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A Network-based Mathematical Programming Approach to Optimal Rostering of Continuous Heterogeneous Workforces

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Keywords: Rostering, Continuous operations, Heterogeneous workforces, Network flows, Mathematical programming

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1. Introduction

Personnel scheduling is one of the most difficult, important, and studied problems in operations research. The optimal choice of the number of employees required to meet customer demand, shift start and stop times, daily lunches and breaks, and the assignment of the adequately skilled employee to the best shift is a classic combinatorial optimization problem. Managers who attempt to manually solve the personnel scheduling problem expend many valuable work hours to find even a feasible solution which has little probability of being optimal based on any objective function. Sub-optimal scheduling increases an industry's tangible costs, not only through the consumption of a manager's time, but also through the misallocation of shifts to meet customer demand and through employees staffing shifts for which they are not qualified. Furthermore, a sub-optimal schedule will increase intangible costs such as lower employee moral from dissatisfaction with a poor work schedule. Eliminating these costs is an easy way to increase the profitability of any business.

Personnel scheduling is a necessary chore associated with any organization employing people to get work accomplished. Some organizations that employ a professional workforce during traditionally scheduled business hours find the personnel scheduling problem a trivial concern. However, industries that rely on a large portion of part-time employees with highly constrained availabilities (work hours) and require staffing for the entire day and evening hours or, in particular, continuous (24-hour) operations have a much more challenging problem. Such industries include fast-food restaurants, hotels and resorts, grocery stores, customer assistance telephone services,

hospitals, and others. The combined salary cost of this segment of the economy is quite large. A timely and optimal solution to their personnel scheduling problem will save managers valuable time, meet customer demand at minimum labor cost, and increase worker moral by matching each employee to the best shift possible. This article presents an extremely fast, computationally efficient, and optimal network-flow based linear programming solution for the extremely challenging integer programming problem of rostering a continuous heterogeneous workforce with realistic constraints.

Personnel scheduling is generally decomposed into more tractable sub-problems. The three primary sub-problems are demand modeling, shift selection, and employee tour scheduling or rostering. Demand modeling determines the number of employees required on duty during a given time interval to satisfy customer needs. Forecasting and queuing theory are the primary tools applied to demand modeling. A typical chart of the number of employees required, illustrating the results of demand modeling, is shown in Figure 1.1.

The second primary personnel scheduling sub-problem is shift selection. Shift selection assigns consecutive hour shifts to satisfy the customer demand while meeting organizational and regulatory requirements for shift lengths, breaks, and mealtime allowances. The objective of shift selection is generally to minimize the number of hours scheduled that exceed customer demand. Figure 1.2 shows an example of a set of shifts that will optimally cover the number of employees required in Figure 1.1.
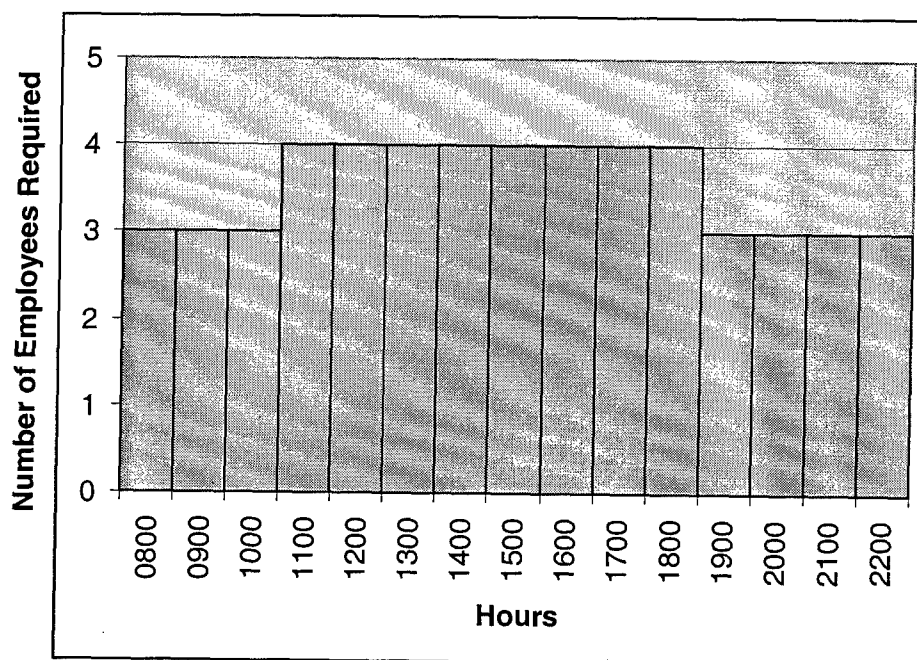
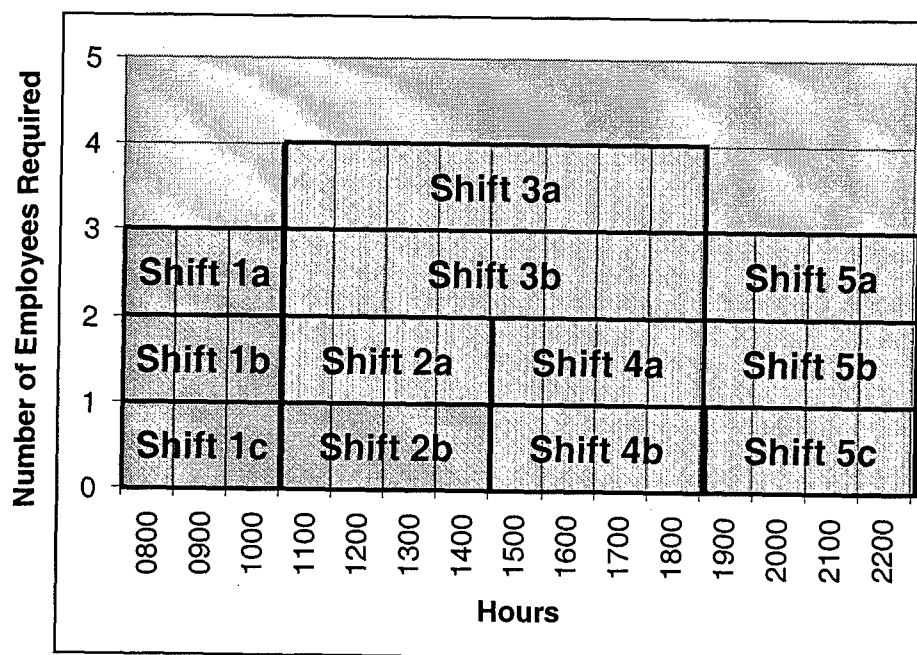Figure 1.1   Employees requirement by hour from demand modeling

