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Air Force Research Laboratory**



**Wave Variable Method to Control Force-Reflecting
Teleoperators with Time Delays: Generalizing the Wave
Variable Method to Multiple Degree-of-Freedom
Systems**

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GENERALIZING THE WAVE VARIABLE METHOD TO MULTIPLE DEGREE-OF-FREEDOM SYSTEMS

I. INTRODUCTION

One of the tasks of the project was to do a literature survey. Various articles were examined, including references [1]-[6]. We examined in detail some of the work done by Wayne Book and Sahgir Munir at Georgia Institute of Technology. In their work, they generalized the wave variable method to multiple degree-of-freedom teleoperation systems by replacing the damping coefficient in the standard wave variable method with a set of scaling matrices. In the following, we will derive a more general set of scaling matrices. In the next section, we review the single degree-of-freedom and multiple degree-of-freedom systems. In Section III, we derive a larger family of scaling matrices and verify that an important subset of scaling matrices results in stable operation of a multiple degree-of-freedom teleoperation system with a fixed but unknown time delay. We then determine the relationship between the extended family of scaling matrices and the family proposed by Munir and Book. Section IV contains a simulation to illustrate the concepts and Section V contains the conclusions.

II. THE WAVE VARIABLE METHOD FOR MULTIPLE DEGREE-OF-FREEDOM SYSTEMS

The wave transformation relations are given by

$$\begin{aligned} u_s(t) &= u_m(t-T) \\ v_m(t) &= v_s(t-T). \end{aligned} \quad (1)$$

The wave transformations for the left wave junction are given by

$$\begin{aligned} u_m(t) &= \frac{b\dot{\theta}_m(t) + \tau_m(t)}{\sqrt{2b}} \\ v_m(t) &= \frac{b\dot{\theta}_m(t) - \tau_m(t)}{\sqrt{2b}} \end{aligned} \quad (2)$$

and that for the right wave junction are given by

$$\begin{aligned} u_s(t) &= \frac{b\dot{\theta}_{sd}(t) + \tau_{pd}(t)}{\sqrt{2b}} \\ v_s(t) &= \frac{b\dot{\theta}_{sd}(t) - \tau_{pd}(t)}{\sqrt{2b}}. \end{aligned} \quad (3)$$

Although the strictly positive parameter b can be chosen arbitrarily, it defines a characteristic impedance associated with the wave variables and directly affects the system behavior [5].

Equations (2) and (3) are for 1 DOF systems. To implement the wave variable method on a system that has more than one degree of freedom, the equations for the transforms must be generalized. Niemeyer and Slotine [5] suggest making b a positive definite matrix. Munir and Book [2]-[4] recommend a more general formulation by writing the transformation equations from before as

$$\begin{aligned} u_m(t) &= A_w \dot{\theta}_m(t) + B_w \tau_m(t) \\ v_s(t) &= C_w \dot{\theta}_s(t) - D_w \tau_{pd}(t) \end{aligned} \quad (4)$$

and

$$\begin{aligned} v_m(t) &= C_w \dot{\theta}_m(t) - D_w \tau_m(t) \\ u_s(t) &= A_w \dot{\theta}_s(t) + B_w \tau_{pd}(t) \end{aligned} \quad (5)$$

where A_w , B_w , C_w , and D_w are $n \times n$ scaling matrices and n is the number of degrees of freedom of the teleoperation system. These matrices cannot be chosen arbitrarily; it is necessary to determine conditions for the scaling matrices to guarantee passivity [6]. To accomplish this we will define the power-flow at each side to be

$$\dot{\theta}_m^T \tau_m = \frac{1}{2} u_m^T u_m - \frac{1}{2} v_m^T v_m \quad (6)$$

for the master side and

$$\dot{\theta}_s^T \tau_{pd} = -\frac{1}{2} u_s^T u_s + \frac{1}{2} v_s^T v_s \quad (7)$$

for the slave side. Substituting equations (4) and (5) into equation (6) or (7), expanding, and matching matrix coefficients yields the requirements

$$\begin{aligned} A_w^T A_w &= C_w^T C_w \\ B_w^T B_w &= D_w^T D_w \end{aligned} \quad (8)$$

and also that

$$I = \frac{1}{2} (2A_w^T B_w + 2C_w^T D_w). \quad (9)$$

Munir and Book [2]-[4] derive conditions on A_w and B_w to ensure that (8) and (9) are satisfied. In particular, they note that the scaling matrices must be nonsingular and consider the special case

$$\begin{aligned} A_w &= C_w \\ B_w &= D_w \end{aligned} \quad (10)$$

so that equation (8) is satisfied. Using this relationship, equation (9) reduces to

$$I = 2A_w^T B_w. \quad (11)$$

Munir and Book then restrict A_w to be symmetric but not necessarily positive definite and prove that the resulting family of scaling matrices results in a stable system by showing that passivity is ensured using the norm of the scattering matrix.

