters (PPT) and six micro-Pulsed Plasma Thrusters (μ PPT) was selected for the nominal propulsion module design. The configuration for the propulsion module is shown in Figure 2. The module can be supplemented with two μ PPT thrusters at each end of the gravity gradient boom to provide radial thrusting and/or attitude control in the roll and pitch axes. The bank of four PPTs are aligned to provide

¹ U.S. Government work not protected by U.S. copyright.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE 2000		2. REPORT TYPE		3. DATES COVERED -	
4. TITLE AND SUBTITLE TechSat 21: Formation Design, Control, and Simulation				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Rich Burns; Craig McLaughlin; Jesse Leitner; Maurice Martin				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Research Laboratory, 3550 Aberdeen Ave SE, Kirtland AFB, NM, 87117-5776				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The satellite cluster approach to space missions requires science and technology advances in several key areas. Among these challenges is understanding the dynamics of satellites in close proximity to each other so that a formation can be intelligently designed, controlled, and simulated. An overview of on-going research in this area under the TechSat 21 program along with preliminary findings is provided. Included in this overview is the recent progress made in the design of formations including designs for circular formations, projected circular formations, and J₂ invariant formations. Strategies for formation control are presented as well as the baseline design for the TechSat 21 propulsion system. Fuel expenditure is estimated for various formations using different control strategies. The TechSat 21 mission requires relative position knowledge between satellites to the millimeter level while the radar is transmitting and receiving; concepts for meeting this requirement are also presented. In order to facilitate mission planning and gain confidence in mission success, the Air Force Research Laboratory (AFRL) is building an end to end simulation testbed for the TechSat 21 mission. An overview of the testbed design and functionality is provided. Focus is centered on the dynamics and control module of the testbed. The dynamics and control module utilized high fidelity orbit propagation as the basis of the simulation of the formation dynamics. Through this simulation control algorithms, relative navigation techniques, and the effects of errors in initial conditions and control forces are investigated.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES 8	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

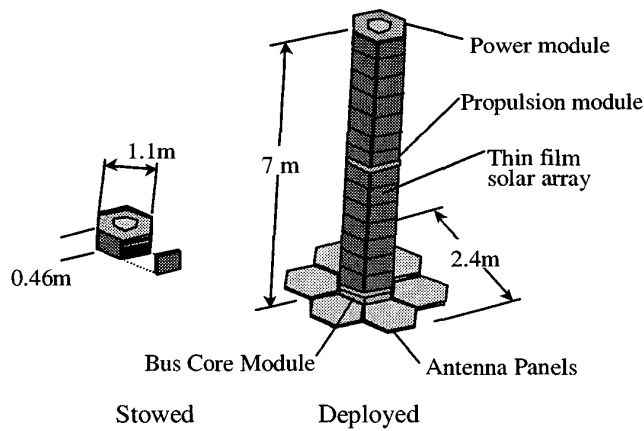


Figure 1 TechSat 21 Design

capability to raise the orbit at the beginning of the mission and deorbit at the end of the mission. These PPTs are also expected to provide most of the thrust required for routine formation maintenance maneuvers during the mission. The six μ PPT thrusters mounted on the perimeter of the propulsion module are designed to provide attitude control in the yaw axis. This will allow the PPTs to be pointed in the proper direction to apply the required formation control thrust. Expected values of PPT and μ PPT specifications are listed in Table 1.

Table 1. Thruster Specifications

Type	Isp (s)	Efficiency	Thrust
PPT	800	10%	2mN
μ PPT	1000	5%	40 μ N

In the case where the initial orbit raise and final deorbit are not required, it may be possible to avoid the use of μ PPTs, whose performance characteristics are not as well established as those of PPTs, in the propulsion module. It is envisioned that six PPTs may be positioned on the perimeter of the module in the manner of the μ PPTs shown in Figure 2. This will result in a mass penalty of slightly more than one kilogram. Additionally, the PPT only propulsion module will result in a fuel expenditure penalty because in general, combinations of thrusters must be used to apply thrust in a specified direction.