Figure 1.2   Example set of shifts that cover the demand in Fig. 1.1

The third personnel scheduling sub-problem is tour scheduling also know as employee rostering. Tour scheduling assigns individuals to specific shifts detailed during shift selection. A heterogeneous workforce is a collection of personnel who have significantly different availabilities, skill sets, and seniorities. Tour scheduling a heterogeneous workforce must consider all of these differences and optimally match the personnel with the best shift possible for which they are eligible. Additionally, considerations for individuals working a minimum and maximum number of shifts per week and insuring adequate rest between shifts for industries with continuous (24-hour) operations are critical to an optimal tour schedule. Heterogeneous and continuous workforces are realistic considerations for many industries, as previously noted. Employee shift preferences and management employee weighting comprise the objective function.

This methodology provides several original contributions to the personnel scheduling literature. The following list summarizes the advancements proposed by this research.

- Provide a solution methodology that simultaneously accounts for variations in employee availabilities, skill sets, and preferences.

- Provide a methodology that rosters employees for continuous operations by allowing for rest hours between shifts.

- Provide a network-flow based formulation that rapidly solves the tour scheduling problem, providing integer answers without any need for branching, bounding, or cutting schemes.

This article is organized in the following manner. Section 2 contains a survey of the pertinent literature and important previous work accomplished in personnel tour scheduling. Section 3 details the tour scheduling problem at Arizona State University for computer lab technicians. Our methodology is general, but we feel that for realism and clarity we will introduce our case study early. Section 4 provides the proposed solution method and model. Section 5 shows the results of the methodology as applied to the case study as well as applied to larger instances of the IP tour scheduling problem. Chapter 6 provides conclusions and recommendations.

2. Literature Survey

A vast amount of scholastic work has been accomplished on the personnel scheduling problem. This work has been documented in a wide array of refereed journals, conference proceedings, and lecture notes. Personnel scheduling spans a wide range of problem instances, specific applications, and solution techniques and methodologies. Many survey papers of the pertinent literature in the area have been written. Some of the most extensive and most recent include Ernst, et. al. (2004a), Ernst, et. al. (2004b), and Alfares (2004).

2.2 Application Areas

Personnel scheduling has a broad range of application. In their article, Ernst, et. al. (2004b) survey over 700 papers in the personnel scheduling literature. They define application areas and catalog prior work in each application area. We refer the reader to this article for additional detail of the categorization. The articles from each

application area may concentrate on all or part of the problem instances described as it pertains to the specific area.

The application areas pertinent to our research are characterized by flexible demand, a heterogeneous or mixed workforce, possible continuous operations, and personnel preference consideration. Alfares (2004) defines a mixed workforce as employees having differing "skill level, learning rate, wage, availability, and work hours." Continuous operations are characterized by demand requirements 24 hours a day. Important application areas of interest as defined by Ernst, et. al. (2004b) are Hospitality and Tourism, Financial Services, and Sales. Although these areas represent a significant proportion of economic activity, they form only a small portion of the personnel scheduling literature, only 16 of the over 700 papers surveyed. Furthermore, the subset of these papers examining the tour scheduling problem is a further reduction of the papers surveyed.

2.2.1 Hospitality and Tourism Tour Scheduling

Hospitality and tourism industries as defined include "hotels, tourist resorts and restaurants" (Ernst, et. al. 2004b). These industries have flexible demands and night and weekend shift requirements as well as employees with varying skill sets. Eveborn and Ronnqvist (2004) propose an elastic set-partitioning model and a branch-and price algorithm to solve the tour scheduling problem. The methodology is imbedded in a general scheduling software package called SCHEDULER. Glover and McMillan (1986) employ a tabu search and Thompson (1996) makes use of simulated annealing. Loucks and Jacobs (1991) begin by using a goal programming approach and then solve the corresponding integer program using a two-phase heuristic. A heuristic approach

combined with a tabu search is developed by Litchfield, Ingolfsson, and Cheng (2003) to roster a restaurant. Finally, Love and Hoey (1990) define a mixed-integer program and solve the tour scheduling problem using a minimum cost network flow simplex algorithm. This paper is of particular interest and will be discussed in further detail.

2.2.2 Financial Services Tour Scheduling

Financial services tour scheduling is applied to staffing "clerical workers in service industries such as banking and insurance." Again, the industry is characterized by flexible demand and by full and part-time workers. Li, Robinson, and Mabert (1991) and Mabert and Raedels (1977) use heuristics to roster workers, the former considering differing skill sets. Mabert and Watts (1982) propose simulation to solve a set-covering formulation and Mould (1996) develops a spreadsheet decision support system (DSS) to allow employers to explore different tour schedules.

2.2.3 Sales Tour Scheduling

Retail sales tour scheduling has received the least amount of attention in the literature. Glover, McMillan, and Grover (1985) develop a DSS that uses a heuristic and Haase (1999) shows a column-generation technique to solve an integer-programming formulation.

