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13. ABSTRACT (Maximum 200 words)

This grant is to support Dr. Yaakov S. Weinstein, a graduate student in the Dept. of Nuclear Science and Engineering at MIT, during his studies of Quantum Computing in the Laboratory of David G. Cory and Timothy F. Havel in the same department, and Seth Lloyd in Mechanical Engineering at MIT.

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Statement of the Problems Studied

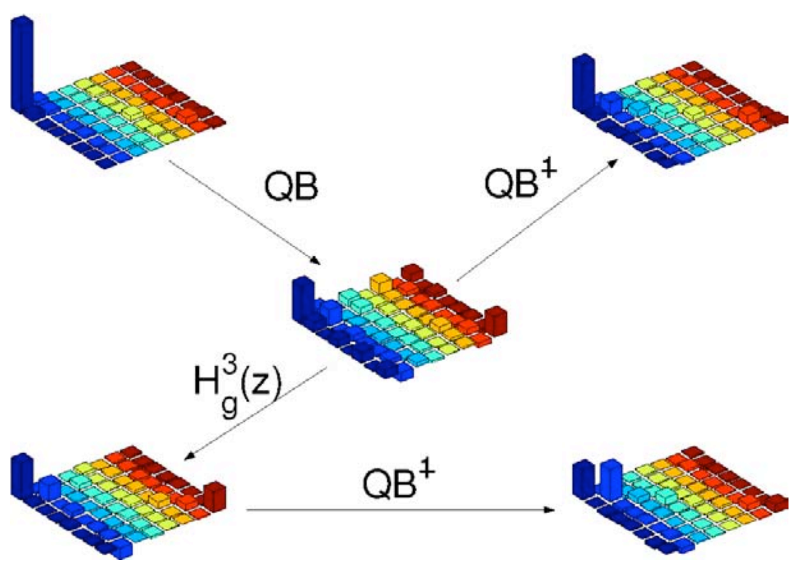
The graduate student on this project, Yaakov S. Weinstein, focussed on the problem of quantifying the level of control over a quantum system, using liquid-state NMR as a test-bed for the development and evaluation of the techniques and metrics he was instrumental in developing. The most direct and complete approach to this problem, quantum process tomography, is also extremely labor-intensive even for the seemingly modest three qubit quantum Fourier transform we chose for this study – which took the entire duration of this grant to complete. At the same time as he was working on this daunting project, he helped to develop other far less laborious methods that could nevertheless give useful information. This led him to study the difficult subjects of quantum chaos and random maps, culminating perhaps in an efficient method of generating pseduo-random unitaries. He is now continuing his work in quantum chaos and random maps as postdoctoral fellow with Stephen Hellberg at the Naval Research Lab.

Summary of the Most Important Results

In the following, we simply reprint the abstracts from each of the peer-reviewed publications listed in the following section, using the same order and reference numbers. Also provided for convenience is the Physics Preprint Server's identification number (http://arxiv.org), when available.

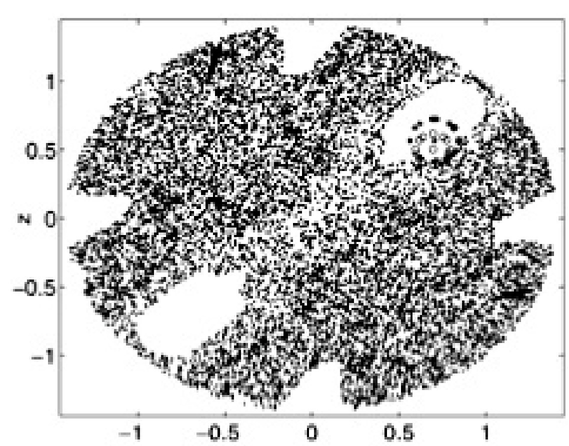
1. Experimental Implementation of the Quantum Baker's Map (quant-ph/0201064)

This paper reports on the experimental implementation of the quantum baker's map via a three bit nuclear magnetic resonance (NMR) quantum information processor. The experiments tested the sensitivity of the quantum chaotic map to perturbations. In the first experiment, the map was iterated forward and then backwards to provide benchmarks for intrinsic errors and decoherence. In the second set of experiments, the least significant qubit was perturbed in between the iterations to test the sensitivity of the quantum chaotic map to applied perturbations. These experiments are used to investigate previous predicted properties of quantum chaotic dynamics.



