

**A complete basis set  
model chemistry  
for excited states**

**George A. Petersson**

*Hall-Atwater Laboratories of Chemistry,  
Wesleyan University,  
Middletown, Connecticut 06459-0180*

# Report Documentation Page

Form Approved  
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE <b>09 FEB 2005</b>		2. REPORT TYPE		3. DATES COVERED -	
4. TITLE AND SUBTITLE <b>A Complete Basis Set Model Chemistry for Excited States</b>				5a. CONTRACT NUMBER <b>F04611-03-C-0015</b>	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) <b>George Petersson</b>				5d. PROJECT NUMBER <b>BMSB</b>	
				5e. TASK NUMBER <b>R2FT</b>	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Spectral Sciences Incorporated, 4 Fourth Avenue, Burlington, MA, 01803-3304</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT <b>N/A</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES <b>31</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

# Outline

## I. Background

- a. John Pople's Model Chemistries

## II. Excited state Models

- a. CCSD(T) vs FCI
- b. CASSCF and CAS-CISD vs FCI

## III. CBS Extrapolations

- a. A Systematic Sequence of Basis Sets
- b. The CASSCF CBS Limit
- c. The CASSCF-CISD CBS Limit

## IV. Results

- a. Geometry
- b. Vibrational Frequencies
- c. Excitation energies
- d. Dissociation Energies

## V. Acknowledgment

# John Pople's Model Chemistries

any

of every

of  
we can

theory,  
es.

## The CBS-Q//B3 Model

**Geometry:** B3LYP/6-311G(2d,d,p)  
**ZPE & Thermal:** B3LYP/6-311G(2d,d,p)  
**SCF:** UHF/6-311+G(3d2f,2df,2p)  
**CBS(extrap) - MP2:** UHF/6-311+G(3d2f,2df,2p)  
**MP3, MP4(SDQ):** MP4(SDQ)/6-31+G(d(f),d,p)  
**MP5 - CCSD(T):** CCSD(T)/6-31+G(d')

### Empirical Corrections:

$$-5.79 \text{ mE}_h |S|_{ii}^2 [\Sigma_i C_{\mu}^2]_{ii} \quad (2e^-)$$

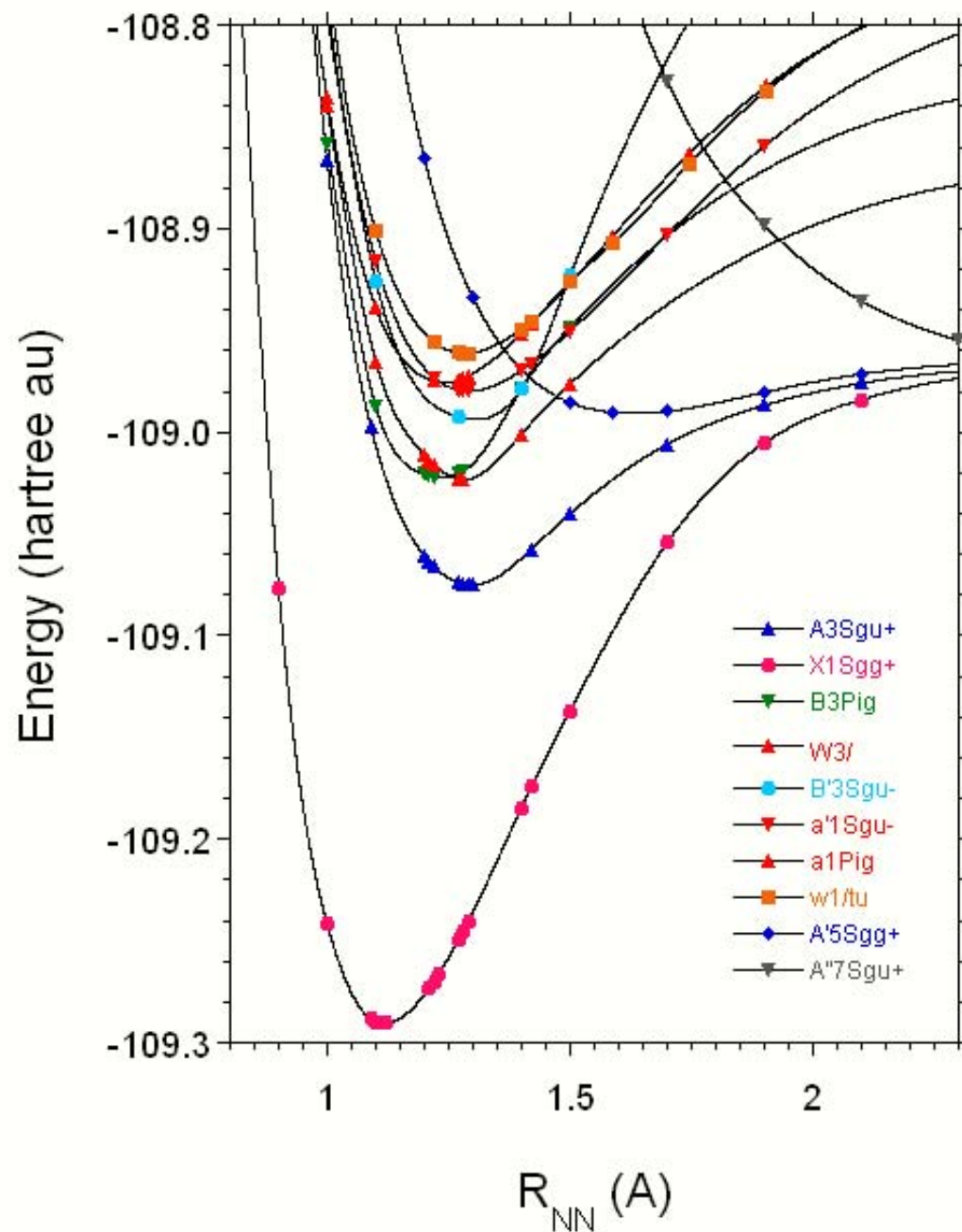
$$-9.54 \text{ mE}_h \Delta(S^2) \quad (\text{Spin Contam.})$$

$$E(\text{core}) \sim 3.92 \text{ mE}_h Q_{\text{Na}} + 2.83 \text{ mE}_h Q_{\text{Na}}^2$$