Mission Orbital Parameters

TechSat 21 is scheduled for launch in late 2003 to mid 2004. For that time period, the formation must achieve a 500-550 km altitude, near circular orbit to accommodate a one year mission lifetime. For significantly longer mission lifetime, it is recommended that the formation move to a circular orbit 700-800 km in altitude in order to move to a region where atmospheric drag is significantly reduced. In order to achieve adequate ground coverage, orbital inclinations from 50-90 degrees are being considered. Nominal formation sizes from 6 m to 5 km are being considered for the Techsat 21 mission.

2. DYNAMICS AND CONTROL RESEARCH

There are several ongoing efforts aimed at directly supporting the TechSat 21 mission. Among these are three efforts in the area of dynamics and control of formation flying. Two universities, Stanford and Texas A&M, have received Air Force Office of Scientific Research (AFOSR)

grants to work in this area while there is a coordinated internal effort at AFRL. These efforts address four main areas of interest to formation flying missions such as TechSat 21: 1) formation configuration and evolution, 2) formation control, 3) relative navigation, and 4) attitude determination and control.

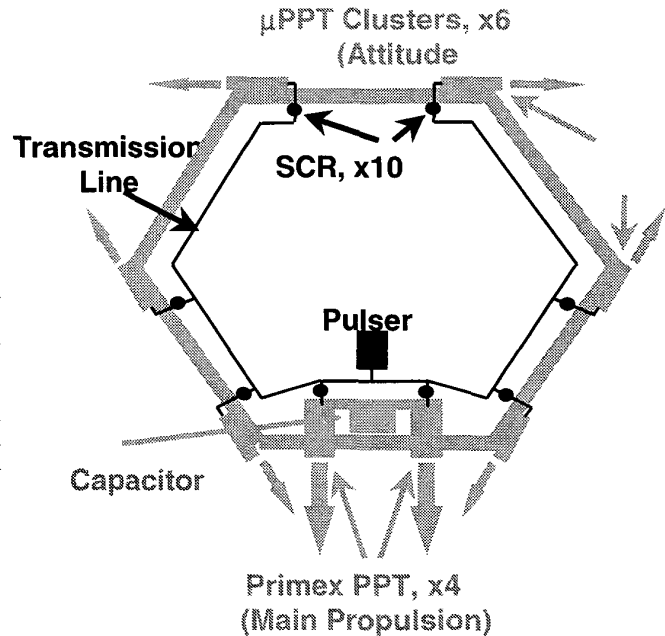


Figure 2 Propulsion Module

Formation Configuration and Evolution

The configuration in which the TechSat 21 satellites will fly is central to all other aspects of the formation flying mission.

The design of the TechSat 21 configuration will likely be some compromise between the capabilities of the formation control system and the requirements of the radar payload. Although the requirements of the radar payload with respect to the formation configuration have not yet clearly been defined, it is apparent that considerations such as a static or rigid formation will be unacceptable from a formation control perspective. That is, these types of configurations would require constant thrust at levels unacceptable given TechSat 21's limited fuel budget. In order to minimize fuel expenditure, it is imperative that the formation accommodates the natural orbital motion of its individual members.

The well known Hill's equations [11]:

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = f_x$$

$$\ddot{y} + 2\omega\dot{x} = f_y$$

$$\ddot{z} + \omega^2 z = f_z$$

describe relative motion of two satellites which are both in close proximity to one another and in nearly circular orbits, where ω is the orbital frequency for the reference satellite,

(x,y,z) are (small) displacements of the “chase” satellite’s position relative to the reference, and (f_x, f_y, f_z) are externally applied forces in the (x,y,z) directions, respectively, as shown in Figure 3. The x direction is radial from the center of the Earth to the reference satellite, y is orthogonal x in the orbital plane of the reference satellite and in the same sense as the velocity vector, while z is normal to the orbital plane and completes right handed triad.

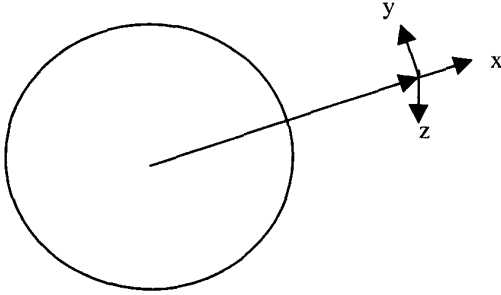


Figure 3 Hill's Frame

simple harmonic oscillation in the out of plane direction. To reiterate, the two assumptions implicit in Hill's equations are (1) nearly circular reference orbit and (2) the displacements (x,y,z) are small compared to the magnitude of the reference position vector. That is, the equations are truncated at order 0 in eccentricity and order 1 in (x,y,z) . This linearization facilitates the closed form solution:

$$\begin{aligned} x(t) &= \frac{\dot{x}_0}{\omega} \sin(\omega t) - (3x_0 + \frac{2\dot{y}_0}{\omega}) \cos(\omega t) \\ &\quad + (4x_0 + \frac{2\dot{y}_0}{\omega}) \\ y(t) &= (6x_0 + \frac{4\dot{y}_0}{\omega}) \sin(\omega t) + \frac{2\dot{x}_0}{\omega} \cos(\omega t) \\ &\quad - (6\omega x_0 + 3\dot{y}_0)t + (y_0 - \frac{2\dot{x}_0}{\omega}) \\ z(t) &= z_0 \cos(\omega t) + \frac{2\dot{z}_0}{\omega} \sin(\omega t). \end{aligned}$$

Clearly, for a formation of satellites to remain close to one another, the secular term in the $y(t)$ equation must be zero. Setting the coefficient of t to zero and solving yields the following relationship for periodic motion in y or along track direction:

$$\dot{y}_0 = -3\omega x_0.$$

Note that this is equivalent to matching energy, and consequently the semi-major axis and period, of the “chase” satellite with that of the reference satellite to first order in the small displacements (x,y,z) , which results in bounded motion for the formation.

Sedwick *et al.* [10] consider the baseline TechSat 21 mission, which is synthetic aperture radar for ground moving target indication (GMTI). The mission is assumed to require a cluster of satellites flying at approximately 800 km altitude

in nearly polar orbits. Hill's equations [11] are reviewed to show the decoupled in-plane and out-of-plane periodic motions and are used to design initial conditions and simulate the motion of the formation in orbit. The quantification of orbital perturbations is presented to baseline the ΔV requirements and mission life. Perturbations due to gravity, atmospheric drag, solar pressure, and electromagnetic force are all quantified; it is shown that gravity perturbations have the largest effect. Finally, several formations and their ΔV requirements were considered. Steered planar circular clusters have the plane containing the satellites always pointing toward the area being observed. A steered non-planar circular cluster no longer constrains the satellites to be co-planar. No propellant is expended in a projected circular cluster to force the satellites out of their natural orbit. It is concluded that formation-flying strategies that require propellant expenditure beyond simple housekeeping are not viable for the TechSat 21 system.

Sabol *et al.* [12] consider a richer dynamical model in the analysis of formation flying missions. The fully non-linear equations of motion are integrated in the presence of a realistic force model. One conclusion states that the major perturbation influencing the dispersion of the formation comes from the dominant term in the geopotential, J_2 , which models the earth's oblateness. Issues such as atmospheric drag and tesseral resonance are also shown to be a major source of dispersion for formations in certain orbits. The importance of designing and controlling a formation using mean elements rather than osculating elements is noted. Mean elements allow better insight into long term behavior since short periodic effects have been eliminated. Initial conditions are defined in terms of mean elements and the numerical studies are carried out using a mean element propagator with the capability to recover short periodic effects. For the formations useful to TechSat 21 (circular and projected circular) fuel expenditure was estimated to be 20-30 m/s/year for TechSat 21 formation sizes (500m-1km).

Schaub and Alfriend [13] have identified families of formations that are invariant to the dominant cause of formation dispersion, the J_2 perturbation in the geopotential. Brouwer's first order solution to the Main Problem of Artificial Satellite Theory [14,15], which considers the geopotential through J_2 , is utilized to enforce two constraints on the satellites in formation: (1) equal nodal periods and (2) equal latitude rates ($d/dt(l+g)$), in Delaunay variables, where l is the mean anomaly and g is the argument of perigee. These constraints provide two relationships among inclination, eccentricity, and semi-major axis, which effectively cancel the formation dispersion due to J_2 to first order. As in Sabol *et al.* [12], mean elements are utilized to gain a better perspective on the long term motion of the system.