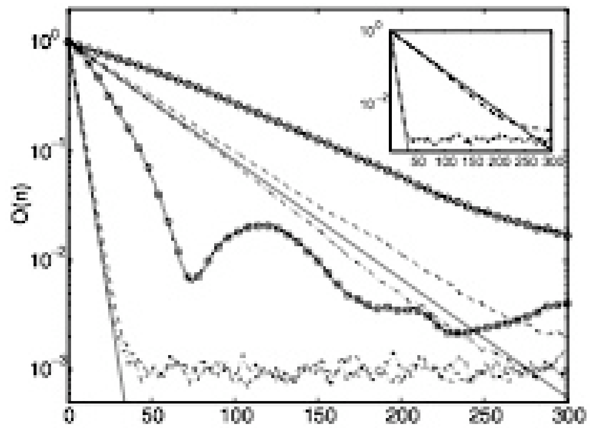
2. Border between Regular and Chaotic Quantum Dynamics (quant-ph/0206039)

We identify a border between regular and chaotic quantum dynamics. The border is characterized by a power law decrease in the overlap between a state evolved under chaotic dynamics and the same state evolved under a slightly perturbed dynamics. For example, the overlap decay for the quantum kicked top is well fitted with $[1+(q -1) (t/\tau)^2]^{1/(1-q)}$ (with the nonextensive entropic index q and τ depending on perturbation strength) in the region preceding the emergence of quantum interference effects. This region corresponds to the edge of chaos for the classical map from which the quantum chaotic dynamics is derived.



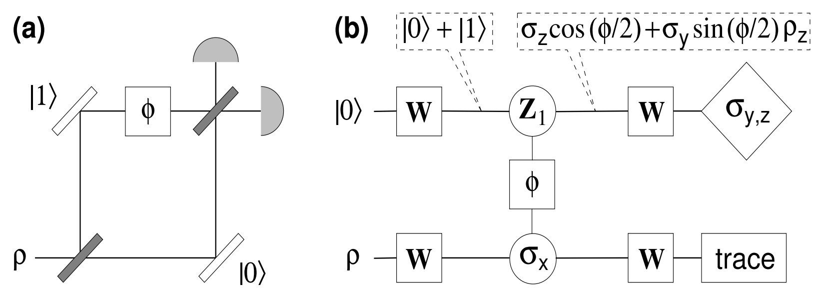
3. Fidelity Decay as an Efficient Indicator of Quantum Chaos (quant-ph/0207099)

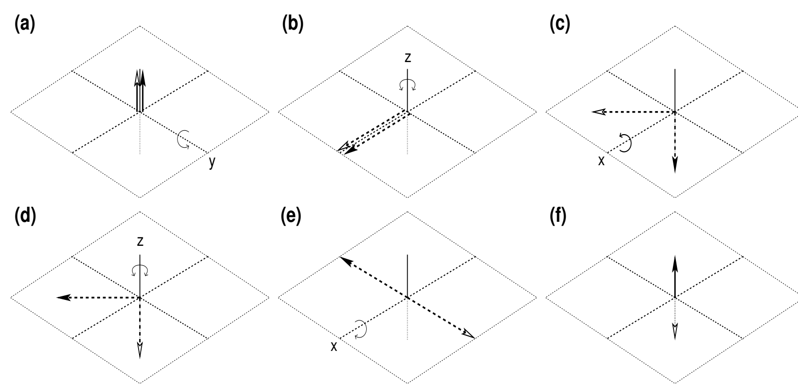
Recent work has connected the type of fidelity decay in perturbed quantum models to the presence of chaos in the associated classical models. We demonstrate that a system's rate of fidelity decay under repeated perturbations may be measured efficiently on a quantum information processor, and analyze the conditions under which this indicator is a reliable probe of quantum chaos and related statistical properties of the unperturbed system. The type and rate of the decay are not dependent on the eigenvalue statistics of the unperturbed system, but depend on the system's eigenvector statistics in the eigenbasis of the perturbation operator. For random eigenvector statistics the decay is exponential with a rate fixed precisely by the variance of the perturbation's energy spectrum. Hence, even classically regular models can exhibit an exponential fidelity decay under generic quantum perturbations. These results clarify which perturbations can distinguish classically regular and chaotic quantum systems.