Although this is more general than what was proposed in [5], because of the specific choices made, only a restricted class of scaling matrix is determined. In this article, we will extend the family of scaling matrices that result in stability and discuss the significance of this extension in terms of the wave variables themselves.

III. DERIVATION OF A LARGER FAMILY OF SCALING MATRICES

To extend the family of scaling matrices originally proposed by Munir and Book, we will first derive the whole family of matrices satisfying (8) and (9). From equation (8) it is clear that A_w^T and C_w^T have the same column space and that B_w and D_w have the same row space. This observation along with equation (9) implies that all four scaling matrices must be nonsingular. Since all square root decompositions of nonsingular square matrices such as those given in (8) are related by pre-multiplication by an orthogonal matrix, it follows that

$$\begin{aligned} C_w &= Q_1 A_w \\ D_w &= Q_2 B_w \end{aligned} \quad (12)$$

where Q_1 and Q_2 are $n \times n$ orthogonal matrices. Now this is not enough to guarantee that (9) is satisfied. We will say that A_w and B_w are *compatible* with respect to (8) and (9) if there are orthogonal matrices Q_1 and Q_2 so that A_w , B_w , $C_w = Q_1 A_w$, and $D_w = Q_2 B_w$ satisfy (8) and (9).

Substituting (12) into (9) and performing the required manipulations gives the following necessary and sufficient condition for A_w and B_w to be compatible in this sense:

$$A_w B_w^T + B_w A_w^T = I. \quad (13)$$

With some algebraic manipulation, it can then be shown that the characterizing condition for A_w , B_w , C_w , and D_w to satisfy (8) and (9) is that the following hold:

1. A_w is nonsingular.
2. $B_w = \frac{1}{2}(I + S)A_w^{-T}$ where S is any $n \times n$ skew-symmetric matrix.
3. $C_w = QA_w$ where Q is any $n \times n$ orthogonal matrix.
4. $D_w = \frac{1}{2}Q(I - S)A_w^{-T}$.

Note that these four conditions guarantee that all four matrices are nonsingular.

Next, we identify a subfamily of scaling matrices that result in stable operation. To do this, we need to determine the scattering matrix. The transfer function for the input-output relationship across the communication link can be shown to be [2]

$$G_w(s) = \begin{bmatrix} \frac{(1 - e^{-2sT})}{1 + e^{-2sT}} B_w^{-1} A_w & \frac{-2e^{-sT}}{1 + e^{-2sT}} I \\ \frac{2e^{-sT}}{1 + e^{-2sT}} I & \frac{(1 - e^{-2sT})}{1 + e^{-2sT}} A_w^{-1} B_w \end{bmatrix}. \quad (14)$$

Setting $s = j\omega$ and simplifying gives

$$G_w(j\omega) = \begin{bmatrix} B_w^{-1} A_w \tanh(j\omega T) & -\operatorname{sech}(j\omega T) I \\ \operatorname{sech}(j\omega T) I & A_w^{-1} B_w \tanh(j\omega T) \end{bmatrix}. \quad (15)$$

Passivity can then be demonstrated using the scattering matrix

$$S(j\omega) = [G_w(j\omega) - I][G_w(j\omega) + I]^{-1}. \quad (16)$$

While calculations involving the inversion of partitioned matrices are generally difficult to do in closed form, (15) has enough mathematical structure to allow this. It can be shown that the expression for $S(j\omega)$ is

$$\begin{bmatrix} [M^2 - I] \sinh(j\omega T) & -2M \\ 2M & -[M^2 - I] \sinh(j\omega T) \end{bmatrix} D(j\omega T) \quad (17)$$

where $M = B_w^{-1} A_w$ and where $D(j\omega T)$ is given in block diagonal form as

$$D(j\omega T) = \operatorname{diag}(D_0(j\omega T), D_0(j\omega T)) \quad (18)$$

with

$$D_0(j\omega T) = [2M \cosh(j\omega T) + (M^2 + I) \sinh(j\omega T)]^{-1}. \quad (19)$$