Formation Control

Control of the TechSat 21 has a nominal requirement to maintain the nominal separations within the formation to approximately 10% of their values. Formations with separations on the order of 1 km will be controlled to 100 m, while for separations on the order of 100 m, separation distance control will be to 10 m. This is a truly staggering requirement when the scale of the problem is considered. The semi-major axis for TechSat 21 orbits will be near 7000 km, so that one can easily see that controlling to this level of precision will require minute corrections to the orbital elements of the individual members of the formation affected by appropriately small thrusts.

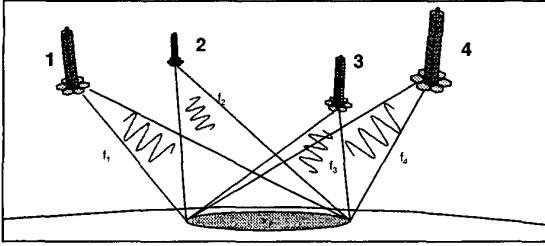


Figure 4 Sparse Array Operations Concept

Schaub *et al.* [16] describe the design and performance of two nonlinear feedback control laws applied to formations designed using the concepts of J_2 invariance. The control laws are designed to feedback mean elements and Cartesian position and velocity states, respectively. For both control laws, the authors present numerical results that reduce and maintain tracking errors in the formation to the meter level. This is accomplished using variable, continuous, low level thrust. The total fuel expenditure was similar for both control laws, but the nature of the application of control forces varied somewhat. With mean element feedback, the control forces tended toward a more impulsive application, while the Cartesian feedback produced a more even force application profile. This due to the fact that mean element feedback favored application of control force at particular points in the orbit to maximize influence on particular orbital elements (e.g., at the equator to change inclination). Numerical simulations of both approaches yielded similar fuel expenditures in the 7m/s range for a period of just 4 orbits; however, the majority of this is accounted for by the corrections made for errors introduced into the initial conditions for the formation. Most expensive was the correction to the inclination as one might expect.

Vadali *et al.* [17] formulated and solved the initial condition determination and fuel optimal control problem for formation flying. In particular they examined how to initialize and maintain formations that have out of plane motion that maximize system performance for most applications. They examined the effects of errors in the initial conditions of a formation and the velocity increment required for correcting those errors for both impulsive and low thrust propulsion systems. The inclination errors are shown to be the most expensive to correct as in Schaub *et al.* [16].

Inalhan, Busse, and How [18] are adopting a Linear Quadratic Regulator (LQR) approach to the formation control design applied to the Orion [19,20] formation flying experiment on MightySat II.2. An example Orion experiment initializes two spacecraft to a 50 m in-track separation and uses the control system to maintain separation to within a 0.5x2.5x2.5 m radial x in-track x cross-track box.

Relative Navigation

The TechSat 21 mission calls for the collection and combination of signals irradiated from each satellite and reflected from the target as shown in Figure 5. This defines a requirement for knowledge of the relative positions of the members of the formation to the millimeter level. Because the signals themselves may be used in this relative position determination, the requirement for the relative navigation

system using measurements external to the radar signals may be relaxed to the centimeter level.

Inalhan, Busse, and How [18] have shown that centimeter level accuracy is indeed achievable through the use of Carrier-phase Differential GPS. The authors have demonstrated accuracy levels of 2-5 cm in the aforementioned Orion formation simulation. A Kalman filter is used to initially estimate Carrier-phase Differential GPS integer biases, and subsequent relative positions are estimated using another Kalman filter in real time.

In order to obtain millimeter level accuracy it is necessary to use phase information from the radar itself while imaging. Using the precise knowledge obtained from GPS as a starting point and the interferometry work of Greenaway [21] it should be possible to obtain the millimeter relative position knowledge required.

Attitude Determination and Control

In order to maintain precise formation control it is necessary to examine the attitude determination and control system. Beck [22] and Beck and Hall [23] give an examination of the coupling between the orbit and attitude dynamics for a rigid satellite. This coupling is enhanced when active orbit maintenance is desired because any error in the attitude will be translated into an error in the direction of the control force application. Wang and Hadaegh [24,25] examine the problem of controlling both the orbit and attitude for a formation of rigid satellites.

TechSat 21 is currently envisioned as a gravity gradient stabilized spacecraft with magnetic torquers providing control. A concept is also being examined to use μ PPTs or PPTs for attitude control in the yaw axis. The attitude control design requirement for the GMTI mission has been specified at 5 deg. It is possible that the requirements to point the PPT thrusters to effectively make formation maintenance maneuvers may drive the attitude control requirement lower. The attitude determination system is currently expected to include a combination of GPS antennae, star trackers, and Inertial Measurement Units (IMU).