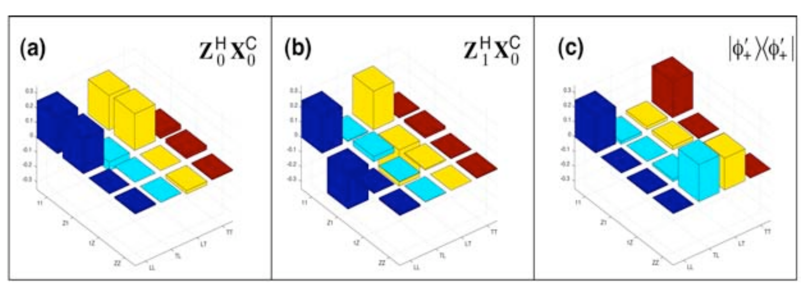


4. Quantum Information Processing by Nuclear Magnetic Resonance Spectroscopy (N/A)

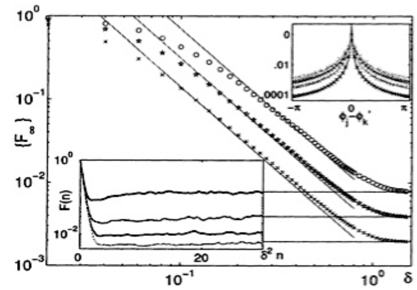
Nuclear magnetic resonance (NMR) is a direct macroscopic manifestation of the quantum mechanics of the intrinsic angular momentum of atomic nuclei. It is best known for its extraordinary range of applications, which include molecular structure determination, medical imaging, and measurements of flow and diffusion rates. Most recently, liquid-state NMR spectroscopy has been found to provide a powerful experimental tool for the development and evaluation of the coherent control techniques needed for quantum information processing. This burgeoning new interdisciplinary field has the potential to achieve cryptographic, communications, and computational feats far beyond what is possible with known classical physics. Indeed, NMR has made the demonstration of many of these feats sufficiently simple to be carried out by high school summer interns working in our laboratory. In this paper the basic principles of quantum information processing by NMR spectroscopy are described, along with several illustrative experiments suitable for incorporation into the undergraduate physics curriculum. These experiments are spin–spin interferometry, an implementation of the quantum Fourier transform, and the quantum simulation of a harmonic oscillator.