2.3 Solution Techniques and Methodologies

The most important aspect of this literature review is to detail the prior work in methodology to solve personnel scheduling problems. Ernst, et. al. (2004b) show a break down of solution techniques and methodologies as was shown for various application areas. The following table shows the techniques and the number of papers using that methodology. Papers employing more than one solution technique appear in

multiple categories. The methodology of particular interest to our research in this article is network flow modeling. The following sections will show the application of the network flow modeling literature to the personnel scheduling problem instances with a concentration on the tour scheduling problem.

2.3.1 Network Flow Tour Scheduling (Airline, Mass Transit, Nurse Applications)

Network flow models have been used extensively in the airline industry. Yan and Chang (2002) and Barnhart, et. al. (1994) solve shortest-path network problems in a column generation formulation to pair airline crews to flight schedules. Column generation using network models to price new columns is proposed by Mason and Smith (1998). Mellouli (2001) uses a "state-expanded aggregated time-space network" to solve airline and rail crew scheduling. Nicoletti (1975) uses assignment sub-problems within a constructive heuristic, and Tingley (1979) sequentially solves assignment and matching problems to roster airlines.

Public and mass transit systems are also well addressed in the tour scheduling literature. Patrikalakis and Xerocostas (1992) develop an approach that uses a network flow problem with side constraints to schedule vehicles for the Athens Area Urban Transport Organization. Carraresi and Gallo (1984) combine a matching approach and Lagrangean relaxation for crew rostering in mass transit. Borndorfer et. al. (2001) apply a column generation technique using network flow sub-problems for duty scheduling in European public transit. Ball and Benoit-Thompson (1988) sequentially solve shortest path and matching problems within a Lagrangean relaxation based heuristic. Banihashemi and Haghani (1991) propose to solve mass transit crew scheduling using a multi-commodity flow model.

Another popular application area for network flow modeling is nurse scheduling. Dowsland and Thompson (2000) mix knapsack, network flow, and tabu search techniques to develop nurse rosters. Jaumard, Semet, and Vovor (1998) solve shortest-path network sub-problems within a column generation integer program. Millar and Kiragu (1998) develop cyclic and acyclic work schedules for nurses using a shortest-path problem with side constraints. Finally, Moz and Pato (2004) solve a nurse re-rostering problem using multi-commodity flow models.

Although the use of network modeling has been well explored in the literature for airline, mass transit, and nurse tour scheduling, these problems differ in nature from our problems of interest. In general, airlines, mass transit, and nursing applications are not characterized by flexible demand and additionally employ a homogeneous workforce where each employee has similar availabilities, skill-sets, and work hours. These differences dramatically affect the formulation of the tour scheduling problem. Additional constraints needed to model employee availability, minimum and maximum work hours, consecutive shift restrictions, and skill-set eligibility are examples of the dissimilarities.

2.3.2 Network Flow Tour Scheduling with a Heterogeneous Workforce

Tour scheduling a work force with varying daily and weekly availabilities, skill-sets, minimum and maximum work hours, and wage rates has not been given a proportionate amount of attention in the personnel scheduling literature. Furthermore, only Love and Hoey (1990) attack this problem from a network flow approach. Love and Hoey define a mixed-integer linear program (MILP) for shift scheduling and tour scheduling for a fast-food restaurant. They then decompose the MILP into two sub-

problems. This is the same approach that we will take. The first sub-problem defines the shifts needed to cover demand. The second sub-problem minimizes the rest of the objective function. As Love and Hoey (1990) state: "each column has at most two nonzero entries and these equations can be constructed such that each column with the two nonzero entries has one +1 and one -1, so this second sub-problem can be solved as a minimum cost network flow problem." They propose solving these sub-problems using network simplex algorithms.

Although Love and Hoey (1990) show that a tour scheduling problem can be solved with a minimum cost network flow, there are limitations to the complexity of the heterogeneous problem that can be solved with their formulation. As acknowledged by the authors, this formulation does not perform well for continuous operations. Additionally, there is no discussion of employees who have differing skill sets or the more complex case of employees with overlapping and differing skill sets. A more robust tour schedule must take into account the specific skill-sets of the employees and the skills required per shift.

Our methodology uses an expanded network model to allow for varying skill sets and specialized side constraints to manage continuous operations. As we will see, the expanded network formulation with specialized side constraints forms a linear program (LP) whose solutions are integer.

3. A Case Study - ASU Computer Lab Technician Tour Scheduling

There are four computer labs at Arizona State University(ASU): the Atrium, BAC, GWC, and ECG. Each lab is staffed by technical experts (typically enrolled ASU

science and engineering students), who assist other students using the computers in the lab. Currently, the Technology Support Analyst Principal (TSAP) is tasked with scheduling the technicians to meet the historic demand for each lab based upon time of day. Each technician has a subset of the four labs in which they are qualified to work. The TSAP currently schedules over fifty technicians to the various labs by hand. He must take into consideration the demand for technicians as a function of day and time, availability and non-availability of the technicians, shift restrictions and maximum hours per week, minimum hours per week, as well as the preferences and qualifications of the technicians for particular shifts. Each such effort requires 2-3 days and a satisfactory solution is the first feasible one found.

Each lab has its own operating hours and staffing requirements. For example, Table 3.1 depicts the operating hours and staff requirements for the Atrium. Although there must be no staff during closed hours, and at least one technician working each operating hour, there is some flexibility regarding the staffing levels during operating hours. Staffing the lab in excess of the requirements in the table will result in excess costs to the university. Staffing the labs at a lower level is not allowed.

	7a	8a	9a	10a	11a	12p	1p	2p	3p	4p	5p	6p	7p	8p	9p	10p	11p
Sunday			3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Monday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Tuesday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Wednesday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Thursday	3	3	3	4	4	5	5	5	4	4	3	3	4	4	4	4	3
Friday	3	3	3	4	4	5	5	5	4	4	3	3	4				
Saturday	3	3	3	4	4	4	4	4	4	4	4	3	3	3	3		

Table 3.1 Lab technician requirements (Atrium) based on hourly demand

Each technician has his/her own availability profile and preferences among those hours. The minimum and maximum shifts per week are developed by the TASP after his consultation with each technician.