Since the calculation of $D_0(j\omega T)$ requires a matrix inversion of

$$\begin{aligned} & K_1 \cos(\omega T) + jK_2 \sin(\omega T) \\ & = 2M \cos(\omega T) + j[M^2 + I] \sin(\omega T), \end{aligned} \quad (20)$$

it must be shown that (20) is nonsingular for all ω . This verification for the whole class of scaling matrices satisfying (8) and (9) requires more work than the subfamily proposed by Munir and Book [2]-[4]. The approach we have taken is to show that (13) imposes restrictions on the eigenvalues of $B_w^{-1} A_w$ that preclude (20) from being singular. This was done by showing that if (20) is singular then $B_w^{-1} A_w$ necessarily has an eigenvalue with zero real part. However, it can be shown that for our family of scaling matrices the eigenvalues of $B_w^{-1} A_w$ necessarily have positive real parts.

Now if (20) is singular then so is its product with its complex conjugate:

$$\begin{aligned} & [K_1 \cos(\omega T) + jK_2 \sin(\omega T)][K_1 \cos(\omega T) - jK_2 \sin(\omega T)] \\ &= K_1^2 \cos^2(\omega T) + K_2^2 \sin^2(\omega T) + j[K_2 K_1 - K_1 K_2] \sin(\omega T) \cos(\omega T) \\ &= \sin^2(\omega T)(B_w^{-1} A_w)^4 + [2 + 2 \cos^2(\omega T)](B_w^{-1} A_w)^2 + \sin^2(\omega T)I. \end{aligned} \quad (21)$$

Note that the resulting imaginary part is zero precisely because $K_1 = 2M$ and $K_2 = M^2 + 1$ commute; otherwise (21) would be complex. We will show that this matrix is nonsingular and in the process prove that (20) is also nonsingular. If $\sin(\omega T) = 0$ then (21) becomes $4(B_w^{-1} A_w)^2$ which is nonsingular. Suppose that $\sin(\omega T) \neq 0$. In order for (21) to be singular at least one of the four roots of

$$p(\lambda) = \sin^2(\omega T)\lambda^4 + [2 + 2 \cos^2(\omega T)]\lambda^2 + \sin^2(\omega T) \quad (22)$$

must be an eigenvalue of $B_w^{-1} A_w$. The roots of this polynomial (which is a quadratic in λ^2) are given by

$$\lambda = \pm \sqrt{\frac{-2 - 2 \cos^2(\omega T) \pm \sqrt{(2 + 2 \cos^2(\omega T))^2 - 4 \sin^4(\omega T)}}{2 \sin^2(\omega T)}}. \quad (23)$$

Since $(2 + 2 \cos^2(\omega T))^2$ dominates $4 \sin^4(\omega T)$, the inner square root is real. Furthermore, $-2 - 2 \cos^2(\omega T)$ dominates the inner square root so the number inside the outer square root is real and nonpositive. Therefore, the roots of $p(\lambda)$ are purely complex (i.e., have zero real part).

Now the A_w and B_w matrices satisfying (8) and (9) also satisfy (13). Because A_w and B_w are invertible, (13) can be written as

$$B_w^{-1} A_w + A_w^T B_w^{-T} = B_w^{-1} B_w^{-T}. \quad (24)$$

This says that the symmetric part of $B_w^{-1} A_w$ is positive definite implying that the real parts of the eigenvalues of $B_w^{-1} A_w$ are strictly positive so that none of the eigenvalues of $B_w^{-1} A_w$ are roots of the polynomial $p(\lambda)$. Therefore (20) is nonsingular implying that $D_0(j\omega)$ is well defined.

Munir [2] demonstrates that the system is stable for their family of scaling matrices by showing that the norm of the scattering matrix

$$\|S\| = \sup_{\omega} \lambda^{1/2}(S^*(j\omega)S(j\omega)) \quad (25)$$

is equal to 1 where $S^*(j\omega)$ denotes the complex conjugate transpose of $S(j\omega)$. This was done by showing that

$$S^*(j\omega)S(j\omega) = I. \quad (26)$$

However, for this calculation to work out, it must be assumed that $M = B_w^{-1} A_w$ is symmetric, which is true for the scaling matrices in [2]-[4] by the symmetry of A_w along with equation (11).