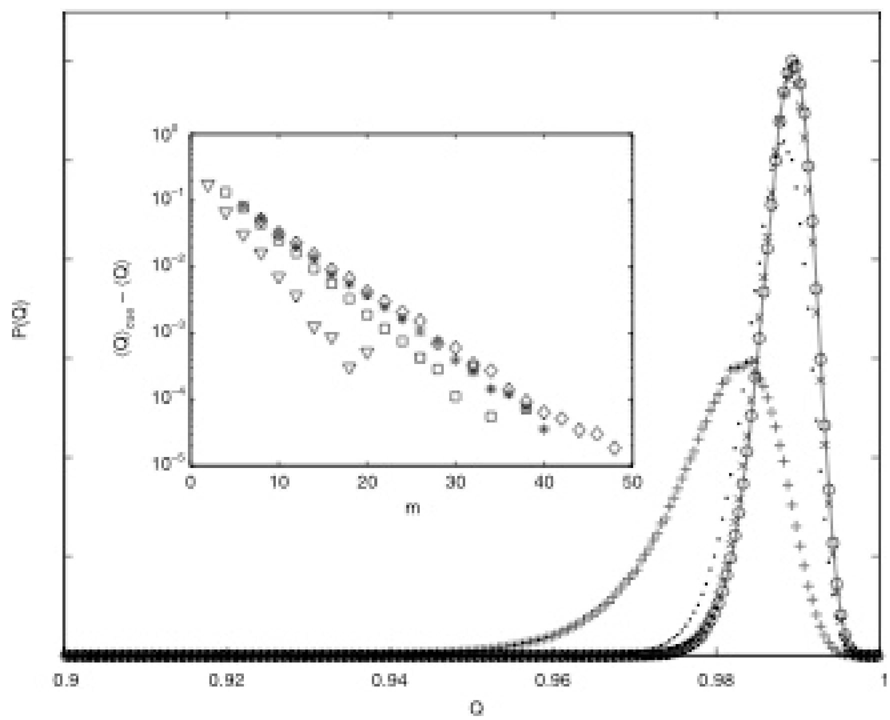


5. Fidelity Decay Saturation Level for Initial Eigenstates (quant-ph/0210063)



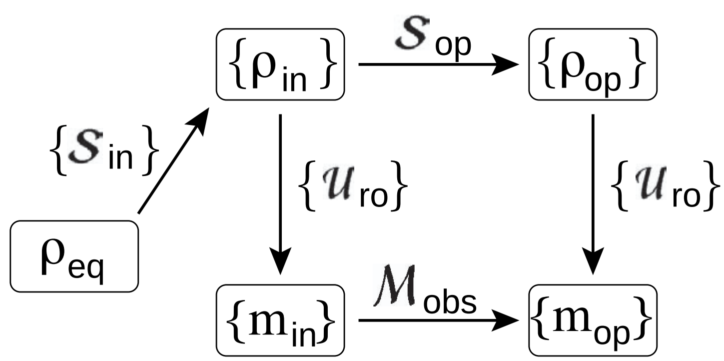
We show that the fidelity decay between an initial eigenstate state evolved under a unitary chaotic operator and the same eigenstate evolved under a perturbed operator saturates well before the 1/N limit, where N is the size of the Hilbert space, expected for a generic initial state. We provide a theoretical argument and numerical evidence that, for intermediate perturbation strengths, the saturation level depends quadratically on the perturbation strength.

6. Pseudo-Random Unitary Operators for Quantum Information Processing (N/A)



In close analogy to the fundamental role of random numbers in classical information theory, random operators are a basic component of quantum information theory. Unfortunately, the implementation of random unitary operators on a quantum processor is exponentially hard. Here we introduce a method for generating pseudo-random unitary operators that can reproduce those statistical properties of random unitary operators most relevant to quantum information tasks. This method requires exponentially fewer resources, and hence enables the practical application of random unitary operators in quantum communication and information processing protocols. Using a nuclear magnetic resonance quantum processor, we were able to realize pseudorandom unitary operators that reproduce the expected random distribution of matrix elements.

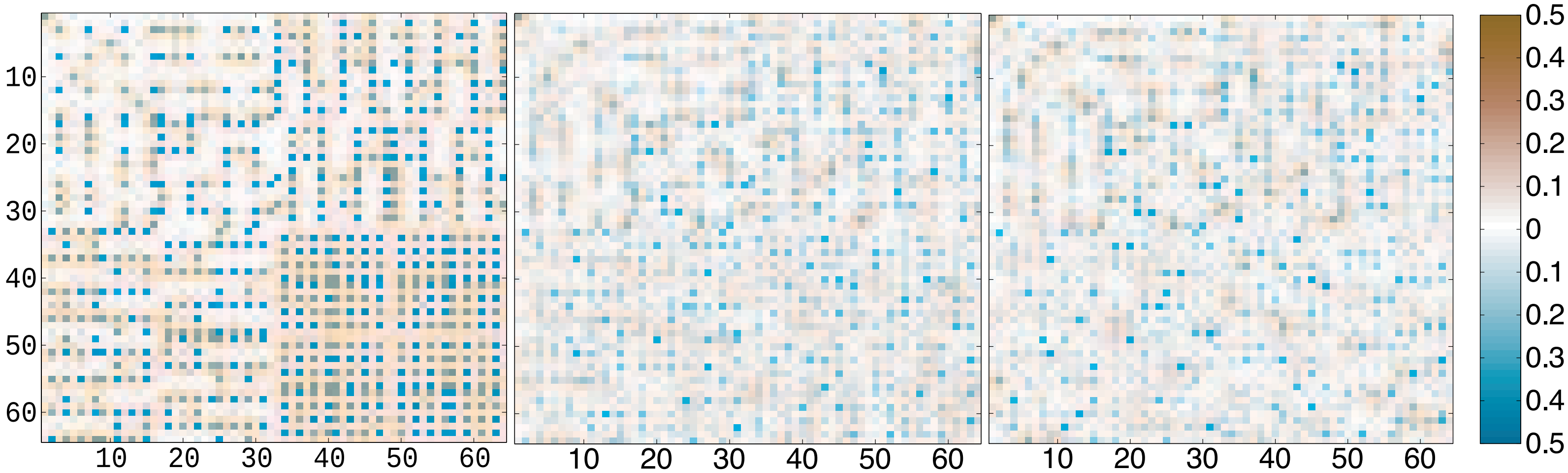
7. Quantum Process Tomography of the Quantum Fourier Transform (quant-ph/0406239)



$$\mathcal{U}_{\text{ro}}(\rho) \equiv \int dq\, U_{\text{ro}}^{(q)} \rho\, U_{\text{ro}}^{(q)\dagger}$$

$$\mathrm{m}_{\text{xx}} \equiv tr(\mathcal{U}_{\text{ro}}(\rho_{\text{xx}})\sigma_-)$$