A two-phase approach to the tour scheduling problem for ASU computer labs is proposed as depicted in Figure 3.1. In Phase 1, we select a set of shifts to cover the staffing requirement for each of the four labs. The scheduler can indicate shift length priorities in order to bias the model in favor of specific shift lengths. For example the scheduler can specify a preference for 6-hour shifts as opposed to 3-hour shifts by assigning them different weights. Additionally, benefits arise from the fact that selecting the shifts for a single lab has no effect on which shifts should be selected for the other three. This independence allows for the selection of shifts for each of the four labs to be done individually, resulting in four smaller Phase 1 models.
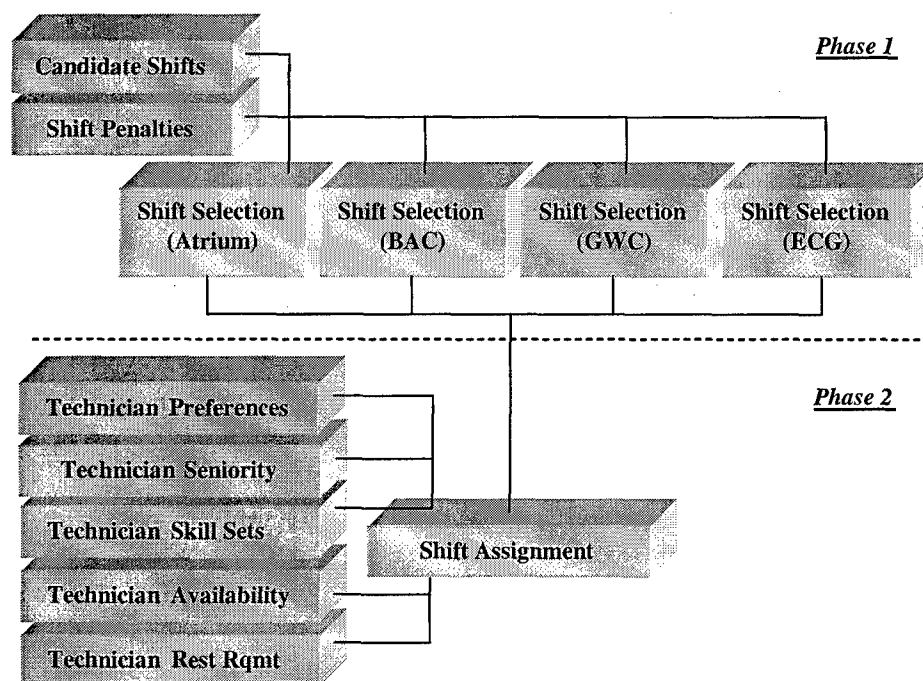
Figure 3.1 Two-phase methodology for shift selection and rostering

In Phase 1, we consider all reasonable shift lengths over the planning horizon. The objective of this phase is to select a set of shifts that covers the lab requirements, as well as minimizes the sum of "shift penalties". Shift penalties are assigned based on the scheduler's preference for a particular shift length. A penalty of 1.0 signifies that no extra weight is given to the hours in a given shift length. Phase 1 then minimizes the total cost of selecting individual shifts, where the cost of each shift is the product of the number of hours and the scheduler's penalty factor for the shift. The penalties for shift lengths solicited from the TSAP for our case study are contained in Table 3.2.

Shift Length	Penalty Factor	Cost
3	1.05	3.15
4	1.00	4.00
5	1.00	5.00
6	1.00	6.00
7	1.10	7.70
8	1.15	9.20

Table 3.2 Shift penalties used to in Phase 1 set covering problem

For example, consider the two possible shift selections with regard to Saturday mid-day lab requirements in Figure 3.2 below. The solutions show the shift lengths chosen as well as the number of employees on each shift. The value of solution 1 is 53.45, whereas the value of solution 2 is 59.9. Solution 1 happens to be the optimal

solution. Both solutions cover the demand profile exactly, but Solution 1 uses shift lengths preferred by the manager.

Saturday	3	3	3	4	4	4	4	4	4	4	4	3	3	3	3		
solution 1	shift 1 (3)			shift 2 (4)				shift 3 (4)				shift 4 (3)					
solution 2	shift 1 (3)							shift 2 (3)									
			shift 3 (1)														

Figure 3.2 Example of effect of preference on two distinct shift definitions

The Phase 1 problem is not computationally challenging. A set-covering integer program (IP) is directly used to solve Phase 1. It selects the optimal set of shifts to cover customer demand based on shift length preferences. In Phase 2, we must assign employees to these shifts, subject to employee preferences by hour, availabilities, skills, rest periods as well as some notion of employee preference by management. This is a very difficult problem to solve efficiently and is the focus of this article.

4. Proposed Tour-Schedule Solution Method and Model

The tour-scheduling problem is inherently a binary set-covering problem. Therefore, as problem size increases, the efficiency of the associated IP decreases rapidly. However, we have developed a formulation of the problem as a minimum cost network-flow, using an arc capacity method, so that the resulting network structure always provides integer binary answers. The network structure can easily be written as an LP and solved using fast solution algorithms such as the CPLEX interior point method. Solutions of this formulation do not require any branching, bounding, or cutting