Experimental Atomic Spin-Orbit Interaction

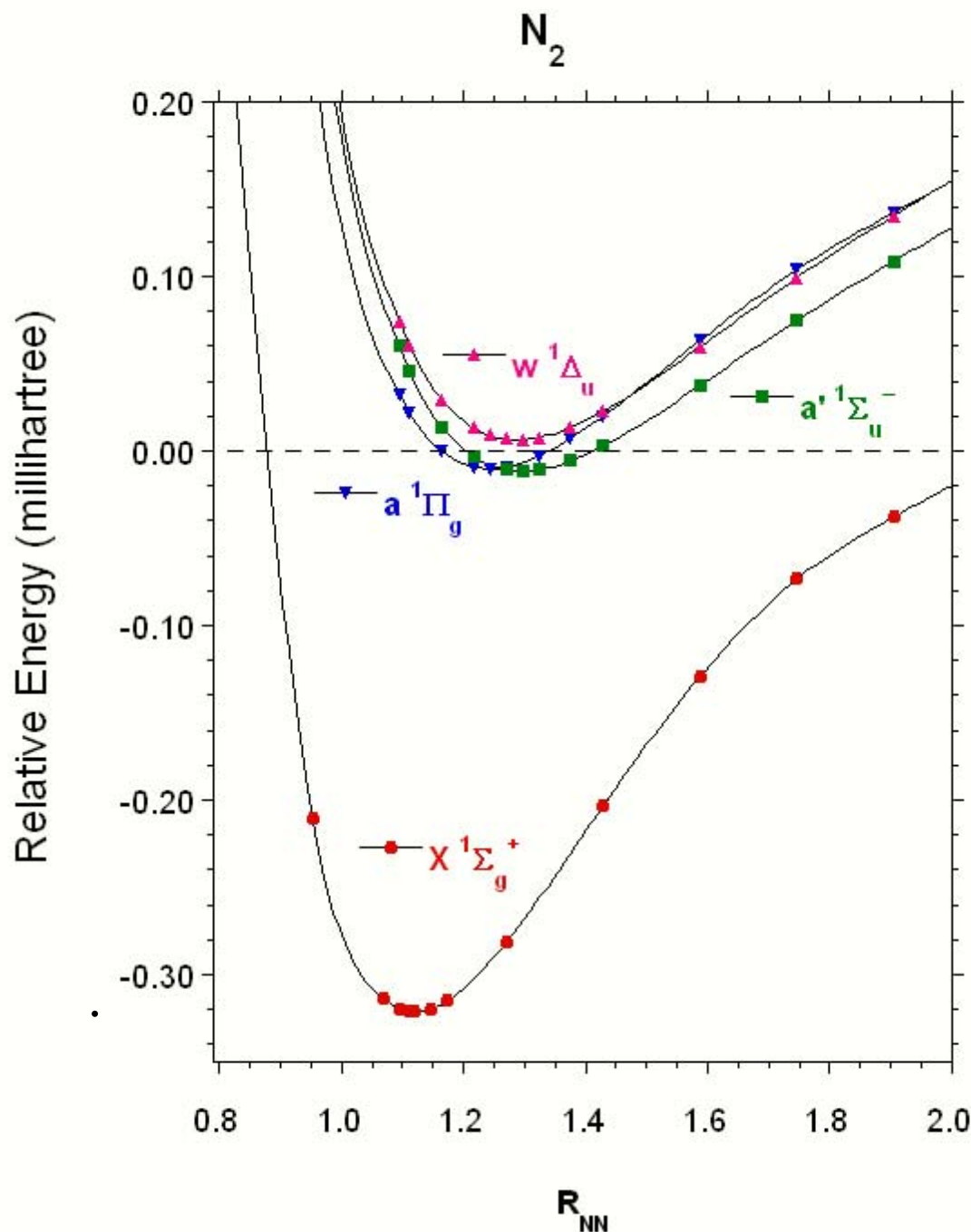
## Test Case

The  $X^1\Sigma_g^+$  ground state, along with the  $a^3\Sigma_u^+$ ,  $A'^5\Sigma_g^+$ , and  $A''^7\Sigma_u^+$  excited states of  $N_2$  dissociates to two  $^4S_{3/2}$  ground state N atoms. Other low-lying singlet and triplet states dissociate to N atoms in  $^2D$  and  $^2P$  excited states.



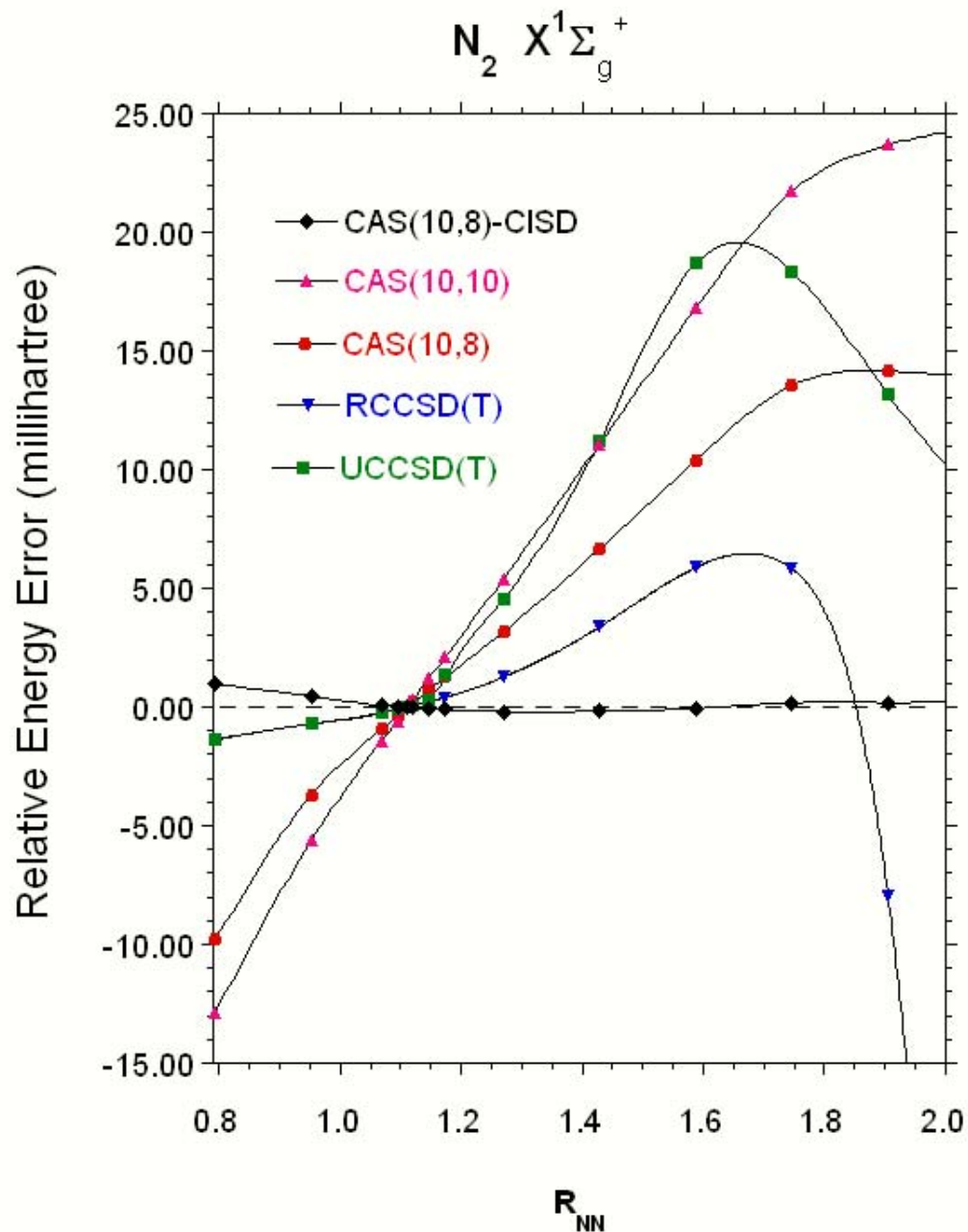
## Benchmarks

The **FCI/cc-pVDZ** energies of low-lying states of  $\text{N}_2$  [H. Larsen, J. Olsen, P Jørgensen, and O. Christiansen, *J. Chem. Phys.* **113**, 6677 (2000)] provide benchmarks for calibration of model chemistry candidates.