If we constrain the family of matrices derived earlier that satisfy equations (8) and (9) to have the property that $B_w^{-1} A_w$ is symmetric, the same calculations used by Munir [2] to ensure passivity for his family of scaling matrices would apply to give (26) for our constrained family of matrices. This will of course reduce the size of our family of scaling matrices. To see this, note that

$$B_w^{-1} A_w = \left[\frac{1}{2}(I + S)A_w^{-T} \right]^{-1} A_w = 2A_w^T(I + S)^{-1} A_w. \quad (27)$$

Now $B_w^{-1} A_w$ is symmetric if and only if its inverse is. This inverse is given by

$$(B_w^{-1} A_w)^{-1} = \frac{1}{2} A_w^{-1}(I + S)A_w^{-T} = \frac{1}{2} A_w^{-1} A_w^{-T} + \frac{1}{2} A_w^{-1} S A_w^{-T}. \quad (28)$$

The matrix $\frac{1}{2}A_w^{-1}A_w^{-T}$ is symmetric while the matrix $\frac{1}{2}A_w^{-1}SA_w^{-T}$ is skew-symmetric so in order for (28) to be symmetric we have that $S=0$. This is the necessary and sufficient condition for $B_w^{-1}A_w$ to be symmetric for the class of matrices satisfying (8) and (9). Thus the following family of scaling matrices can be used for applying the wave variable method to a multiple degree-of-freedom system:

1. A_w is nonsingular.
2. $B_w = \frac{1}{2}A_w^{-T}$.
3. $C_w = QA_w$ where Q is any $n \times n$ orthogonal matrix.
4. $D_w = \frac{1}{2}QA_w^{-T}$.

Note that setting $A_w = \sqrt{b/2}$ and $Q=1$ for the scalar case results in exactly the same solution as (2) and (3).

Since choosing a set of scaling matrices requires the selection of an $n \times n$ nonsingular matrix A_w and an $n \times n$ orthogonal matrix Q , there are a total of $n^2 + n(n-1)/2 = 3n^2/2 - n/2$ degrees of freedom in choosing the scaling matrices. However, because the collection of scaling matrices presented in [2]-[4] are uniquely defined by the selection of a single symmetric matrix A_w , the family of scaling matrices in [2]-[4] is only $n(n+1)/2$ -dimensional.

It is then natural to ask how this extension of the scaling matrices affects the wave variables. To see this, we first consider the effect of Q . The v wave variables are given in equations (4) and (5). Substituting in the expressions for C_w and D_w yields

$$\begin{aligned} v_m(t) &= QA_w \dot{\theta}_m(t) - \frac{1}{2}QA_w^{-T} \tau_m(t) \\ &= Q[A_w \dot{\theta}_m(t) - \frac{1}{2}A_w^{-T} \tau_m(t)], \end{aligned} \quad (29)$$

which clearly demonstrates that Q merely applies an orthogonal transformation to the v -variable, i.e., it will merely rotate and/or reflect the v -variable. The same holds for $v_s(t)$. This will clearly have no effect on the power flow equations (6) and (7).

Next, consider the effect of allowing the matrix A_w to be nonsymmetric. There is a well-known result in matrix theory called the polar decomposition that states that any square matrix M can be written as a product $M = PU^T$ where P is a symmetric positive definite matrix and U is an orthogonal matrix. Setting $M = A_w^T$ gives that $A_w = UP_w$ for some suitable orthogonal matrix U and symmetric positive definite matrix P_w . The u wave variable then becomes

$$\begin{aligned} u_m(t) &= QP_w \dot{\theta}_m(t) + \frac{1}{2}(QP_w)^{-T} \tau_m(t) \\ &= Q[P_w \dot{\theta}_m(t) + \frac{1}{2}P_w^{-T} \tau_m(t)] \end{aligned} \quad (30)$$

so that the u -variable is merely rotated and/or reflected. A similar statement holds for the v -variable.

IV. SIMULATION RESULTS

The performance of the wave variable method was tested using a 2 degree-of-freedom linear model. The equations of motion for both the master and slave manipulators were given by

$$\tau_{pd} = J_s \ddot{\theta} + B_s \dot{\theta} \quad (31)$$

where τ_{pd} is the input torque, J_s is the desired 2×2 constant symmetric positive definite inertia matrix, and B_s is the damping matrix with the same qualities as J_s . With the wave variable parameters and the equations of motion necessary to complete the system, we can simulate a 2 degree-of-freedom bilateral teleoperation system.

As can be seen in Fig. 2, the behavior of the linear 2 degree-of-freedom slave is similar to the behavior of the master. In this simulation, the input torques were 3 and -4 N.m, respectively. The slave manipulator was able to match the performance of the master manipulator with respect to torque, position, and velocity. Also, in less than 4 seconds the output torques had reached their steady state values. Due to the nature of the wave variable method, it can be shown that the system will stabilize for any amount of constant time delay.