schemes to find integer solutions – the integer solutions are a consequence of the formulation method. Therefore, very large problem instances can be solved in a computationally insignificant amount of time compared to the corresponding IP solution. Figure 4.1 shows our formulation of our generalized tour-scheduling problem as a minimum cost network-flow.
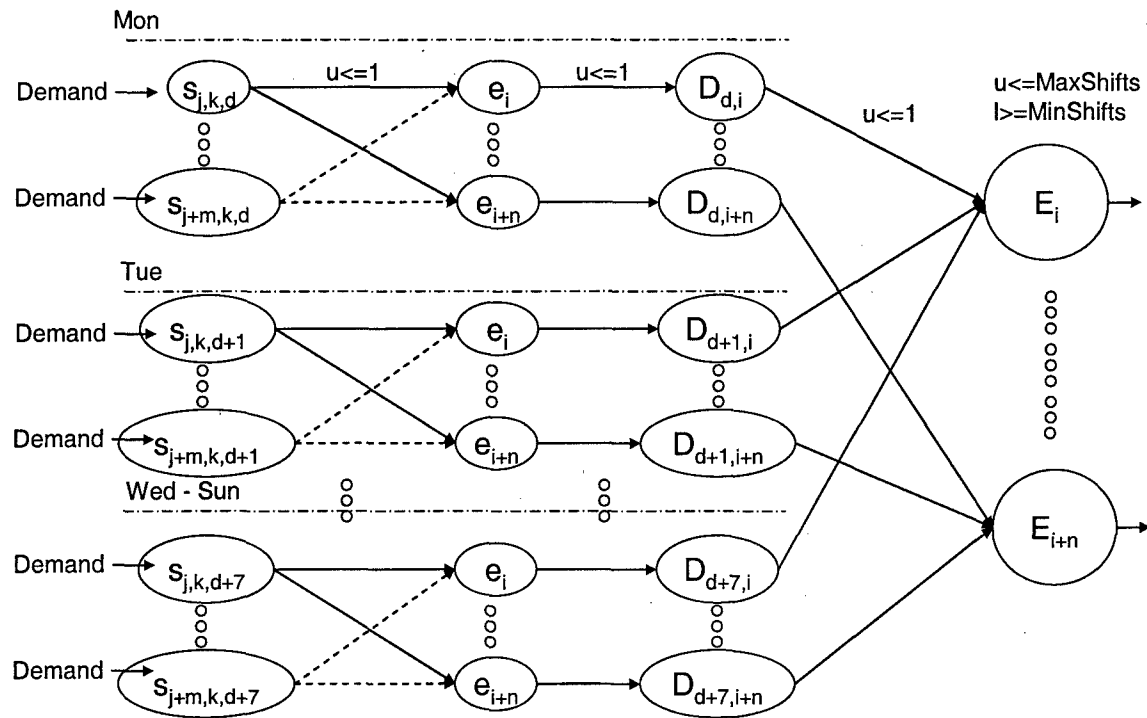

Figure 4.1 Network representation of the Phase 2 tour-scheduling problem

Here, $s_{j,k,d}$ denotes shift number j requiring skill set k and on day d, e_i is employee number i, $D_{d,i}$ is the total number of shifts, D, on day d for employee i, and E_i is the total number of shifts per week, E, for employee i.

The number of employees needed for each shift node flows from the Demand arc. Each shift node has an edge to each qualified and available employee node. The

shift-to-employee arc is capacitated at 1, meaning only one of the required staffing for a shift can be assigned to a single employee. Each employee then flows their daily work assignment to the employees' daily-total-node, D. This arc is capacitated at 1, meaning each employee can work only one shift per day. Finally, the employees' daily-total-nodes channel the flow to the employees' weekly-total-node which contains the capacities to enforce min and max shifts per week.

The balance equations for the above network flow formulation are:

$$\sum_i s_{j,k,d} e_i = demand \qquad \text{for all } j,k,d \qquad (4.1)$$

$$\sum_{j,k} s_{j,k,d} e_i = D_{d,i} \qquad \text{for all } i,d \qquad (4.2)$$

$$\sum_d D_{d,i} \leq \max number\_of\_shifts \qquad \text{for all } j \qquad (4.3)$$

$$\sum_d D_{d,i} \geq \min number\_of\_shifts \qquad \text{for all } j \qquad (4.4)$$

$$D_{d,i} \leq 1 \qquad \text{for all } i,d \qquad (4.5)$$

Constraint set 1 requires that the demand for a number of employees for each type of shift is met. Set 2 totals the shifts per employee for each day and requires that the total be less than or equal to 1 in constraint set 5 (no employee works more than one shift per day, but remember that the shift length is a variable). Constraint sets 3 and 4 require that the minimum and maximum numbers of shifts per week are met for each employee. The following three sub-sections describe how our model incorporates all the constraints needed to create a tour-schedule for a real world business, including a heterogeneous work force and incorporate consecutive shift restrictions.

4.1 Consecutive Shift Restrictions

Adequate rest between consecutive shifts is a requirement for any realistic tour-schedule of continuous operations. Industries that are only open for an 8-hour day have

tour-schedules that implicitly contain adequate rest. However, when scheduling an industry with continuous or 24-hour operations, restrictions on consecutive shifts is a requirement for any realistic tour-scheduling formulation. Love and Hoey (1990) and Loucks and Jacobs (1991) note the need for consecutive shift restrictions and state the lack of such restrictions as limitations to their tour-scheduling models. Our formulation allows for specialized side constraints to accommodate continuous operations.

As input in any specific application, we use the appropriate industry standard for how much time is required between the start of shift j and the start of the next shift. This leads to a matrix of shifts, J, that conflict with each other in the sense that an employee assigned to work shift j cannot be scheduled to work any shift conflicted with shift j. The following constraint is added to constraints 1-5 above to accommodate consecutive shift restrictions.

$$\sum_{j \in J} s_{j,k,d} e_i \leq 1 \qquad \text{for all conflicted shifts} \qquad (4.6)$$

Surprisingly, all of our empirical experience indicates that these side constraints do not destroy the integrality of the solution to the minimum cost network flow.

4.2 Employee Availabilities and Varying Skill Sets

Many industries have employees with heterogeneous availabilities. For example, service industries often employ younger, part-time employees who have restrictions on their time due to schooling or other activities. Our model can be modified to accommodate such heterogeneous availabilities. We simply remove the network arc from a shift to an employee who is not available during the hours encompassing that shift. No flow units can travel from that shift to the employee and consequently the employee is not assigned to that shift.