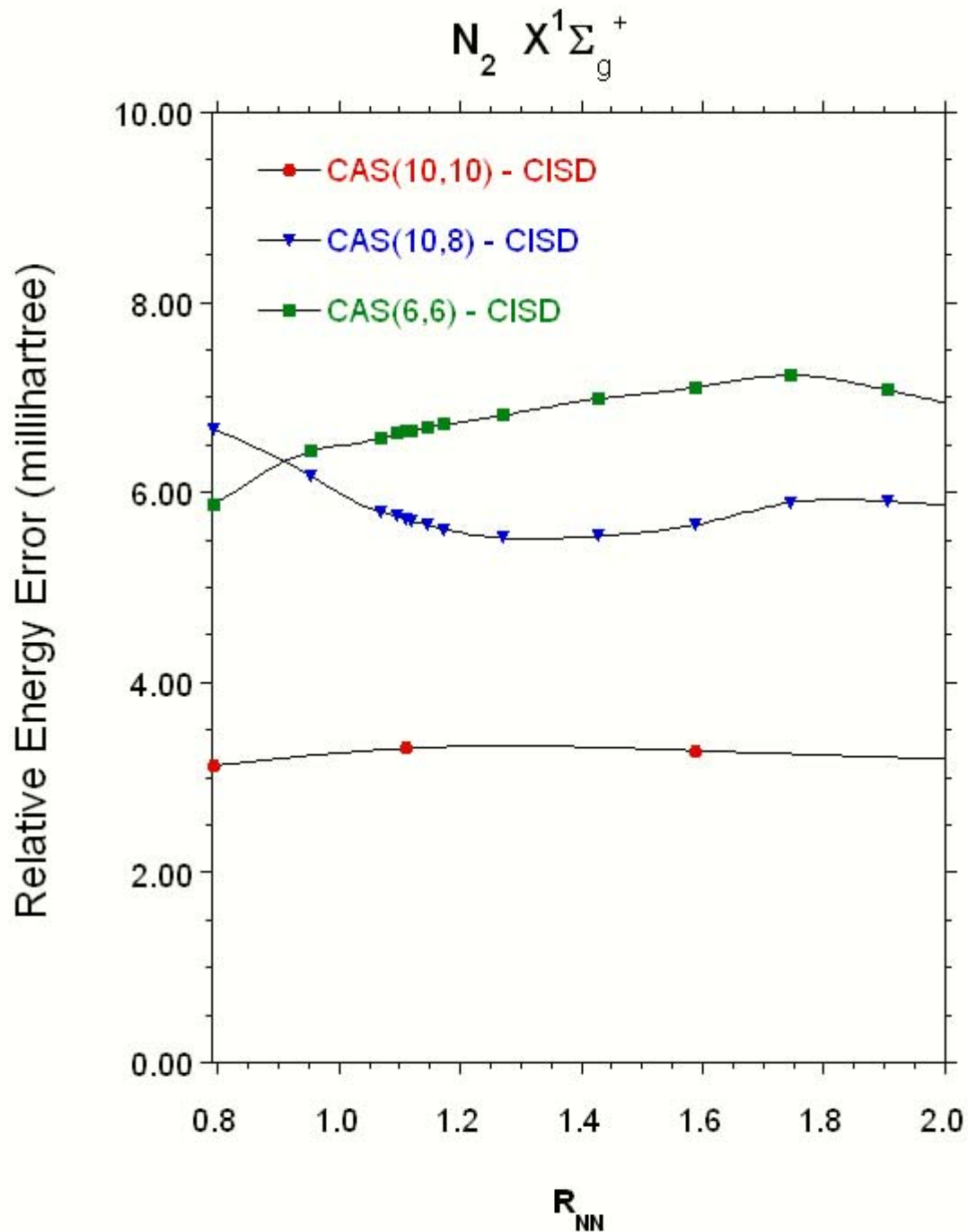


# Model

The **error** in the energy of the ground state of  $N_2$ , relative to the energy at the equilibrium bond length. Note that the Multi-reference **CAS-CISD** is far more accurate than **CCSD(T)**.



The error in the energy of the ground state of  $N_2$ , relative to the Full CI energy is insensitive to bond length, but a **constant size-consistency correction** must be made for comparisons with atoms.



**Table I.** The effect of **state averaging** on CASCF and CASSCF-CISD

State	$R_{NN}$ (Å)	Basis Set	$N_{\text{root}}=1$ ( $E_h$ )	$N_{\text{root}}=6$ ( $E_h$ )	$\Delta E$ ( $mE_h$ )
<b>CAS(6<sub>e-</sub>,6<sub>Orb</sub>)</b>					
$X^1\Sigma_g^+$	1.10	2ZaP	-109.1045238	-109.0781418	26.3820
		3ZaP	-109.1251169	-109.0979344	27.1825
$A^3\Sigma_u^+$	1.27	2ZaP	-108.8740932	-108.8606554	13.4378
		3ZaP	-108.8898435	-108.8761258	13.7177
<b>CAS(6<sub>e-</sub>,6<sub>Orb</sub>) – CISD</b>					
$X^1\Sigma_g^+$	1.10	2ZaP	-109.2896344	-109.2885919	<b>1.0425</b>
$A^3\Sigma_u^+$	1.27	2ZaP	-109.0736612	-109.0733565	<b>0.3047</b>

**Table II.** Calculated N<sub>2</sub> cc-pVDZ **excitaton energies** (mE<sub>h</sub>).

RNN(Å)	Excitation	CAS(6,6)	CAS(6,6)- CISD	FCI
<b>1.1113</b>	$X^1\Sigma_g^+ \rightarrow a^1\Pi_g$	<b>404.8</b>	<b>345.6</b>	<b>343.2</b>
	$\rightarrow a'^1\Sigma_u^-$	<b>375.9</b>	<b>367.9</b>	<b>367.0</b>
	$\rightarrow w^1\Delta_u$	412.1	383.5	381.7
<b>1.2700</b>	$X^1\Sigma_g^+ \rightarrow a^1\Pi_g$	313.8	274.6	272.2
	$\rightarrow a'^1\Sigma_u^-$	280.6	271.6	270.6
	$\rightarrow w^1\Delta_u$	336.9	290.3	288.2
<b>1.4288</b>	$X^1\Sigma_g^+ \rightarrow a^1\Pi_g$	252.8	226.7	223.4
	$\rightarrow a'^1\Sigma_u^-$	215.9	207.0	206.2
	<b>RMS Error</b>	<b>35.3</b>	<b>2.0</b>	

# Outline

## VI. Background

- a. John Pople's Model Chemistries

## VII. Excited state Models

- a. CCSD(T) vs FCI
- b. CASSCF and CAS-CISD vs FCI

## VIII. CBS Extrapolations

- a. A Systematic Sequence of Basis Sets
- b. The CASSCF CBS Limit
- c. The CASSCF-CISD CBS Limit