3. TECHSAT 21 SIMULATION FACILITY

In order to facilitate system trade studies and gain confidence that an acceptable level of performance can be achieved, AFRL has initiated a program to construct an end-to-end simulation facility, called the Distributed Architecture Testbed (DAT), for the TechSat 21 program. In order to simulate the entire TechSat 21 system, the DAT structure has been modularized by functionality; these modules include: 1) dynamics and control, 2) radar (payload), 3) command and data handling, 4) flight software implementation, and 5) simulation control.

The dynamics and control module (DCM) has the task to simulate the natural motion of the satellites in six degrees of freedom (orbital and attitude). Additionally, DCM will host the orbit and attitude determination and control systems. The DCM will reside in a parallel computing environment allowing each satellite in the formation to reside on its own processor. This will reduce simulation time and more accurately represent the actual formation flying mission. The DCM is described pictorially in Fig. 4.

At the core of the DCM resides the Draper Semi-analytic Satellite Theory (DSST) orbital propagator [26-28]. The

DSST is a mean element propagator with accurate short periodic effect recovery. This allows accurate simulation of satellite motion as well as insight into the long term motion of the satellites, which is useful to the control algorithms.

The purpose of the DCM is to facilitate system level analysis and trade studies that affect changes to the structure, stability, or performance of the orbital and attitude control systems. Additionally, the DCM provides critical input to the other modules of the DAT which enable evaluation of mission performance metrics. Key areas for trade studies using the DCM are discussed in subsequent paragraphs.

Formation design

Different formation configurations will be tested for evaluation of fuel expenditure as well as mission performance. Tested formations will include in-track, circular, projected circular, and J_2 invariant formations. Variations on all the formation configurations will be examined for use in different potential applications of formation flying missions including the radar mission and geolocation. The goal will be to find formations that minimize fuel use and optimize system performance for a given application.

Formation Control Strategies

Simple and advanced strategies for formation control will be tested to evaluate their effects on fuel expenditure and mission performance. Control methodologies to be tested include PID, linear quadratic regulator/gaussian, feedback linearization, and nonlinear adaptive methods. These methods will be used to examine the control of the formation when the payload is not active, when the payload is active, initializing the formation, and changing formations during the mission. Potential formation control strategies will come from Texas A&M [16,17], Stanford [18], AFRL [29-33], and probably others.

Error Analysis

Errors in formation initialization will be simulated to determine the effect on the long term motion of the formation and on mission effectiveness. Errors in the application of control forces will be simulated to investigate the stability of each control methodology. The control forces will experience errors in both magnitude and direction. The magnitude errors are attributable to variations in the PPT thrusters while directional errors arise from both errors in the attitude determination system and attitude motion during the application of the control forces. This leads to an interesting coupling between the attitude and translational motions. This coupling must be examined to accurately determine formation control requirements and performance.

Propulsion System

Trade studies will be conducted for the combination PPT and μ PPT design and the PPT only design to determine the fuel cost and mission performance differences between the different systems. The studies will also examine the possibility and the need to control the attitude with either μ PPTs or PPTs.

4. SUMMARY AND CONCLUSIONS

With the arrival of miniaturized technology, small satellites have achieved a high level of capability that is enabling missions of satellite formations with distributed functionality. The AFRL TechSat 21 will demonstrate the

ability of satellite formations to perform the mission of a much larger, single satellite at greatly reduced cost. Several ongoing research efforts seek to resolve key issues related to the dynamics and control of such a satellite formation. Additionally, the simulation testbed at AFRL will provide end-to-end simulation capability. This will allow system level analysis and trade studies for the TechSat 21 program and other formation flying programs. In the area of dynamics and control these studies will include an examination of formation design, formation control strategies, error analysis, and propulsion system requirements.