Similarly, it is often the case that many service industries have different skill requirements needed during a particular shift. If an employee does not possess the required skill for a particular shift, the arc from the shift to the employee is removed from the network. If the employee has multiple skill sets, the arc from the employee to all shifts within their set is contained within the network.

This is an important aspect to the efficiency of the tour-schedule. Many examples within the tour-scheduling literature schedule only one department within the industry at a time. Therefore, the employees assigned to that department can be scheduled only within that department and there is no mechanism to schedule that employee across departments. This could lead a department to over-staffing costing a business extra payroll. We have no such restriction.

Removal of the two types of arcs described above are easy to program. They have no effect on the integrality of solutions from the LP-network solution. This is because the resulting network is still of minimum cost flow form.

4.3 The Objective Function - Employee and Manager Preferences and a Perturbation

The objective function of the constraint set described above can now be defined. First, each employee scores each shift, on a preference scale, based upon their desire to work at that time. Any scale could be used, but we have found in practice that one based on a scale of (0,100) allows for adequate granularity. The preliminary objective function is then:

$$\text{MAX} \quad \sum_{j,k,d,i} s_{j,k,d} e_i \bullet pref(s_{j,k,d} e_i) \qquad (4.7)$$

Additionally, we allow the manager to express their preferences of employees on shifts taking such factors as seniority or job performance into account. We call these preferences "rewards". The larger the reward on a scale of (0,100), the more likely the model will select that employee for desired shifts (and it certainly helps to break ties). The objective function becomes:

$$\text{MAX} \sum_{j,k,d,i} s_{j,k,d} e_i \bullet \left[pref(s_{j,k,d} e_i) + \text{Re}\,ward(s_{j,k,d}, e_i)\right] \qquad (4.8)$$

Finally, it is possible for the algorithm to find an alternate optimal solution that contains fractional employee assignments (there is always an integer solution). This occurs when the preferences of two employees for a particular shift are equal and neither is violating a consecutive shift restriction or a maximum number of shifts per week constraint. To remove this non-integer alternate optimal solution effect from the view of manager we add to the objective function a small random value, ε, whose magnitude is chosen to range from (0,1) based on the magnitude of the preference and reward scales of (0,100). The random value is not reported as part of the final objective function value, merely used to remove the possibility of non-integer alternative optimal solutions

$$\text{MAX} \quad \sum_{j,k,d,i} s_{j,k,d} e_i \bullet \left[pref(s_{j,k,d} e_i) + \text{Re}\,ward(e_i)\right] + \varepsilon \qquad (4.9)$$

Data organization to support these calculations are straightforward. A spreadsheet is used as the input data file. A Visual Basic macro then writes the linear program in CPLEX .lp input format. This macro runs almost instantaneously. The output file is then be given to CPLEX for optimization. The ease of changing data in the

input data file, the speed with which the output file is written, and the speed with which the problem is optimized (more on this in the next section) allows the user to study a wide variety of problem instances and conduct sensitivity analysis. Figure 4.2 shows an example of the input data file format. The data file for the ASU computer lab problem has 195 rows of shifts with 50 columns of preferences (the sum of employee and manager values).

Day	Shift ID	Shift Number	Com Lab	Shift Demand	Start Time	Emp Number 1 / All High	2 / All High	3 / All High	4 / All High	5 / All High	6 / All High	7 / All High	8 / All High	9 / All High	10 / All High
						Employee Priority									
1	56	1	Atr	1	9	0	0	0	0	0	0	0	0	0	0
	57	2	Atr	1	9	0	0	0	0	0	0	0	0	0	0
	58	3	Atr	1	9	0	0	0	0	0	0	0	0	0	0
	64	4	Atr	1	10	0	0	0	0	0	0	0	0	0	0
	75	5	Atr	1	12	264	242	270	221	231	292	275	206	221	277
	81	6	Atr	1	13	299	228	295	260	295	269	232	203	291	286
	88	7	Atr	1	14	227	297	251	269	248	257	273	291	248	200
	99	8	Atr	1	16	226	205	299	256	279	276	201	253	274	288
	111	9	Atr	1	18	257	260	240	225	214	221	269	248	245	300
	118	10	Atr	1	19	226	213	267	229	282	273	276	217	218	250
	123	11	Atr	1	20	285	245	300	281	276	278	243	272	277	242
	129	12	Atr	1	21	243	255	228	284	223	206	238	209	230	206

Figure 4.2 Input Data File

In the next section, our model of the ASU Computer Labs Phase 2 tour-schedule as well as other larger problem instances are solved – in orders of magnitude less time than the corresponding IP.

5. Computational Results

5.1 Tour-Schedule for ASU Computer Labs

The model described in Section 4 was applied to the real world example of scheduling student technicians at ASU Computer Labs. We worked closely with a

scheduler using the data for shift requirements for the spring semester of 2003. The four labs combined have 195 distinct shifts that must be covered. In order to protect student privacy, student technician preferences and availabilities are generated from typical behaviors. We generated a class schedule for 50 students, which is the current number of technicians employed. The class schedules consist of either morning classes (unavailable from 0800-1200), afternoon classes (unavailable from 1200 – 1600), or evening classes (unavailable from 1600-2000). Each student technician is assumed to attend a typical student-worker load of three different classes per week, with each class meeting twice a week. Therefore, each student is unavailable for six time periods per week and available all weekend. We ran five distinct models, where each model adds one more of the real world constraints described in Section 4. Solutions used CPLEX 8.1 on a 2.4 GHz PC with parallel processors and 1 Mb of RAM.

Model	CPU Time (sec)	Optimal Objective
Complete Availability, Technician Preferences	0.03	38625
Restricted Availability, Technician Preferences	0.04	38495
Restricted Availability, Technician Preferences, Consecutive Shift Restrictions	0.29	38452
Restricted Availability, Technician Preferences, Consecutive Shift Restrictions, Varying Skill Sets	0.70	38091
Restricted Availability, Technician Preferences, Consecutive Shift Restrictions, Varying Skill Sets, Preference + Reward	0.93	43088

Table 5.1 Results of the Tour-Schedule Model for ASU Computer Labs

In each case, a feasible solution to the tour-scheduling problem is solved in a fraction of a second, rather than seconds or minutes. Why is this computational improvement important? In general, it means that variations on optimization problems may be run in real-time that allow the manager to see the effect of changes of interest. In the ASU Computer Lab case study, the following are examples of such uses:

- Re-rostering when someone is unavailable, or whose schedule changes (dropping and adding classes after the initial schedule, for example).