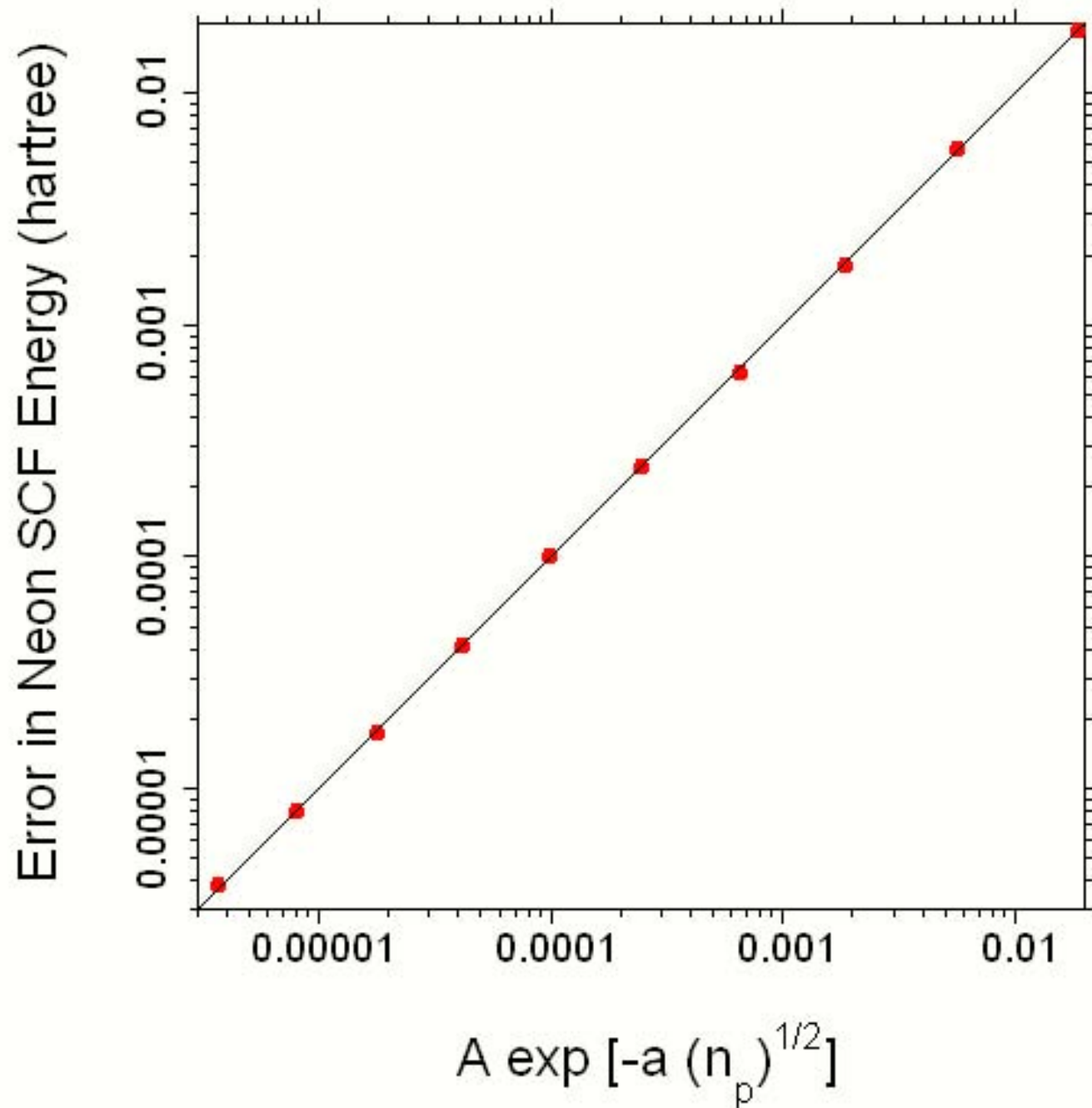
## IX. Results

- a. Geometry
- b. Vibrational Frequencies
- c. Excitation energies
- d. Dissociation Energies

## X. Acknowledgment

"Approved for public release; distribution unlimited"

The **SCF** energy converges to the **CBS** (complete basis set) limit as the exponential of the square-root of the number of Gaussian basis functions.



$$E_{SCF \text{ Limit}} \cong E(n_2) + \{ \exp[a(n_1)^{1/2} - a(n_2)^{1/2}] - 1 \}^{-1} \{ E(n_2) - E(n_1) \}$$

**Table III.** Linear SCF extrapolation parameters for the neon atom.

$n_1$	$n_2$	$\{ \exp[a(n_1)^{1/2} - a(n_2)^{1/2}] - 1 \}^{-1}$
5	6	0.426
6	7	0.490
7	8	0.551
8	9	0.609
<b>6</b>	<b>8</b>	<b>0.132</b>
<b>8</b>	<b>10</b>	<b>0.178</b>
<b>10</b>	<b>12</b>	<b>0.220</b>
<b>12</b>	<b>14</b>	<b>0.262</b>

**Table IV.** The convergence of the N<sub>2</sub> SCF anisotropy energy (hartree) with the number of polarization functions in the basis set.

Basis Set	E <sub>SCF</sub>	ΔE <sub>SCF</sub> (n)	E <sub>SCF</sub> Error	Extrapolation Factor <sup>a</sup>
<b>16s10p</b>	-108.910331			
<b>16s10p1d</b>	-108.981649	-0.071318	-0.011416	
<b>16s10p2d1f</b>	-108.991667	-0.010018	-0.001398	<b>0.139</b>
<b>16s10p3d2f1g</b>	-108.992847	-0.001180	-0.000218	<b>0.184</b>
<b>16s10p4d3f2g1h</b>	-108.993025	-0.000178	-0.000040	<b>0.22</b>
<b>16s10p5d4f3g2h1i</b>	-108.993057	-0.000032	-0.000008	<b>0.25</b>
SCF Limit = -108.993065				

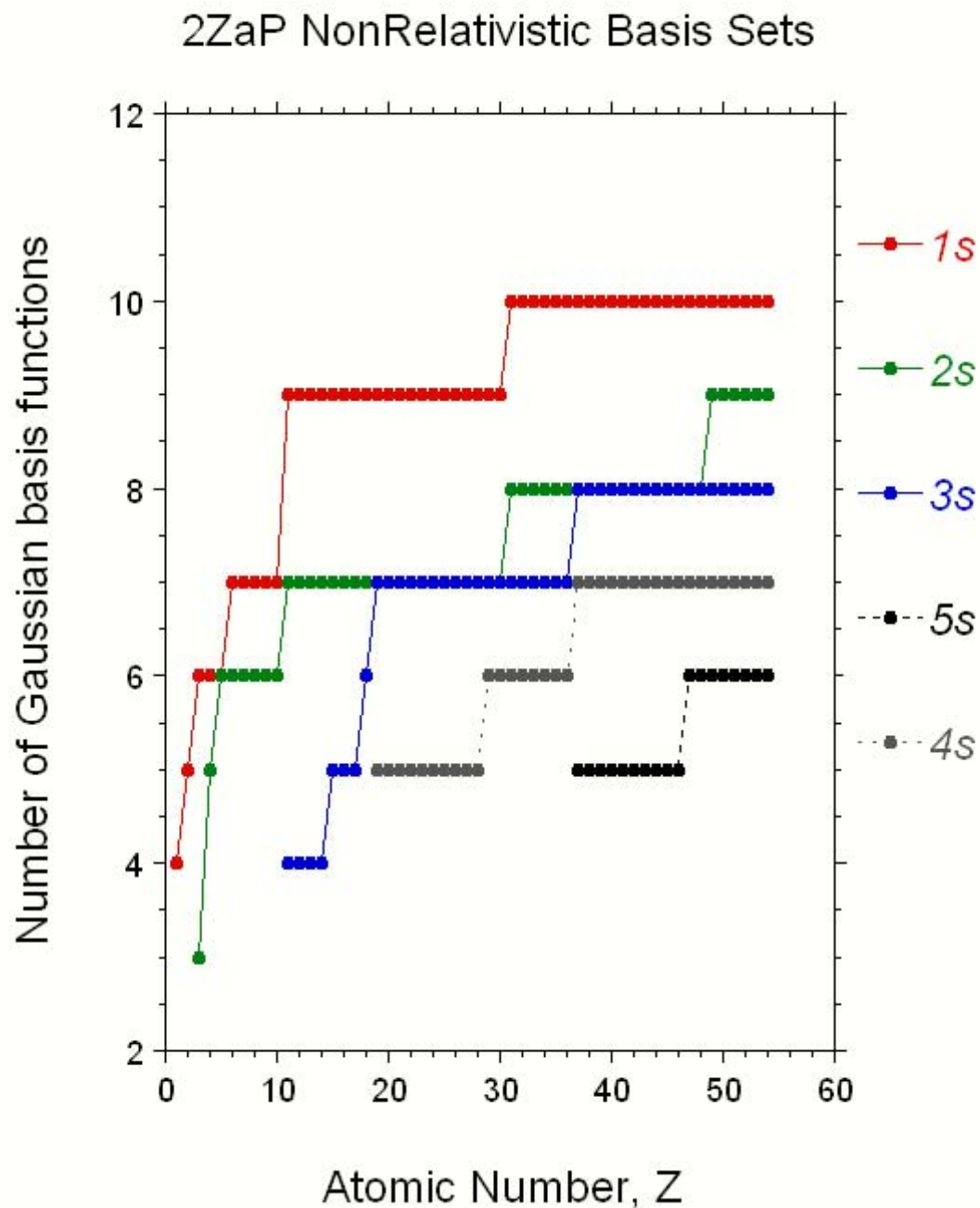
a. Factor = E<sub>SCF</sub> Error(n) / ΔE<sub>SCF</sub>(n)