The results of quantum process tomography on a three-qubit nuclear magnetic resonance quantum information processor are presented, and shown to be consistent with a detailed model of the system-plus-apparatus used for the experiments. The quantum operation studied was the quantum Fourier transform, which is important in several quantum algorithms and poses a rigorous test for the precision of our recently-developed strongly modulating control fields. The results were analyzed in an attempt to decompose the implementation errors into coherent (overall systematic), incoherent (microscopically deterministic), and decoherent (microscopically random) components. This analysis yielded a superoperator consisting of a unitary part that was strongly correlated with the theoretically expected unitary superoperator of the quantum Fourier transform, an overall attenuation consistent with decoherence, and a residual portion that was not completely positive - although complete positivity is required for any quantum operation. By comparison with the results of computer simulations, the lack of complete positivity was shown to be largely a consequence of the incoherent errors during the quantum process tomography procedure. These simulations further showed that coherent, incoherent, and decoherent errors can often be identified by their distinctive effects on the spectrum of the overall superoperator. The gate fidelity of the experimentally determined superoperator was 0.64, while the correlation coefficient between experimentally determined superoperator and the simulated superoperator was 0.79; most of the discrepancies with the simulations could be explained by the cummulative effect of small errors in the single qubit gates.



List of Publications

Papers in peer-reviewed journals

1. Y. S. Weinstein, S. Lloyd, J. Emerson and D. G. Cory, “Experimental implementation of the quantum baker’s map,” *Phys. Rev. Lett.* **89**(**15**), **157902** (**2002**).

2. Y. S. Weinstein, S. Lloyd and C. Tsallis, “Border between regular and chaotic quantum dynamics,” *Phys. Rev. Lett.* **89**(**21**), **214101** (**2002**).

3. J. Emerson, Y. S. Weinstein, S. Lloyd and D. G. Cory, “Fidelity decay as an efficient indicator of quantum chaos,” *Phys. Rev. Lett.* **89**(**28**), **284102** (**2002**).

4. T. F. Havel, D. G. Cory, N. Boulant, E. M. Fortunato, M. A. Pravia, G. Teklemariam, Y. S. Weinstein, A. Bhattacharyya and J. Hou, “Quantum information processing by nuclear magnetic resonance,” *Am. J. Phys.* **70**(**3**), **345–62** (**2002**).

5. Y. S. Weinstein, J. Emerson, S. Lloyd and D. G. Cory, “Fidelity decay saturation level for initial eigenstates,” Quantum Information Processing **1**(**6**), **439–48** (**2002**).

6. J. Emerson, Y. S. Weinstein, M. Saracenos, S. Lloyd and D. G. Cory, “Pseudo-random unitary operators for quantum information processing,” *Science Mag.* **302**(**5653**), **2098–2100** (**2003**).

7. Y. S. Weinstein, T. F. Havel, J. Emerson, N. Boulant, M. Saracenos, S. Lloyd and D. G. Cory, “Quantum process tomography of the quantum Fourier transform,” *J. Chem. Phys.* **121**(**13**), **6117–33** (**2004**).

Papers in non-peer-reviewed conference proceedings

8. Y. S. Weinstein, C. Tsallis and S. Lloyd, “On the emergence of nonextensivity at the edge of quantum chaos,” in *Decoherence and Entropy in Complex Systems*, *Lect. Notes Phys.* **633**, **385–397**, (Springer-Verlag, überalles, **2003**).

Papers presented at conferences but not published

9. Y. S. Weinstein, J. Emerson, M. Saracenos, N. Boulant, S. Lloyd and D. G. Cory, “Quantum process tomography of the quantum Fourier transform,” *Quantum Computing Program Review*, Nashville, TN (August, **2003**).

Participating Personnel and Degrees Granted

This grant was to support Dr. Yaakov S. Weinstein, a graduate student in the Dept. of Nuclear Science and Engineering at MIT, during his studies of Quantum Computing in the Laboratory of David G. Cory and Timothy F. Havel in the same department, and Seth Lloyd in Mechanical Engineering at MIT. Dr. Weinstein received his Ph.D. from MIT in December, 2004.

Titles of Inventions

N/A

Bibliography

See above List of Publications.