REFERENCES

- [1] David C. Folta, Lauri Kraft Newman, and Tom Gardner, "Foundations of Formation Flying for Mission to Planet Earth and New Millennium," *Proceedings of the AIAA/AAS Astrodynamics Conference*, San Diego, CA, July 29-31, 1996, AIAA-96-3645.
- [2] Kim Luu *et al.*, "University Nanosatellite Distributed Satellite Capabilities to Support TechSat 21," *Proceedings of the 13th Annual AIAA/USU Conference on Small Satellites*, Logan, UT, August 23-26, 1999.
- [3] Kim Luu *et al.*, "Microsatellite and Formation Flying Technologies on University Nanosatellites," *Proceedings of the AIAA Space Technology Conference and Exposition*, Albuquerque, NM, September 28-30, 1999, AIAA 99-4535.
- [4] Alain Roux, "Cluster Regroups for Relaunch," *Aerospace America*, pp. 48-51, August 1998.
- [5] W. M. Folkner *et al.*, "LISA Orbit Selection and Stability," *Classical and Quantum Gravity* **14**, 1405-1410, 1997.
- [6] John Koss, Stephen Pohlig, Milton Trichel, and William Jeffrey, "Precision Radar Surveillance and Mapping with Discoverer II," *Proceedings of the AIAA Space Technology Conference and Exposition*, Albuquerque, NM, September 28-30, 1999, AIAA 99-4570.
- [7] Alok Das, Richard Cobb, and Michael Stallard, "TechSat 21: A Revolutionary Concept in Distributed Space Based Sensing," *Proceedings of the AIAA Defense and Civil Space Programs Conference and Exhibit*, Huntsville, AL, October 28-30, 1998, AIAA 98-5255.
- [8] Maurice Martin and Michael J. Stallard., "Distributed Satellite Technologies - The TechSat 21 Program," *Proceedings of the AIAA Space Technology Conference and Exposition*, Albuquerque, NM, September 28-30, 1999, AIAA 99-4479.
- [9] John H. Schilling, Ronald A. Spores, and Gregory G. Spanjers, "Micropropulsion Options for the TechSat 21 Space-Based Radar Flight," submitted to *Journal of Propulsion and Power*.
- [10] R. J. Sedwick, E. M. C. Kong, and D. W. Miller, "Exploiting Orbital Dynamics and Micropropulsion for Aperture Synthesis Using Distributed Satellite Systems: Applications to TechSat 21," *Proceedings of the AIAA Defense and Civil Space Programs Conference and Exhibit*, Huntsville, AL, October 28-30, 1998, AIAA 98-5289.