## The nZaP Basis Sets

The number of Gaussian primitives in the 2ZaP basis sets was selected to give a constant, *1 millihartree, SCF error per electron*.

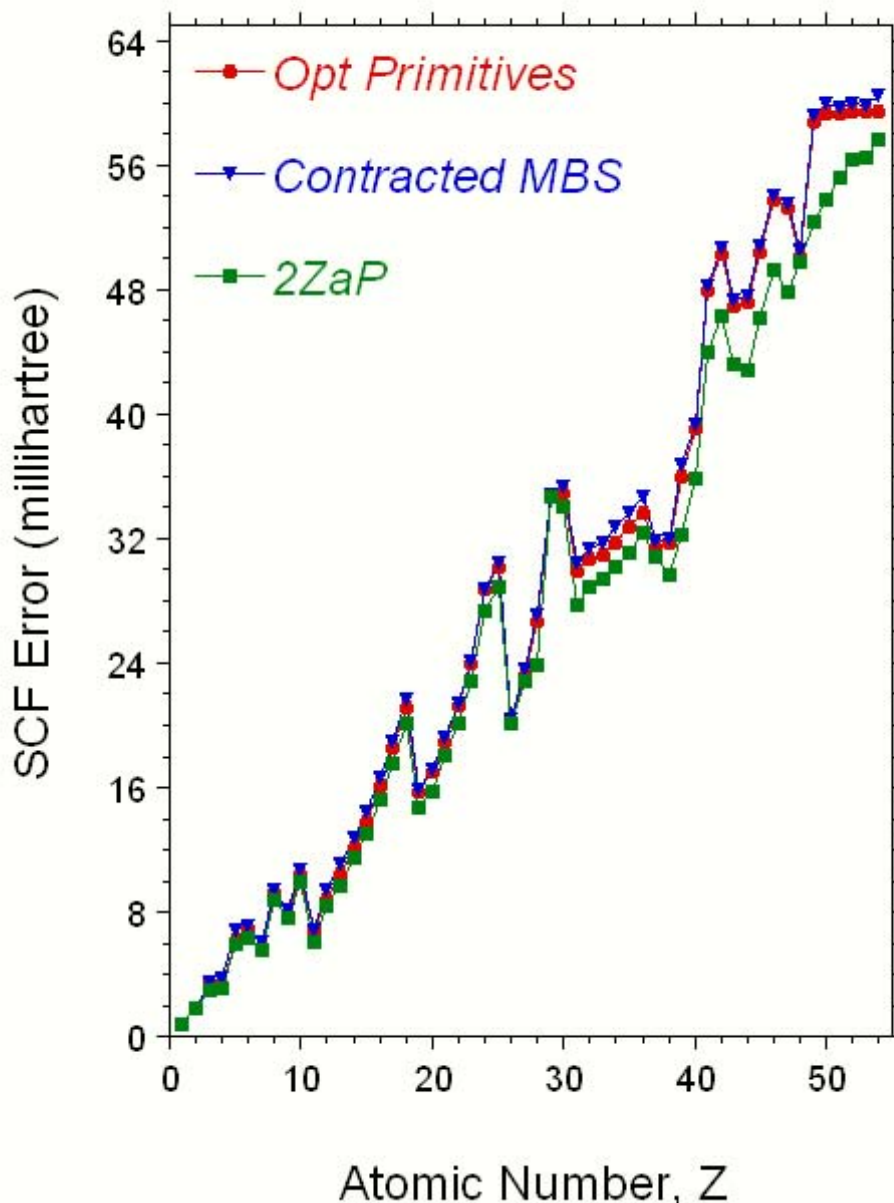
The 3ZaP basis sets each contain two more primitives of each angular momentum.

The 4ZaP basis sets include an additional two, etc.