- Introduction of the student confidential information, after an initial basic schedule is built by developers.

- Determination of which types of employees are needed to improve coverage and even staff sizing (which we cover in detail in Section 5.3).

- Study the effects of rewards chosen by the manager and preferences selected by the students.

These types of analyses are essentially impossible in the manual system, yet require only seconds for a re-roster up to hours for a sensitivity analysis. These time periods are a small fraction of the 2-3 days spent just to get one feasible schedule manually using the current practice. They are also a fraction of the time needed if solutions were obtained by a brute-form IP as we can see from Table 5.1. The benefits of our method increase as problem size grows larger. The ASU example has about 10,000 binary variables and 1,000 constraints, but is in no way a challenge to our method. In the next sub-section we look at the computational behavior of larger problem instances. Those larger problems are randomly generated problems, not from our case study, but containing the same types of features.

5.2 Computational Efficiency for Larger Problem Instances

Four larger problem instances were solved using realistic yet nominal data. The computational effort is summarized in Table 5.2 below. The ASU computer lab problem is on the order of the third problem listed. Our formulation continues to solve the tour-scheduling problem in an insignificant amount of computer time even for very large problem instances of 420 distinct shifts (about the number of shifts in three months with a granularity of 4 hours on shift definition) and 80 employees. Our formulation is completely adaptable to scheduling additional time periods and, as we can see below, the formulation will solve these larger problems very quickly.

Number of Shifts	Number of Employees	Network-Based LP			Integer Program		
		Variables (continuous)	Constraints	CPU Time (sec)	Variables (binary)	Constraints	CPU Time (sec)
35	10	420	205	0	350	90	0.02
70	20	1540	430	0	1400	180	0.07
175	50	9100	1225	0.06	8750	450	1.53
420	100	42700	3020	0.5	42000	1040	135.98

Table 5.2 Computational Results for Larger Problem Instances

Figures 5.1 and 5.2 illustrate the emerging gap in computational time required to get a solution between our formulation, Network-based Linear Program (NBLP), and a brute-force IP. Our formulation will be able to solve much larger problems before the computational time becomes excessive. This will allow businesses to schedule months, quarters, or years at a time.
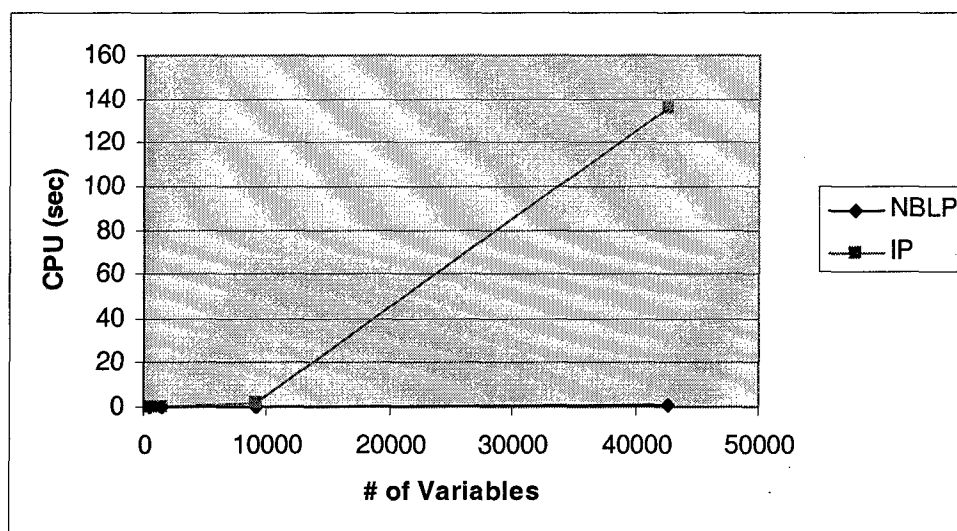
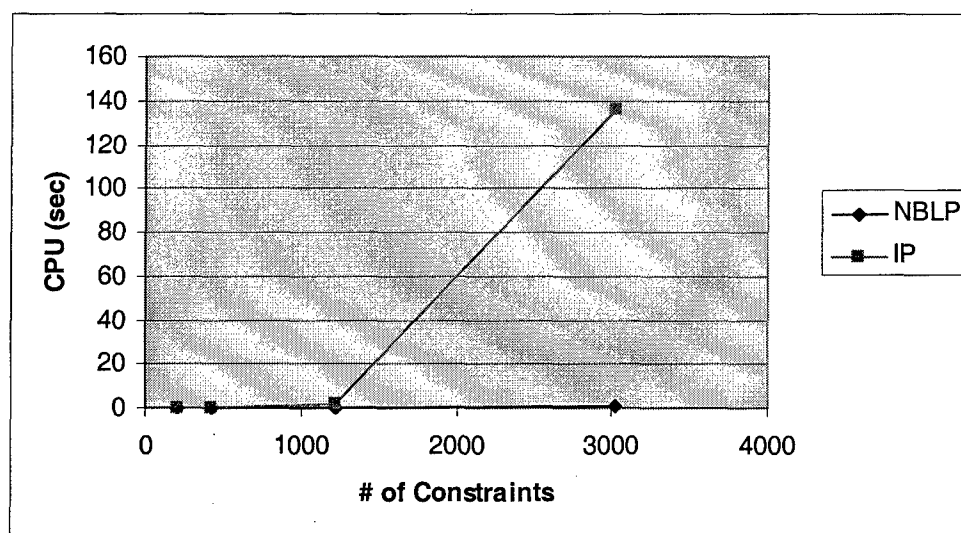Figure 5.1 Computational Time Comparison versus the Number of Variables

Figure 5.2 Computational Time Comparison versus the Number of Constraints

5.3 Staff Sizing Application

The speed at which the formulation can be solved and the ease with which real world constraints such as employee availability, consecutive shift restrictions and varying skills sets are incorporated, allows for an additional important result for industries with heterogeneous workforces. Industries that are looking to streamline their

employee payrolls or are opening a new facility can use the algorithm to predict the minimum number of staff required to fill shifts based on customer demand.