- [11] G. W. Hill, "Researches in the Lunar Theory," *American Journal of Mathematics* **1**, 5-26, 1878.
- [12] Chris Sabol, Richard Burns, and Craig McLaughlin, "Satellite Formation Flying Design and Evolution," *Proceedings of the AAS/AIAA Space Flight Mechanics Meeting*, Volume 102 in *Advances in the Astronautical Sciences*, 1999, AAS-99-121.
- [13] Hanspeter Schaub and Kyle T. Alfriend, "J₂ Invariant Relative Orbits for Spacecraft Formations," *Proceedings of the 1999 Flight Mechanics Symposium*, NASA Goddard Space Flight Center, Greenbelt, MD, pp. 125-139, May 1999, NASA/CP-1999-209235.
- [14] Dirk Brouwer, "Solution of the Problem of Artificial Satellite Theory without Drag," *Astronomical Journal* **64**, 378-397, 1959.
- [15] Dirk Brouwer and Gerald M. Clemence, *Methods of Celestial Mechanics*, New York: Academic Press, 1961.
- [16] Hanspeter Schaub, Srinivas R. Vadali, John L. Junkins, and Kyle T. Alfriend, "Spacecraft Formation Flying Control Using Mean Orbital Elements," *Proceedings of the AAS/AIAA Astrodynamics Specialists Conference*, Girdwood, AK, August 1999, AAS 99-310.
- [17] S. R. Vadali, H. Schaub, and K. T. Alfriend, "Initial Conditions and Fuel-Optimal Control for Formation Flying of Satellites," *Proceedings of the AIAA Guidance, Navigation and Control Conference*, Portland, OR, August 1999, AIAA 99-4265.
- [18] Gokhan Inalhan, Franz D. Busse, and Jonathon P. How, "Precise Formation Flying Control of Multiple Spacecraft Using Carrier-Phase Differential GPS," Submitted to 2000 AAS/AIAA Space Flight Mechanics Meetings, Clearwater, FL.
- [19] Jonathon P. How, Robert Twiggs, David Weidow, Kathy Hartman, and Frank Bauer, "Orion: a Low-Cost Demonstration of Formation flying in Space Using GPS," *Proceedings of the AIAA/AAS Astrodynamics Conference*, Boston, MA, August 1998, AIAA 98-4398.
- [20] Andrew Robertson, Gokhan Inalhan, and Jonathon P. How, "Spacecraft Formation Flying Control Design for the Orion Mission," *Proceedings of the AIAA Guidance, Navigation and Control Conference*, Portland, OR, August 1999, AIAA 99-4266.
- [21] A. H. Greenaway, "Alternative Approaches to Adaptive Optics," *Proceedings of the NATO Advanced Summer Institute on Adaptive Optics*, Cargese, Corsica, 1993.
- [22] Jeffrey A. Beck, *Relative Equilibria of a Rigid Satellite in a Central Gravitational Field*, Ph. D. Thesis, Air Force Institute of Technology, Wright-Patterson AFB, Ohio, September 1997.
- [23] Jeffrey A. Beck and Christopher D. Hall, "Relative Equilibria of a Rigid Satellite in a Circular Keplerian Orbit," *Journal of the Astronautical Sciences* **46**, 215-247, 1998.
- [24] P. K. C. Wang and F. Y. Hadaegh, "Coordination and Control of Multiple Microspacecraft Moving in Formation," *Journal of the Astronautical Sciences* **44**, 315-355, 1996.
- [25] P. K. C. Wang, F. Y. Hadaegh, and K. Lau, "Synchronized Formation Rotation and Attitude Control of Multiple Free-Flying Spacecraft," *Journal of Guidance, Control, and Dynamics* **22**, 28-35, 1999.
- [26] Wayne D. McClain, *A Semianalytic Artificial Satellite Theory Volume 1: Application of the Generalized Method of Averaging to the Artificial Satellite Problem*, 1992.
- [27] D. A. Danielson, et al., *Semianalytic Satellite Theory*, Technical Report NPS-MA-95-002, Naval Postgraduate School, 1994.
- [28] Martin E. Feiger, *An Evaluation of Semi-analytic Satellite Theory Against Long Arcs of Real Data for Highly Eccentric Orbits*, Masters Thesis, Department of Aeronautics and Astronautics, MIT, January 1987.
- [29] Vikram Kapila, Andrew Sparks, and Jim Buffington, "Spacecraft Formation Flying: Dynamics and Control," *Proceedings of the American Control Conference*, San Diego, CA, June 1999.
- [30] Andrew Sparks, "Satellite Formationkeeping Control in the Presence of Gravity Perturbations," Submitted to the 2000 American Control Conference, Chicago, IL.
- [31] R. K. Yedavalli and Andrew Sparks, "Satellite Formation Flying Control Design Based on Hybrid Control System Stability Analysis," Submitted to the 2000 American Control Conference, Chicago, IL.
- [32] Hsi-Han Yeh and Andrew Sparks, "Geometry and Control of Satellite Formations," Submitted to the 2000 American Control Conference, Chicago, IL.
- [33] Wei Kang and Andrew Sparks, "Coordinated Control of Multi-Satellite Systems," Submitted to the 2000 American Control Conference, Chicago, IL.

Rich Burns is a research engineer at the Air Force Research Laboratory Space Vehicles Directorate specializing in formation flying, satellite theory, and nonlinear dynamic systems. He is a current Ph.D. candidate at the University of Cincinnati in the Engineering Mechanics Department. He received his MS from Stanford University and his BS from the University of Notre Dame.

Craig McLaughlin is a research aerospace engineer at the Air Force Research Laboratory Space Vehicles Directorate specializing in GPS orbit determination, geolocation, and formation flying. He received his Ph.D. and MS in Aerospace Engineering Sciences from the University of Colorado and his BSAE from Wichita State University.

Jesse Leitner received the BS degree in Aerospace Engineering from the University of Texas and the MS and Ph.D. degrees from Georgia Tech. He is currently an Aerospace Engineer with NASA at Goddard Space Flight Center in Greenbelt, MD. His current areas of interest are in nonlinear, adaptive control, and neural networks.

Maurice Martin is the TechSat 21 program manager at the Air Force Research Laboratory Space Vehicles Directorate.