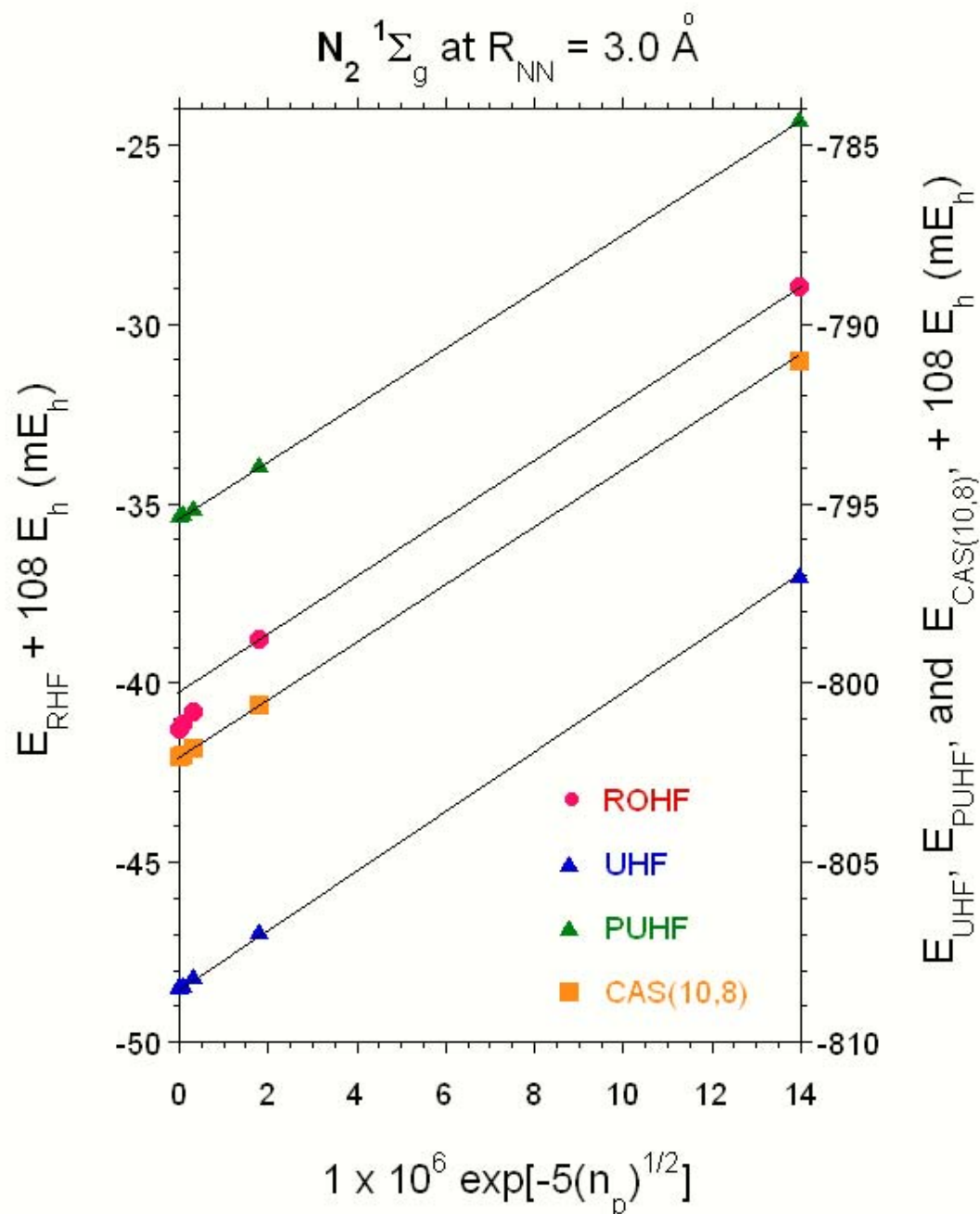


## The nZaP Basis Sets

The contracted nZaP basis sets are similar in structure to the cc-pVnZ basis sets, but with *diffuse valence functions added*. The number of primitives in each contracted function is sufficient to prevent contraction from significantly raising the SCF energy. All “correlation primitives” (*valence and polarization*) were optimized at the MP2 level.



**UHF** energies  
 mimic the  
 basis set  
 convergence  
 of **CASSCF**  
 energies, but  
**ROHF**  
 energies  
 do not.



**Table V.** Basis set convergence of CASSCF and UHFSCF as a function of geometry.

$R_{NN}$ (Å)	Basis Set	CAS(6,6) ( $E_h$ )	$\Delta E$ ( $mE_h$ )	$\Delta(\Delta E)_R$ ( $mE_h$ )	UHF ( $E_h$ )	$\Delta E$ ( $mE_h$ )	$\Delta(\Delta E)_R$ ( $mE_h$ )
1.09	2ZaP	-109.1035124			-108.9687667		
	3ZaP	-109.1246270	-21.1146	0.5215	-108.9903071	-21.5404	0.4654
	4ZaP	-109.1279935	-3.3665	0.1042	-108.9935983	-3.2912	0.0966
	5ZaP	-109.1288265	-0.8330	0.0213	-108.9944093	-0.8110	0.0198
	6ZaP				-108.9945297	-0.1204	0.0013
1.10	2ZaP	-109.1045238			-108.9674580		
	3ZaP	-109.1251169	<b>-20.5931</b>	<b>0.4819</b>	-108.9885330	<b>-21.0750</b>	<b>0.4264</b>
	4ZaP	-109.1283792	<b>-3.2623</b>	<b>0.0978</b>	-108.9917276	<b>-3.1946</b>	<b>0.0898</b>
	5ZaP	-109.1291909	<b>-0.8117</b>	<b>0.0204</b>	-108.9925188	<b>-0.7912</b>	<b>0.0189</b>
	6ZaP				-108.9926379	-0.1191	0.0011
1.11	2ZaP	-109.1049575			-108.9655455		
	3ZaP	-109.1250687	-20.1112	0.4447	-108.9861941	-20.6486	0.3898
	4ZaP	-109.1282332	-3.1645	0.0917	-108.9892989	-3.1048	0.0833
	5ZaP	-109.1290245	-0.7913	0.0195	-108.9900712	-0.7723	0.0181
	6ZaP				-108.9901892	-0.1180	0.0011

$$\begin{aligned}
E_{CAS}(CBS) &\cong E_{CAS}(3ZaP) \\
&+ 1.2 \times [E_{UHF}(4ZaP) - E_{UHF}(3ZaP)] \\
&\times \left[ \frac{E_{CAS}(3ZaP) - E_{CAS}(2ZaP)}{E_{UHF}(3ZaP) - E_{UHF}(2ZaP)} \right]
\end{aligned}$$

**Table VI.** Calculated Full Valence CASSCF energies (hartree au).

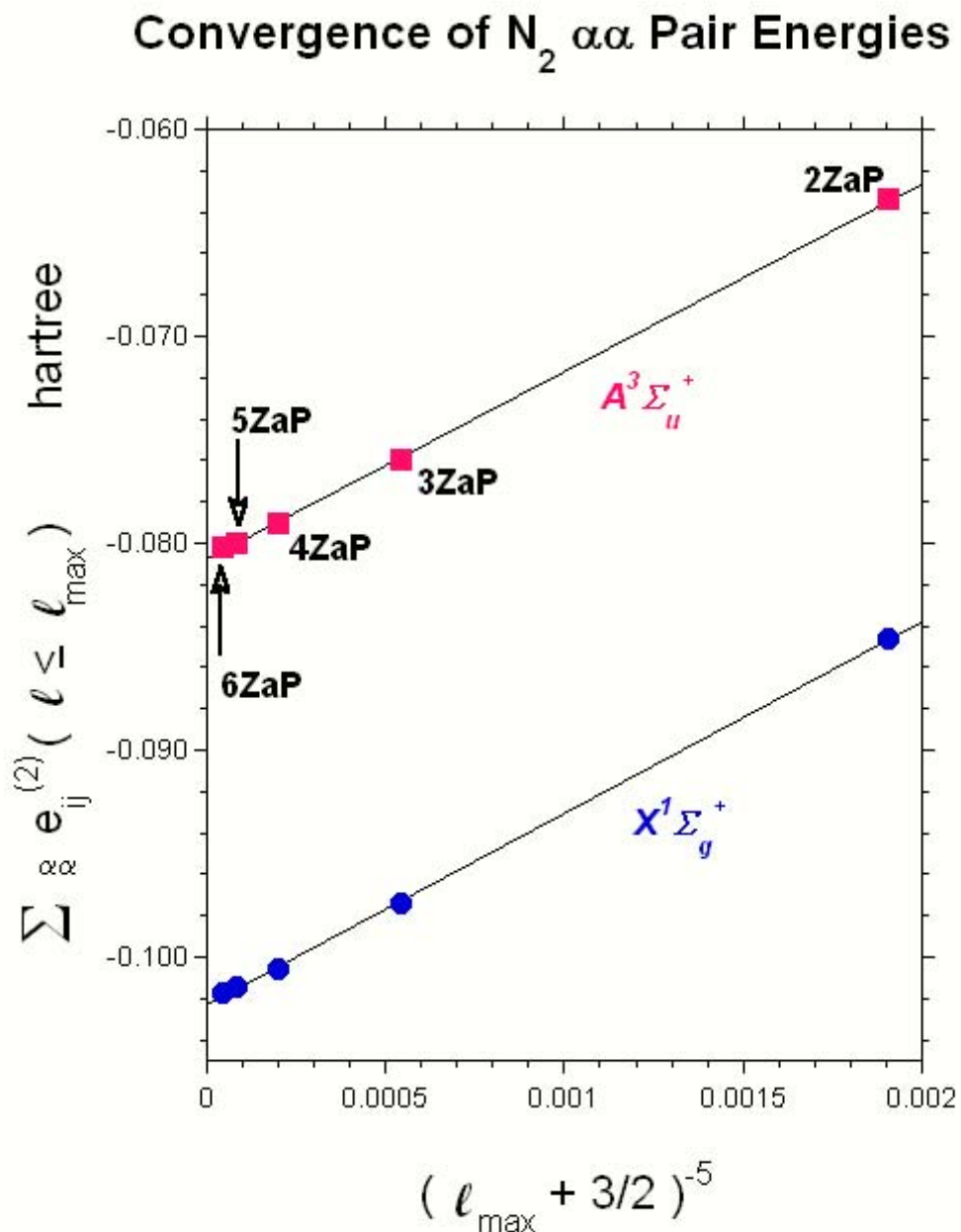
	<b>2ZaP</b>	<b>3ZaP</b>	<b>4ZaP</b>	<b>5ZaP</b>	<b>6ZaP</b>	<b>Limit</b>
<b>C<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup></b>	-75.6233832	-75.6412898	-75.6437447	-75.6442121	-75.6442908	-75.6443188
<b>C<sub>2</sub> a<sup>3</sup>Π<sub>u</sub></b>	-75.6073350	-75.6230710	-75.6251470	-75.6255495	-75.6256266	-75.6256541
<b>C<sub>2</sub> b<sup>3</sup>Σ<sub>g</sub><sup>-</sup></b>	-75.5777931	-75.5941905	-75.5962600	-75.5966964	-75.5967840	-75.5968152
<b>C<sub>2</sub> A<sup>1</sup>Π<sub>u</sub></b>	-75.5627890	-75.5788475	-75.5809697	-75.5813798	-75.5814615	-75.5814906
<b>CN X<sup>2</sup>Σ<sup>+</sup></b>	-92.3544025	-92.3730997	-92.3761806	-92.3766716	-92.3767524	-92.3767812
<b>CO X<sup>1</sup>Σ<sup>+</sup></b>	-112.8946223	-112.9181416	-112.9218328	-112.9224855	-112.9226137	-112.9226594
<b>N<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup></b>	-109.1166132	-109.1373292	-109.1406653	-109.1414826	-109.1415931	-109.1416324
<b>NO X<sup>2</sup>Π<sub>i</sub></b>	-129.3892421	-129.4106758	-129.4138042	-129.4143983	-129.4144941	-129.4145282
<b>O<sub>2</sub> X<sup>3</sup>Σ<sub>g</sub><sup>-</sup></b>	-149.7411441	-149.7681917	-149.7713131	-149.7719475	-149.7720640	-149.7721055
<b>O<sub>2</sub> a<sup>1</sup>Δ<sub>g</sub></b>	-149.7059304	-149.7327122	-149.7358092	-149.7364403	-149.7365522	-149.7365920
<b>O<sub>3</sub> X<sup>1</sup>A<sub>1</sub></b>	-224.5478659	-224.5938866	-224.5991383	-224.6002546	-224.6004950	-224.6005806
<b>OH X<sup>2</sup>Π</b>	-75.4325178	-75.4440618	-75.4456305	-75.4458720	-75.4459251	-75.4459441
<b>OH A<sup>2</sup>Σ<sup>+</sup></b>	-75.2670507	-75.2808979	-75.2825953	-75.2828559	-75.2829216	-75.2829450
<b>RMS Error</b>	<b>0.0224147</b>	<b>0.0030328</b>	<b>0.0005829</b>	<b>0.0001180</b>	<b>0.0000310</b>	