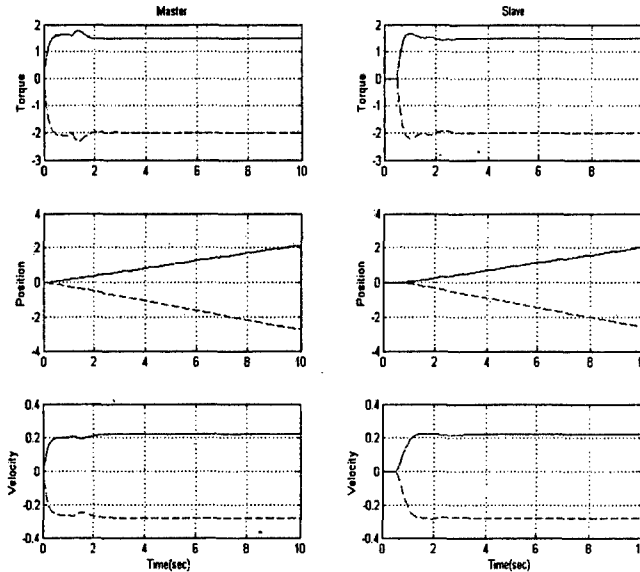


Fig. 2: The response of a 2-DOF system with a $2T = 1$ sec total time delay.

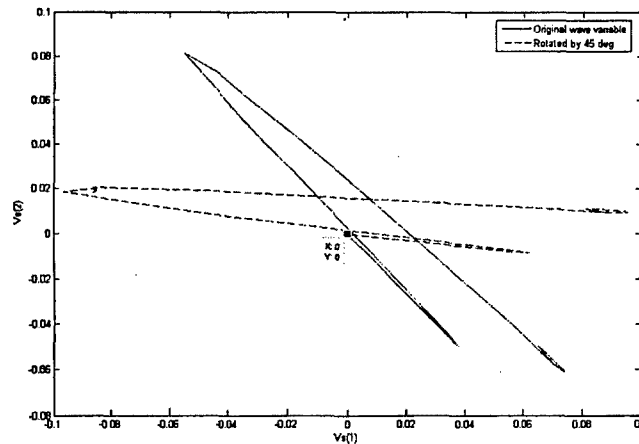


Fig. 3: A plot of the wave variable v_s for the 2-DOF teleoperation system for two different Q matrices.

Fig. 2 represents the simulation results for a given fixed A_w where Q is allowed to be any orthogonal matrix. No matter which Q is used, the overall system output will not change. What do change are the wave variables that are sent across the communication line. Fig. 3 shows a plot of the wave variable v_s for the cases when Q is the identity matrix and when it is a rotation

matrix corresponding to a rotation of 45 degrees. The figure clearly shows that v_s will be rotated by the same amount that C_w and D_w are. Also, when looking at $P_w = U^T A_w$, which is a rotated version of A_w , the same statement from before can be made again. This time however both u_m and v_s will be rotated, and again there will be no change to the system output.

V. CONCLUSIONS

In summary, we have presented the derivation of an extension of the wave variable method to multiple degree-of-freedom systems. We have shown that the collection of scaling matrices determined in [2]-[4] to preserve passivity is not complete and have determined a larger family of feasible scaling matrices. We have also shown how the new scaling matrices relate to those proposed in [2]-[4].

VI. REFERENCES

- [1] R.J. Anderson and M.W. Spong, "Bilateral control of teleoperators with time delay," *IEEE Transactions on Automatic Control*, vol. 34, no. 5, May 1989, pp. 494-501.
- [2] S. Munir, "Internet-based teleoperation," Ph.D. Thesis, Dept. of Mechanical Engineering, Georgia Institute of Technology, March 2001.
- [3] S. Munir and W.J. Book, "Wave-based teleoperation with prediction," *American Controls Conference*, Arlington, VA, June 25-27, 2001, pp. 4605-4611.
- [4] S. Munir and W.J. Book, "Internet-based teleoperation using wave variables with prediction," *IEEE/ASME Transactions on Mechatronics*, vol. 7, no. 2, June 2002, pp. 124-133.
- [5] G. Niemeyer and J.-J.E. Slotine, "Stable adaptive teleoperation", *IEEE Journal of Oceanic Engineering*, Vol. 16, No. 1, January 1991, pp 152-162.
- [6] J.-J.E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Inc., New Jersey, 1991