Given a level of employee availability, the industry standard for rest between consecutive shifts, and the number of skills required, a set of employee data can be generated. The formulation can then be solved for this case and the minimum number of employees that covers that definition of demand can be found by running the model until a feasible solution. Repeating this for several sets of employee data, an actual minimum number of employees needed to staff the business will become apparent.

Below is an example of how our formulation can be used to generate the minimum number of employees required. The example problem instance contains 49 shifts (7 shifts per day for a week) and 15 employees. The table shows employee requirements for a nominal industry as employee availability mix reduces from full (100%) availability to half time (50%) availability. The availabilities are generated randomly for this example, so 50% availability would mean each employee is randomly unavailable for half of the hours each week. In Table 5.3, below each availability is the minimum number of employees required for that problem instance. An average, standard deviation, and a minimum employee to shift ratio is calculated.

Availability		100%	90%	80%	70%	60%	50%
Minimum	Trial 1	10	11	12	13	15	15
Number	Trial 2		12	13	14	14	15
of	Trial 3		11	12	14	14	16
Employees	Trial 4		11	13	14	15	15
Average Minimum		10	11.25	12.5	13.75	14.5	15.25
Standard Deviation			0.500	0.577	0.500	0.577	0.500
Employee/Shift Ratio		0.204	0.230	0.255	0.281	0.296	0.311

Table 5.3 Employee Requirements as a Function of Employee Availability

As expected, as the time each employee has available decreases the minimum number of employees required to staff the demanded number of shifts increases. This formulation leads to an employee/shift ratio. Given the parameters of the proposed business, the entrepreneur can look at varying scenarios, chose as conservative an estimate as desired and find the ratio of employees to shifts from a graph like the one shown below. If circumstances change as the business moves closer to opening or in streamlining processes, the graph can be consulted to determine a new level of staffing ratio based upon any new level of employee availability. Similar graphs could be constructed varying consecutive shift restrictions or skill sets.
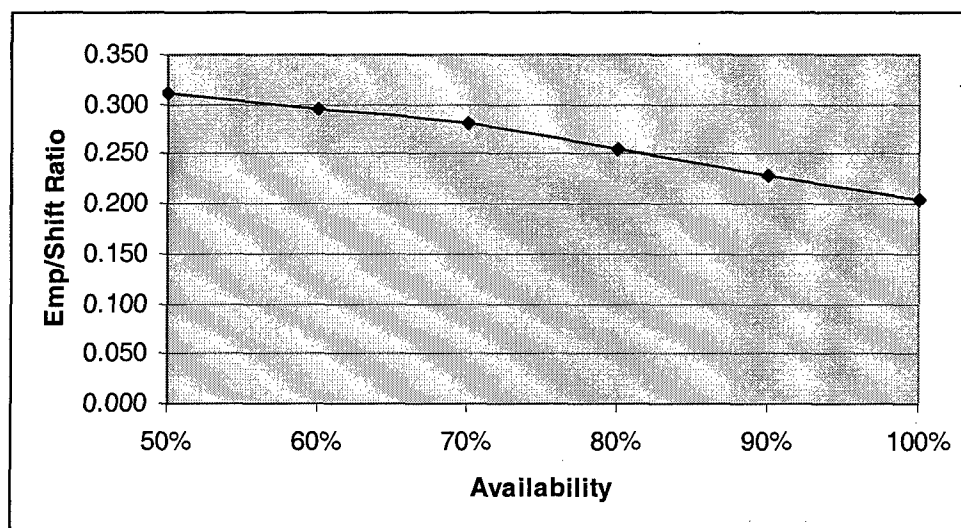

Figure 5.3 Employee to Shift Ratio Based Upon Employee Availability

6. Conclusions and Recommendations

Our formulation of the tour-scheduling problem allows a scheduler to incorporate many of the real world constraints inherent in an implementable tour-schedule. The scheduler can consider heterogeneous employee availabilities, varying skill sets,

consecutive shift restrictions, and seniority or job performance incentives. Additionally, the formulation solves very large problem instances in a computationally insignificant amount of time compared to the corresponding IP solution. This allows the scheduler to reschedule often or consider new information that would require additional tweaking of the tour-schedule.

The formulation was applied successfully to a real world example involving a very heterogeneous workforce and continuous operations. The Technology Support Analyst Principal (TSAP) for the ASU computer labs indicated that it took him 2 to 3 days per semester to schedule the computer labs. Due to the high constraint on the availabilities of the technicians it is a very laborious process that the TSAP does manually with the aid of a spreadsheet for recording the tour schedule. By hand, it is very difficult to ensure that students are given a proper amount of rest between shifts. Shifts that occur consecutively or within a minimum number of hours are deemed to be conflicted and not scheduled to the same technician. The proposed model can accommodate any minimum window of rest required by the scheduling organization and can, therefore, schedule businesses with continuous operations.

Finally, we took advantage of the computational efficiency of the model and solved many problem instances to generate a minimum employee to shift ratio. Such an analysis would be very valuable to any business seeking to cut payroll or to an entrepreneur who is looking to open a new business and needs to determine appropriate staffing levels dynamically as conditions change, or in the planning stages of a business that does not exist yet.

In all cases we considered, the inclusion of heterogeneous workers and mandated rest breaks in a continuous schedule add realism to the tour scheduling problem.

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