**Table VII.** Extrapolated Full Valence CASSCF energies (hartree au).

	<b>3ZaP</b>	<b>Error</b> <b>4ZaP</b>	<b>5ZaP</b>
<b>C<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup>(Re)</b>	-0.0001705	0.0000255	-0.0000059
<b>C<sub>2</sub> a<sup>3</sup>Π<sub>u</sub>(Re)</b>	-0.0001753	0.0000582	0.0000012
<b>C<sub>2</sub> b<sup>3</sup>Σ<sub>g</sub><sup>-</sup>(Re)</b>	-0.0002094	0.0000064	-0.0000040
<b>C<sub>2</sub> A<sup>1</sup>Π<sub>u</sub>(Re)</b>	-0.0000459	0.0000176	0.0000026
<b>CN X<sup>2</sup>Σ<sup>+</sup>(Re)</b>	-0.0001536	0.0000366	-0.0000031
<b>N<sub>2</sub> X<sup>1</sup>Σ<sub>g</sub><sup>+</sup>(Re)</b>	-0.0005054	-0.0001428	-0.0000276
<b>NO X<sup>2</sup>Π<sub>i</sub>(Re)</b>	-0.0001124	-0.0001120	-0.0000175
<b>O<sub>2</sub> X<sup>3</sup>Σ<sub>g</sub><sup>-</sup>(Re)</b>	0.0000564	-0.0001285	-0.0000186
<b>O<sub>2</sub> a<sup>1</sup>Δ<sub>g</sub>(Re)</b>	-0.0000230	-0.0001280	-0.0000168
<b>OH X<sup>2</sup>Π(Re)</b>	-0.0000524	-0.0000353	-0.0000136
<b>OH A<sup>2</sup>Σ<sup>+</sup>(Re)</b>	0.0000168	-0.0000603	-0.0000194
<b>RMS Error</b>	<b>0.0002327</b>	<b>0.0000956</b>	<b>0.0000245</b>
<b>Before Extrap</b>	<b>0.0030328</b>	<b>0.0005829</b>	<b>0.0001180</b>

## MP2 CBS Limit

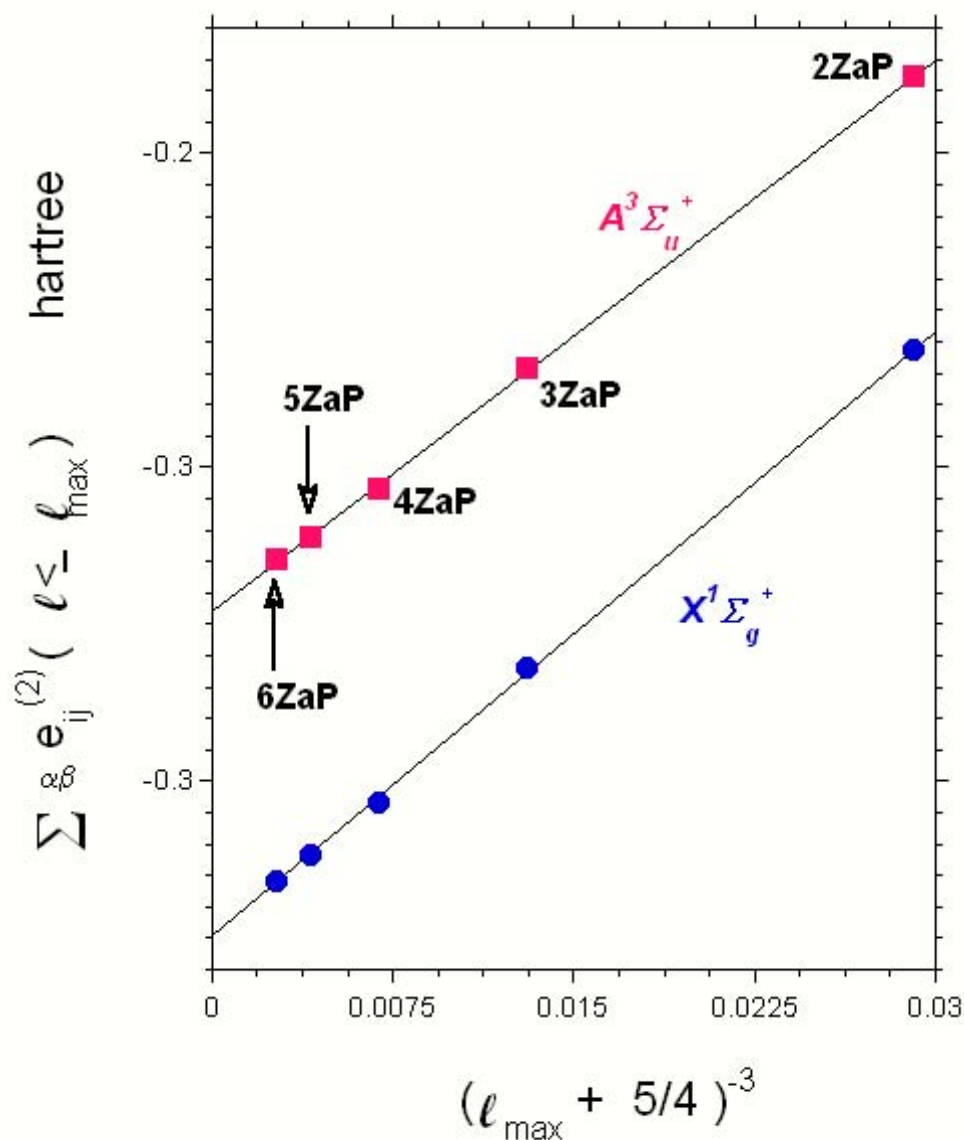
The  $\alpha\alpha$  second-order Møller-Plessett electron pair correlation energies converge to the CBS limit as the inverse **fifth** power of the maximum angular momentum included in the basis set.



## MP2 CBS Limit

The  $\alpha\beta$  second-order Møller-Plessett electron pair correlation energies converge to the CBS limit as the inverse **third** power of the maximum angular momentum included in the basis set.

### Convergence of $N_2$ $\alpha\beta$ Pair Energies



**Table VIII.** The basis set convergence of dynamic correlaton energy.

State	Basis Set	$E^{(2)}$ ( $mE_h$ )	$\Delta E^{(2)}$ ( $mE_h$ )	$E_{\text{CAS-CISD}} - E_{\text{CAS}}$ ( $mE_h$ )	$\Delta(E_{\text{CAS-CISD}} - E_{\text{CAS}})$ ( $mE_h$ )
$X^1\Sigma_g^+$ 1.10Å	2ZaP	-316.0189		-181.9089	
	3ZaP	-379.3504	<b>-63.3315</b>	-244.4998	<b>-62.5909</b>
	4ZaP	-403.7559			
	5ZaP	-413.3317			
	6ZaP	-417.4648			
	CBS Limit	-426.90			
$A^3\Sigma_u^+$ 1.27Å	2ZaP	-250.9989		-195.6356	
	3ZaP	-310.1058	<b>-59.1069</b>	-255.0882	<b>-59.4526</b>
	4ZaP	-332.4087			
	5ZaP	-341.1121			
	6ZaP	-344.7652			
	CBS Limit	-353.59			

$$\begin{aligned}
E_{CAS-CISD}(CBS) &\cong E_{CAS-CISD}(3ZaP) \\
&+ \left[ E^{(2)}(CBS) - E^{(2)}(3ZaP) \right] \\
&\times \left[ \frac{E_{CAS-CISD}(3ZaP) - E_{CAS-CISD}(2ZaP)}{E^{(2)}(3ZaP) - E^{(2)}(2ZaP)} \right]
\end{aligned}$$

# Outline

## XI. Background

- a. John Pople's Model Chemistries

## XII. Excited state Models

- a. CCSD(T) vs FCI
- b. CASSCF and CAS-CISD vs FCI

## XIII. CBS Extrapolations

- a. A Systematic Sequence of Basis Sets
- b. The CASSCF CBS Limit
- c. The CASSCF-CISD CBS Limit

## XIV. Results

- a. Geometry
- b. Vibrational Frequencies
- c. Excitation energies
- d. Dissociation Energies

## XV. Acknowledgment

"Approved for public release; distribution unlimited"

**Table IX.** Calculated CAS-CISD **geometries** ( $R_e$  in Å) for low-lying excited states of  $N_2$ .

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1 \Sigma_g^+$	1.1175	1.1039	1.0991	1.0977
$A^3 \Sigma_u^+$	1.3066	1.2951	1.2905	1.2866
$B^3 \Pi_g$	1.2314	1.2201	1.2152	1.2126
$W^3 \Delta_u$	1.2972	1.2859	1.2815	
$B'^3 \Sigma_u^-$	1.2955	1.2845	1.2800	1.2784
$a'^1 \Sigma_u^-$	1.2914	1.2805	1.2759	1.2755
$a^1 \Pi_g$	1.2390	1.2271	1.2213	1.2203
$w^1 \Delta_u$	1.2837	1.2728	1.2684	1.268
<b>RMS Error</b>	<b>0.0181</b>	<b>0.0066</b>	<b>0.0020</b>	

**Table X.** Calculated CAS-CISD harmonic **vibrational frequencies** ( $\omega_e$  in  $\text{cm}^{-1}$ ) for low-lying excited states of  $\text{N}_2$ .

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1 \Sigma_g^+$	2319.1	2345.2	2358.1	2358.57
$A^3 \Sigma_u^+$	1418.5	1439.7	1450.8	1460.64
$B^3 \Pi_g$	1707.4	1715.3	1727.4	1733.39
$W^3 \Delta_u$	1482.0	1499.1	1508.9	1501.4
$B'^3 \Sigma_u^-$	1494.5	1509.9	1518.9	1516.88
$a'^1 \Sigma_u^-$	1508.6	1528.4	1538.8	1530.25
$a^1 \Pi_g$	1648.2	1686.1	1696.4	1694.21
$w^1 \Delta_u$	1543.5	1561.3	1571.3	1559.26
<b>RMS Error</b>	<b>31.1</b>	<b>12.1</b>	<b>7.3</b>	

**Table XI.** Calculated CAS-CISD **excitation energies** (Ev) of N<sub>2</sub>.

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1\Sigma_g^+ \rightarrow$				
$A^3\Sigma_u^+$	5.843	6.093	6.192	6.224
$B^3\Pi_g$	7.275	7.397	7.447	7.392
$W^3\Delta_u$	7.233	7.364	7.432	7.415
$B'^3\Sigma_u^-$	8.067	8.230	8.265	8.217
$a'^1\Sigma_u^-$	8.427	8.471	8.407	8.450
$a^1\Pi_g$	8.547	8.653	8.562	8.590
$w^1\Delta_u$	8.932	9.008	8.927	8.939
<b>RMS Error</b>	<b>0.176</b>	<b>0.065</b>	<b>0.037</b>	

**Table XII.** Calculated CAS-CISD dissociation energies (Ev) of N<sub>2</sub>.

<b>State</b>	<b>2ZaP</b>	<b>3ZaP</b>	<b>CBS limit</b>	<b>Exp.</b>
$X^1\Sigma_g^+ \rightarrow$				
$A^3\Sigma_u^+$	5.857	6.093	6.192	6.224
$B^3\Pi_g$	7.256	7.397	7.447	7.392
$W^3\Delta_u$	7.301	7.364	7.432	7.415
$B'^3\Sigma_u^-$	8.063	8.230	8.265	8.217
$a'^1\Sigma_u^-$	8.417	8.471	8.407	8.450
$a^1\Pi_g$	8.537	8.653	8.562	8.590
$w^1\Delta_u$	8.914	9.008	8.927	8.939
<b>RMS Error</b>	<b>0.167</b>	<b>0.065</b>	<b>0.037</b>	

# Acknowledgments

Dr. Michael J. Frisch

Dr. Matthew Braunstein

Dr. David K. Malick

# Support

Gaussian Inc.™

Wesleyan University

Department of Defense